QCD at non-zero temperature from the lattice

Harvey Meyer

Lattice 2015, Kobe, Japan, 18 July 2015

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

Overview

- **Overview of equilibrium properties**
  transition temperature, thermodynamics, hadron resonance gas model, static screening masses

- **News on the Columbia plot**
  Order of the thermal transition as a function of $(m_{u,d}, m_s)$

- **Near-equilibrium (real-time) properties**
  pion quasiparticle, heavy-quark momentum diffusion coefficient, vector channel for light quarks, nucleon channel, quarkonium

Motivation

Strongly interacting matter at temperatures $T = 100 - 500\,\text{MeV}$

- probed in heavy-ion collisions
- state of matter for the first microsecond after Big Bang

Thermal physics:

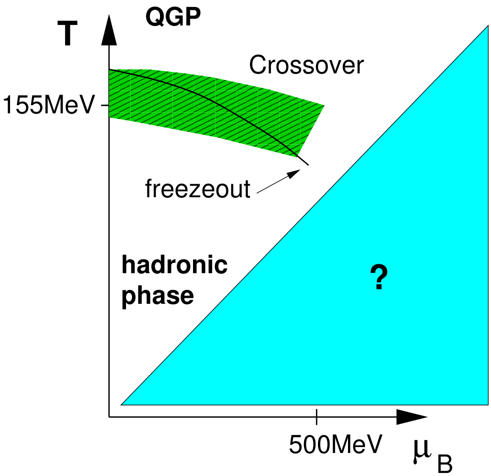
$$\langle A \rangle = \frac{1}{Z} \mathrm{Tr}\,\{e^{-\beta H} A\}, \qquad Z = \mathrm{Tr}\,\{e^{-\beta H}\}$$

Matsubara formalism particularly well-suited for **equilibrium physics:**
path integral formulation

- imaginary time direction of length $\hbar/(k_B T)$.
- boson fields have periodic, fermion fields antiperiodic boundary conditions.

$\longrightarrow$ particularly well suited for lattice QCD: $Z = \int DU\, D\bar{\psi}\, D\psi\, e^{-S}$.

QCD phase diagram at $\mu_B \simeq 0$



- freezeout curve: heavy-ion collision phenomenology.

Semi-quantitative expectations for QCD at $T > 0$

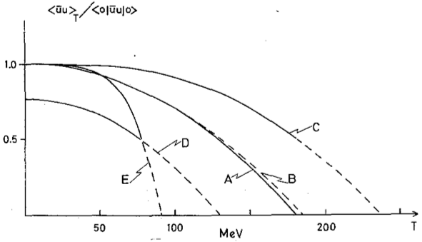


Fig. 1. The fermion condensate as a function of temperature for three quark flavours of equal mass. The various curves correspond to different choices of the box size L and of the quark mass – parameterized through the corresponding value of M_π. A includes the two-loop contribution (2); B, ..., E are given at one-loop accuracy (24). A and B: $M_\pi = 0, L = \infty$; C: M_π = 135 MeV, $L = \infty$; D: M_π = 135 MeV, L = 2.5 fm; E: M_π = 135 MeV, $L = 1/T$.

Chiral condensate

Gasser, Leutwyler, PLB 184:83, 1987

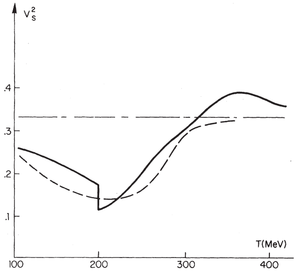


FIG. 7. Crude estimate of sound velocity versus temperature.

Speed of sound

Bjorken PRD27:140-151, 1983

The pseudocritical temperature at physical (u, d, s) quark masses

$\triangle$ Inflection point of $m(\langle\bar{\psi}\psi\rangle|_0^T)/T^4$:

Staggered fermions: $155(2)(3)$MeV BW 1005.3508

$\triangle$ From the chiral susceptibility:

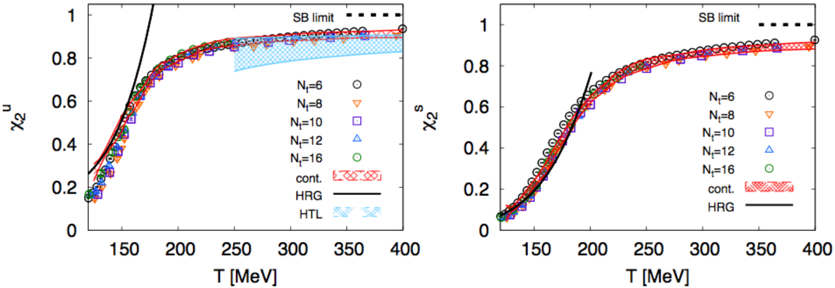
Staggered fermions: 147(2)(3)MeV BW 1005.3508

154(8)(1) MeV HotQCD, 1111.1710

Domain wall fermions: 155(1)(8) MeV HotQCD, 1402.5175

- Good agreement among staggered fermion calculations
- Now also good agreement with domain-wall fermions (and soon Wilson fermions?).

Deconfinement: does it coincide with chiral restoration?



- Not a completely sharp question.
- Light-quark number susceptibility: suggests that deconfinement occurs practically at the same temperature as chiral restoration.
- strangeness fluctuations: rise delayed by about $\Delta T = 20$MeV.
- Successful predictions of the hadron resonance gas model (HRG).

Fig. from S. Borsanyi et al. 1112.4416

Thermodynamic potentials

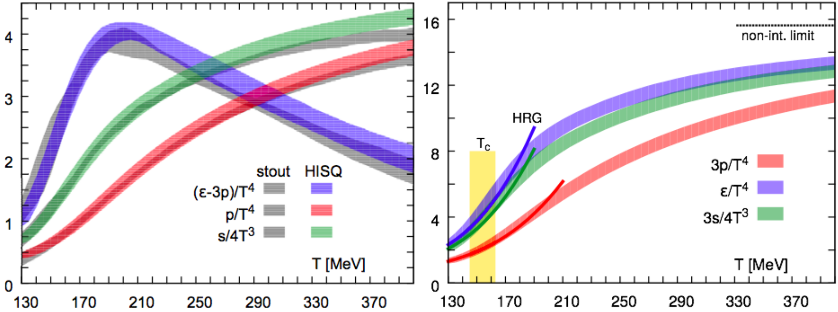
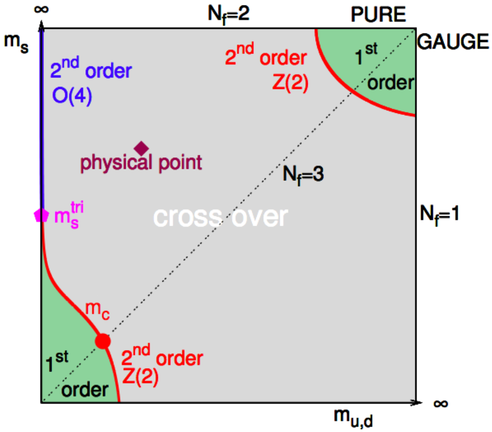


Fig. from review by Soltz et al. 1502.02296

- at $T = 260$MeV, $p_{\rm norm} \equiv p/p_{\rm SB} \approx 1/2$:
  far from weakly interacting quarks and gluons.
- $(e-3p)/[\frac{3}{4}(e+p)] \approx 1/3$: large departure from a scale-invariant system.
- HRG model works well up to $T = 160\,$MeV.

The 'Columbia plot'



- nature of the thermal phase transition as a function of the quark masses
- the situation at small $m_{u,d}$ is not settled yet; e.g. the phase transition in the massless $N_{\rm f} = 2$ theory could be 1st order.

Fig. from review by Ding, Karsch & Mukherjee 1504.05274

The pseudocritical temperature as a function of the quark masses

Using $r_0 = 0.50000$fm for the purpose of the comparison.

Pure gauge theory (1$^{\text{st}}$ order, Z(3) center symmetry):
$T_c = 294(2)\text{MeV}$ Francis et al. 1503.05652

$N_f = 2$ **QCD at** $m_\pi > m_\pi^{\text{phys}}$ **(crossover)**:
- O(a) improved Wilson, $N_\tau = 16$ [Brandt et al. 1310.8326]

$m_\pi = 295$ MeV $T_c = 211(5)$ MeV
$m_\pi \approx 220$ MeV $T_c = 193(7)$MeV

- Twisted-mass QCD, continuum, $N_\tau \leq 12$ [Burger et al. 1412.6748]

$m_\pi = 333$ MeV $T_c = 180(12)$ MeV
Some tension here; the authors use $r_0 = 0.462$fm $\Rightarrow$ quote $T_c = 195(13)$MeV.

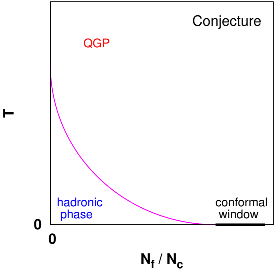
$N_f = 2 + 1$ **O(a) improved Wilson at** $m_\pi > m_\pi^{\text{phys}}$ **(crossover)**
continuum results [Borsanyi et al. 1504.03676]
downward trend of T_c clearly seen from the chiral condensate.

Expanding one's horizons

Does T_c gradually become smaller as N_f is increased until the conformal window is reached?

Lombardo, Miura, Nunes da Silva, Pallante 1506.05946; 1507.00375 Kogut & Sinclair (sextet fermions)

NB. larger N_f gives more statistical weight to baryons in the HRG



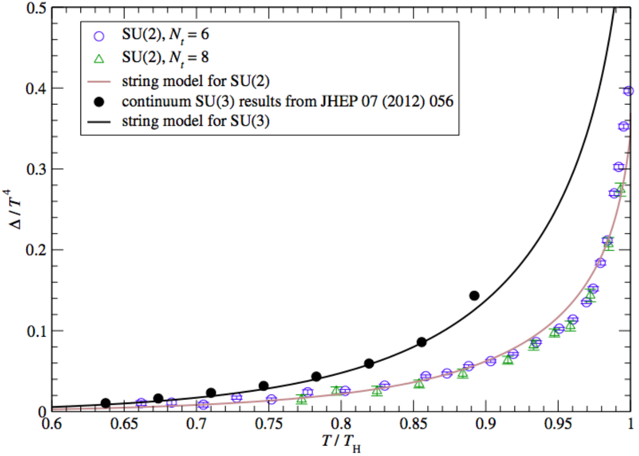
- study in pure gauge theory $2 \leq N_c \leq 8$:
  - $p_{\rm norm}(SU(N_c), x) \equiv (p/p_{\rm SB})(T = xT_c)$ is almost indep. of N_c for $x > 1.1$
    $\Rightarrow$ the multiplicity of the physical degrees of freedom $\propto$ nb. gluons
  - $p_{\rm norm}(SU(N_c), 1.6) \approx 1/2$

  Bringoltz & Teper hep-lat/0506034; Panero 0907.3719; Datta & Gupta 1006.0938

- exceptional gauge groups, G(2)
  - color singlet asymptotic states, no center symmetry
  - $p_{\rm norm}(G(2), x)$ is consistent with $p_{\rm norm}(SU(N_c), x)$

  Bruno et al. 1409.8305; see also Pepe & Wiese hep-lat/0610076, Cossu et al. 0709.0669.

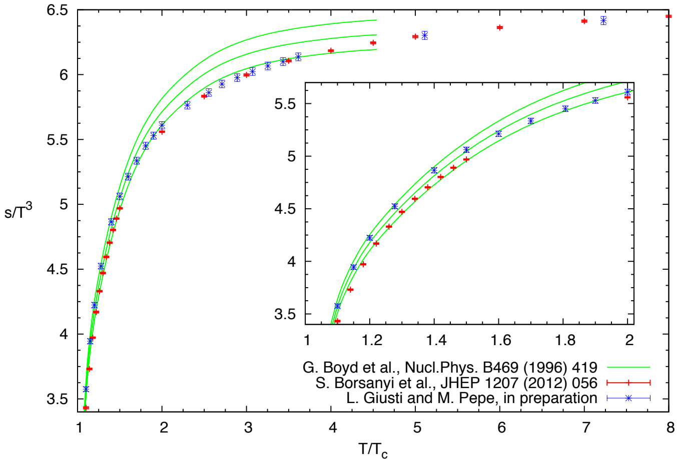
High-precision thermodynamics in pure gauge theory: low-T



- good agreement between 'glueball gas' model with spectrum taken from the closed Nambu-Goto string up to very close to the phase transition.
- SU(2) has 'fewer states': only charge conjugation + states.

Fig. from Caselle et al. 1505.01106; see also HM, 0905.4229 and Borsanyi et al. 1204.6184

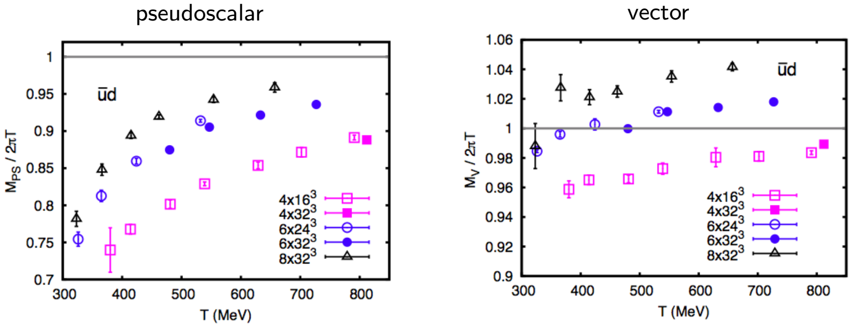
High-precision thermodynamics in SU(3) gauge theory: high-T



- use of shifted boundary conditions [see Giusti & HM 1211.6669]
- small, but statistically significant differences between recent calculations (accuracy $\approx 0.5\%$)

Giusti, Pepe, preliminary.

Static screening masses in light quark sector at high T



- pseudoscalar mass remains below $2\pi T$, vector rises above around $T = 2T_c$
- weak coupling prediction: $m/2\pi T = 1 + c\,g^2$, $c > 0$
  Laine, Vepsalainen hep-ph/0311268.

Figs. from Cheng et al. 1010.1216 ($N_f = 2+1$, p4 staggered); see also S. Gupta & Karthik 1302.4917 ($N_f = 2$ staggered); Brandt et al. ($N_f = 2$, Wilson) 1310.8326; M. Müller et al. 1311.3889 quenched. Talks about flavor-singlet screening masses: **J. Weber** and **A. Pasztor**.

Portrait of QCD at finite temperature

From the lattice:

- low-T phase: hadron resonance gas model describes equilibrium properties very well
- chiral + deconfinement crossover transition around $T = 155$MeV
- high-T phase: multiplicity of degrees of freedom consistent with quarks+gluons
- ...but many quantities far from weak-coupling predictions at least until $T \approx 2.5T_c$.

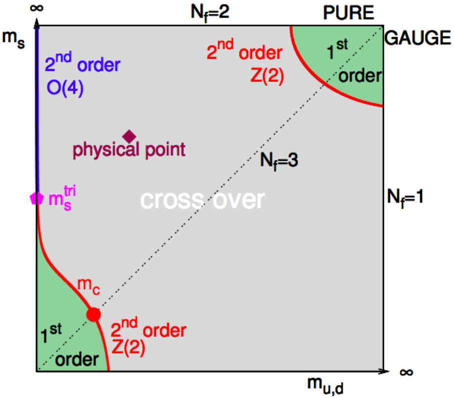
In addition, heavy-ion phenomenology points to a medium with very small shear viscosity/entropy density in the range $T_c \lesssim T \lesssim 2.5T_c$, e.g.

$$\eta/s \approx \begin{cases} 0.12 & \text{RHIC} \\ 0.2 & \text{ALICE} \end{cases}$$

Gale, Jeon, Schenke 1301.5893; White Paper 1502.02730

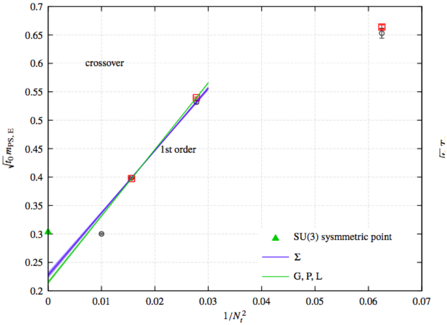
All this indicates that the partonic degrees of freedom are strongly correlated.

News on the Columbia plot at this conference



Size of the 1st order region in lower-left corner of the Columbia plot

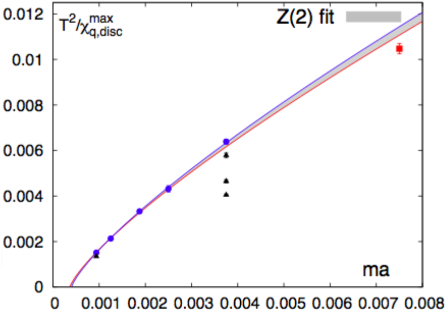
- in the SU(3) chiral limit, $\exists$ strong arguments that the transition is 1st order [Pisarski Wilczek, PRD29 (1984) 338]
- confirmed by lattice simulations, but huge variations on $m_\pi^{\rm crit}$ among $N_t = 4$ and $N_t = 6$ results: finer lattice spacing $\rightsquigarrow$ lower $m_\pi^{\rm crit}$



- Published result of continuum extrapolation from finite-size scaling at $N_t = 6$ and 8: $m_\pi^{\rm crit} = 304(7)(14)(7)$MeV (O($a$) improved Wilson, Iwasaki gauge action, scale setting with t_0)
- likely to be reduced further at larger N_t...to be followed.

See X.-Y. Jin et al. 1411.7461 and talk by **Y. Nakamura**. Curvature of critical surface: **S. Takeda**

Size of the 1st order region (II)

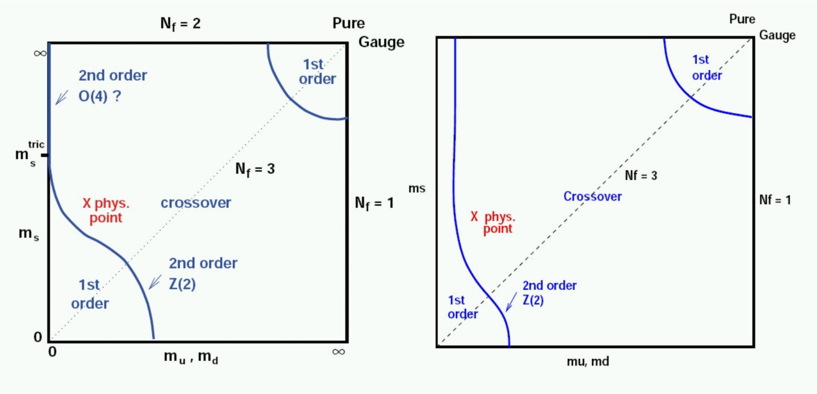


- 'infinite-volume' method with HISQ action at $N_t = 6$
- data down to $m_\pi = 80\text{MeV}$
- fit with Z(2) exponents: $m_\pi^{\text{crit}} \lesssim 50\text{MeV}$.

Talk by **H.-T. Ding** (Bielefeld-BNL-CCNU collaboration).

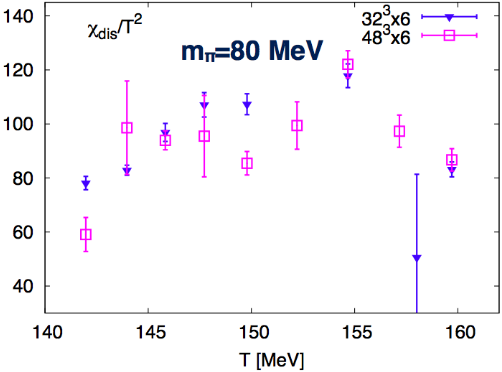
Order of the phase transition in the $m_{u,d} \to 0$ limit

Alternative scenario for the Columbia plot:



Ways to address the question: take $m_{u,d} \to 0$ at $m_s = m_s^{\rm phys}$ or at $m_s = \infty$.

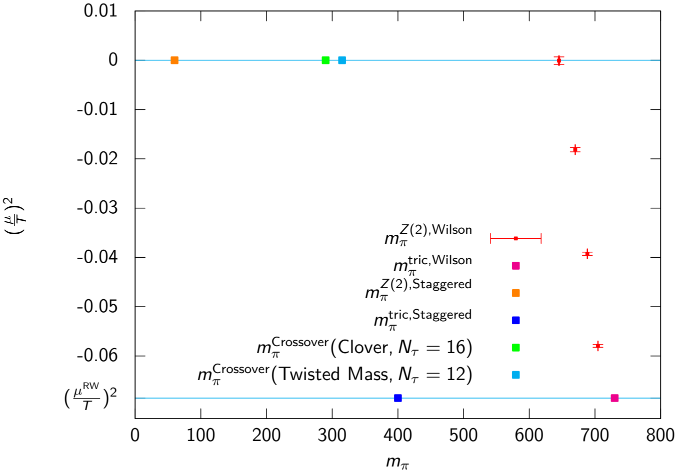
Order of the phase transition in the $m_{u,d} \to 0$ limit at $m_s = m_s^{\rm phys}$



- HISQ action, $N_t = 6$: no sign of 1st order transition at $m_\pi = 80$MeV
- $f(m_{u,d}, T) = h^{1+1/\delta} f_{\rm sing}(z) + {\rm regular}$, $z \equiv t/h^{1/\beta\delta}$, $t = (T - T_c)/T_c$, $h \propto m_{u,d}/m_s$
- good fit obtained with O(2) exponents (taste splitting); well consistent with the standard scenario.

HotQCD 1312.0119 + 1302.5740 and Talk by **H.-T. Ding**.

Order of the phase transition in the $m_{u,d} \to 0$ limit in $N_f = 2$ QCD



- large cutoff effects on coarse Wilson ensembles.

Talk by **Ch. Pinke**. Method based on imaginary chemical potential: Bonati et al., 1408.5086.

Near-equilibrium properties

- the pion quasiparticle in the low-temperature phase of QCD
- the heavy-quark momentum diffusion constant
- spectral functions in the vector channel
- the nucleon channel
- quarkonium.

Formalism

- Relation between the Euclidean correlator and the spectral function:

$$G(x_0,\mathbf{p}) = \int d^3x\; e^{-i\mathbf{p}\cdot\mathbf{x}}\, \langle J(x) J(0)\rangle \overset{\star}{=} \int_0^\infty \frac{d\omega}{2\pi}\, \rho(\omega,\mathbf{p}) \frac{\cosh[\omega(\beta/2 - x_0)]}{\sinh[\omega\beta/2]}.$$

- Relation of the spectral function to Wightmann correlator:

$$\rho(\omega,\mathbf{p}) = (1-e^{-\beta\omega}) \int_{-\infty}^{\infty} dt \int d^3x\; e^{i\omega t - i\mathbf{p}\cdot\mathbf{x}}\; \frac{1}{Z}\mathrm{Tr}\,\{e^{-\beta\hat{H}}\; \hat{J}(t,\mathbf{x})\; \hat{J}(0)\}.$$

NB. $\hat{J}(t) = e^{i\hat{H}t}\hat{J}\,e^{-i\hat{H}t}$.

- Analogous object to the 'hadronic tensor' in $\gamma^* N \to X$:

$$W^{\mu\nu}(q,p) = \frac{1}{2}\sum_{\sigma_N} \int d^4x\; e^{-iq\cdot x}\; \langle N|\hat{J}^\mu(x)\,\hat{J}^\nu(0)|N\rangle.$$

$\star$ inverse problem for $\rho(\omega,\boldsymbol{p})$

One conceptual point

- Inserting a complete set of states in Euclidean correlator:

$$G(x_0, \mathbf{p}) = \frac{1}{Z} \sum_{n,m} |\langle n|J(\boldsymbol{p})|m\rangle|^2 \; e^{-(\beta - x_0)E_m} e^{-E_n x_0}$$

NB. the $|n\rangle$ are eigenstates of the Hamiltonian.

- Correspondingly, formal expression for the spectral function

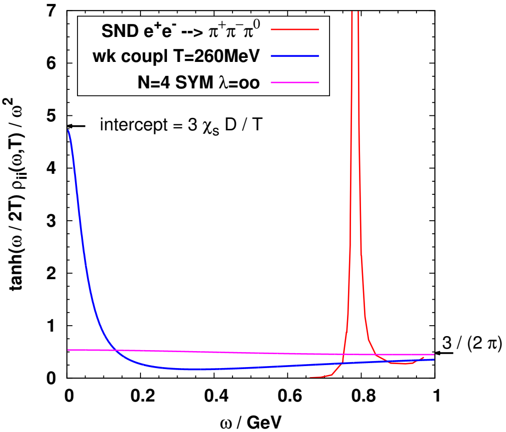
$$\rho(\omega) = \frac{4\pi}{Z} \sinh(\beta\omega/2) \sum_{n,m} |\langle n|J(\boldsymbol{p})|m\rangle|^2 \; e^{-\beta(E_n+E_m)/2} \delta(\omega - (E_n - E_m)).$$

- These expressions are useful to prove formal relations, such as the connection with the Minkowski-space correlators.

- ... but what we are after are the collective excitations of the medium (= frequency-poles of the correlator in infinite volume), which depend on the temperature and are not related in a simple way to the $|n\rangle$, e.g.
  - hydrodynamic excitations (associated with conserved currents): have to be there.
  - quasiparticle = pole at $(\omega = \omega_{\boldsymbol{p}})$ with $\mathrm{Im}(\omega_{\boldsymbol{p}}) \lesssim \mathrm{Re}(\omega_{\boldsymbol{p}})$;
    $v_{\mathrm{g}} = \frac{d\omega_{\boldsymbol{p}}}{d|\boldsymbol{p}|}$ is its group velocity. There are media with no quasiparticles.

Motivation: expected thermal changes in spectral functions

Isoscalar vector channel: spectral fct. of $J_i = \frac{1}{\sqrt{2}}(\bar{u}\gamma_i\bar{u} + \bar{d}\gamma_i d)$



- presence of weakly coupled quasiparticles $\Rightarrow$ transport peak at $\omega = 0$; is it really there at $T \approx 260$MeV ?

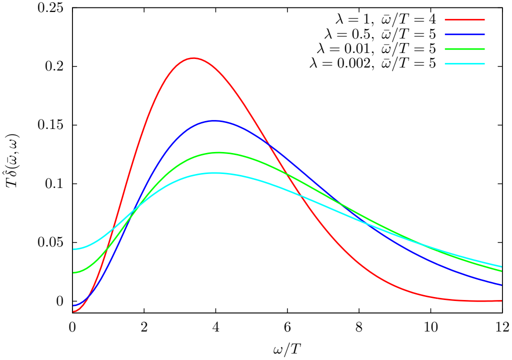
SND hep-ex/0305049

$D =$ diffusion coefficient; $\chi_s =$ static susceptibility.

Some basics on the inverse problem

$$\text{Linearity:} \qquad \sum_{i=1}^{n} c_i(\bar\omega)\, G(t_i) = \int_0^\infty d\omega\, \rho(\omega) \underbrace{\sum_{i=1}^{n} c_i(\bar\omega)\, \frac{\cosh[\omega(\beta/2 - t_i)]}{\sinh[\omega\beta/2]}}_{\widehat{\delta}(\bar\omega,\omega)}$$

- choose the coefficients $c_i(\bar\omega)$ so that the 'resolution function' $\widehat{\delta}(\bar\omega, \omega)$ is as narrowly peaked around a given frequency $\bar\omega$ as possible (idea behind the Backus-Gilbert method, [used in Robaina et al. 1506.05732])



Resolution function at $\bar\omega = 4$ for $N_t = 24$, $t_i/a = 5, \dots 12$.

Some basics on the inverse problem

$$\text{Linearity:}\quad \sum_{i=1}^{n} c_i(\bar\omega)\, G(t_i) = \int_0^\infty d\omega\, \rho(\omega)\, \underbrace{\sum_{i=1}^{n} c_i(\bar\omega)\, \frac{\cosh[\omega(\beta/2 - t_i)]}{\sinh[\omega\beta/2]}}_{\widehat{\delta}(\bar\omega,\omega)}$$

- For given $\{t_i\}$, a certain resolution in frequency can be achieved; however, the required c_i are strongly oscillating (ill-posed problem)
- $\Rightarrow$ finite accuracy of data further limits the resolution
- if you *know* a priori that the spectral function is slowly varying on the scale $\Delta\omega \sim T$ the problem is again well posed.
- problem: whether there is a narrow transport peak or narrow quasiparticle peaks is precisely what we want to know.

Methods used: fit ansatz; maximum entropy method (MEM); new Bayesian method [Burnier & Rothkopf 1307.6106], talk by **S. Kim**; stochastic methods, talks by **H. Ohno** and **H.-T. Shu**.

The pion quasiparticle in the low-temperature phase

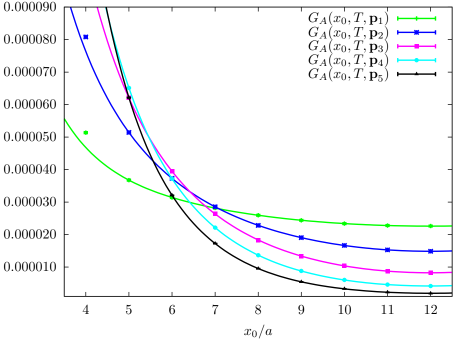
- Chiral symmetry is spontaneously broken for $T < T_c$: $-\langle\bar\psi\psi\rangle > 0$.
- Goldstone theorem $\Rightarrow$ a divergent spatial correlation length exists in the limit $m \to 0$.
- somewhat less obvious: a massless real-time excitation exists, the pion quasiparticle.
- dispersion relation: $\omega_{\boldsymbol{p}} = u\sqrt{m_\pi^2 + \boldsymbol{p}^2} + \ldots$; m_π = screening mass(!)
  [Son and Stephanov, PRD 66, 076011 (2002)]
- pion dominates Euclidean two-point function of A_0 and of P at $x_0 = \beta/2$

$$
\begin{array}{lcc}
T = 0: & \multicolumn{2}{c}{\text{pion mass} = 267(2)\,\text{MeV}} \\
 & \swarrow & \searrow \\
T = 169\text{MeV}: & \text{quasiparticle mass} = 223(4)\text{MeV} & \text{screening mass} = 303(4)\text{MeV}.
\end{array}
$$

Implications for the hadron resonance gas model!?

Robaina et al. 1406.5602; 1506.05732

Pion quasiparticle: test of the dispersion relation



- also the residue in two-point function of A_0 and of P are predicted
- dispersion relation & residue compatible with correlators at small $\mathbf{p} \neq 0$.

$$G_A(x_0, \mathbf{p}) = \frac{1}{3} \int d^3x\, e^{i\mathbf{p}\cdot\mathbf{x}} \langle A_0^a(x) A_0^a(0) \rangle = \int_0^\infty \frac{d\omega}{2\pi}\, \rho^A(\omega, \mathbf{p}) \frac{\cosh[\omega(\beta/2 - x_0)]}{\sinh[\omega\beta/2]}.$$

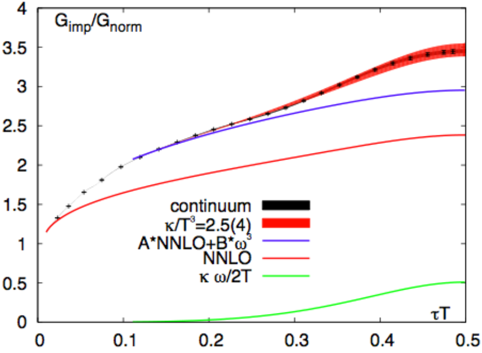
$$\text{Ansatz}: \rho^A(\omega, \mathbf{p}) = a_1(\mathbf{p})\delta(\omega - \omega_{\mathbf{p}}) + a_2(\mathbf{p})(1 - e^{-\omega\beta})\theta(\omega - c).$$

24×64^3 thermal ensemble, $T = 169$MeV, $m_\pi|_{T=0} = 270$MeV   1506.05732.

Heavy quark momentum diffusion coefficient κ

$$G(\tau) = \frac{\left\langle \mathrm{Re\,Tr}\left(U(\beta,\tau) g E_k(\tau,\mathbf{0}) U(t,0) g E_k(0,\mathbf{0})\right)\right\rangle}{-3\,\langle \mathrm{Re\,Tr}\, U(\beta,0)\rangle} = \int_0^\infty \frac{d\omega}{2\pi}\,\rho(\omega)\,\frac{\cosh[\omega(\beta/2-\tau)]}{\sinh[\omega\beta/2]}$$

- color parallel transporters $U(t_2,t_1)$ are propagators of static quarks
- (Lorentz) force-force correlator on the worldline of the quark.

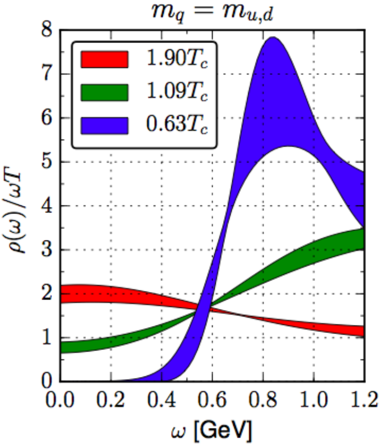


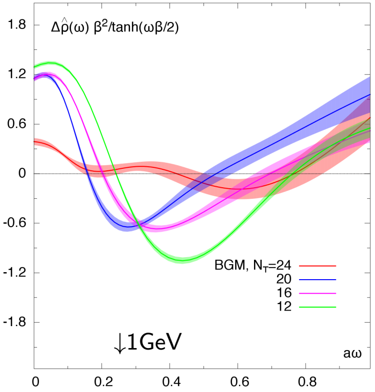
$$\kappa = \lim_{\omega\to 0} \frac{T}{\omega}\rho(\omega), \qquad D = 2T^2/\kappa.$$

NNLO calculation available:
$\rho(\omega) =$ smooth function $\overset{\omega\to\infty}{\sim} g^2\omega^3$.

Kaczmarek et al. 1409.3724; see also Caron-Huot, Laine, Moore 0901.1195;
HM 1012.0234; Banerjee et al. 1109.5738;

The isovector vector channel





- shift of spectral weight from the ρ to low frequency region as T increases.

Left: Aarts et al. (($N_f = 2 + 1$), also strange current included) 1412.6411;

Right: Francis et al. ($N_f = 2$), 1212.4200 and in preparation; see also talk by **Fl. Meyer**.

Spectral sum rules for $\Delta\rho(\omega,\boldsymbol{k},T) \equiv \rho(\omega,\boldsymbol{k},T) - \rho(\omega,\boldsymbol{k},0)$

$$\int_{-\infty}^{\infty} d\omega\,\omega\,\Delta\rho_V^L(\omega,\boldsymbol{k},T) = 0, \qquad \forall\boldsymbol{k} \qquad [1107.4388]$$

$$\int_{-\infty}^{\infty} \frac{d\omega}{\omega}\Delta\rho_V^L(\omega,\boldsymbol{k},T) = \chi_s - \kappa_l \boldsymbol{k}^2 + \mathrm{O}(|\boldsymbol{k}|^4),$$

$$\int_{-\infty}^{\infty} \frac{d\omega}{\omega}\Delta\rho_V^T(\omega,\boldsymbol{k},T) = \kappa_t \boldsymbol{k}^2 + \mathrm{O}(|\boldsymbol{k}|^4),$$

$$\int_{-\infty}^{\infty} d\omega\;\omega\;\Delta\rho_{\mathrm{A}}^L(\omega,\boldsymbol{k},T) = -m\langle\bar\psi\psi\rangle\Big|_0^T, \qquad \forall\boldsymbol{k} \qquad [1406.5602]$$

$$\vdots$$

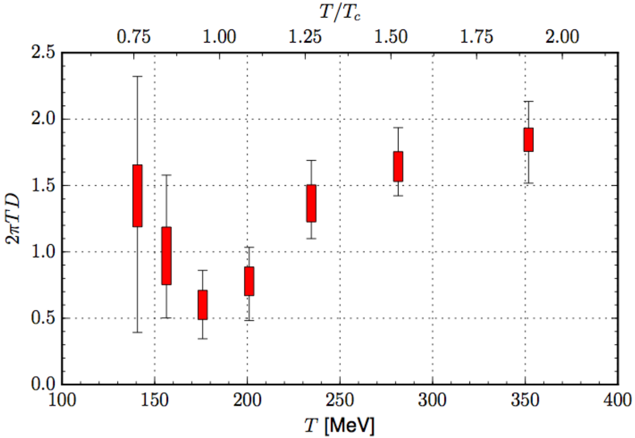
$\exists$ interpretation of κ_l and κ_t in terms of screening/antiscreening of electric probe charges and currents placed in the medium Brandt et al. 1310.5160

$$\frac{1}{3}\int d^3x\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\,\langle V_0^a(x)V_0^a(0)\rangle = \int_0^\infty d\omega\,\rho_V^L(\omega,\boldsymbol{k},T)\,\frac{\cosh\omega(\beta/2-x_0)}{\sinh\omega\beta/2},$$

$$-\tfrac{1}{6}\left(\delta_{il} - \frac{k_i k_l}{\boldsymbol{k}^2}\right)\int d^3x\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\,\langle V_i^a(x)V_l^a(0)\rangle = \int_0^\infty d\omega\,\rho_V^T(\omega,\boldsymbol{k},T)\,\frac{\cosh\omega(\beta/2-x_0)}{\sinh\omega\beta/2},$$

$$\frac{1}{3}\int d^3x\, e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\,\langle A_0^a(0)A_0^a(x)\rangle = \int_0^\infty d\omega\,\rho_{\mathrm{A}}^L(\omega,\boldsymbol{k},T)\,\frac{\cosh(\omega(\beta/2-x_0))}{\sinh(\omega\beta/2)}$$

The diffusion coefficient D



- lattice results consistent with a strongly coupled scenario
- but a narrow transport peak cannot presently be excluded, which would yield larger D.

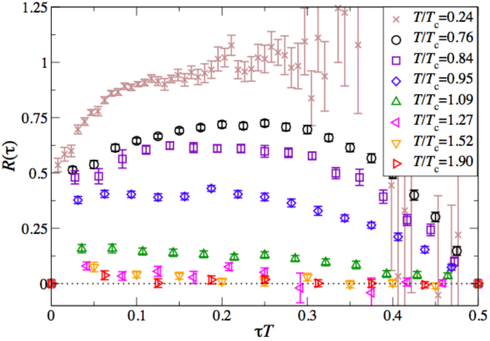
Aarts et al. 1412.6411.

Fermionic correlators new!

- Nucleon interpolating operator (parity +): $O_{N+} = \frac{1}{2}(1+\gamma_0)\,\epsilon_{abc}(u_a C\gamma_5 d_b)u_c$

$$
\begin{aligned}
G(x_0) &= \int d^3x \, \langle O_{N+}(x)\ \bar{O}_{N+}(0)\rangle \\
&= \int_0^\infty \frac{d\omega}{2\pi}\ \frac{1}{1+e^{\omega/T}}\ \left[\rho_+(\omega)e^{-\omega x_0} - \rho_-(\omega)e^{-\omega(\beta-x_0)}\right]
\end{aligned}
$$

- Chiral symmetry restored ⇒ parity doubling: $G(\beta - x_0) = G(x_0)$.



$$R(x_0) = \frac{G(x_0)-G(\beta-x_0)}{G(x_0)+G(\beta-x_0)}$$

parity doubling occurs at $T \approx T_c$

⇝ full spectral analysis underway.

Aarts et al. 1502.03603

Quarkonium in the high-temperature phase

- for m_Q large, $\bar{Q}Q$ tightly bound $\rightarrow$ survives as a quasiparticle in medium
- but Debye screening of $\bar{Q}Q$ potential and Landau damping $\Rightarrow$ $\mathrm{Im}(\omega_{\boldsymbol{p}})$ grows and its residue (wrt $\bar{c}\boldsymbol{\gamma}c$) is reduced.
- at what temperature different $\bar{c}c$ and $\bar{b}b$ 'states' 'melt' provides a thermometer in heavy-ion collisions; p-wave bound states melt before before s-wave bound states etc.
- formulate the problem in NRQCD; advantages:
  - $G(\tau) = \int_{-2m_Q}^{\infty} \frac{d\omega}{2\pi}\, e^{-\omega\tau}\, \rho(\omega)$
  - $\rho(\omega)$ softer in the UV than in the relativistic theory
  - $\rho(\omega)$ does not contain a transport peak.

Bottomomium: NRQCD and pNRQCD studies

- ground state Υ survives at least up to $2T_c$, χ_{b1} melts immediately above T_c [Harris et al. 1402.6210, $N_f = 2+1$, $m_\pi = 400$MeV].
- Kim et al. [1409.3630] find χ_{b1} survives for some time.

Charmonium: studies in relativistic formulation, recently also in NRQCD.

Talks by **S. Kim**, **A. Ikeda**, **H. Ohno** and **H.-T. Shu**.

Interpretation of screening masses: static and non-static

Consider perturbating the Hamiltonian,

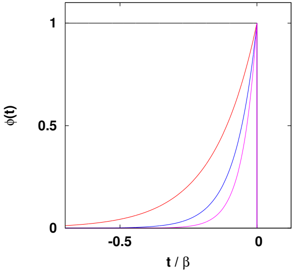
$$\hat{H}_\phi(t) = \hat{H} - \int d^3y\, \phi(t,\boldsymbol{y})\hat{J}(t,\boldsymbol{y}),$$

with the external perturbation given by

$$\phi(t,\boldsymbol{y}) = \delta(\boldsymbol{y})e^{\omega t}\theta(-t), \qquad \omega \geq 0.$$

Linear response $\Rightarrow$

$$\delta\langle J(t=0,\boldsymbol{x})\rangle = \underbrace{G_E^{JJ}(\omega_n,\boldsymbol{x})}_{\text{Euclidean corr.}}, \qquad \text{for } \omega = \omega_n = 2\pi T n.$$



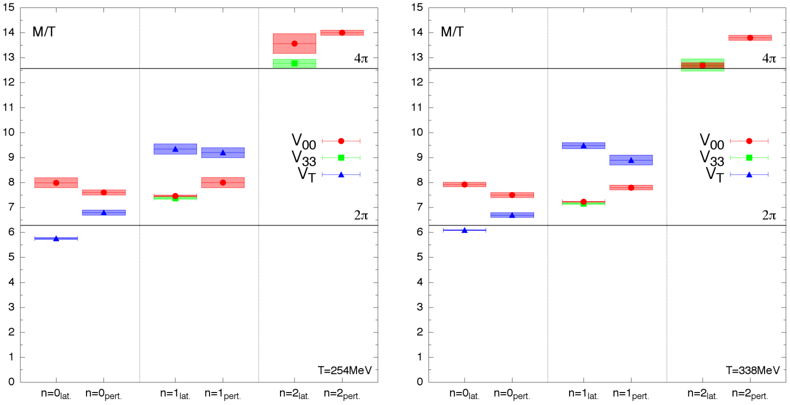
Correlation length in Matsubara sector ω_n = length scale over which a perturbation with the time dependence $e^{\omega_n t}$ is screened ($n \geq 0$).

Screening masses at high temperatures

Weak-coupling picture of flavor-non-singlet screening masses:

- fermions have an effective mass of at least $\pi T \Rightarrow$ dimensional reduction
- they form non-relativistic, 2+1d bound states of size O(m_E^{-1})
  Laine, Vepsalainen hep-ph/0311268
- expect bound state to be described by a Schrödinger equation in 2+1d.
- Non-static sector: potential has a connection with an effective potential used in the calculation of the dilepton production rate
  [Aurenche, Gelis, Moore, Zakaret hep-ph/0211036; Caron-Huot 0811.1603; Panero, Rummukainen, Schäfer 1307.5850].

Vector screening masses: lattice vs. EFT



$T = 254$ MeV $\qquad$ $T = 340$ MeV

Satisfactory agreement between lattice QCD and the EFT predictions.

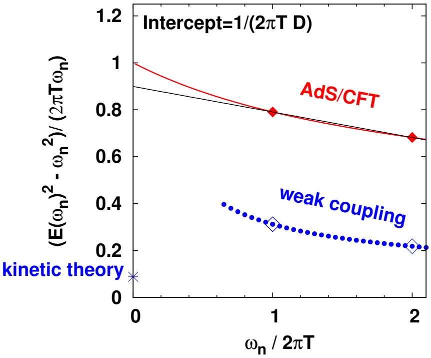
Brandt et al. 1404.2404; $N_t = 16$ and $N_t = 12$, $N_s = 64$; $m_\pi|_T = 0 = 270$MeV

Non-static screening masses and transport coefficients

Linear response along with a constitutive equation for the vector current $\boldsymbol{J} \Rightarrow$

$$G_E^{J_0 J_0}(\omega_n, k) \overset{\omega_n, k\to 0}{=\!=\!=} \frac{\chi_s D k^2}{\omega_n + D k^2} \qquad \Rightarrow \qquad E(\omega_n)^2 \overset{\omega_n \to 0}{\sim} \frac{\omega_n}{D}.$$

χ_s = static susceptibility, D = diffusion coefficient, $E(\omega_n)$ = screening mass in sector ω_n



In the limit $T \to \infty$, extrapolating the screening masses in the lowest Matsubara sectors to $\omega_n = 0$ gives the correct result, $1/(T\,D) = 0$.

Brandt, Francis, Laine, HM 1408.5917; Kinetic theory: Arnold, Moore & Yaffe hep-ph/0111107

Conclusion

- many equilibrium quantities known in the continuum limit (mostly from staggered fermion calculations, now universality checks with other actions)
- new impetus to put the topology of the Columbia plot on solid footing, and to be more quantitative.
- progress in near-equilibrium quantities
  - $N_t \approx 24$, few-permille precision on correlation functions, quenched continuum results
  - theory support (effective field theory and sum rules, advantageous reformulation of the problem,...)

Topics not covered: external magnetic fields (K. Szabo LAT13); $U_A(1)$ aspects; many talks on finite-density at this conference.

Backup slides