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Lattice QCD Input to Axion Cosmology

Evan Berkowitz
Lawrence Livermore National Laboratory

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arXiv:1505.07455 – E. Berkowitz, M. Buchoff, E. Rinaldi

LLNL-PRES-669910
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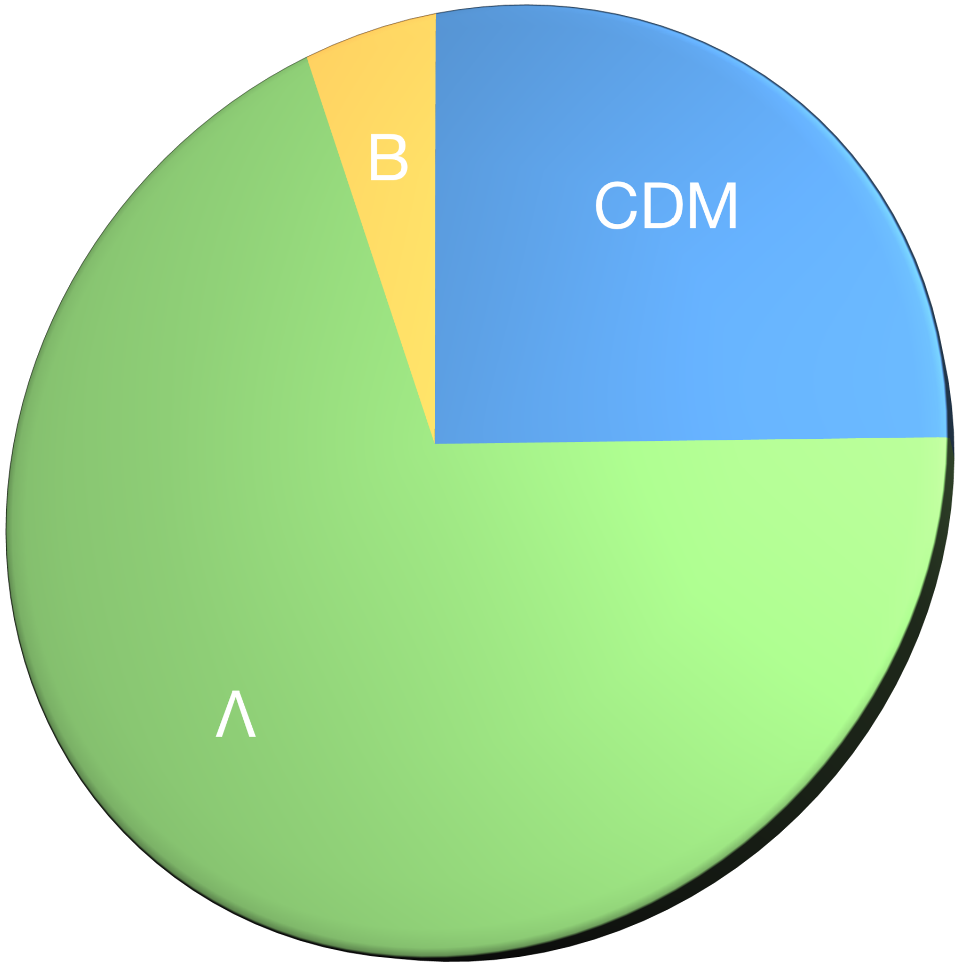
Big Idea

- Axions were originally proposed to deal with the Strong CP Problem, also form a plausible DM candidate.
  - Calculating the axion energy density requires nonperturbative QCD input.
- Being sought in ADMX (LLNL, UW) & CAST (CERN), and (soon) IAXO with large discovery potential in the next few years.
- Requiring $\Omega_a \leq \Omega_{CDM}$ yields a lower bound on the axion mass today.
  Preskill, Wise & Wilczek, Phys Lett B **120** (1983) 127-132

The Economist, 19 Dec 2006



Ω_{tot} = 1.000(7)
PDG 2014 via
P.A.R. Ade, et al., (Planck Collab. 2013 XVI), arXiv: 1303.5076v1.

QCD Theta Term

- QCD has a parameter, θ.
    - Controls QCD CP violation.
    - Topological.
- θ can take any value in (-π,π].
- Neutron EDM ≲ 3 10^{-26}e•cm

Baker et al., PRL 97, 131801 (2006) / hep-ex/0602020

- $\Longrightarrow$ |θ| ≲ 10^{-10}

$$\mathcal{L}_{\mathrm{QCD}} \ni \theta \, \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

CP Violating

$$Q = \frac{1}{32\pi^2} \int d^4x \; \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} \in \mathbb{Z}$$

$$e^{iS} \propto e^{iQ\theta}$$

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  - $\Longrightarrow$ $|\theta| \lesssim 10^{-10}$

$$\mathcal{L}_{\rm QCD} \ni \theta \, \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

CP Violating

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$$e^{iS} \propto e^{iQ\theta}$$

Strong CP Problem:
Why is θ so small?

Axions

Peccei & Quinn: PRL **38** (1977) 1440, PR **D16** (1977) 1791

- Couple to topological charge

$$\mathcal{L}_{\text{axions}} = \frac{1}{2}\left(\partial_\mu a\right)^2 + \left(\frac{a}{f_a} + \theta\right) \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

- Otherwise have shift symmetry.

$$a \to a + \alpha$$

- Amenable to effective theory treatment

$$V_{\text{eff}} \sim \cos\left(\theta + c\langle a\rangle\right)$$

$$m_a^2 f_a^2 = \left.\frac{\partial^2 F}{\partial \theta^2}\right|_{\theta=0}$$

- PQ symmetry can break before or after inflation.

Axions

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- Otherwise have shift symmetry.

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- Amenable to effective theory treatment

$$V_{\text{eff}} \sim \cos\left(\theta + c\langle a \rangle\right)$$

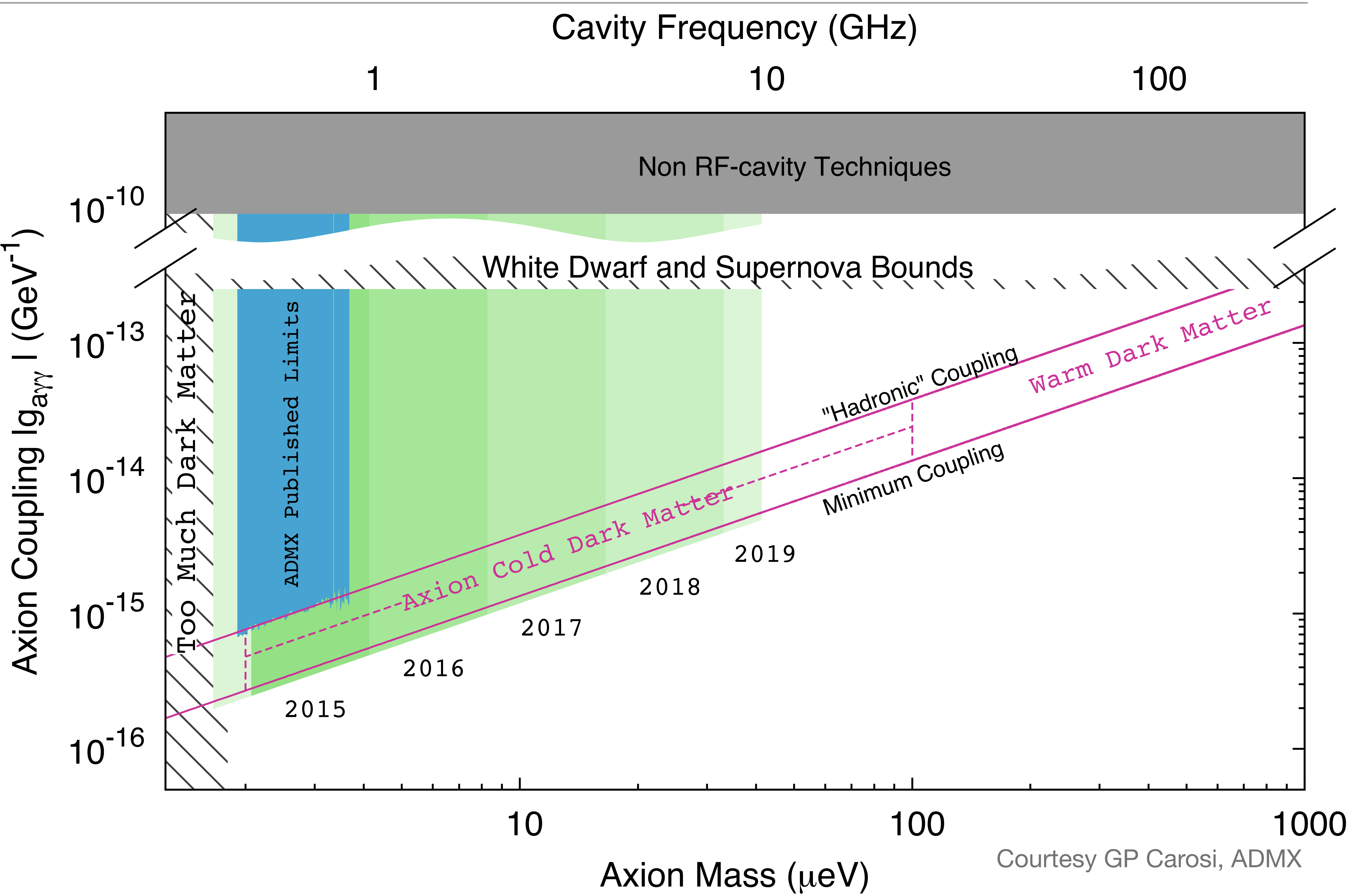
$$m_a^2 f_a^2 = \chi$$

Axion physics

QCD Topological Susceptibility

- PQ symmetry can break before or after inflation.

Current Axion Constraints



Courtesy GP Carosi, ADMX

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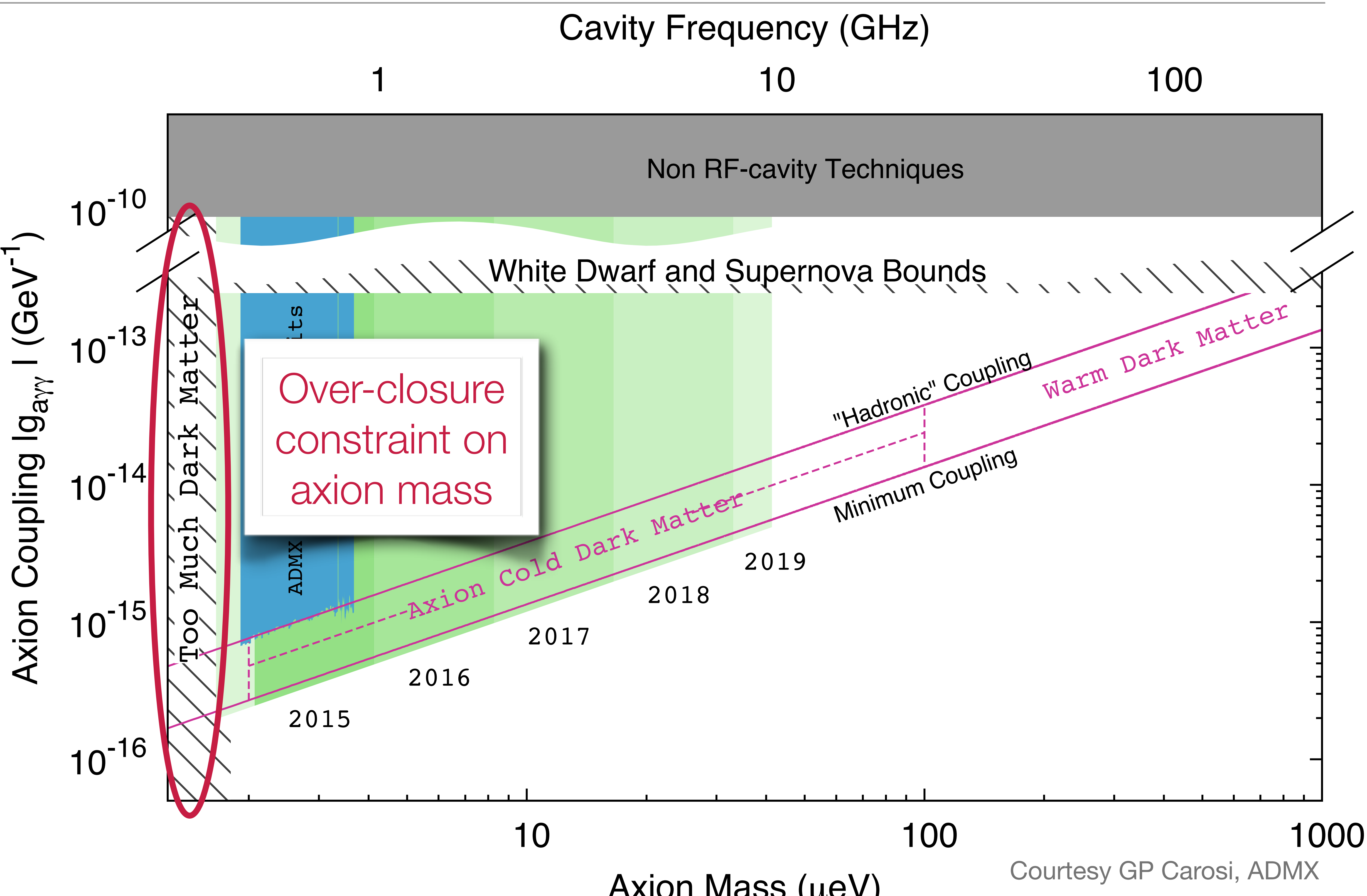

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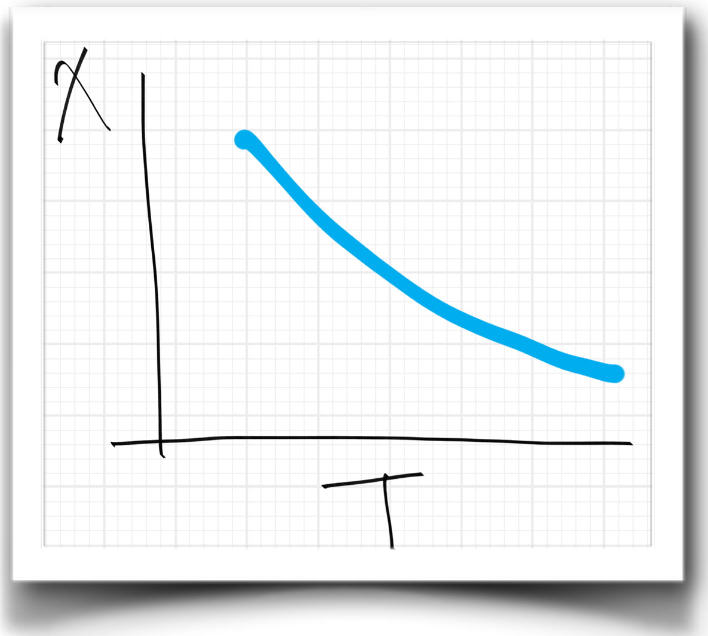
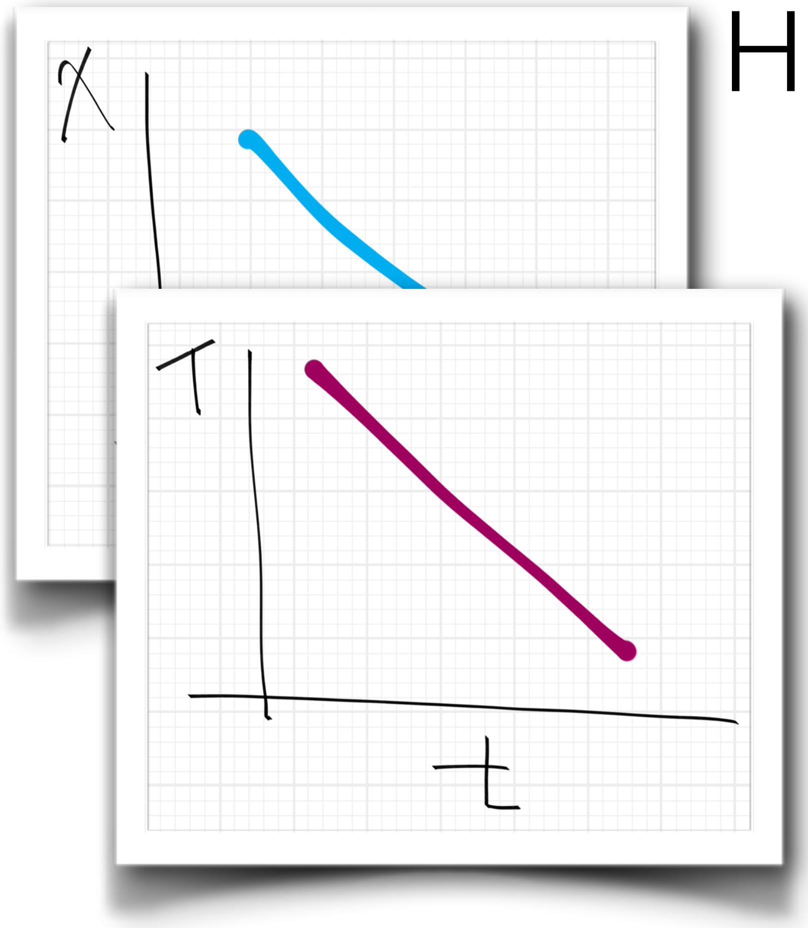
The Over-Closure Bound



High temperature arguments imply χ vanishes as $T \to \infty$

The Over-Closure Bound

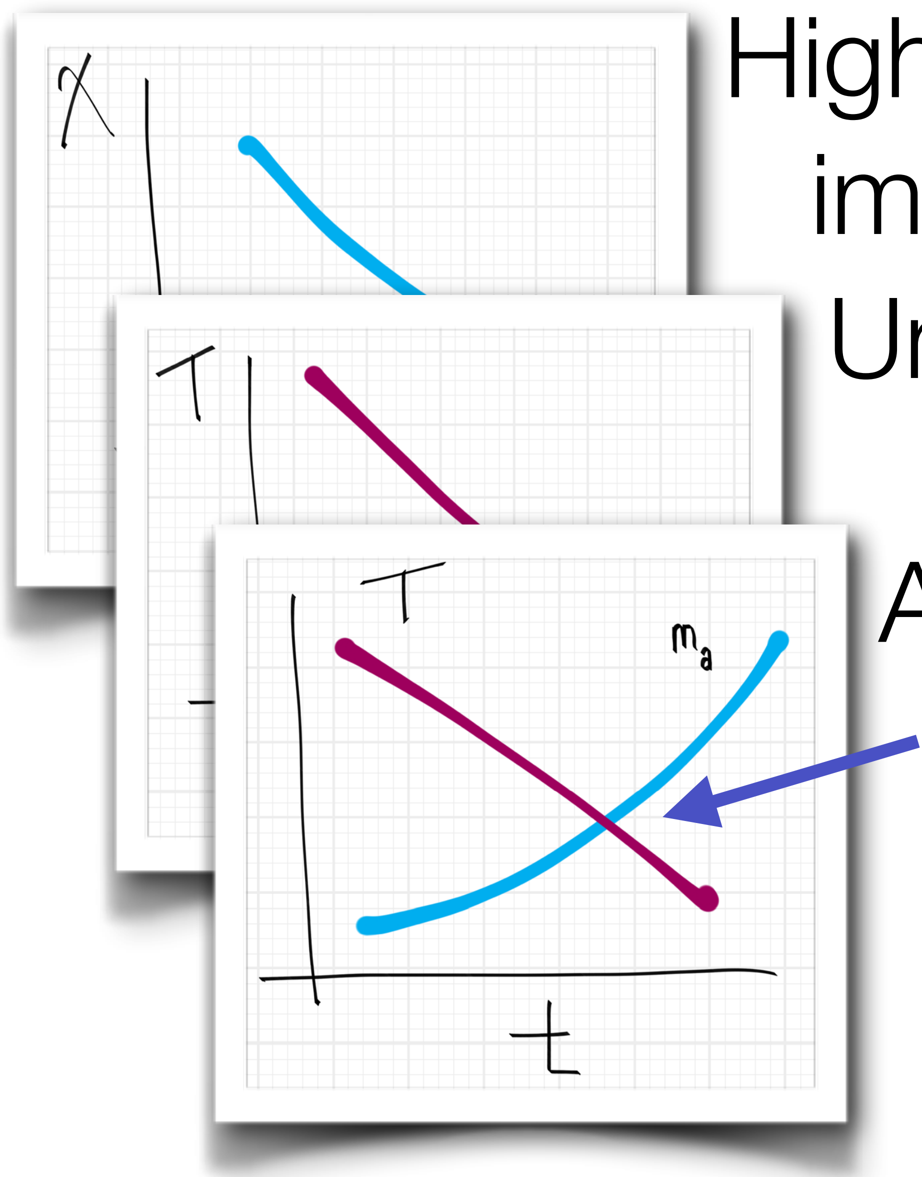


High temperature arguments

imply χ vanishes as $T \to \infty$

Universe cools as it expands

The Over-Closure Bound



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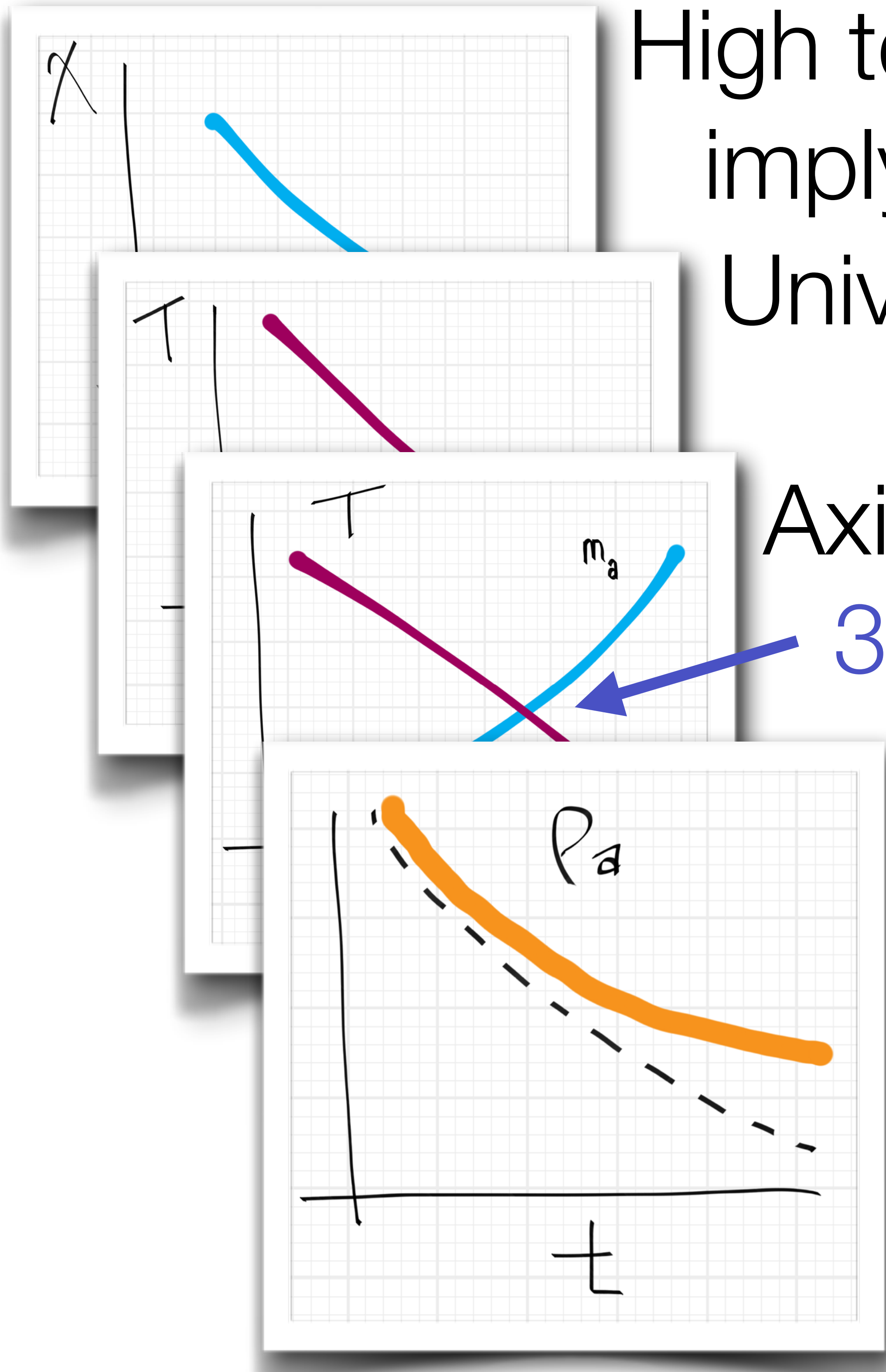
Universe cools as it expands

Axion number fixed when

$3H \sim m \leftrightarrow T_1 \approx 5.5\ T_c$

H: Hubble constant

The Over-Closure Bound



High temperature arguments imply χ vanishes as T→∞

Universe cools as it expands

Axion number fixed when

3H ~ m ↔ $T_1 \approx 5.5\ T_c$

H: Hubble constant

Axion mass continues to grow as universe cools

$$\frac{\rho(t)R^3}{m_a(t)} = \#\text{ axions in a fixed comoving volume}$$

Axion Density

$$\frac{\rho(t)R^3}{m_a(t)} = \#\text{ axions in a fixed comoving volume}$$

$$\rho(T_\gamma) = \rho(T_1)\frac{m_a(T_\gamma)}{m_a(T_1)}\left(\frac{R(T_1)}{R(T_\gamma)}\right)^3$$

$$T_\gamma = 2.73\mathrm{K}$$

$$T_1 = T_1(f_a, \chi)$$

$$m_a(T_1) = \frac{\sqrt{\chi(T_1)}}{f_a}$$

$$m_a(T_\gamma) = \frac{1}{f_a}\frac{\sqrt{m_u m_d}}{m_u + m_d} f_\pi m_\pi$$

χPT

$$R(T)$$

from cosmology

$$\rho(T_1) = \frac{1}{2}m_a^2 f_a^2 \theta_1^2$$

θ_1 random in a (cosmologically) small volume if PQ-breaking is after inflation.

$$\langle \theta_1^2 \rangle = \frac{\pi^2}{3}$$

(eg. BICEP?)

Axion Density

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$$T_1 = T_1(f_a, \chi) \qquad m_a(T_1) = \frac{\sqrt{\chi(T_1)}}{f_a}$$

$$m_a(T_\gamma) = \frac{1}{f_a}\frac{\sqrt{m_u m_d}}{m_u + m_d} f_\pi m_\pi$$ χP

Lattice determination of χ(T) required

$R(T)$ from cosmology

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χP

Lattice determination of χ(T) required

$R(T)$ from cosmology

small

flation.

BICEP?)

Punchline: given χ(T)

$\rho(T_\gamma)$ = a function of f_a only

Axion Density

$\rho(T_\gamma)$ ↑ with f_a

$\Omega_a \leq \Omega_{\rm CDM}$ ➡ upper bound on f_a

$$m_a(T_\gamma) = \frac{1}{f_a}\frac{\sqrt{m_u m_d}}{m_u + m_d} f_\pi m_\pi$$ (χPT) ➡ lower bound on m_a today

Axion Density

$\rho(T_\gamma)$ ↑ with f_a

$\Omega_a \leq \Omega_{\rm CDM}$ ➡ upper bound on f_a

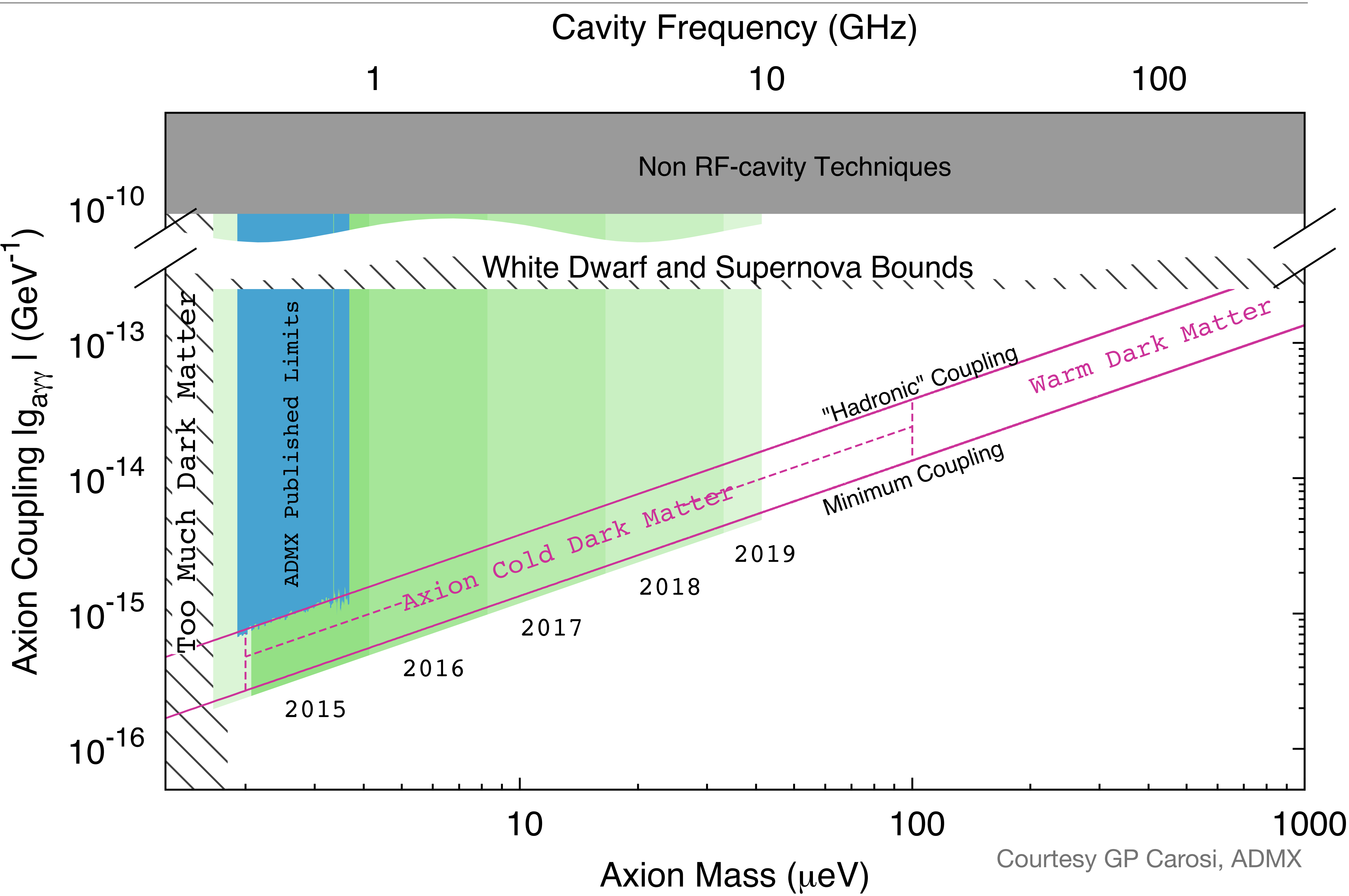
$$m_a(T_\gamma) = \frac{1}{f_a}\frac{\sqrt{m_u m_d}}{m_u + m_d} f_\pi m_\pi \quad (\chi\text{PT})$$

➡ lower bound on m_a today

Punchline: given χ(T) we get m_a lower bound

Current Axion Constraints

Wantz & Shellard, arXiv:0910.1066



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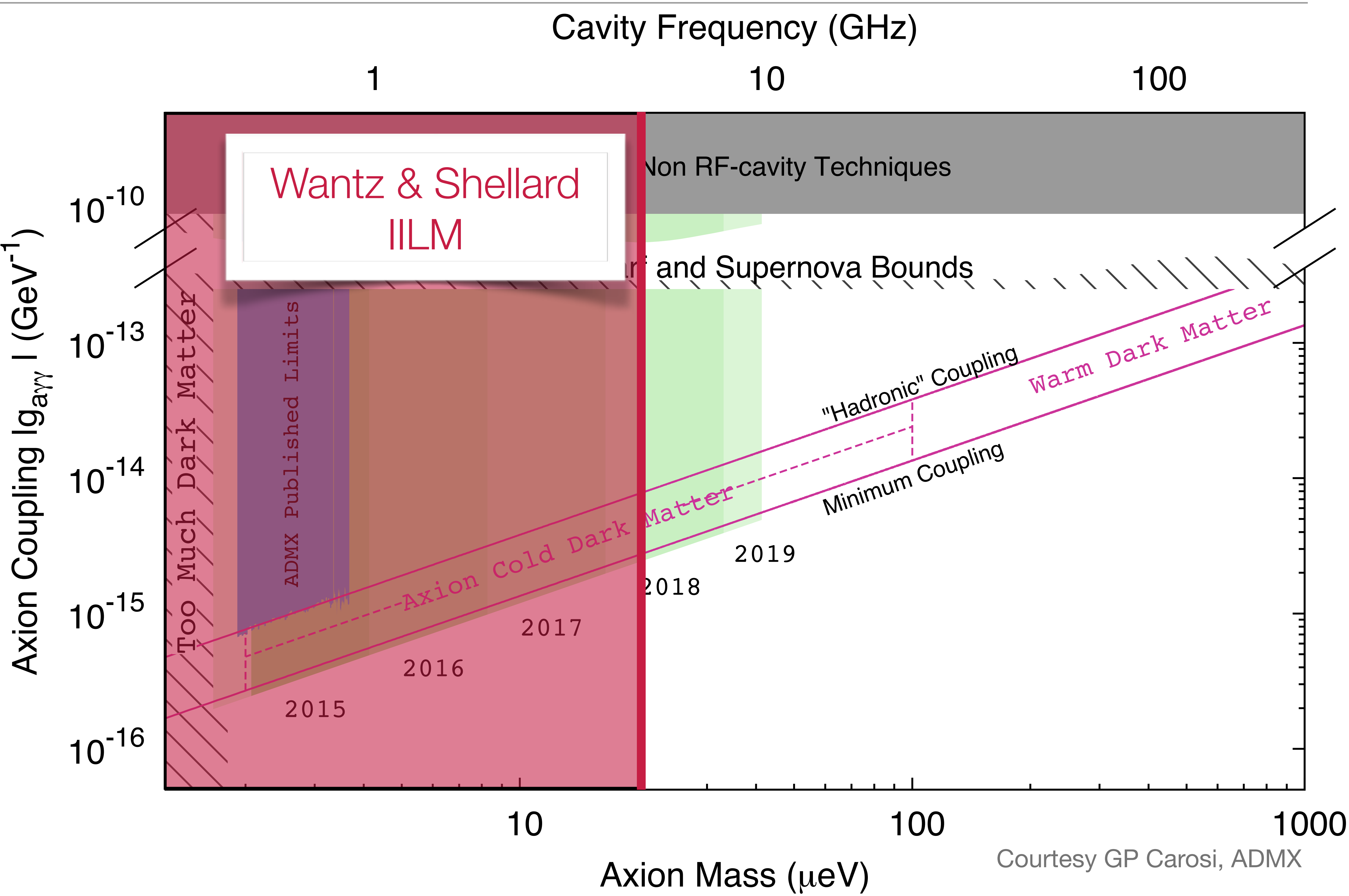

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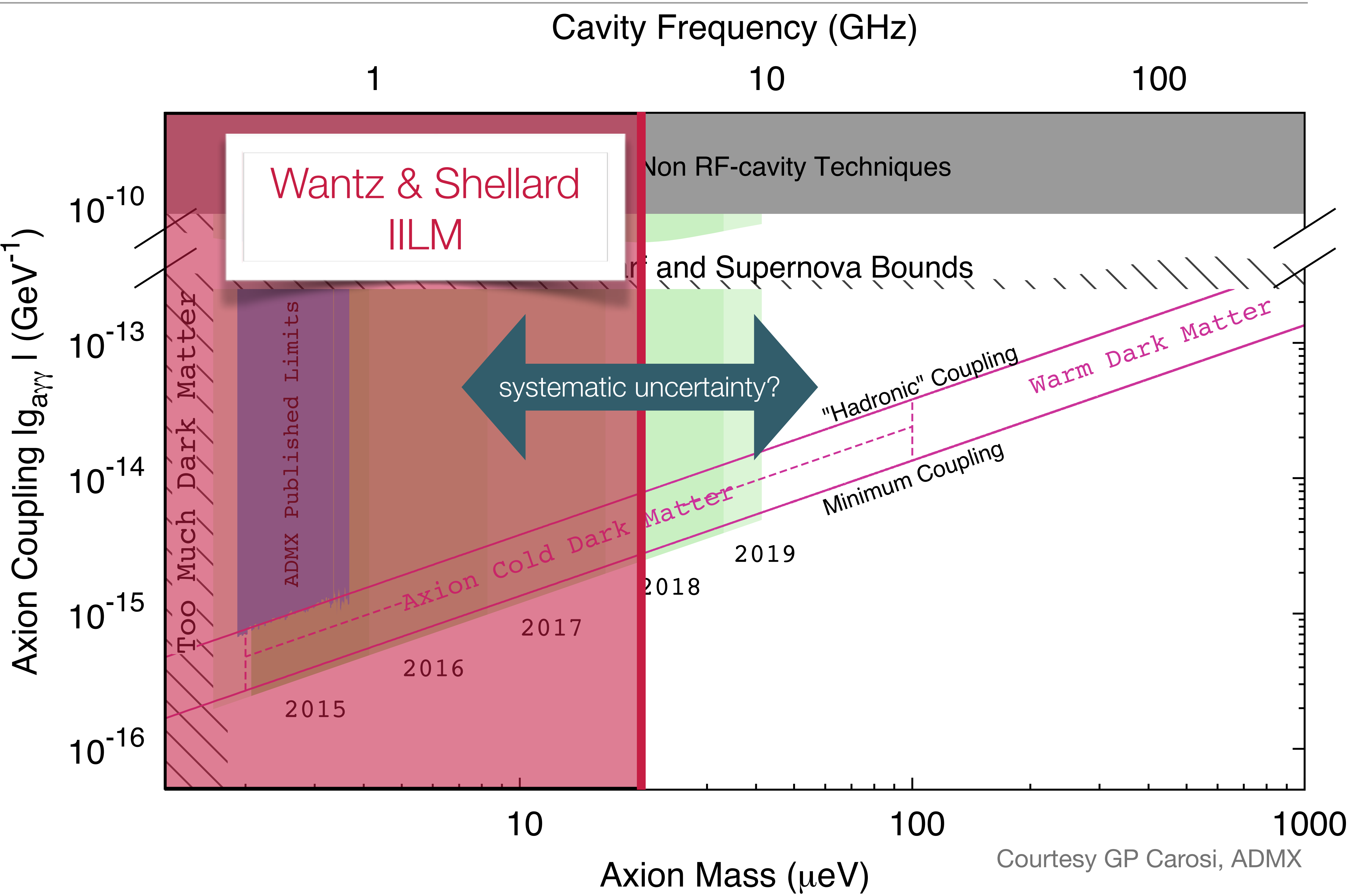

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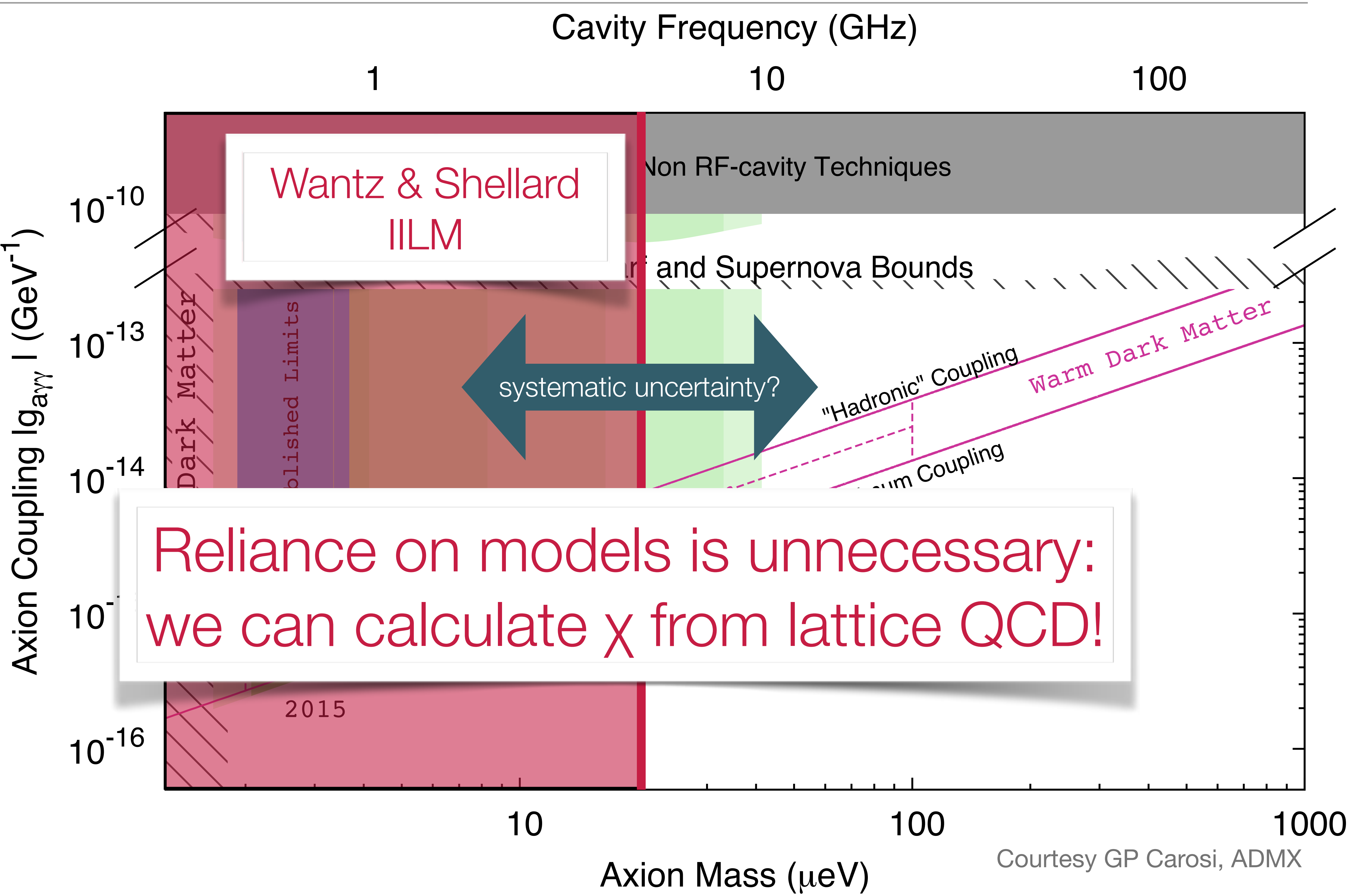

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CAVEAT

We study pure Yang-Mills, and not yet full QCD.

- Dramatically more efficient algorithms enable huge statistics and volumes, shorter autocorrelation times.
- T_c is ~284 MeV, compared to 154 MeV in QCD.
- High temperature tends to suppress quark loops.
  - What counts as high temperature?
  - Unclear if this holds true for topological observables.
- Lower bound relies on late PQ breaking.
  - Early PQ breaking: the same lattice data constrains f_a (or m_a) and initial θ.

Overview of Lattice Ensembles & Measurements

Berkowitz, Buchoff, and Rinaldi, arXiv:1505.07455

- SU(3) YM with Wilson plaquette action
- T between 1.2 and 2.5
- N_σ between 48 and 144 (larger at higher T)
- N_τ either 6 or 8
- Between 14000 and 52000 measurements
  - Combined hot & cold starts
  - Cut of 2000 cfg.s for thermalization
  - 10 compound sweeps of 1 heatbath step and 8 over-relaxation steps

$$\frac{1}{32\pi^2}\sum_x \epsilon^{\mu\nu\rho\sigma}\Box_{\mu\nu}\Box_{\rho\sigma}$$

$Q_{\mathbb{R}}$ raw measurement

$Q_{\mathbb{Z}}$ naïve rounding

Q_a artifact corrected
Lucini & Teper, hep-lat/0103027

Q_f globally fit
del Debbio *et al.*, hep-th/0204125

Q_{OV} overlap

Q_{WF} Wilson Flow

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T/T_c	β	$a\sqrt{\sigma}$	N_τ	N_σ	N_{meas}	central value $\chi^{1/4}/T_c \pm \delta\chi^{1/4}/T_c$ statistical error for							
						$\chi_{\mathbb{R}}$		$\chi_{\mathbb{Z}}$		χ_a		χ_f	
1.2	6.001	0.2161	6	64	14000	0.3880	0.0012	0.3814	0.0012	0.3871	0.0012	**0.4192**	**0.0013**
1.31	6.053	0.1979	6	48	15600	0.3495	0.0009	0.3130	0.0009	0.3392	0.0010	0.3691	0.0011
				64	36000	0.3424	0.0006	0.3358	0.0006	0.3402	0.0007	0.3703	0.0007
				80	14000	0.3426	0.0010	0.3389	0.0010	0.3416	0.0010	**0.3735**	**0.0011**
	6.242	0.1484	8	64	33998	0.3634	0.0010	0.3493	0.0010	0.3520	0.0010	0.3687	0.0010
				96	14000	0.3556	0.0015	0.3533	0.0014	0.3537	0.0015	0.3703	0.0015
1.4	6.095	0.1852	6	64	54000	0.3153	0.0005	0.3077	0.0005	0.3095	0.0005	**0.3370**	**0.0005**
1.5	6.139	0.1729	6	64	54000	0.2928	0.0005	0.2833	0.0005	0.2814	0.0005	**0.3068**	**0.0005**
1.6	6.182	0.1621	6	64	53998	0.2721	0.0005	0.2587	0.0005	0.2568	0.0005	**0.2799**	**0.0005**
1.7	6.223	0.1525	6	64	24000	0.2536	0.0008	0.2330	0.0008	0.2369	0.0008	**0.2585**	**0.0008**
1.8	6.263	0.1441	6	64	24000	0.2343	0.0008	0.2005	0.0009	0.2178	0.0008	0.2368	0.0008
				80	32000	0.2320	0.0006	0.2262	0.0006	0.2185	0.0006	**0.2368**	**0.0006**
	6.471	0.1080	8	96	14000	0.2306	0.0016	0.2170	0.0017	0.2236	0.0015	0.2312	0.0016
1.9	6.301	0.1365	6	64	24000	0.2175	0.0009	0.1672	0.0011	0.2019	0.0008	0.2190	0.0009
				80	34000	0.2164	0.0006	0.2095	0.0006	0.2026	0.0006	**0.2189**	**0.0006**
1.99	6.550	0.0973	8	64	14795	0.2013	0.0034	0.1800	0.0036	0.1986	0.0029	0.2013	0.0034
2.0	6.338	0.1297	6	48	15600	0.2040	0.0018	0.1292	0.0027	0.1898	0.0016	0.2042	0.0018
				64	25598	0.2032	0.0010	0.1390	0.0014	0.1893	0.0009	0.2041	0.0010
				80	26000	0.2014	0.0008	0.1920	0.0008	0.1888	0.0007	0.2030	0.0008
				96	14000	0.2004	0.0008	0.1961	0.0008	0.1900	0.0008	**0.2038**	**0.0009**
2.1	6.373	0.1235	6	80	24000	0.1880	0.0009	0.1749	0.0009	0.1774	0.0008	**0.1889**	**0.0009**
2.5	6.502	0.1037	6	128	14000	0.1497	0.0010	0.1479	0.0010	0.1494	0.0008	0.1492	0.0010
				144	15797	0.1525	0.0008	0.1513	0.0008	0.1495	0.0006	**0.1518**	**0.0008**

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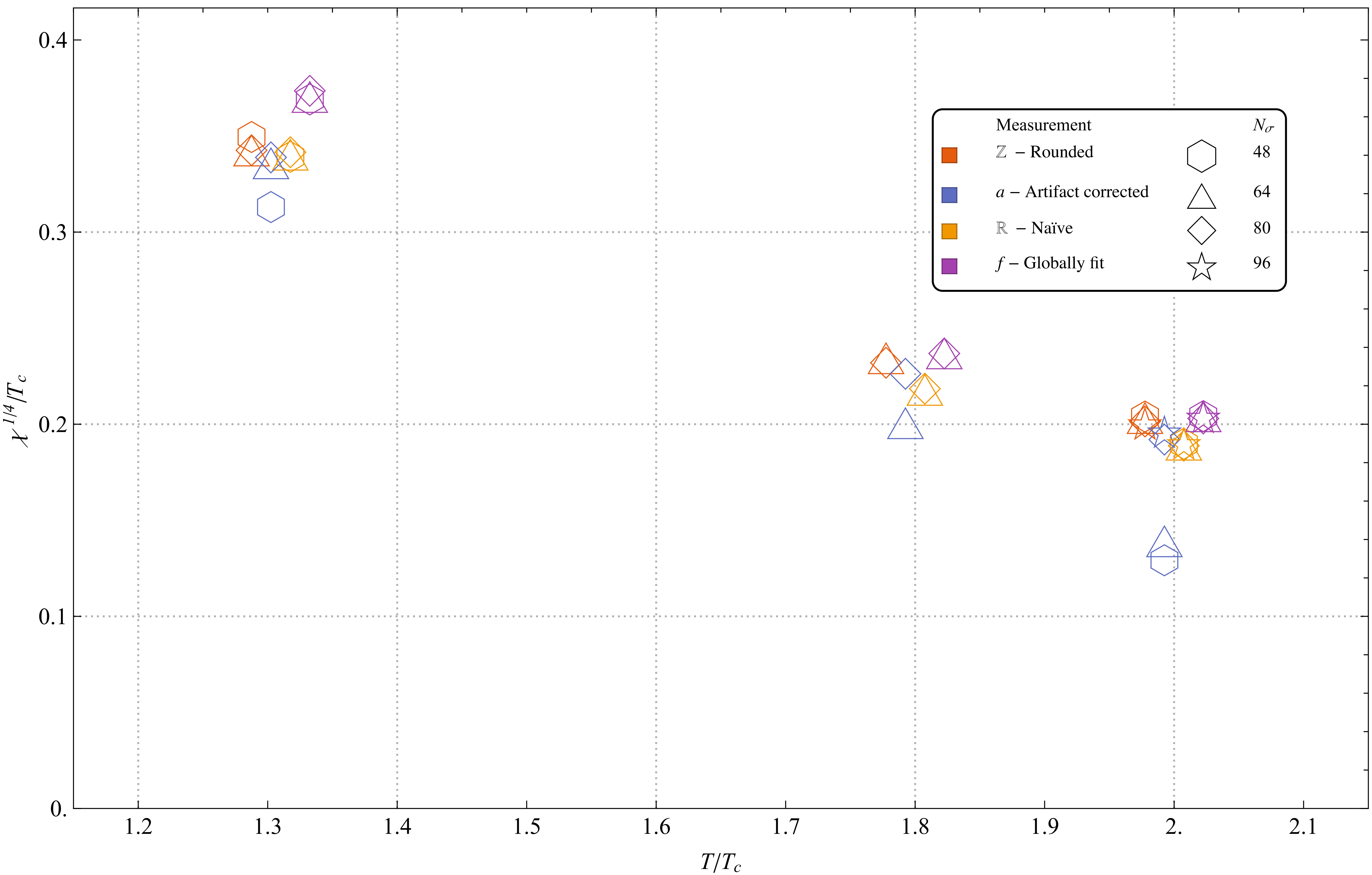
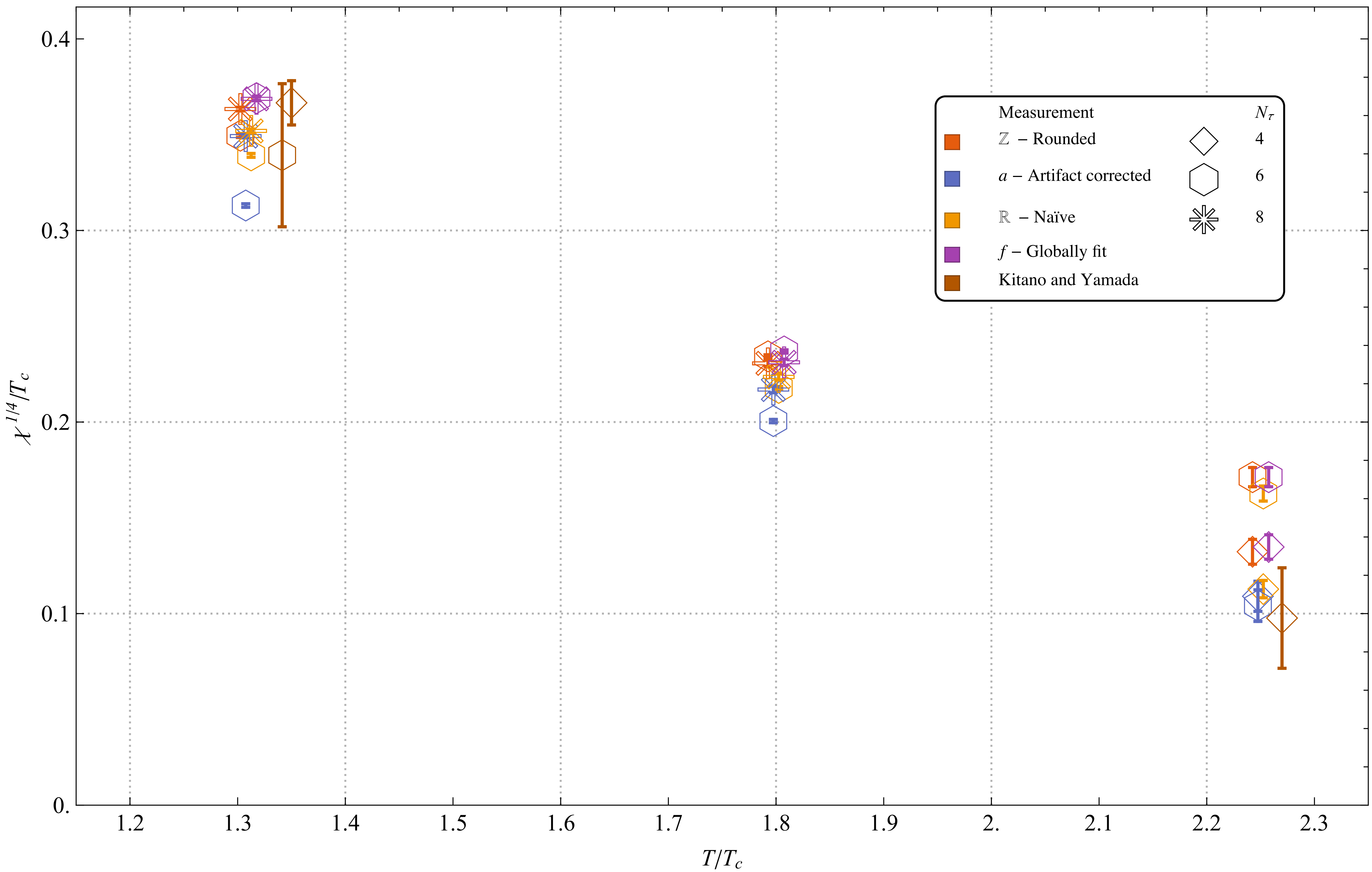
Finite Volume Effects

Berkowitz, Buchoff, and Rinaldi, arXiv:1505.07455



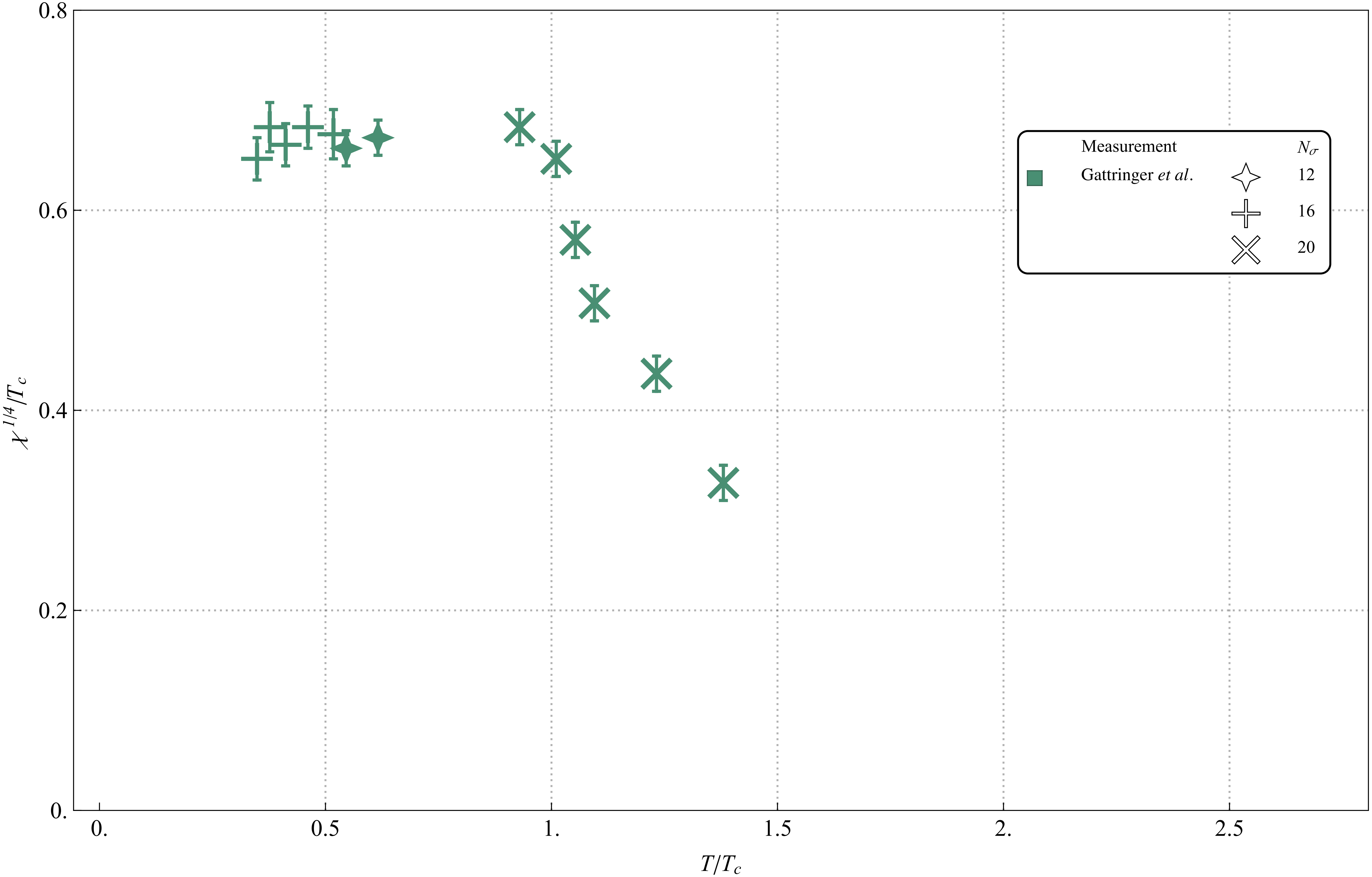
Discretization Effects

Berkowitz, Buchoff, and Rinaldi (arXiv:1505.07455), Kitano & Yamada (arXiv:1506.00370)



Best Lattice Results

Gattringer et al. (arXiv:hep-lat/0203013)



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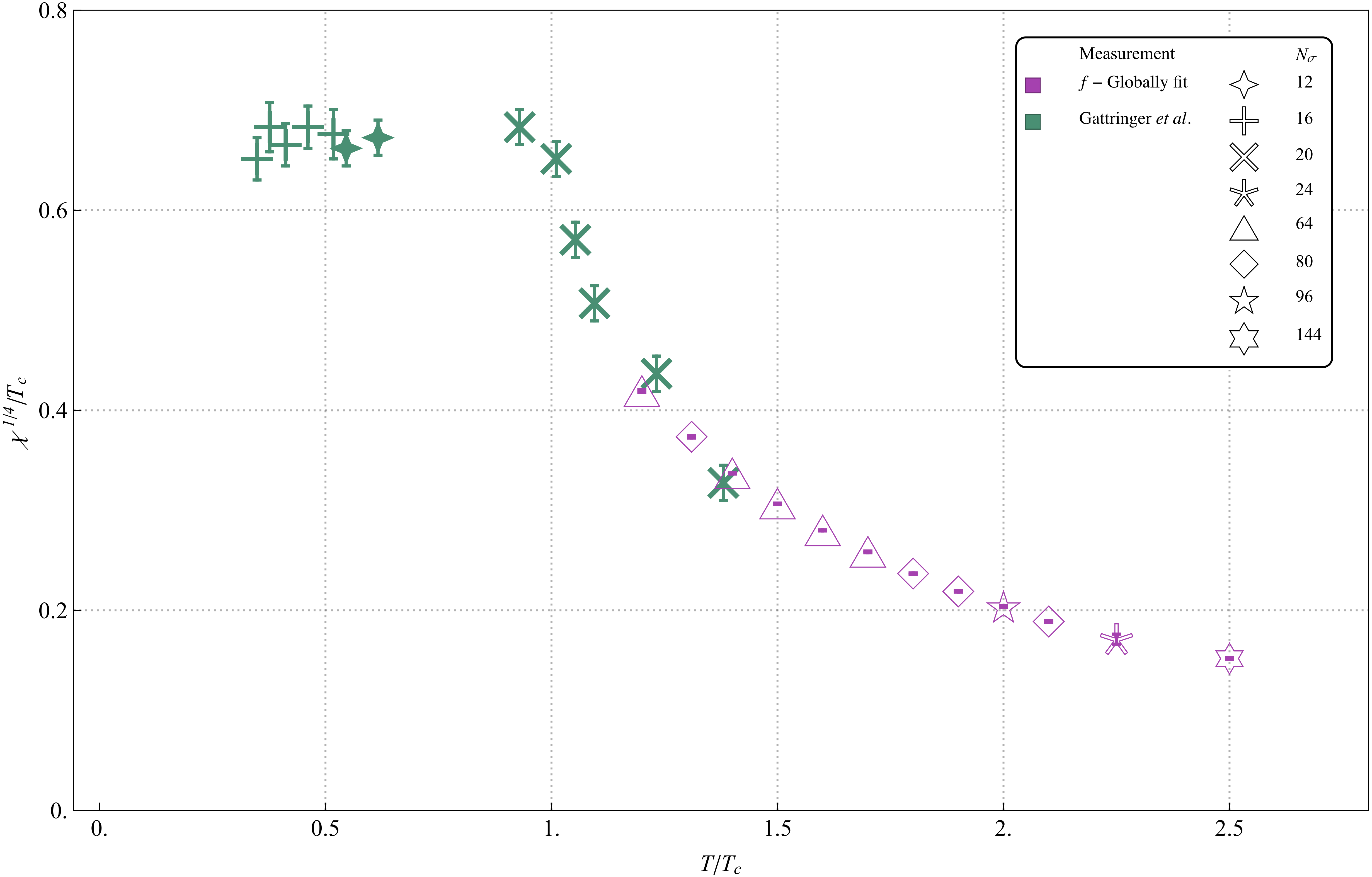

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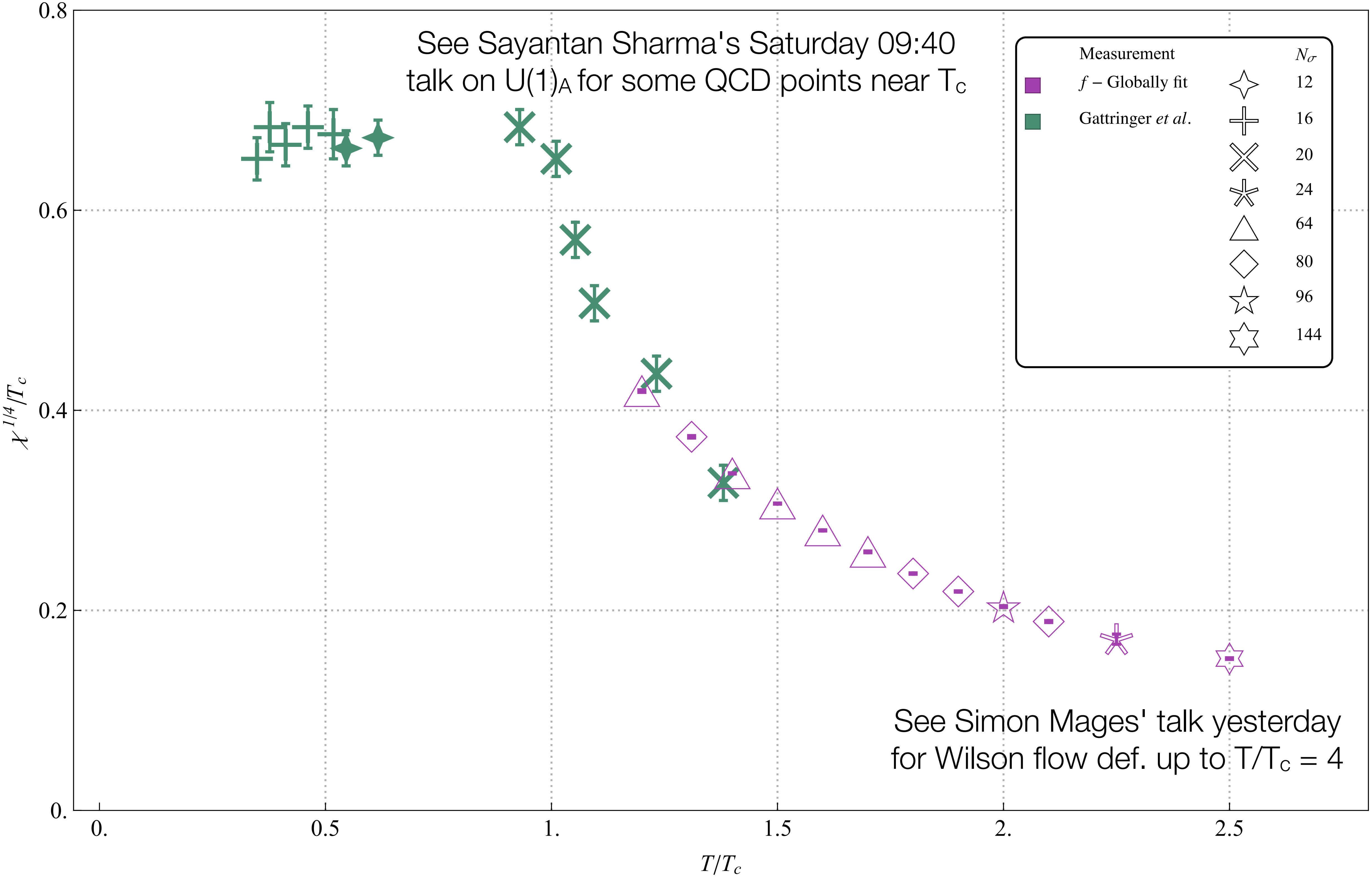
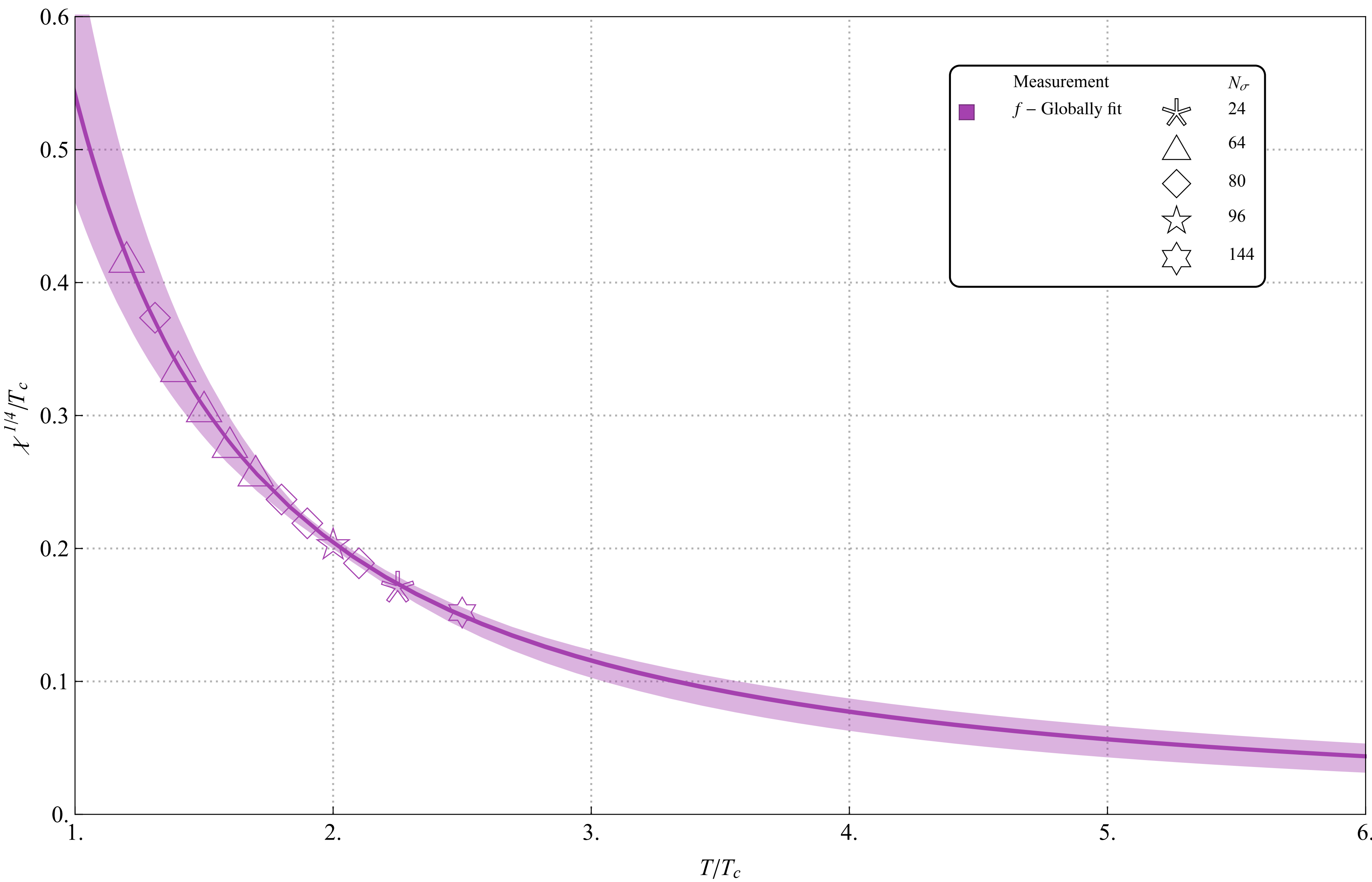
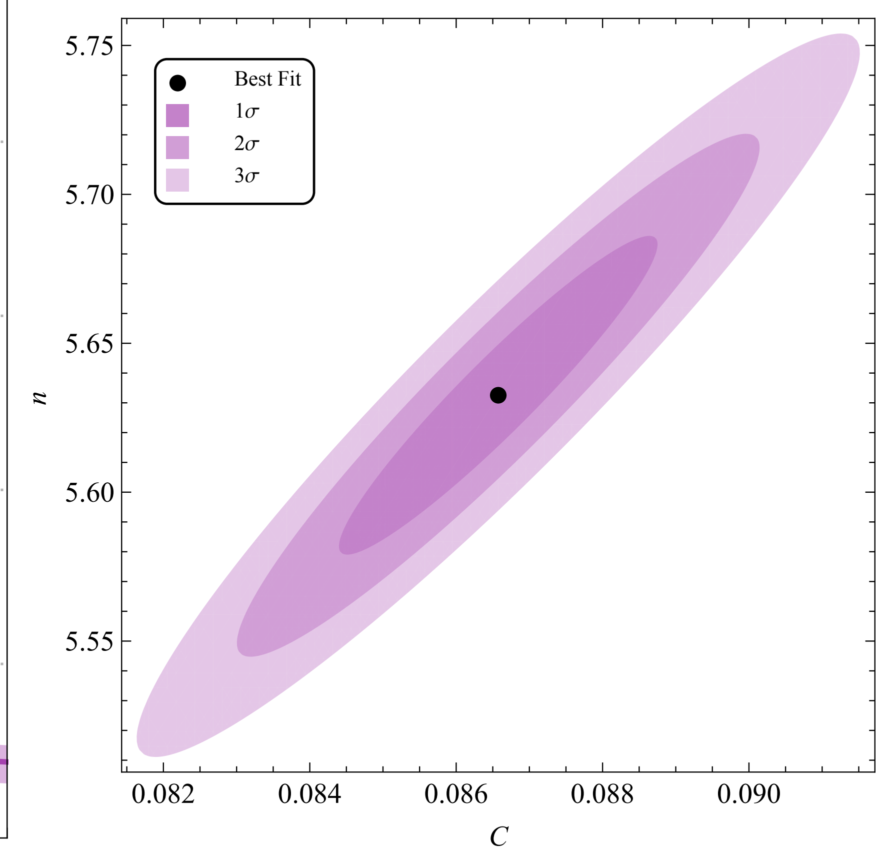


DIGM Best Fit & Extrapolation

Berkowitz, Buchoff, and Rinaldi, arXiv:1505.07455

$$\frac{\chi}{T_c^4} = \frac{C}{(T/T_c)^n}$$

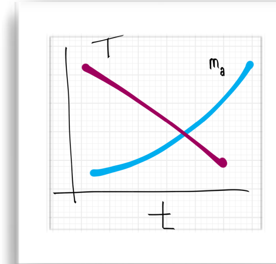




	C	n
Best Fit	0.0865	5.63
Covariance Matrix		
C	$2.\times10^{-6}$	$5.\times10^{-5}$
n	$5.\times10^{-5}$	0.0012

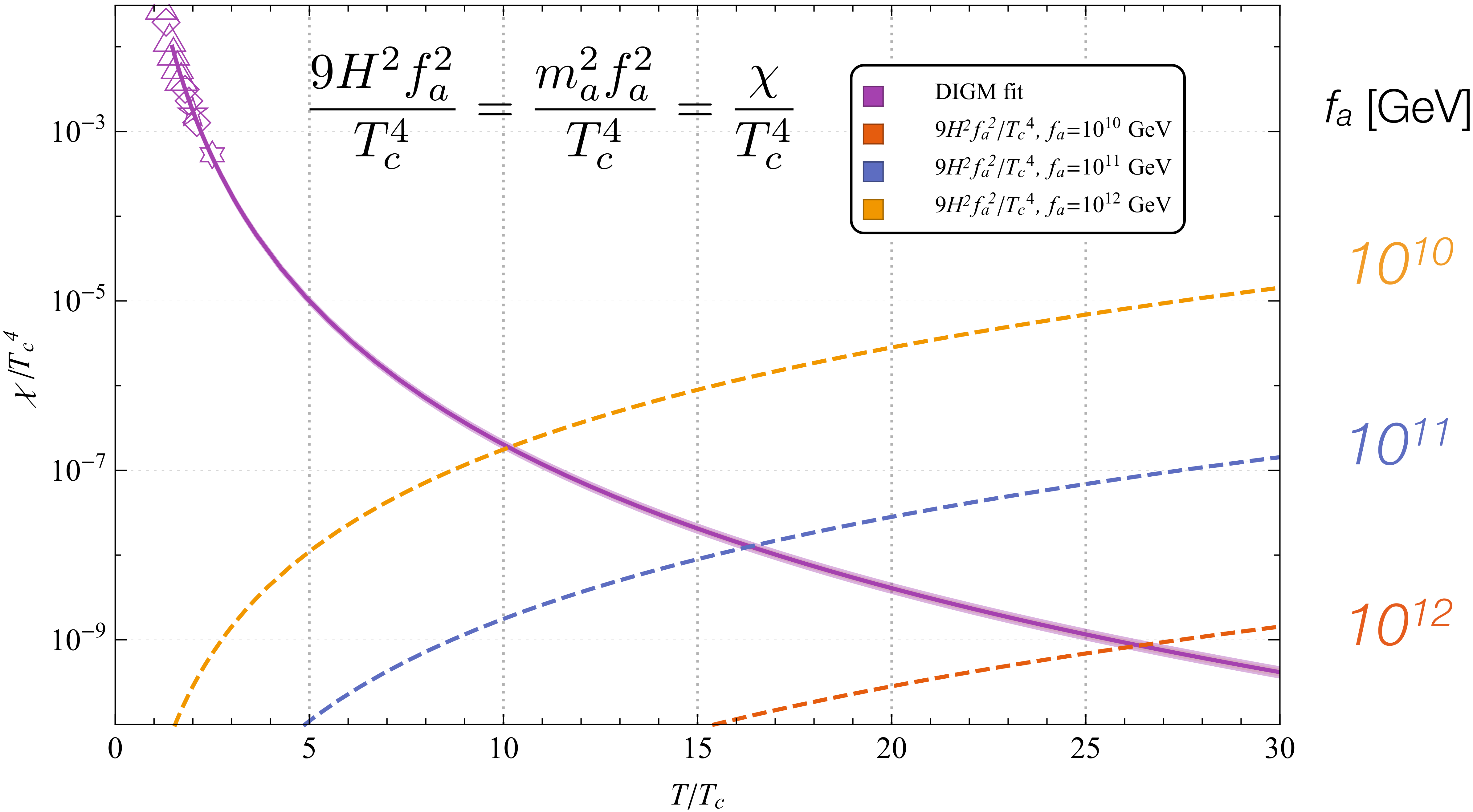
Axion Production Ceases

Berkowitz, Buchoff, and Rinaldi, arXiv:1505.07455



Axion number fixed at T_1

when 3H ~ m



$$H^2 = \frac{\pi^2}{90}\frac{1}{m_P^2}\, g_{*R}(T)\, T^4$$

Axion Density

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$$T_1 = T_1(f_a, \chi)$$

$$m_a(T_1) = \frac{\sqrt{\chi(T_1)}}{f_a}$$

Rely on our lattice calculation

$$m_a(T_\gamma) = \frac{1}{f_a}\frac{\sqrt{m_u m_d}}{m_u + m_d} f_\pi m_\pi$$

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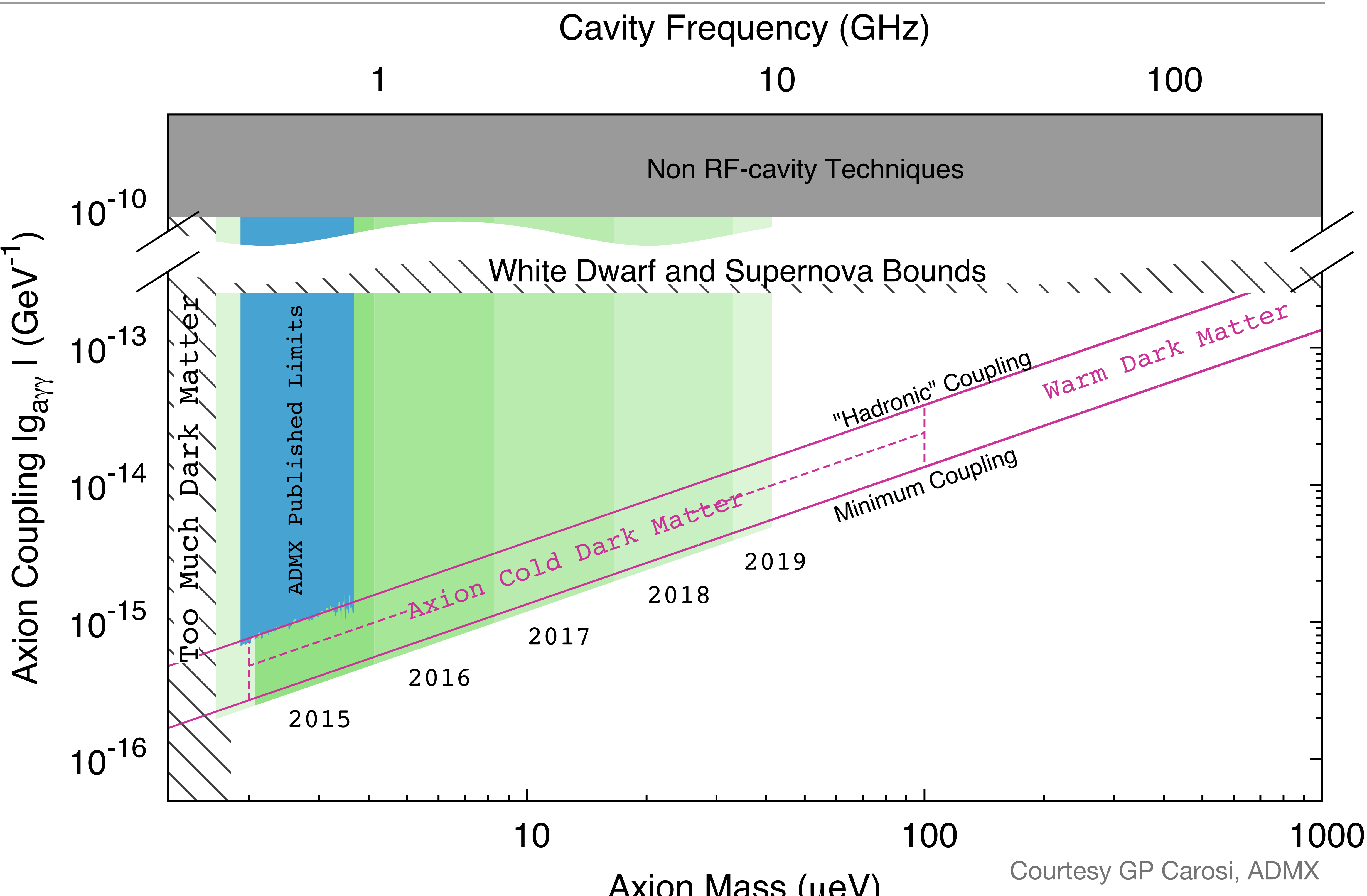

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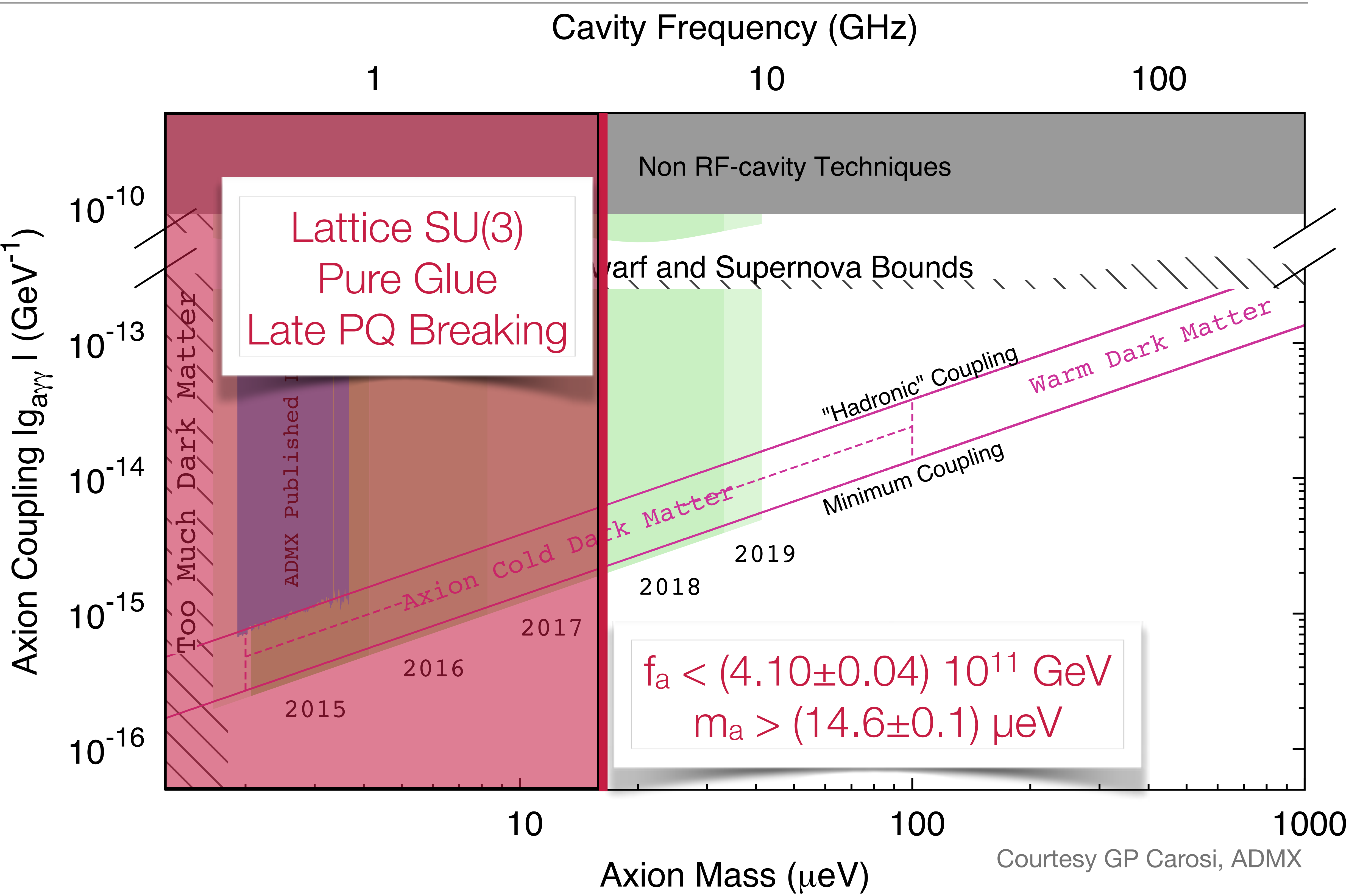
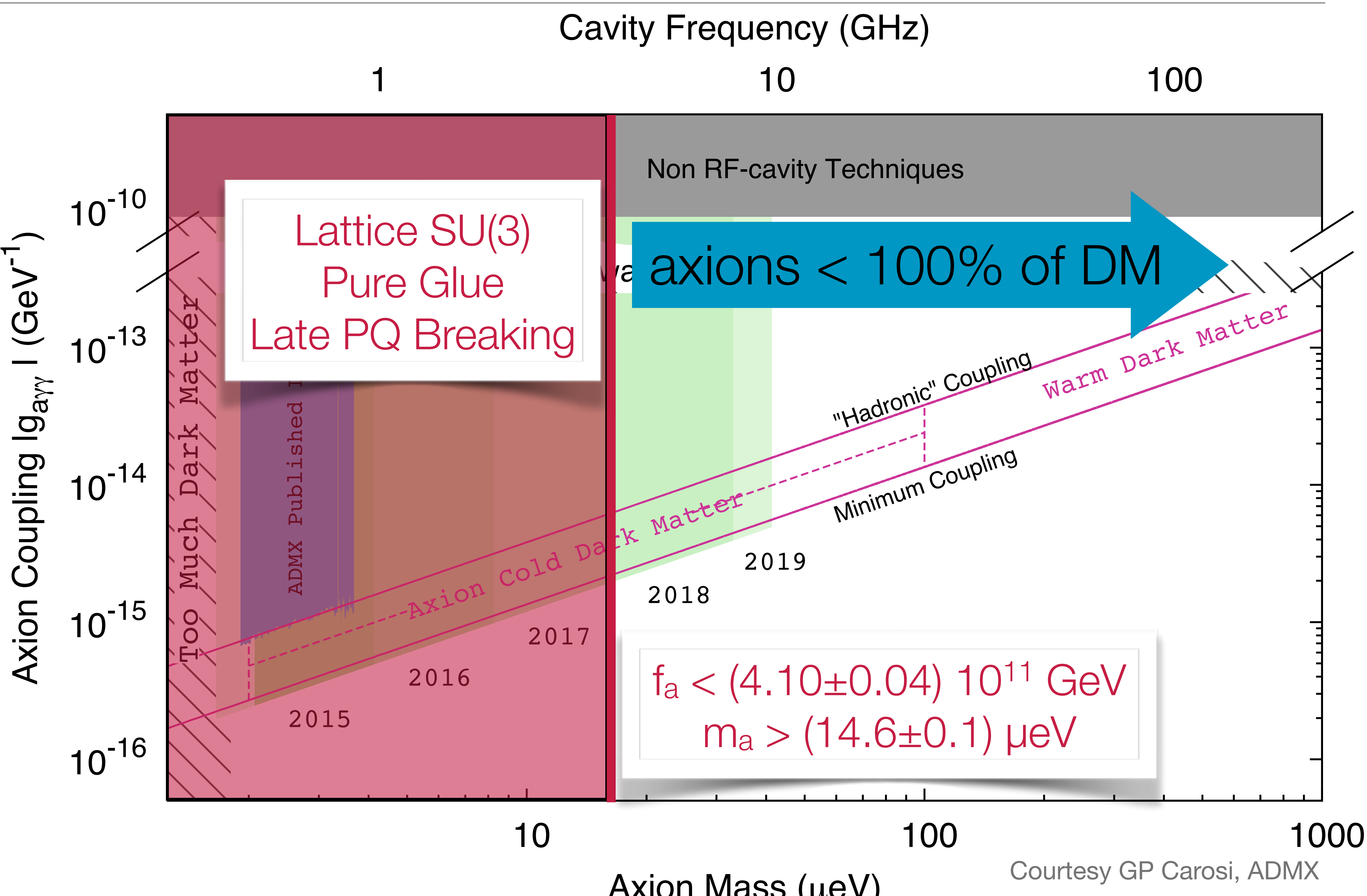


Courtesy GP Carosi, ADMX

The Over-Closure Bound As It Stands Today

Berkowitz, Buchoff, and Rinaldi, arXiv:1505.07455



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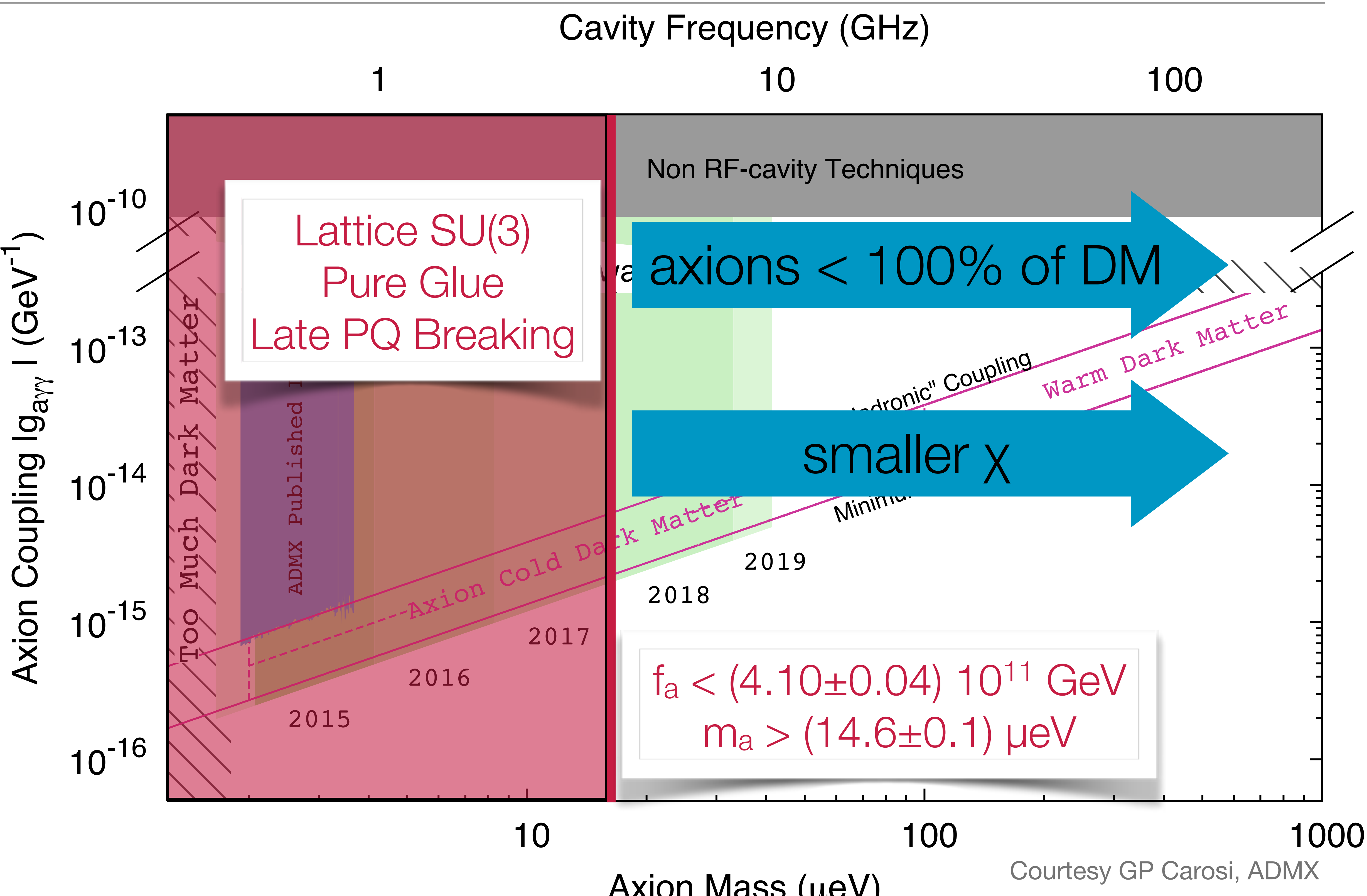
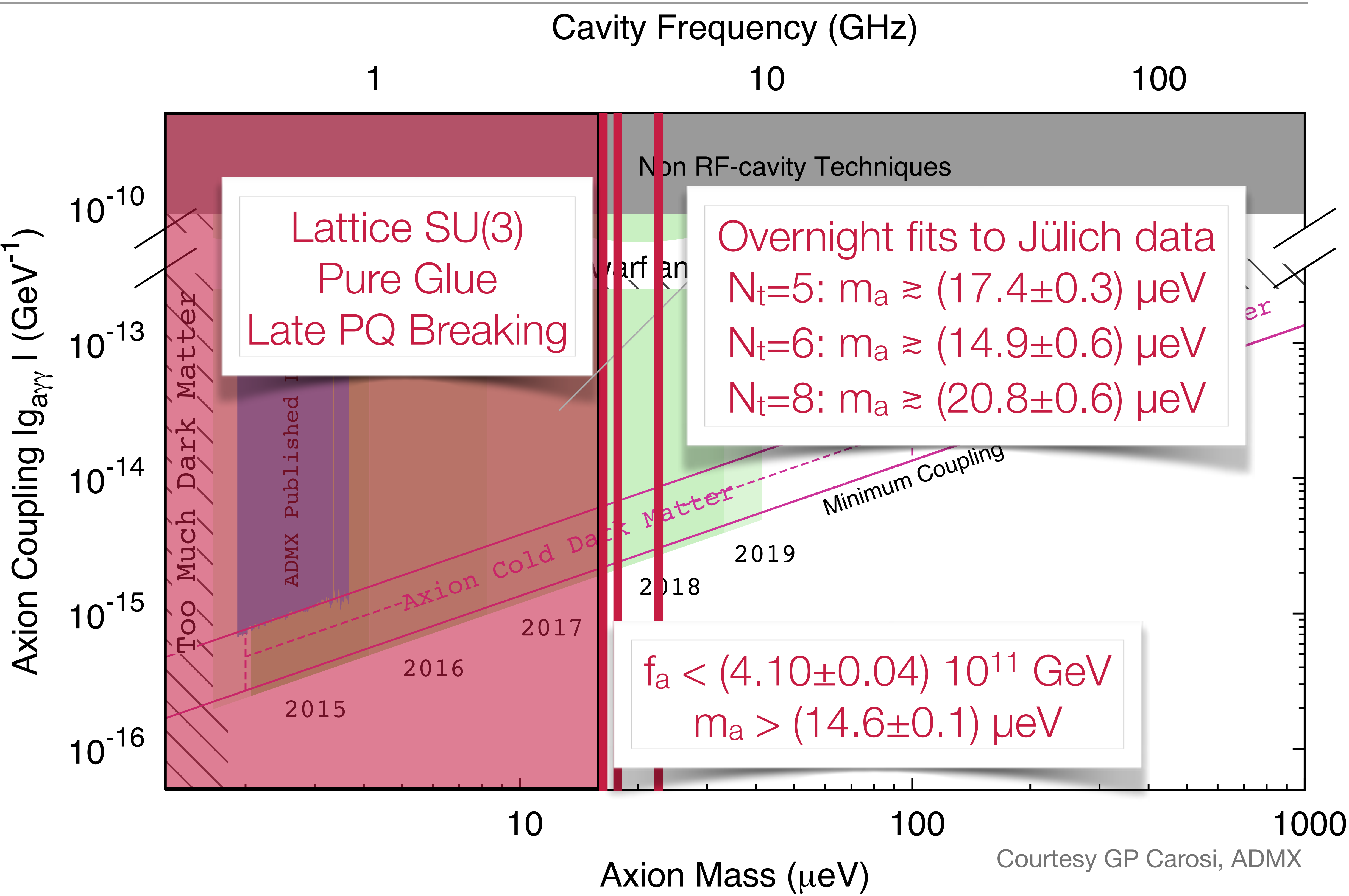


Courtesy GP Carosi, ADMX

The Over-Closure Bound As It Stands Today

Berkowitz, Buchoff, and Rinaldi, arXiv:1505.07455, Simon Mages' talk 15 July 18:10



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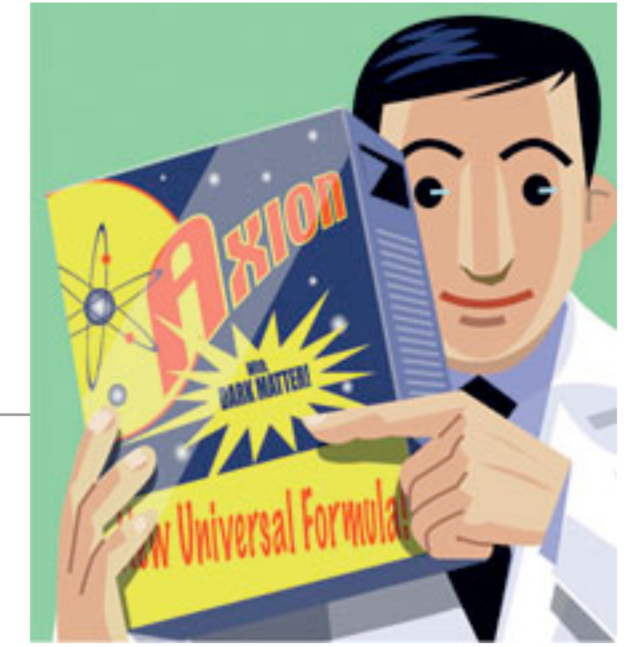
Conclusions & Outlook



The Economist, 19 Dec 2006

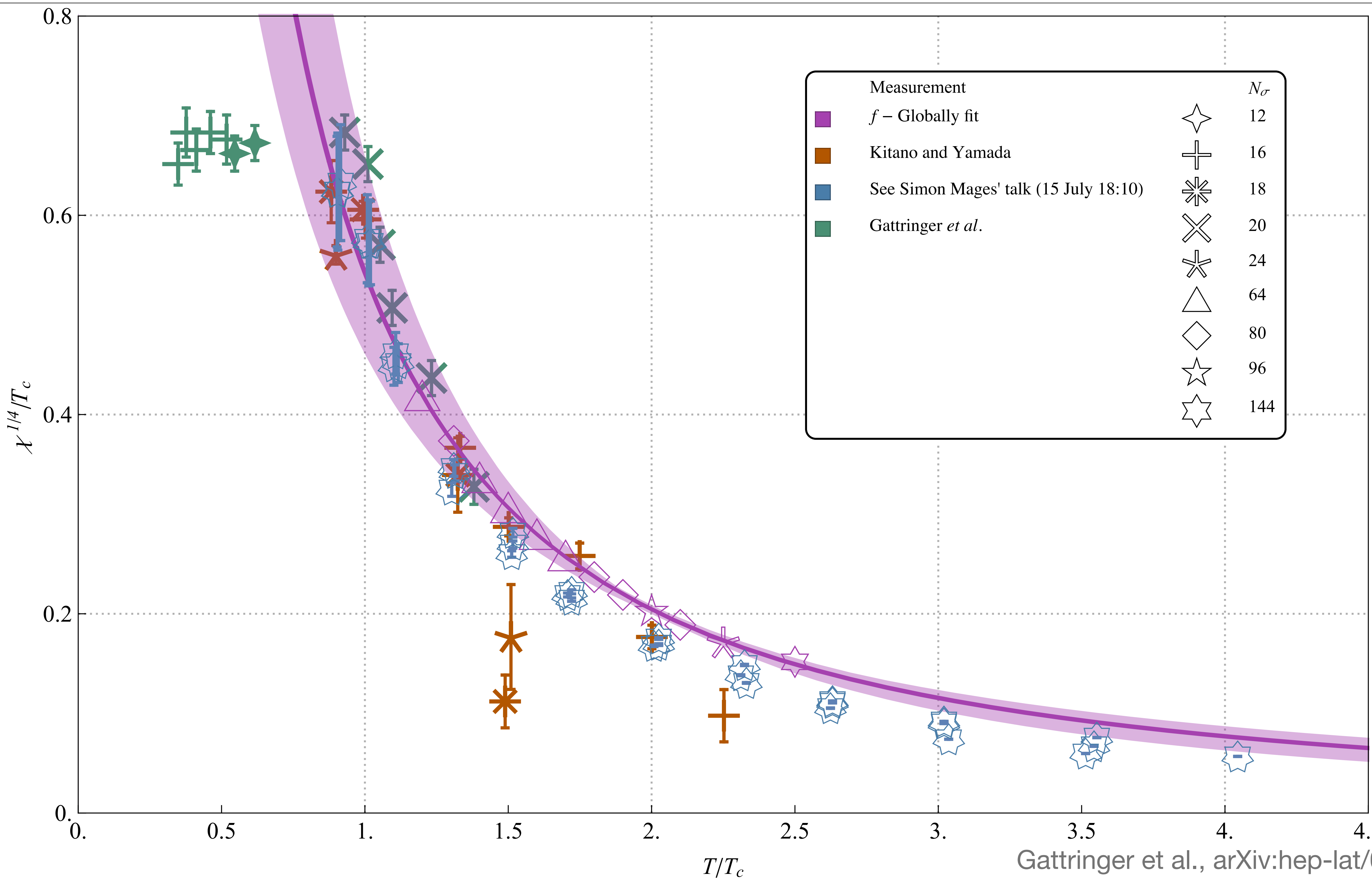
- PQ symmetry:
    - cleans up the Strong CP problem
    - provides a plausible, largely unconstrained DM candidate: the axion.
- Axion searches will search large swaths of interesting parameter space soon.
- Power law (DIGM-inspired) fits outstandingly to pure glue at high temperature.

Lattice QCD can provide important nonperturbative input for calculating Ω_a

Future Steps

- Measure higher moments? May be able to get χ_4, χ_6.
  T=0: Cé, Consonni, Engle & Giusti, arXiv:1506.06052
- Incorporate quarks
- Move to Wilson Flow definition
- Explore fixed topology methods / open boundary conditions at high T.
  Aoki *et al.*, arXiv:0707.0396v2   Lüscher & Schæfer, arXiv:1105.4749
- Finite θ:
  - Imaginary-θ has no sign problem
  - Real, finite θ may be amenable to Langevin methods

Comparison with subsequent work



Gattringer et al., arXiv:hep-lat/0203013
Kitano & Yamada, arXiv:1506.00370
Mages et al. (15 July 18:10)