Determination of $U_A(1)$ restoration from meson screening masses by using the entanglement PNJL model: Toward chiral regime

arXiv: 1504.04463

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Contents of this talk

◆ Determine temperature dependence of $U_A(1)$ symmetry restoration from lattice QCD results on π and a0 meson screening masses by using an effective model

◆ Investigate the Columbia plot near the physical point

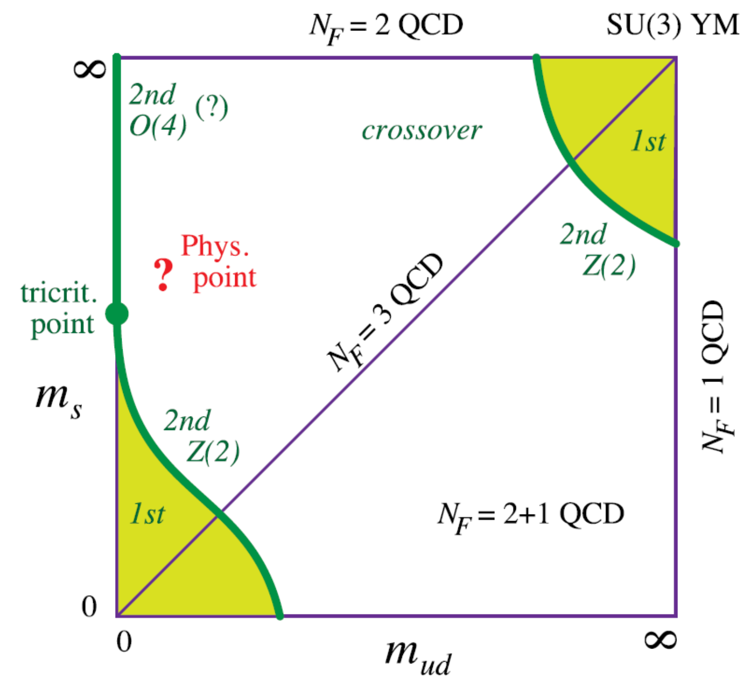
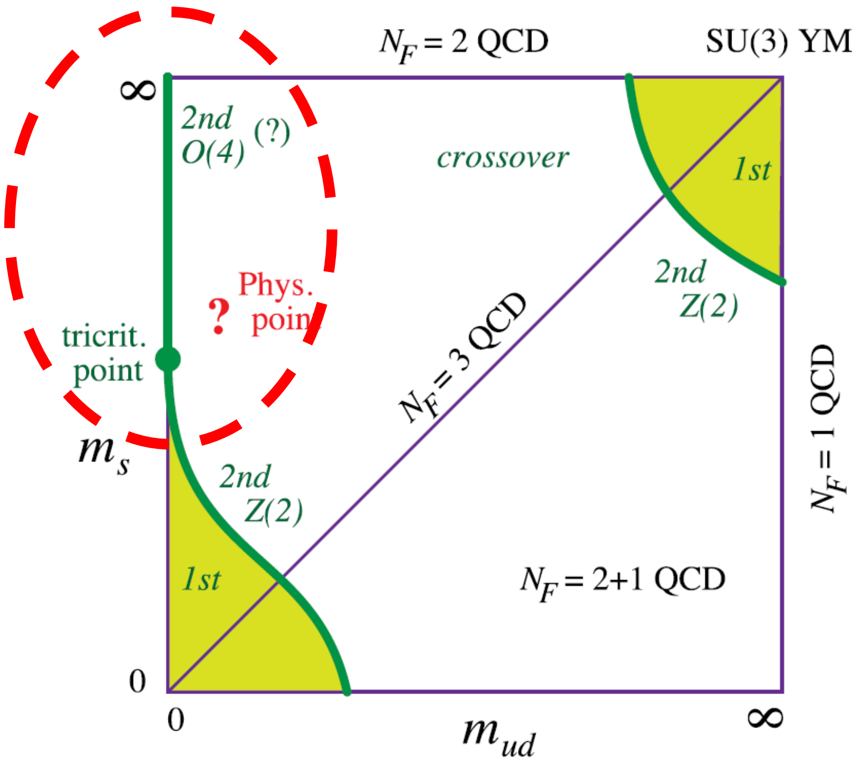


Figure : K. Kanaya, PoS(Lattice 2010)012

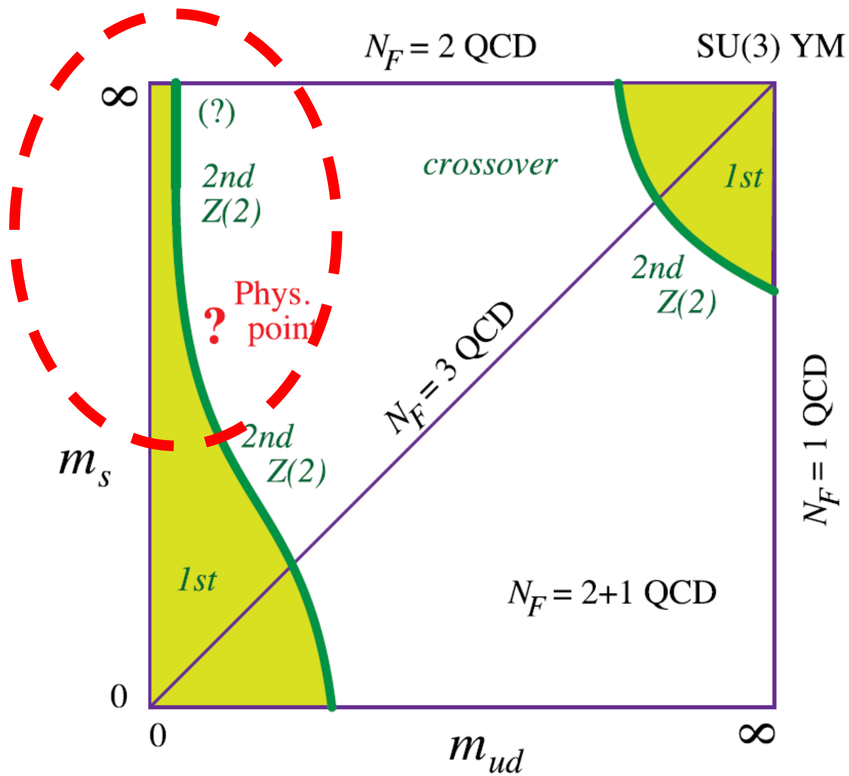
$U_A(1)$ restoration and chiral phase transition

Two possible scenarios in Columbia plot[1]:

$U_A(1)$ symmetry not restored



$U_A(1)$ symmetry restored



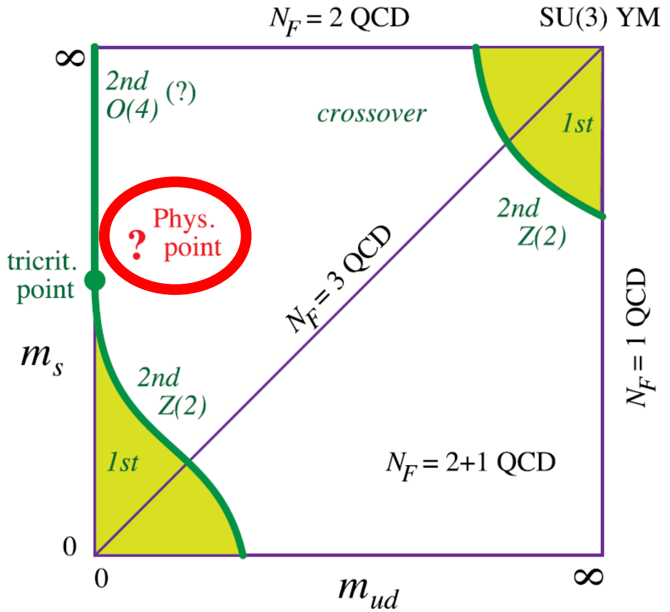
[1] R. Pisarski and F. Wilczek, Phys. Rev. D 29. 338 (1984)

Figure : K. Kanaya, PoS(Lattice 2010)012

Order parameter of $U_A(1)$ restoration

$U_A(1)$ restoration can be determined from the difference between $U_A(1)$ partner's screening masses (For example, π and a_0 mesons)

Recently, these screening masses were measured near the physical point [2]



[2] M. Cheng et al., Eur. Phys. J. C 71, 1564 (2011).

2+1 flavor p4 staggered fermion, $N_S^3 \times N_\tau = 16^3 \times 4, 24^3 \times 6,\ 32^3 \times 8$

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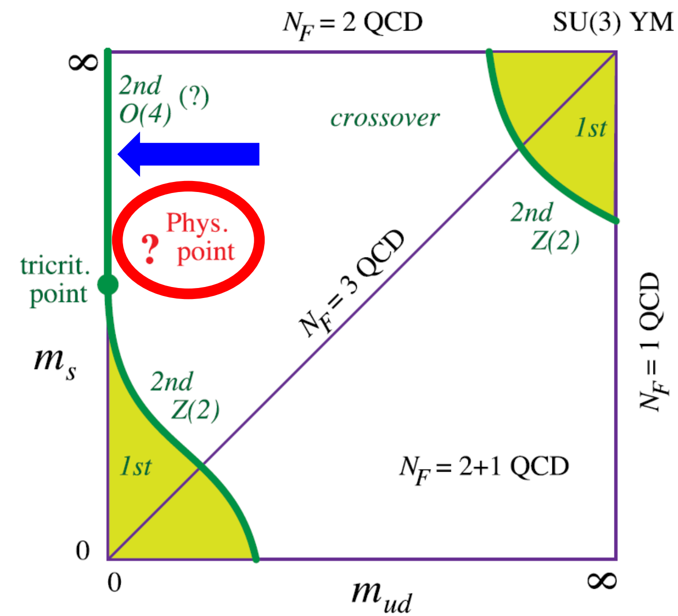
$U_A(1)$ restoration can be determined from the difference between $U_A(1)$ partner's screening masses (For example, π and a_0 mesons)



Recently, these screening masses were measured near the physical point [2]

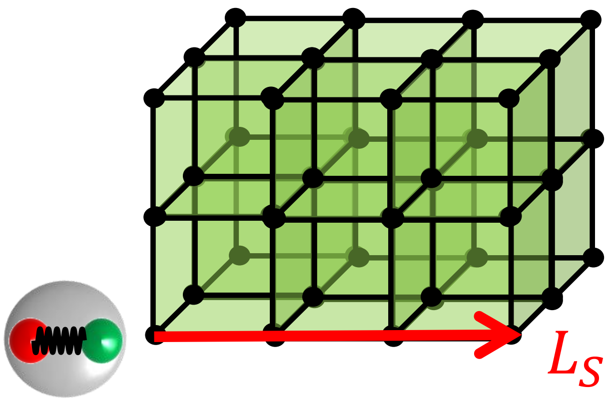
If we can construct the effective model that reproduces the lattice QCD data [2], we can extrapolate the lattice data to light-quark chiral limit by using the effective model

[2] M. Cheng et al., Eur. Phys. J. C 71, 1564 (2011).

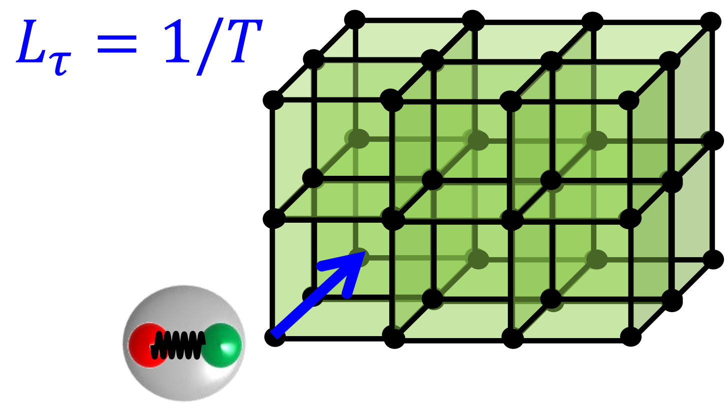
2+1 flavor p4 staggered fermion, $N_S^3 \times N_\tau = 16^3 \times 4, 24^3 \times 6, \ 32^3 \times 8$

Meson screening mass

Exponential damping of meson propagator in the spatial direction



Spatial direction
- Screening mass (M_{scr})



Imaginary-time direction
- Pole mass (M_{pole})

In lattice QCD at finite T, it is relatively easy to measure M_{scr} compared with M_{pole}
(L_s can be taken arbitrarily large (in principle) but $L_\tau = 1/T$.)

Meson screening mass

However, it is not easy to calculate screening mass with effective models.

Very recently, we constructed a new formalism for calculating screening mass in NJL-type effective models [3]

We can calculate the screening mass easily with effective models.

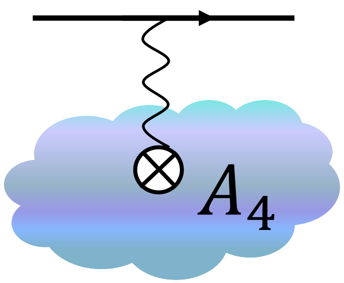
[3]M.I, T. Sasaki, K. Kashiwa, H. Kouno, M. Yahiro, Phys. Rev. D89, 071901(R) (2014)

Polyakov loop extended Nambu－Jona-Lasinio model with entanglement vertex (EPNJL model)

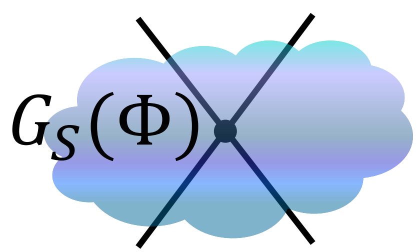
3 flavor PNJL model [5] = 3 flavor NJL model [4] + Polyakov loop

$$\mathcal{L} = \bar{q}(i\gamma_\nu D^\nu - \hat{m}_0)q + G_{\mathrm{s}}(\Phi)\sum_{a=0}^{8}[(\bar{q}\lambda_a q)^2 + (\bar{q}i\gamma_5\lambda_a q)^2]$$

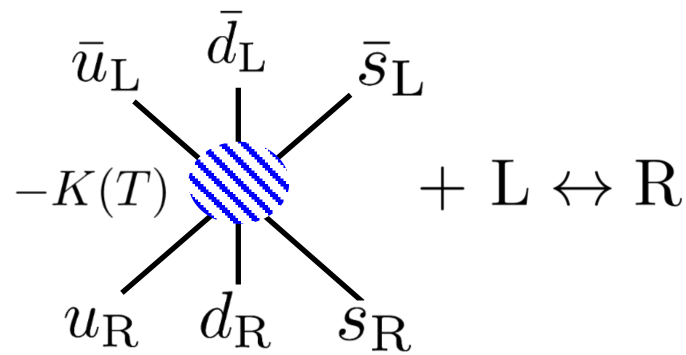
$$-K(T)\left[\det_{ij}\bar{q}_i(1+\gamma_5)q_j + \det_{ij}\bar{q}_i(1-\gamma_5)q_j\right] - \mathcal{U}(\Phi[A],\bar{\Phi}[A],T)$$



gluon: static background



entanglement coupling



$SU_R(3)\times SU_L(3)$ symmetric,
$U_A(1)$ symmetry breaking interaction
(Kobayashi-Maskawa-'t Hooft (KMT) interaction)

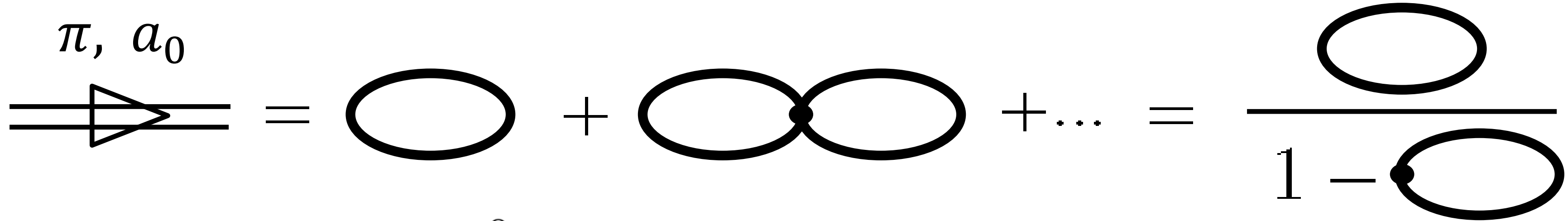
Entanglement coupling

$$G_{\mathrm{s}}(\Phi) = G_{\mathrm{s}}\cdot\left(1-\alpha_1\Phi\bar{\Phi}\right)$$

[4] T. Hatsuda and T. Kunihiro, Phys. Rep, 247,221 (1994)
[5] K. Fukushima, Phys. Lett. B 581

Meson propagator and screening mass

Treat meson propagator as quark-antiquark scattering in the ring approximation;



external momentum: $q_0 = 0$, $\mathbf{q}$

Meson propagator in momentum space:

$$\chi_{\xi\xi}(0,\tilde{q}^2) = \frac{\Pi_{\xi\xi}(0,\tilde{q}^2)}{1 - 2G_\xi \Pi_{\xi\xi}(0,\tilde{q}^2)} \qquad \tilde{q} = \pm|\mathbf{q}|$$

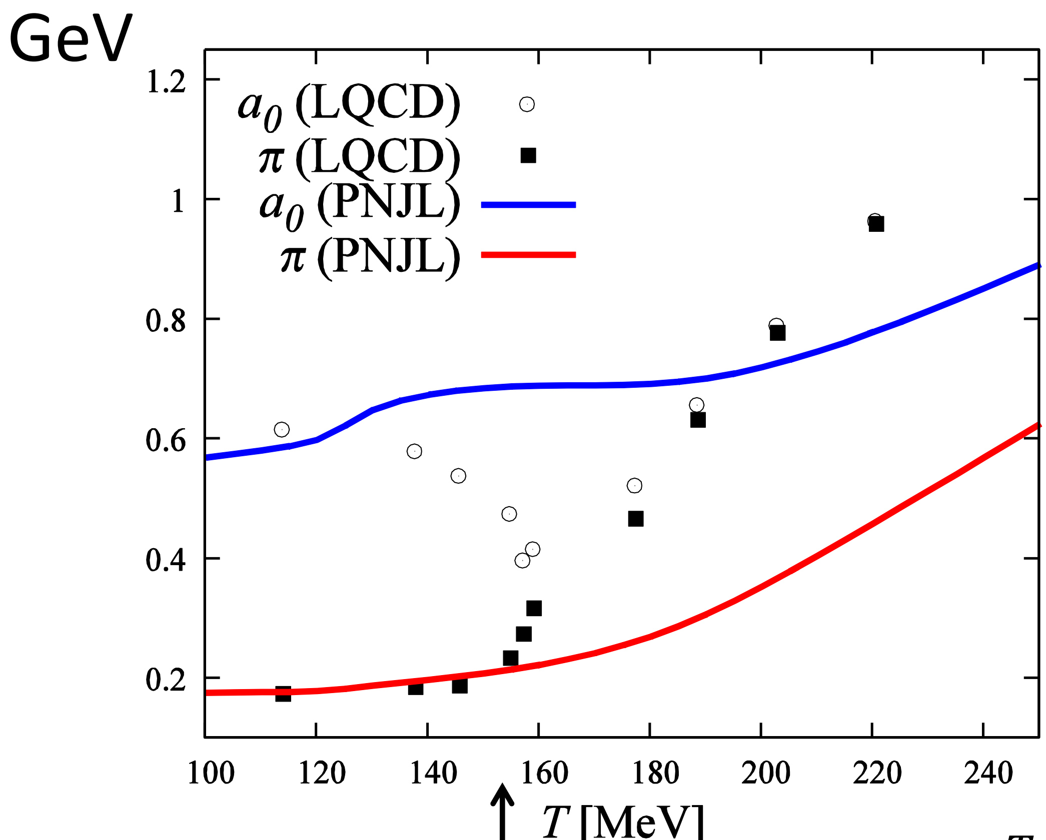
Fourier transformation of χ ➤ Spatial propagator η

$$\eta_{\xi\xi}(r) = \frac{1}{4\pi^2 i r}\int_{-\infty}^{\infty} d\tilde{q}\ \tilde{q}\chi_{\xi\xi}(0,\tilde{q}^2)e^{i\tilde{q}r} \sim \frac{1}{r}e^{-\underline{M_{\xi,\mathrm{scr}}} r}$$

$r \to \infty$

π and a_0 meson screening masses

PNJL model can not reproduce lattice QCD results for $T > T_c$.



$T_c = 154 \pm 9$ MeV [5]

T_c : critical temperature for chiral phase transition

[2]LQCD data M_{scr}: M. Cheng et al., Eur. Phys. J. C 71, 1564 (2011).

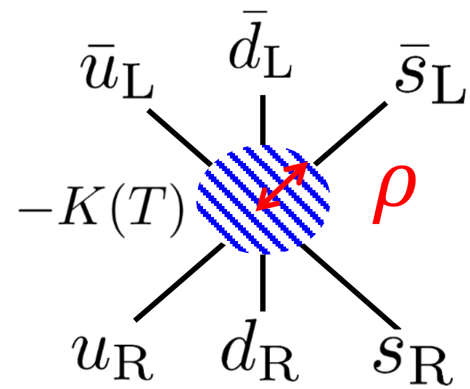
[6]LQCD data T_c: A. Bazavov et al., Phys. Rev. D 85, 054503(2012).

T dependence of KMT interaction $K(T)$

[5] R.D.Pisarski and L.G.Yaffe, Phys. Lett. B97, 110 (1980).
[6] E. Shuryak, Nucl. Phys. B203, 93 (1982); B214, 237 (1983).

KMT interaction is induced by Instantons and Antiinstantons

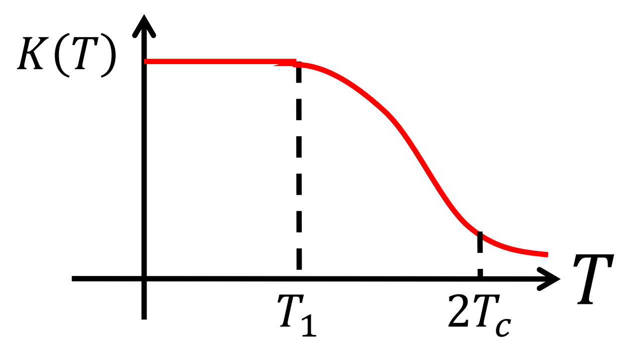
In quark gluon plasma phase ($T \geq 2T_c$) ,
Instanton density is suppressed by Debye screening[5]



$$dn_{\mathrm{inst}}(T) = dn_{\mathrm{inst}}(0) \times \exp\left[-\pi^2\rho^2T^2\left(\frac{2N_c}{3} + \frac{N_f}{3}\right)\right]$$

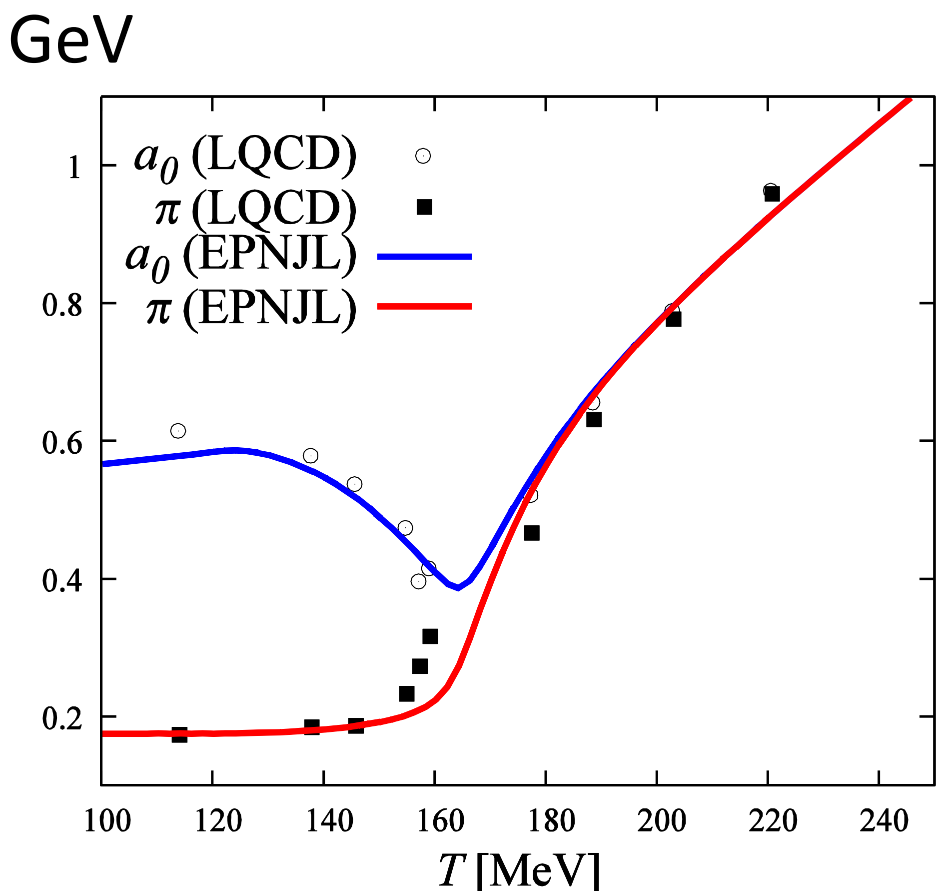
ρ [fm] : instanton radius ($\rho = 1/3$ [fm] in Instanton liquid model [6])

$$K(T) = \begin{cases} K & (T < T_1) \\ K\exp\left[-(T-T_1)^2/b^2\right] & (T \geq T_1) \end{cases}$$



Parameters b, T_1 are determined from π, a_0 meson screening masses

π and a_0 meson screening masses

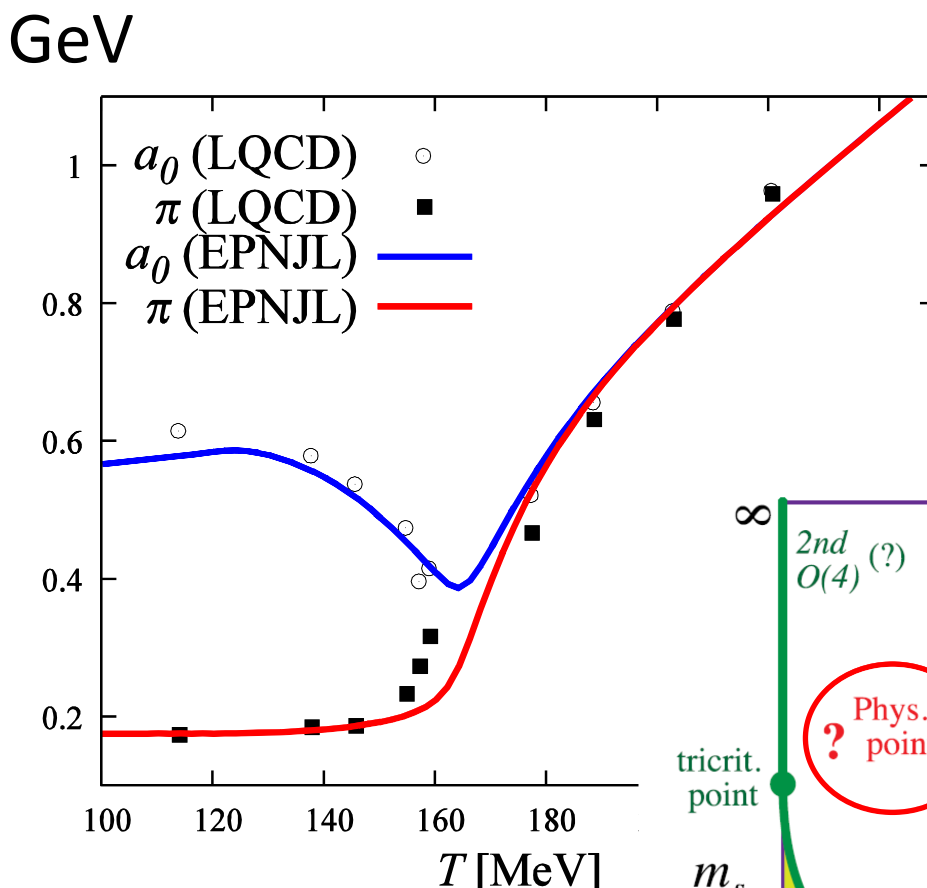


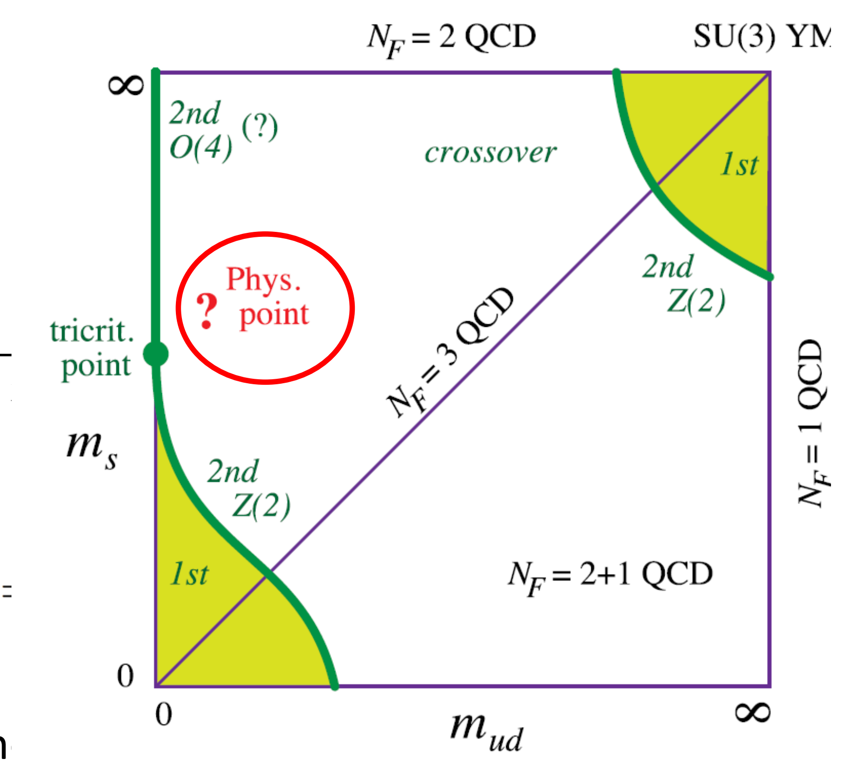
Parameters:
$b = 36$ MeV, $T_1 = 121$ MeV.

$$K(T) = \begin{cases} K & (T < T_1) \\ K \exp\left[-(T-T_1)^2/b^2\right] & (T \geq T_1) \end{cases}$$

[2]LQCD data: M. Cheng et al., Eur. Phys. J. C 71, 1564 (2011).

π and a_0 meson screening masses





Parameters:

$b = 36$ MeV, $T_1 = 121$ MeV.

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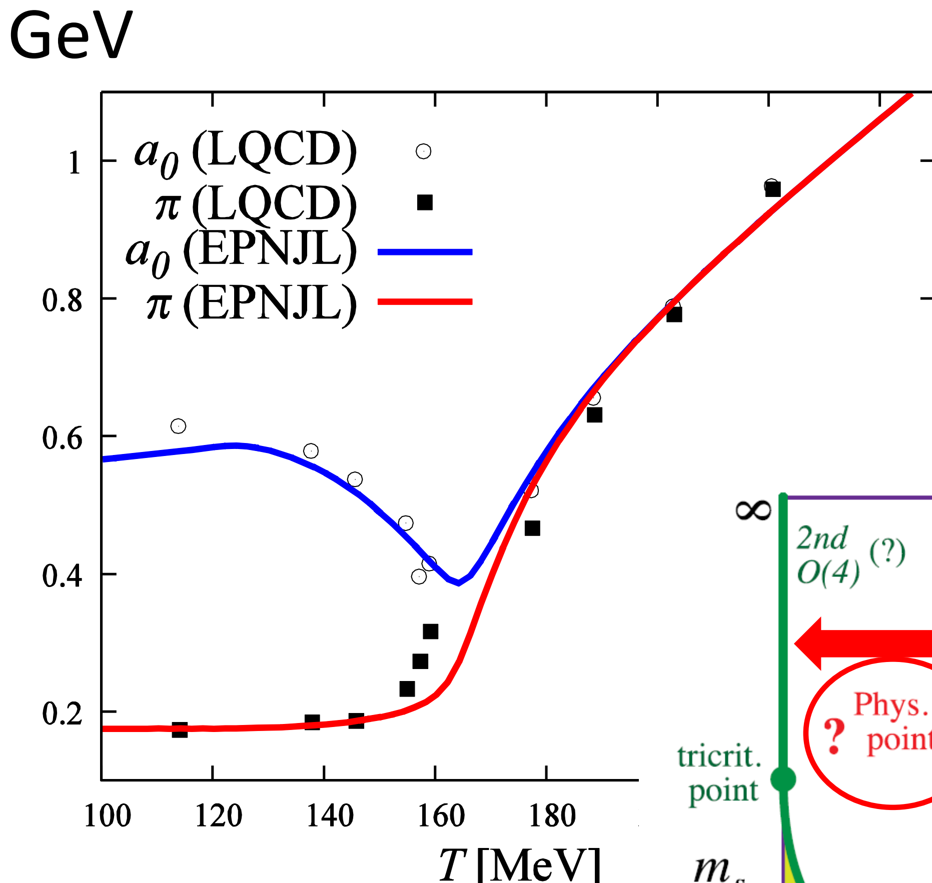
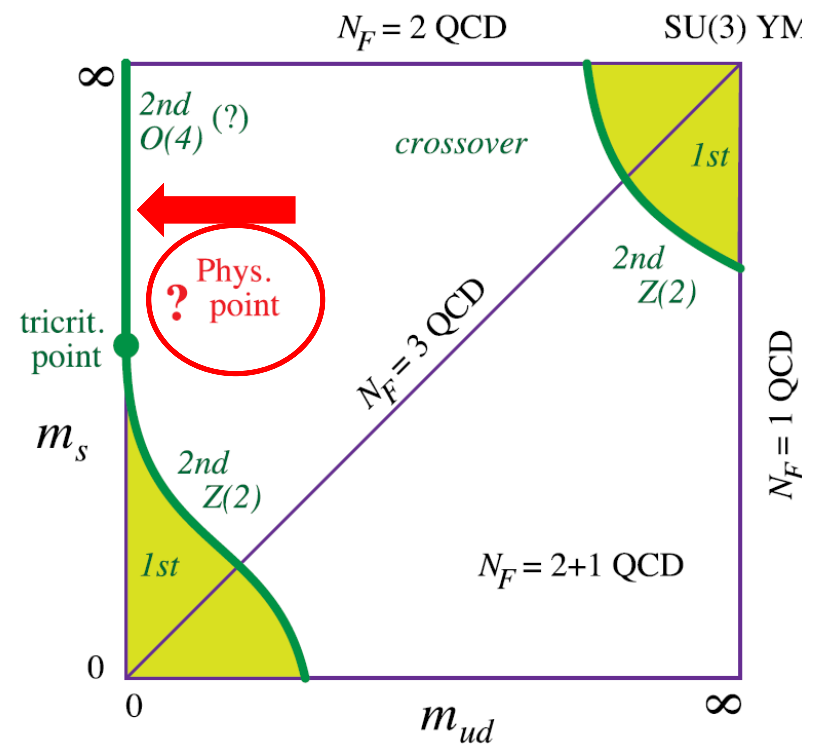
[2]LQCD data: M. Ch

π and a_0 meson screening masses





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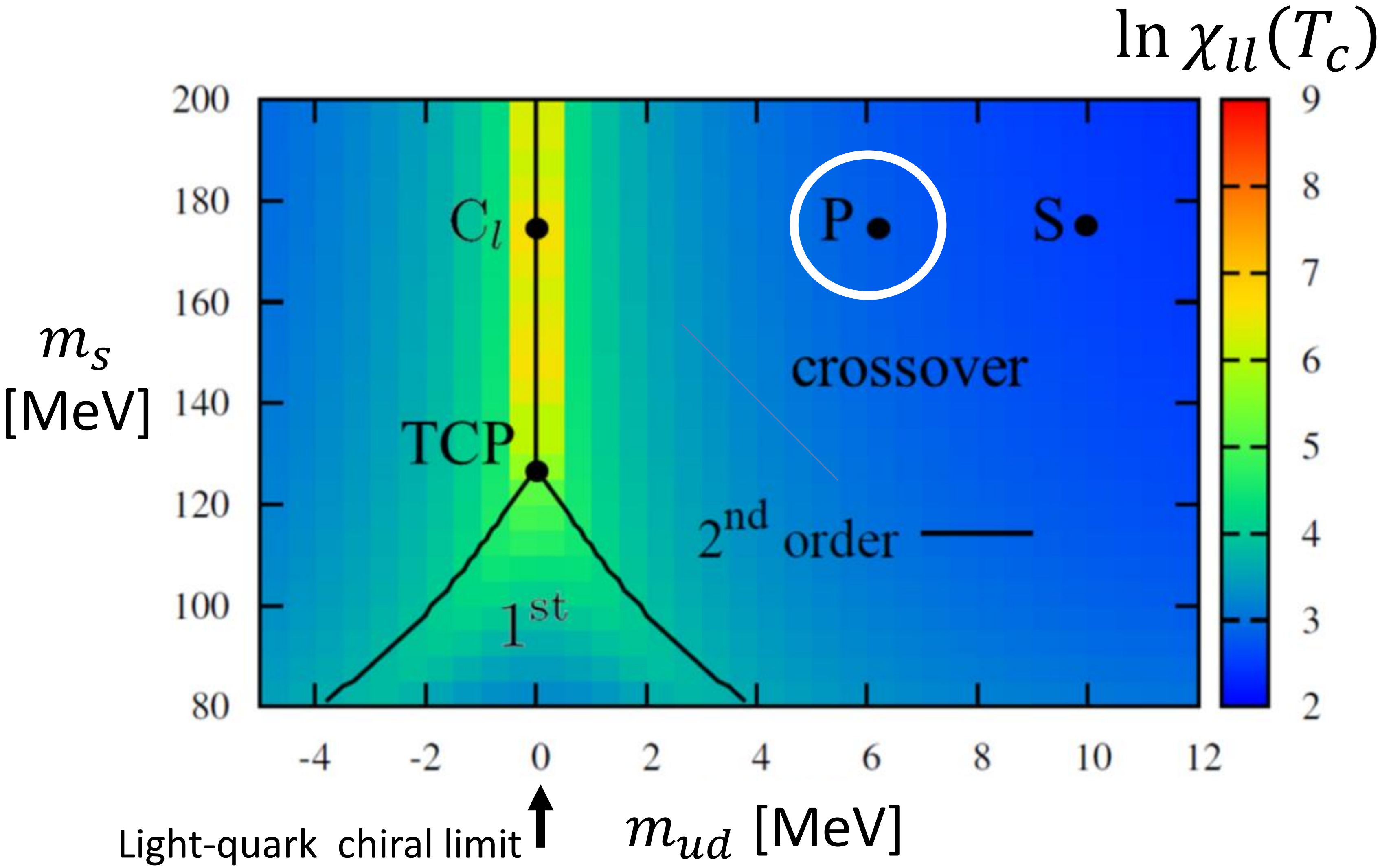
Columbia plot near the physical point (P)



Tricritical point exists at $(m_{ud}, m_s) = (0,127\text{MeV})$

Summary

- Determine temperature dependence of $U_A(1)$ symmetry restoration from lattice QCD results on π and a0 meson screening masses by using an effective model

  - The effective model well reproduces lattice QCD results of π and a_0 meson screening masses (Suppression of KMT interaction is essential)

- Investigate the Columbia plot near the physical point

  - At C_l point, chiral phase transition is 2^{nd} order Tricritical point exists at $(m_{ud}, m_s) = (0{,}127\text{MeV})$

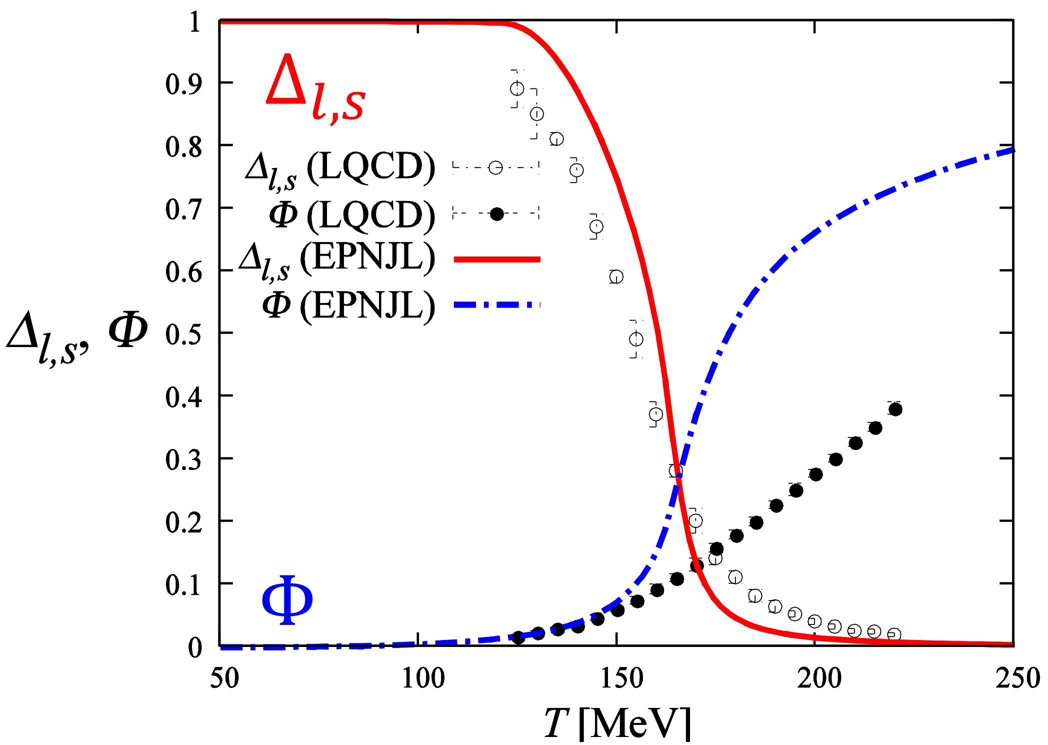
Thank you for your attention!

Backup

Renormalized chiral condensate $\Delta_{l,s}$ and Renormalized Polyakov loop Φ

$$\Delta_{l,s} = \frac{\sigma_l(T) - \frac{m_l}{m_s}\sigma_s(T)}{\sigma_l(0) - \frac{m_l}{m_s}\sigma_s(0)}$$

σ_l, σ_s: Light-, strange-quark chiral condensate



$\Delta_{l,s}$: The effective model and LQCD results agree with each other.

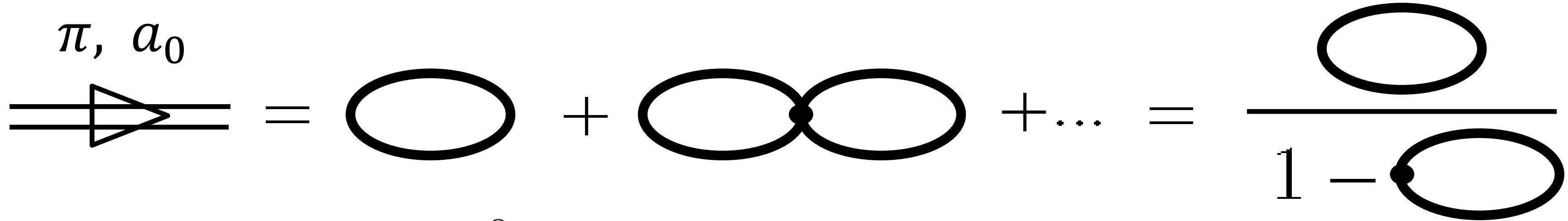
Φ : Φ_{EPNJL} does not contain the renormalization factor.

$T_c^{\mathrm{chiral}} = 163\mathrm{MeV}$, $T_c^{\mathrm{deconf}} = 165\mathrm{MeV}$ are consistent with [6], [7].

[6] A. Bazavov et al., Phys. Rev. D 85, 054503(2012), [7]Y. Aoki et al., J. High Energy Phys. 06 (2009) 088.

Meson propagator in momentum space

- Quark-antiquark scattering in the ring approximation;



external momentum: $q_0 = 0\ ,\ \mathbf{q}$

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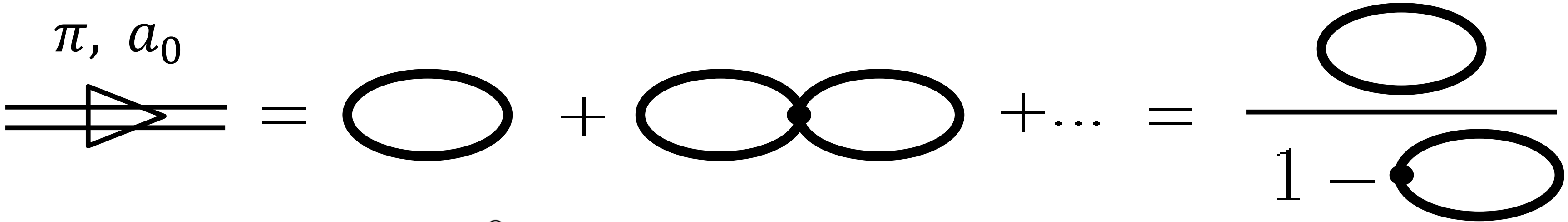
$$\chi_{\xi\xi}(0,\tilde{q}^2) = \frac{\Pi_{\xi\xi}(0,\tilde{q}^2)}{1 - 2G_\xi \Pi_{\xi\xi}(0,\tilde{q}^2)}$$

$\Pi_{\xi\xi}(0,\tilde{q}^2)$: quark loop $\quad \tilde{q} = |\mathbf{q}| \quad$ ξ : mesonic channel

G_ξ : strength of effective $q\bar{q}$ interaction

Meson propagator in momentum space

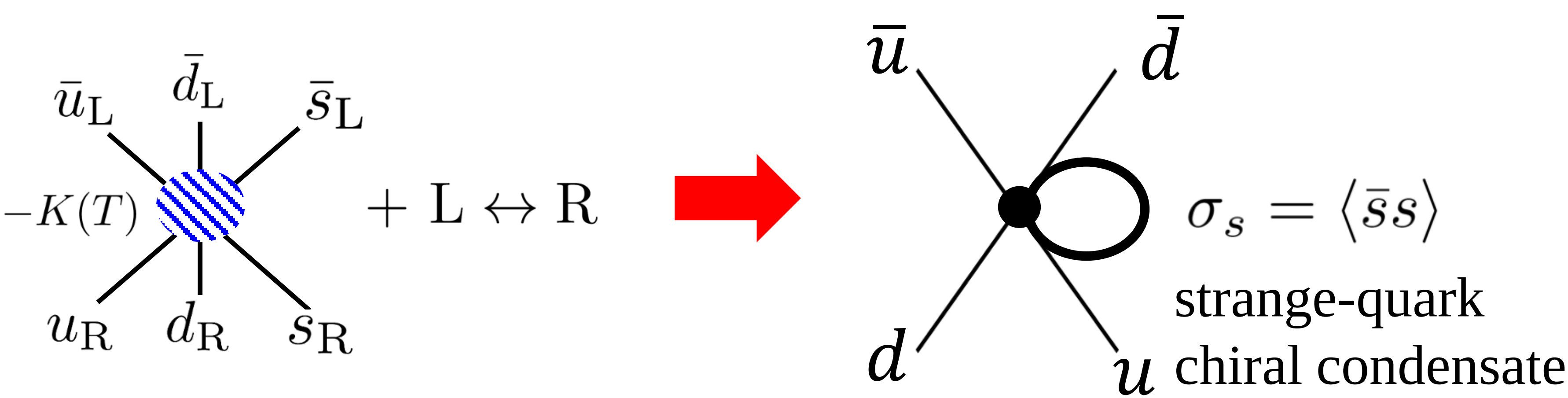
➢ Quark-antiquark scattering in the ring approximation;



external momentum: $q_0 = 0\ ,\ \mathbf{q}$

Mean field approximation

➢ KMT interaction is the effective two-body interaction



Meson propagator in momentum space

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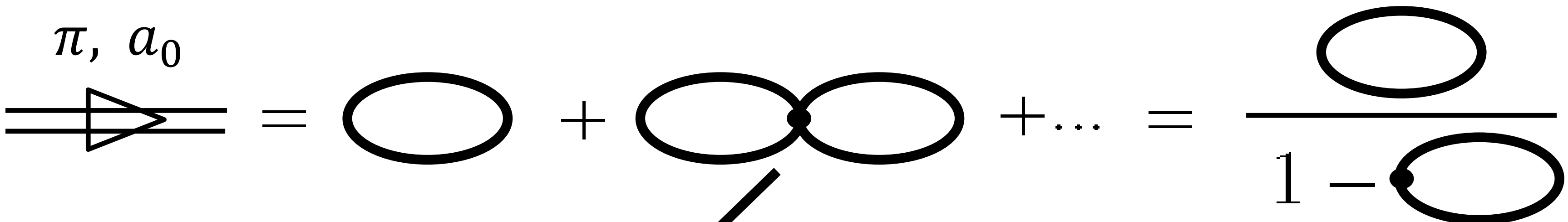

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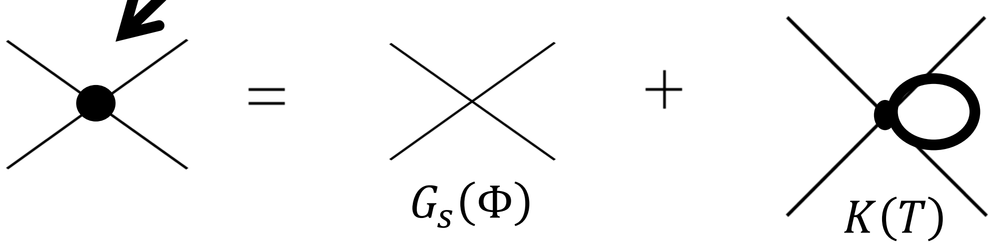


$$\begin{cases} G_\pi = G_s(\Phi) - \frac{1}{2}K(T)\sigma_s \\ G_{a_0} = G_s(\Phi) + \frac{1}{2}K(T)\sigma_s \end{cases}$$

$U_A(1)$ symmetry breaking from KMT interaction and σ_s.

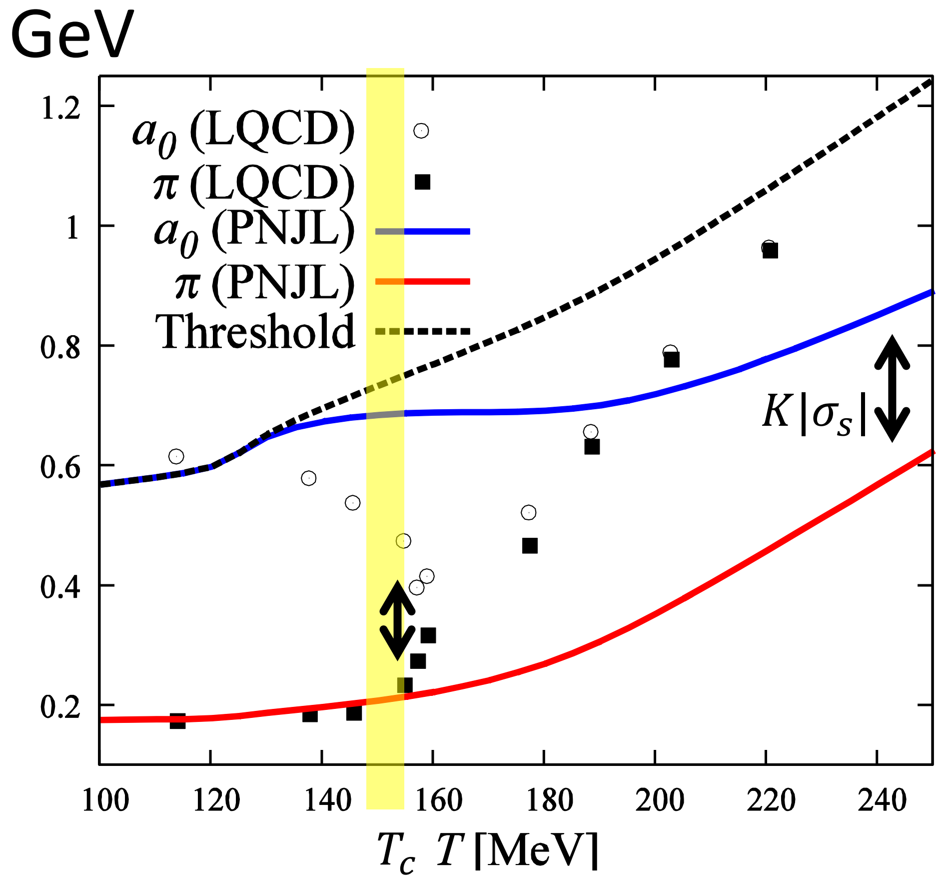
π and a_0 meson screening masses
($\bar{u}d$ meson)

KMT interaction strength is temperature independent.
PNJL model can not reproduce lattice QCD at all.

$T > T_c$:
$U_A(1)$ symmetry breaking from quark mass is negligible.
Main contribution is

$$M_{a_0} - M_\pi \propto K|\sigma_s|$$

- Lattice QCD results indicate the strong suppression of KMT interaction around T_c



[4] M. Cheng et al., Eur. Phys. J. C 71, 1564 (2011).

Meson propagator in the coordinate space and screening mass

Fourier transformation of χ ➢ Spatial propagator η

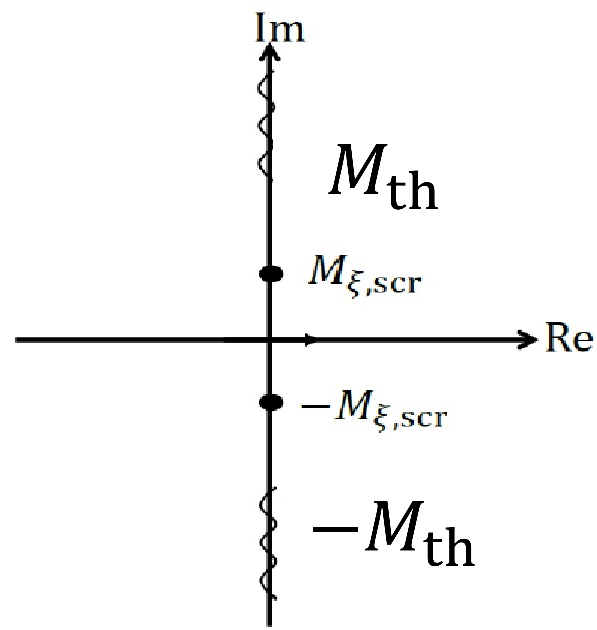
$$\eta_{\xi\xi}(r) = \frac{1}{4\pi^2 i r}\int_{-\infty}^{\infty} d\tilde{q}\ \tilde{q}\chi_{\xi\xi}(0,\tilde{q}^2)e^{i\tilde{q}r} \underset{r\to\infty}{\sim} \frac{1}{r}e^{-M_{\xi,\mathrm{scr}}r}$$

Difficulty:

Evaluation of highly oscillating function

➢ complex $\tilde{q}$ integral

Complex $\tilde{q}$ plane



χ has two poles on the imaginary axis.
Screening mass is a pole if it is located below $M_{\mathrm{th}} \sim 2\pi T$ [4].

$$\eta_{\xi\xi}(r) \underset{r\to\infty}{\sim} \frac{a}{r}\exp(-M_{\xi,\mathrm{scr}}r) + \cancel{\frac{b}{r}\exp(-M_{\mathrm{th}}r)}$$

[4] Phys. Rev. D89, 071901(R) (2014)