Standard-model prediction for direct CP violation in $K \to \pi\pi$ decays

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Introduction and Motivation

Motivation for studying K→ππ Decays

- Likely explanation for matter/antimatter asymmetry in Universe, baryogenesis, requires violation of CP in decays (direct CPV).

- Direct CPV first observed in late 90s at CERN and Fermilab in $K_0 \to \pi\pi$:

$$\eta_{00} = \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)}, \qquad \eta_{+-} = \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)}.$$

$$\mathrm{Re}(\epsilon'/\epsilon) \approx \frac{1}{6}\left(1 - \left|\frac{\eta_{00}}{\eta_{\pm}}\right|^2\right) = 16.6(2.3) \times 10^{-4} \qquad \text{(experiment)}$$

measure of direct CPV

measure of indirect CPV

- In terms of isospin states: $\Delta I=3/2$ decay to I=2 final state, amplitude A_2
  $\Delta I=1/2$ decay to I=0 final state, amplitude A_0

$$\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$$

$$A(K^0 \to \pi^+\pi^-) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2 e^{i\delta_2},$$
$$A(K^0 \to \pi^0\pi^0) = \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} - 2\sqrt{\frac{1}{3}}A_2 e^{i\delta_2}.$$

$$\Rightarrow \quad \epsilon' = \frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\left(\frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right)$$

(δ_I are strong scattering phase shifts.)

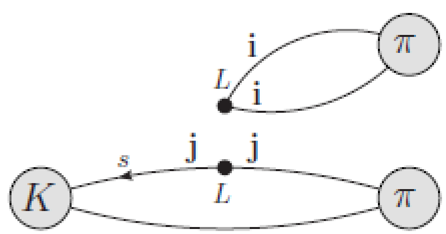
- Amount of direct CPV in Standard Model appears too low to describe measured M/AM asymmetry: tantalizing hint of new physics.

- Small size of ε' makes it particularly sensitive to new direct-CPV introduced by most BSM models.

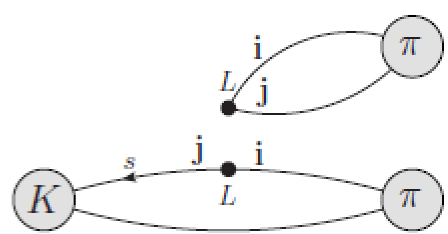
The role of the lattice

- In experiment kaons approx 450x (!) more likely to decay into I=0 pi-pi states than I=2. $$\frac{\mathrm{Re}A_0}{\mathrm{Re}A_2} \simeq 22.5$$ (the ΔI=1/2 rule)
- Perturbative running to charm scale accounts for about a factor of 2. Is the remaining 10x non-perturbative or New Physics?
- The answer is **low-energy *QCD!*** RBC/UKQCD [arXiv:1212.1474, arXiv:1502.00263]

  Strong cancellation between the two dominant contractions



Contraction ①.



Contraction ②.

$$\mathrm{Re}(A_2) \sim ① + ②$$

$$② \approx -0.7①$$

heavily suppressing $\mathrm{Re}(A_2)$.

- SU(3) ChPT unreliable because strange quark is too heavy.
- Lattice techniques allow for calculation completely from first principles and without any model dependence.

Standard Model Physics and Lattice Determination

Weak Effective Theory

- At energy scales $\mu \ll M_W$, $K \to \pi\pi$ decays accurately described by weak effective theory.

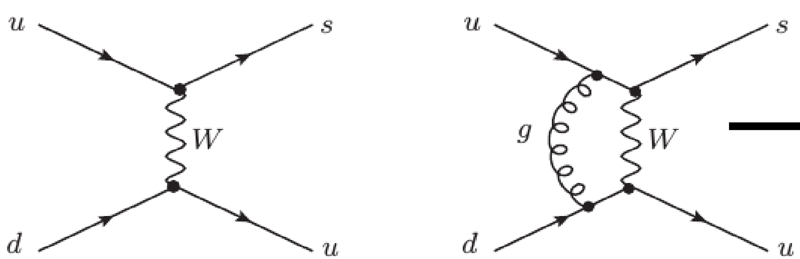
$$H_W^{\Delta S=1} = \frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{j=1}^{10} [z_j(\mu) + \tau y_j(\mu)] Q_j$$

perturbative Wilson coeffs.

$$\tau = -\frac{V_{ts}^* V_{td}}{V_{us}^* V_{ud}} = 0.0014606 + 0.00060408i$$

Imaginary part solely responsible for CPV
(everything else is pure-real)

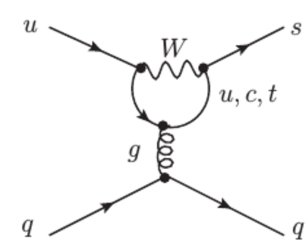
- Q_j are 10 effective four-quark operators:



(a) current-current

$\longrightarrow Q_1, Q_2$

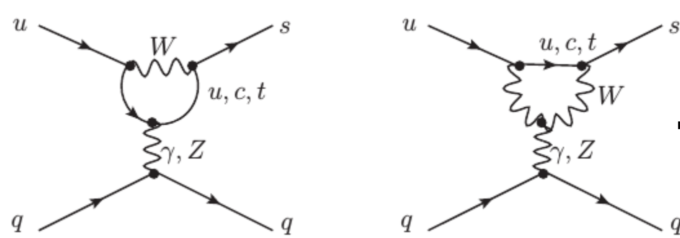
dominate
$\mathrm{Re}(A_0)$, $\mathrm{Re}(A_2)$



(b) QCD penguin

$\longrightarrow Q_3 - Q_6$

Q_4, Q_6 dominate
$\mathrm{Im}(A_0)$



(c) Electro-Weak penguin

$\longrightarrow Q_7 - Q_{10}$

Q_7, Q_8 dominate
$\mathrm{Im}(A_2)$

Lattice Determination of K→ππ

- On the lattice compute $M_j = \langle (\pi\pi)_I | Q_j | K \rangle$
- Operators must be renormalized into same scheme as Wilson coeffs: Use RI-(S)MOM NPR and perturbatively match to MSbar at high scale.
- Mixing under renormalization, hence Z is a matrix.

$$A_{2/0} = F \frac{G_F}{\sqrt{2}} V_{ud} V_{us} \sum_{i=1}^{10} \sum_{j=1}^{7} \left[\Big(z_i(\mu) + \tau y_i(\mu) \Big) Z_{ij}^{\mathrm{lat}\to\overline{\mathrm{MS}}} M_j^{\frac{3}{2}/\frac{1}{2},\mathrm{lat}} \right],$$

- F is finite-volume correction calculated using LL method.
- Important to calculate with physical (energy-conserving) kinematics. With physical masses:

$$2 \times m_\pi \sim 270 \text{ MeV} \qquad\qquad m_K \sim 500 \text{ MeV}$$

we require non-zero relative momentum for the pions.

- This is excited state of the ππ-system. Possibilities:
  - try to perform multi-state fits to very noisy data (esp. A_0 where there are disconn. diagrams) or
  - modify boundary conditions to remove the ground-state

An old homework problem

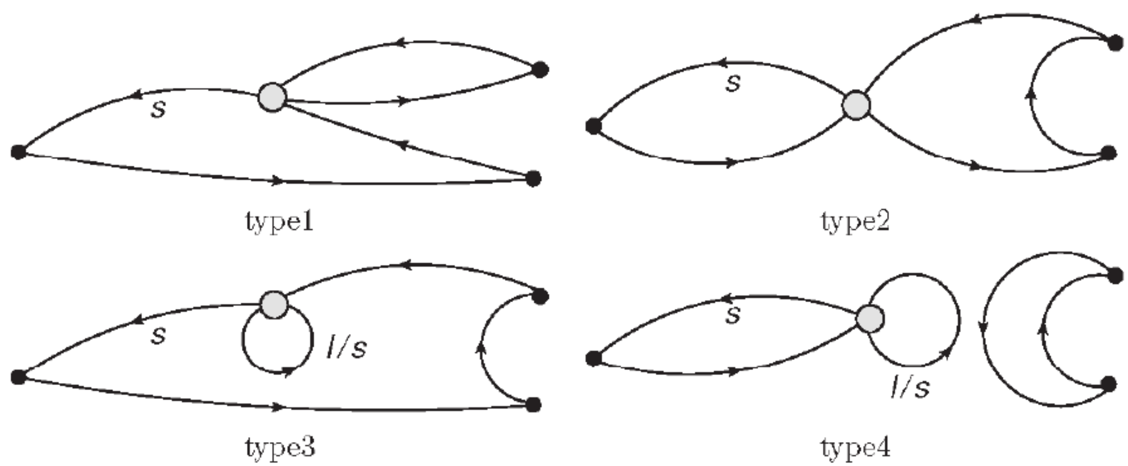
- 1964: CP-violation (indirect) first observed at BNL (Cronin, Fitch et al → 1980 Nobel prize)
- 1973: Framework for Standard Model CPV established (Kobayashi, Maskawa)
- 1993: Publication of first evidence of direct CPV from NA31 expt at CERN.
- 1999: KTeV at FermiLab and NA48 at CERN confirm direct CPV.
- 2001: First quenched calculations of ε' performed by CP-PACS and RBC using single particle amplitudes and LO ChPT to correct for missing pion.
- 2001: Technique established for lattice measurement of decays (Lellouch, Luscher)
- 2011: First full threshold (stationary, unphysically-heavy pions) calc. of A_0 and A_2 using dynamical domain wall fermions performed by RBC/UKQCD.
- 2012: First calculation of A_2 performed by RBC/UKQCD using DWF with physical kinematics, pion masses and large physical volume but single lattice spacing.
- 2015: Continuum calculation of A_2 performed by RBC/UKQCD
- 2015: Full threshold calculation of A_0 and A_2 using Wilson fermions by Ishizuka *et al* [arXiv:1505.05289]
- 2015: (This work) First complete, *ab initio* determination of ε' with physical kinematics and pion masses.

$\Delta I=1/2$ Calculation

arXiv:1505.07863 [hep-lat]

Matrix element calculation

- A_0 obtained via neutral kaon decays $K^0 \to \pi^+\pi^-$ and $K^0 \to \pi^0\pi^0$
- ~50 distinct contraction topologies → 4 classes:



- Type 4 disconn. diagrams dominate noise. Use Trinity-style all-to-all (A2A) propagators:
    - 900 exact low-eigenmodes computed using Lanczos algorithm
    - Stochastic high-modes with full spin/color/flavor dilution
- Measure with 5 different K→π separations (10,12,14,16,18)
- Perform all spatial and temporal translations of both type 3 and type 4 diagrams (cleaner type 1 and 2 measured every 8th timeslice)

Physical Kinematics

- A_2 calculation used APBC on d-quarks, removes stationary **charged pion** state BUT breaks isospin and doesn't work for π^0.
- Solution: Use G-parity BCs:

$$\hat{G} = \hat{C}e^{i\pi \hat{I}_y} \quad : \quad \hat{G}|\pi^\pm\rangle = -|\pi^\pm\rangle \qquad \hat{G}|\pi^0\rangle = -|\pi^0\rangle$$

- As a boundary condition: (i=+, -, 0)

$$\pi^i(x+L) = \hat{G}\pi^i(x) = -\pi^i(x) \longrightarrow |p| \in (\pi/L,\, 3\pi/L,\, 5\pi/L\ldots)$$

(moving ground state)

- At quark level: $\hat{G}\begin{pmatrix} u \\ d \end{pmatrix} = \begin{pmatrix} -C\bar{d}^T \\ C\bar{u}^T \end{pmatrix}$ where C = $\gamma^2\gamma^4$ in our conventions
- Gauge invariance → gauge field must obey charge conjugation BCs; new ensembles needed.
- For stationary kaon we must introduce fictional degenerate partner to the strange quark: s'

$$|\tilde{K}^0\rangle = \left(|\bar{s}d\rangle + |\bar{u}s'\rangle\right)/\sqrt{2}$$ is G-parity even (p=0)

- Coupling of unphysical kaon partner to physical operators exponentially suppressed and can be neglected.

Ensemble

- 32^3x64 Mobius DWF ensemble with IDSDR gauge action at β=1.75. Coarse lattice spacing (a^{-1}=1.378(7) GeV) but large, $(4.6\text{ fm})^3$ box.
- Using Mobius params (b+c)=32/12 and L_s=12 obtain same explicit χSB as the L_s=32 Shamir DWF + IDSDR ens. used for ΔI=3/2 but at reduced cost.
- Utilized USQCD 512-node BG/Q machine at BNL, the DOE “Mira” BG/Q machines at ANL and the STFC BG/Q “DiRAC” machines at Edinburgh, UK.
- Performed 216 independent measurements (4 MDTU sep.).
- Cost is ~1 BG/Q rack-day per complete measurement (4 configs generated + 1 set of contractions).
- G-parity BCs in 3 spatial directions results in close matching of kaon and $\pi\pi$ energies:

$$m_K=490.6(2.4)\text{ MeV}$$

$$E_{\pi\pi}(I=0) = 498(11)\text{ MeV}$$

$$E_{\pi\pi}(I=2) = 573.0(2.9)\text{ MeV}$$

$$E_\pi=274.6(1.4)\text{ MeV} \quad (m_\pi = 143.1(2.0)\text{ MeV})$$

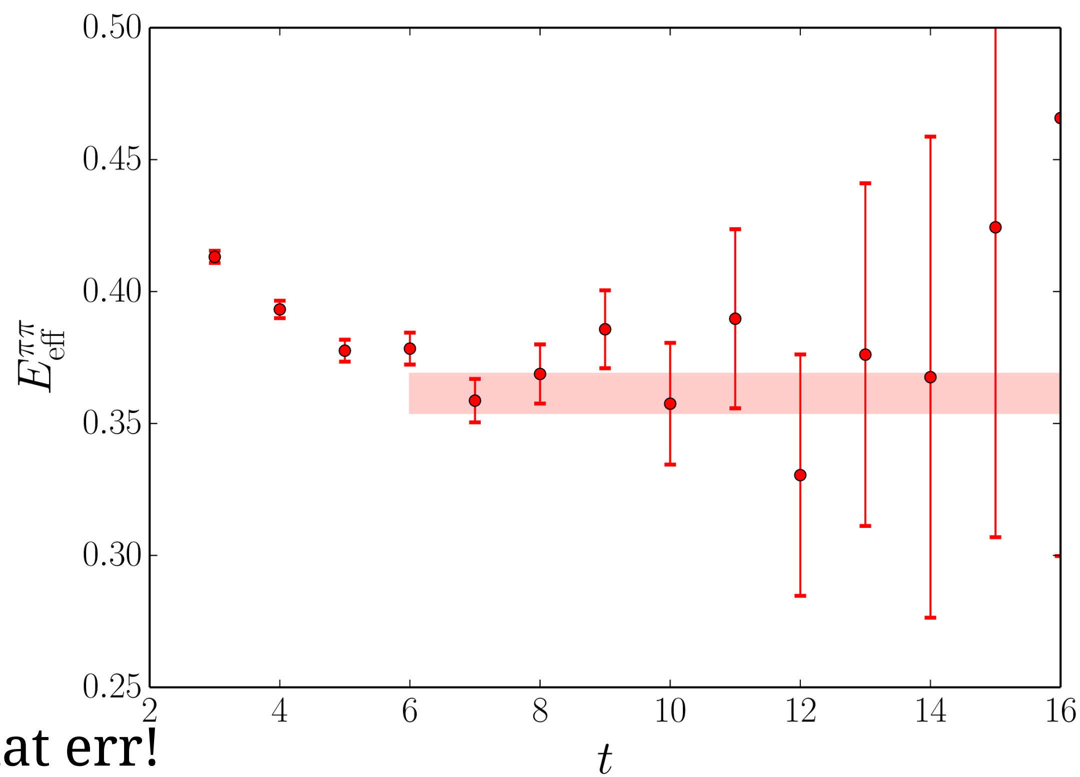
I=0 ππ energy

- Signal/noise deteriorates quickly due to vacuum contrib.
- Difficult to determine plateau start. Performed both 1- and 2-state fits.

$t_{\rm min}$	$E_{\pi\pi}$	$E_{\rm exc}$	$\chi^2/{\rm dof}$
2	0.363(9)	1.04(17)	1.7(7)
3	0.367(11)	1.27(73)	1.8(8)
4	0.364(12)	0.86(39)	1.9(8)

$t_{\rm min}$	$E_{\pi\pi}$	$\chi^2/{\rm dof}$
5	0.375(6)	2.2(9)
6	0.361(7)	1.6(7)
7	0.380(11)	0.9(7)

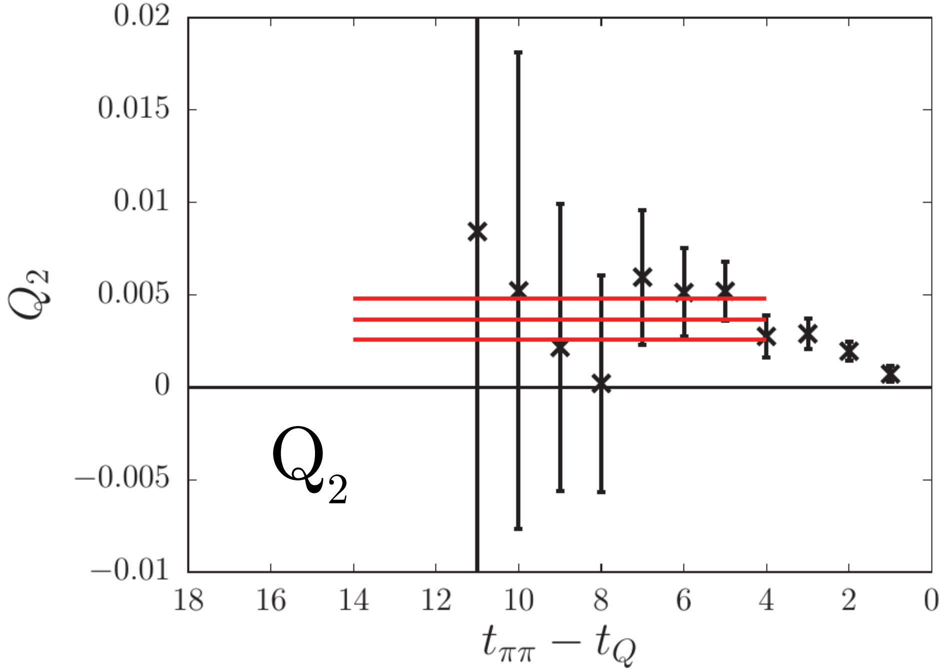
2% stat err!



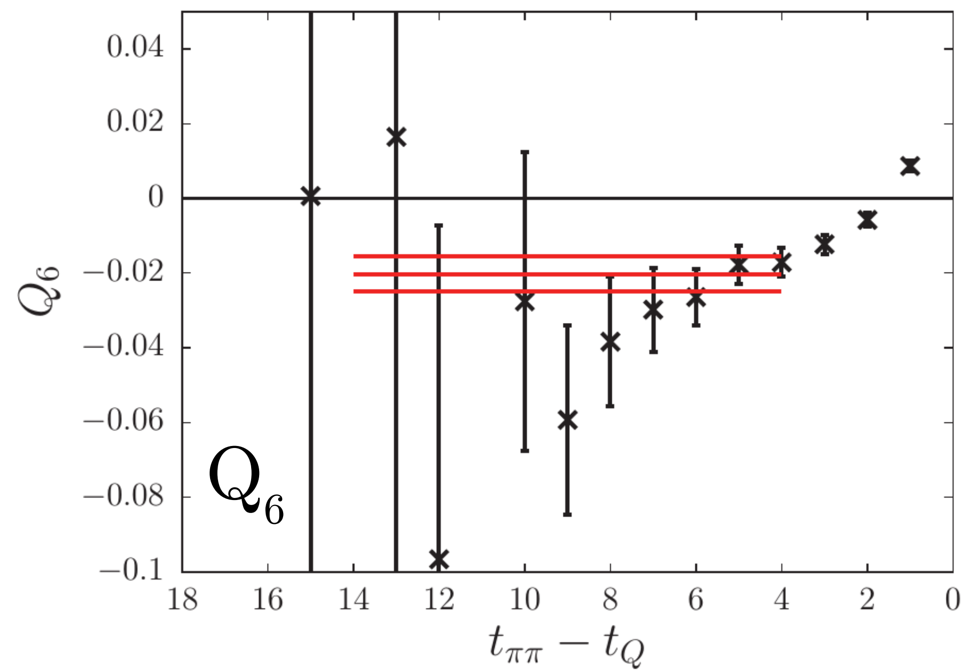
- Our phase shift $\delta_0 = 23.8(4.9)(1.2)^\circ$ lower than most pheno estimates, which prefer $\delta_0 \sim 35^\circ$.
- Luscher formula very steep in $E_{\pi\pi}$: small shifts energy translate to large (fractional) errors in δ_0. More statistics needed to resolve.
- Using 35° → ~3% change in A_0; much smaller than other errs. For consistency we choose to use our lattice value.

Matrix element fits

[Dominant contribution to Re(A_0)]



[Dominant contribution to Im(A_0)]



- Use $t_{min}(\pi \rightarrow Q) = 4$ here rather than 6 as signal quickly decays into noise (40% increase in stat. error with t_{min}=5!).
- However comparison to t_{min}=3 shows no statistically resolvable difference, suggesting excited state contamination small.
- Estimate 5% excited state systematic by comparing single-exp fit result for $\pi\pi$(I=0) amplitude with t_{min}=4 to double-exp fit with t_{min}=3.

Systematic errors

- Errors for each separate operator matrix element:

Description	Error	Description	Error
Finite lattice spacing	8%	Finite volume	7%
Wilson coefficients	12%	Excited states	$\leq 5\%$
Parametric errors	5%	Operator renormalization	15%
Unphysical kinematics	$\leq 3\%$	Lellouch-Lüscher factor	11%
Total (added in quadrature)			26%

- Treat as uncorrelated when combining to form A_0.

- 15% renormalization error dominant due to low, 1.53 GeV renormalization scale. Estimate by comparing two different RI/SMOM intermediate schemes and use the largest observed differences.

- 12% Wilson coefficient error large for same reason. Conservatively estimate as largest observed fractional change between using LO and NLO.

Results for A_0

$$\mathrm{Re}(A_0) = 4.66(1.00)_{\mathrm{stat}}(1.21)_{\mathrm{sys}} \times 10^{-7}\ \mathrm{GeV} \quad \text{(This work)}$$

$$\mathrm{Re}(A_0) = 3.3201(18) \times 10^{-7}\ \mathrm{GeV} \quad \text{(Experiment)}$$

- Good agreement between lattice and experiment for $\mathrm{Re}(A_0)$ serves as test for method.
- $\mathrm{Re}(A_0)$ from expt far more precise, and is dominated by tree-level Q_1 and Q_2 hence unlikely to receive large BSM contributions. Use for computing ε'.

$$\mathrm{Im}(A_0) = -1.90(1.23)_{\mathrm{stat}}(1.04)_{\mathrm{sys}} \times 10^{-11}\mathrm{GeV} \quad \text{(This work)}$$

- ~85% total error on the predicted $\mathrm{Im}(A_0)$ due to strong cancellation between dominant Q_4 and Q_6 contributions:

$$\begin{aligned} \Delta[\mathrm{Im}(A_0), Q_4] &= 1.82(0.62)(0.32) \times 10^{-11} \\ \Delta[\mathrm{Im}(A_0), Q_6] &= -3.57(0.91)(0.24) \times 10^{-11} \end{aligned}$$

despite only 40% and 25% respective errors for the matrix elements.

Results for ε' and concluding remarks

Results for ε'

- Using $\mathrm{Re}(A_0)$ and $\mathrm{Re}(A_2)$ from experiment and our lattice values for $\mathrm{Im}(A_0)$ and $\mathrm{Im}(A_2)$ and the phase shifts,

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\varepsilon}\left[\frac{\mathrm{Im}A_2}{\mathrm{Re}\mathrm{A}_2} - \frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}\right]\right\}$$

$$\boxed{\begin{array}{lr} = \quad 1.38(5.15)(4.43)\times 10^{-4}, & \text{(this work)} \\ \quad 16.6(2.3)\times 10^{-4} & \text{(experiment)} \end{array}}$$

- Find discrepancy between lattice and experiment at the 2.1σ level.

Conclusions and Outlook

- First direct computation of A_0 with controllable errors performed.
- Measured $\mathrm{Re}(A_0)$ in good agreement with experiment.
- 85% total error on $\mathrm{Im}(A_0)$ despite 25% and 40% errors on dominant Q_6 and Q_4 contributions resp., due to strong mutual cancellation.
- On final result, stat. error currently dominant.
- Sys. errors dominated by perturbative truncation errors on the renormalization and Wilson coeffs due to low, 1.53 GeV scale.
- Currently computing NPR running to higher energies in order to reduce this systematic.
- Total error on $\mathrm{Re}(\varepsilon'/\varepsilon)$ is ~3x the experimental error, and we observe a 2.1σ discrepancy. Strong motivation for continued study!
- Hope to achieve O(10%) errors on $\mathrm{Re}(\varepsilon'/\varepsilon)$ on a timescale of ~5 years.
- We hope these results with spur new efforts in the experimental community to reduce the current 15% error on the experimental number.

Thank you!

$\Delta I=3/2$ Calculation

Phys.Rev. D 91 (2015) 7, 074502
[arXiv:1502.00263 [hep-lat]].

Calculation Strategy

- A_2 can be computed directly from charged kaon decay:

$$\langle(\pi\pi)^{I=2}_{I_3=1}|H_W|K^+\rangle = \sqrt{2}A_2e^{i\delta_2}$$

- Remove stationary (charged) pion state using antiperiodic BCs on d-quark propagator: $d(x+L) = -d(x)$ $\rightarrow$ $|p| \in (\pi/L,\, 3\pi/L,\, 5\pi/L\ldots)$

$\pi^+(x+L) = [\bar{u}d](x+L) = -\pi^+(x)$ Moving ground state!

$\pi^0(x+L) = [\bar{u}u - \bar{d}d](x+L) = +\pi^0(x)$ Stationary ground state....

- Use Wigner-Eckart theorem to remove neutral pion from problem

$$\langle(\pi^+\pi^0)_{I=2}|Q^{\Delta I_z=1/2}|K^+\rangle = \frac{\sqrt{3}}{2}\langle(\pi^+\pi^+)_{I=2}|Q^{\Delta I_z=3/2}|K^+\rangle$$

- APBCs on d-quark break isospin symmetry allowing mixing between isospin states: however $\pi^+\pi^+$ is the only charge-2 state with these Q-numbers hence it cannot mix.

- Calculation performed on RBC & UKQCD 48^3x96 and 64^3x128 Mobius DWF ensembles with $(5\text{ fm})^3$ volumes and a=0.114 fm, a=0.084 fm. Continuum limit computed.
- Make full use of eigCG and AMA to translate over all timeslices. Obtain 0.7-0.9% stat errors on all bare matrix elements!
- Results:

$$\mathrm{Re}(A_2) = 1.50(4)_{\mathrm{stat}}(14)_{\mathrm{sys}} \times 10^{-8}\ \mathrm{GeV}$$
$$\mathrm{Im}(A_2) = -6.99(20)_{\mathrm{stat}}(84)_{\mathrm{sys}} \times 10^{-13}\ \mathrm{GeV}$$

10%, 12% total errors on Re, Im!

- Systematic error completely dominated by perturbative error on NPR and Wilson coefficients.