

Beyond Standard Model: Lattice Calculations at the Energy Frontier

**Anna Hasenfratz
University of Colorado Boulder**

Lattice 2015, Kobe



I would like to thank the organizers for an amazing Lattice 2015 Conference, and for giving me the opportunity to review

"Lattice studies for BSM models, especially the strong dynamics"

I also want to thank everyone who sent me details of their work, answered my questions, and gave excellent talks, posters. I learned a lot.

I have to apologize to everyone whose work I cannot mention or review appropriately - and that is pretty much everyone.

Strong Yukawa models : 1989 and 2014

Massive fermions without fermion bilinear condensates

Venkitesh Ayyar and Shailesh Chandrasekharan

Department of Physics, Box 90305, Duke University, Durham, North Carolina 27708, USA

NON-PERTURBATIVE STUDY OF THE STRONGLY COUPLED SCALAR-FERMION MODEL ☆

Anna HASENFRATZ

*Supercomputer Computations Research Institute and Physics Department, The Florida State University,
Tallahassee, FL 32306-4052, USA*

and

Thomas NEUHAUS

Institut für Theoretische Physik, Universität Bielefeld, D-4800 Bielefeld, Fed. Rep. Germany

Received 14 June 1988; revised manuscript received 18 January 1989

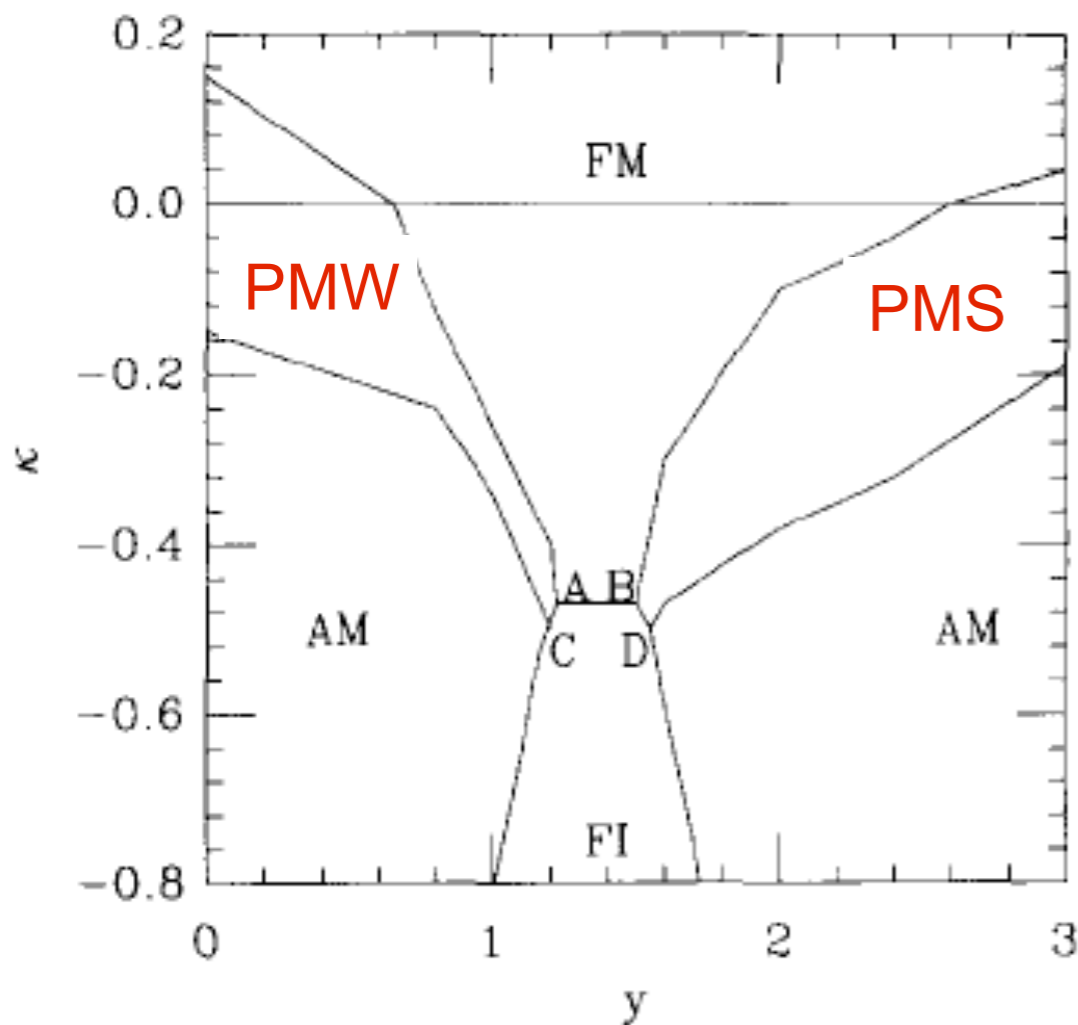
In Memoriam



Thomas Neuhaus 1956-2015

Strong Yukawa models : 1989

Generic phase diagram (scalar hopping vs Yukawa coupling):



Two paramagnetic phases

$$\langle \Phi \rangle = 0 \quad \langle \bar{\psi}_n \psi \rangle = 0$$

PMW : usual perturbative phase

PMS : strong coupling phase with massive fermions :

PMS phase: mass generation without fermion bilinear condensate
(but fermions decouple in any continuum limit)

Strong Yukawa models : 2015

Use 2 copies of (reduced) staggered fermions

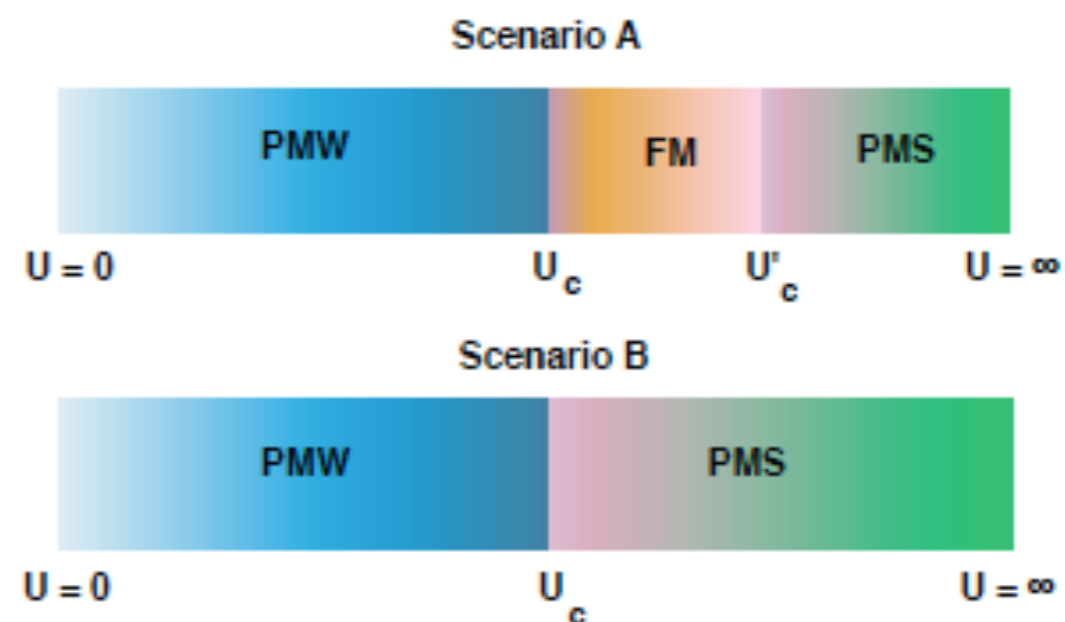
$$S = S_0 - \kappa \sum_{x, \hat{\alpha}} \sigma_x \sigma_{x+\hat{\alpha}} - Y \sum_{i=1,2} \sum_x \sigma_x \bar{\psi}_{x,i} \psi_{x,i}$$

talk by V. Ayyar:
arXiv:1410.6474

Simulations at $\kappa = 0$:

$$S = S_0 - U \sum_x \left\{ \bar{\psi}_{x,1} \psi_{x,1} \bar{\psi}_{x,2} \psi_{x,2} \right\}$$

In 3D Scenario B is realized
with 2nd order phase transition between
PMW and PMS phases
topological vs trivial insulator



Dynamics?

The system has topological “hedgehog” solutions that can condense
similar to 2D XY model!

S. Catterall, in prep

Preliminary results in 4D suggest the same structure

More on Yukawa models

Talks by D. Chu,
O. Akerlund

Yukawa models in the weak coupling region are “trivial”

→ include a Φ^6 interaction and study this as an effective theory (mimics new dynamics)

- What is the effect of the Φ^6 on the IR dynamics ?

The model is not universal, but could show generic properties

The two groups use different (complementary ?) approaches

- Berlin-Taiwan : MC with overlap fermions
- Akerlund, DeForcrand, Steinbauer : effective MF (analytical)

More on Yukawa models

Talks by D. Chu,
O. Akerlund

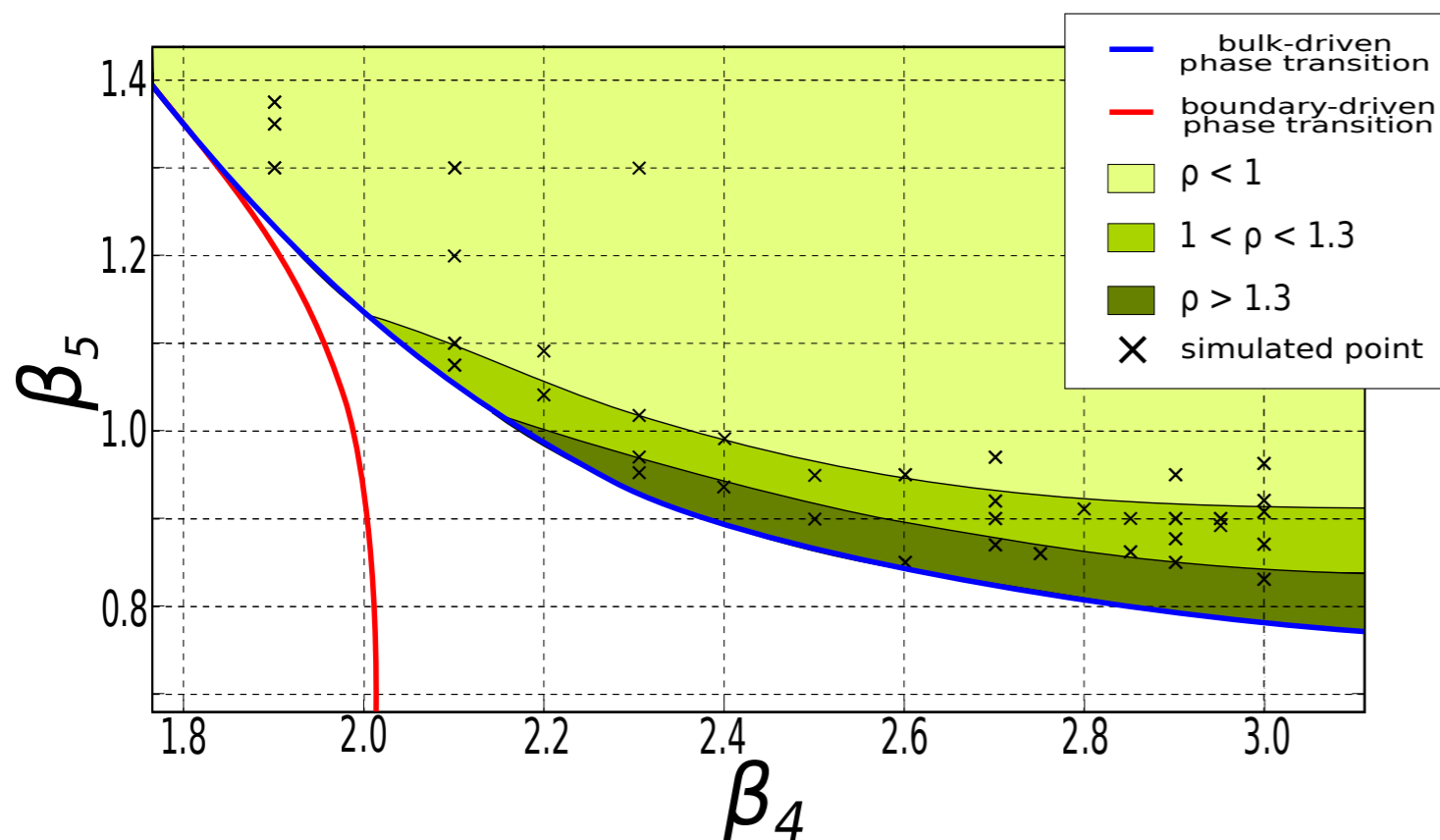
Yukawa models in the weak coupling region are “trivial”

→ include a Φ^6 interaction and study this as an effective theory
(mimics new dynamics)

- What is the effect of the Φ^6 on the IR dynamics
 - vacuum stability problems are relaxed:
we do not sit on the edge
 - Finite T : the EW phase transition can become 1st order
EW baryogenesis is possible
(but might need to consider gauge-scalar interactions)

Gauge-Higgs unification on 5D orbifold

Idea: identify the Higgs field with the five-dimensional components of a gauge field



ρ = ratio of SM Higgs to W
dark area: $\rho > 1.3$

- dimensional reduction via localization:

Alberti, Irges, Knechtli and Moir,
arXiv:1506.06035; talk by G. Moir

- study of warped fifth dimension

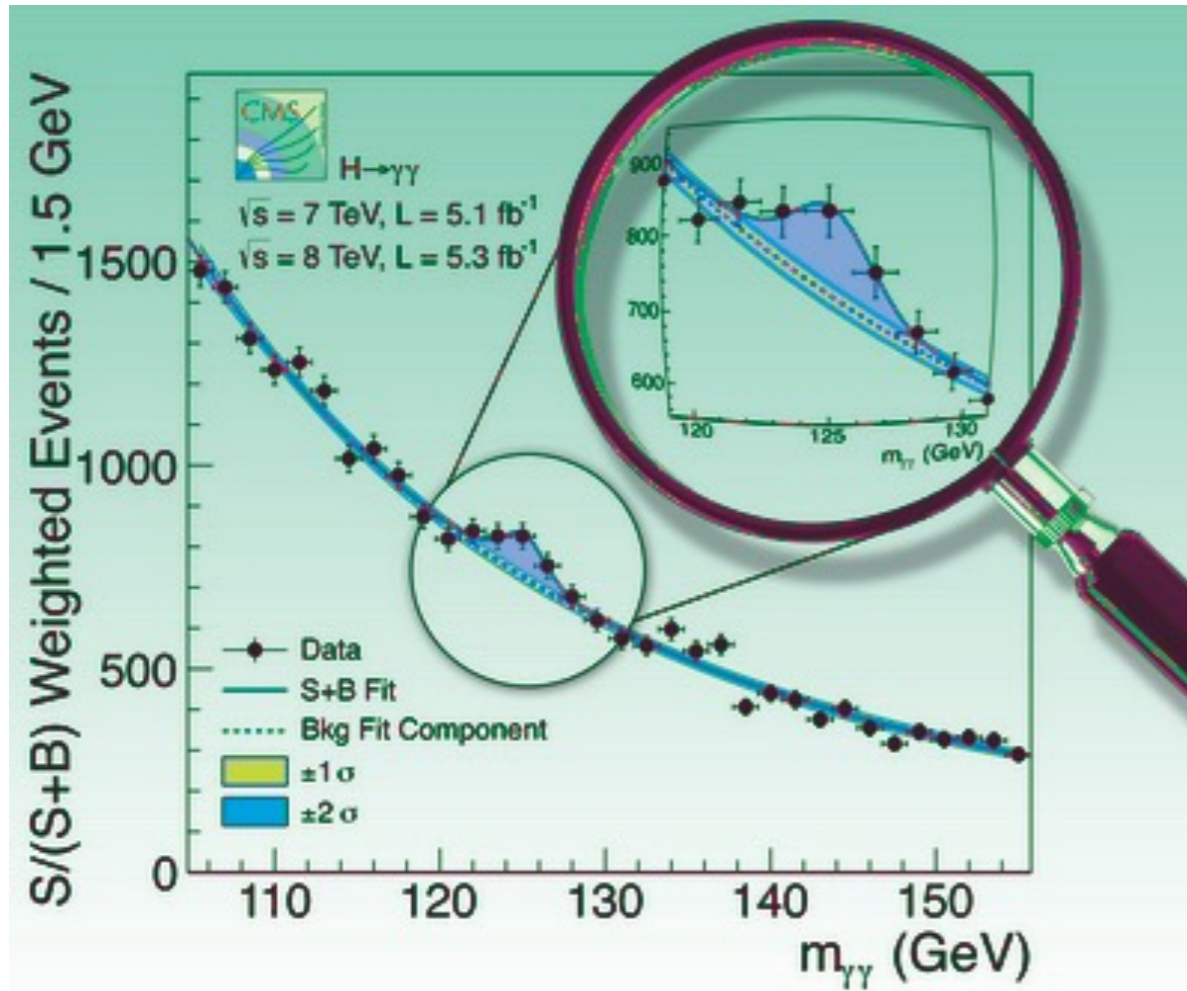
talk by E Lambrou

Strong dynamics - Composite Higgs models

Challenges & progress

Higgs era of particle physics

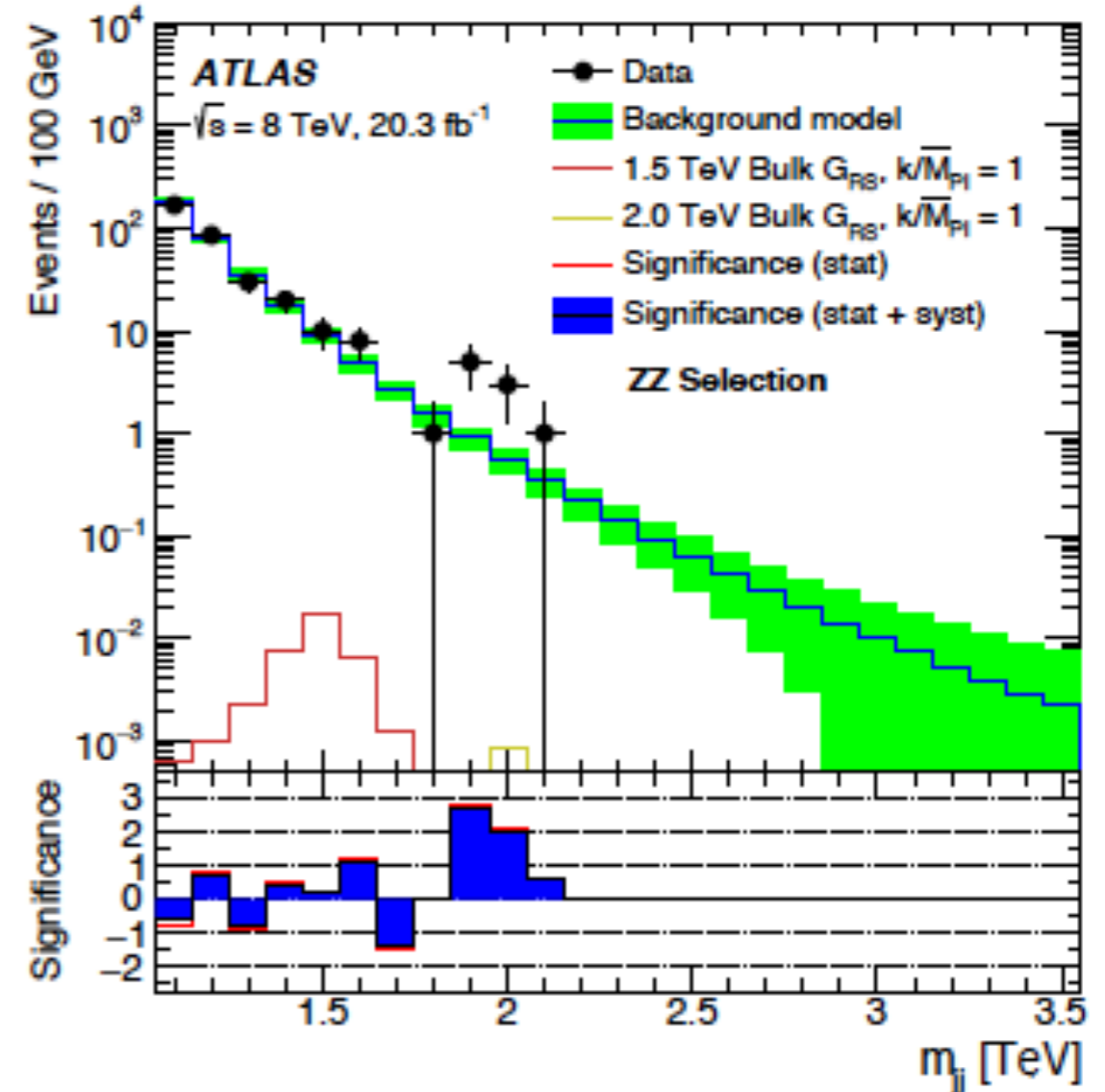
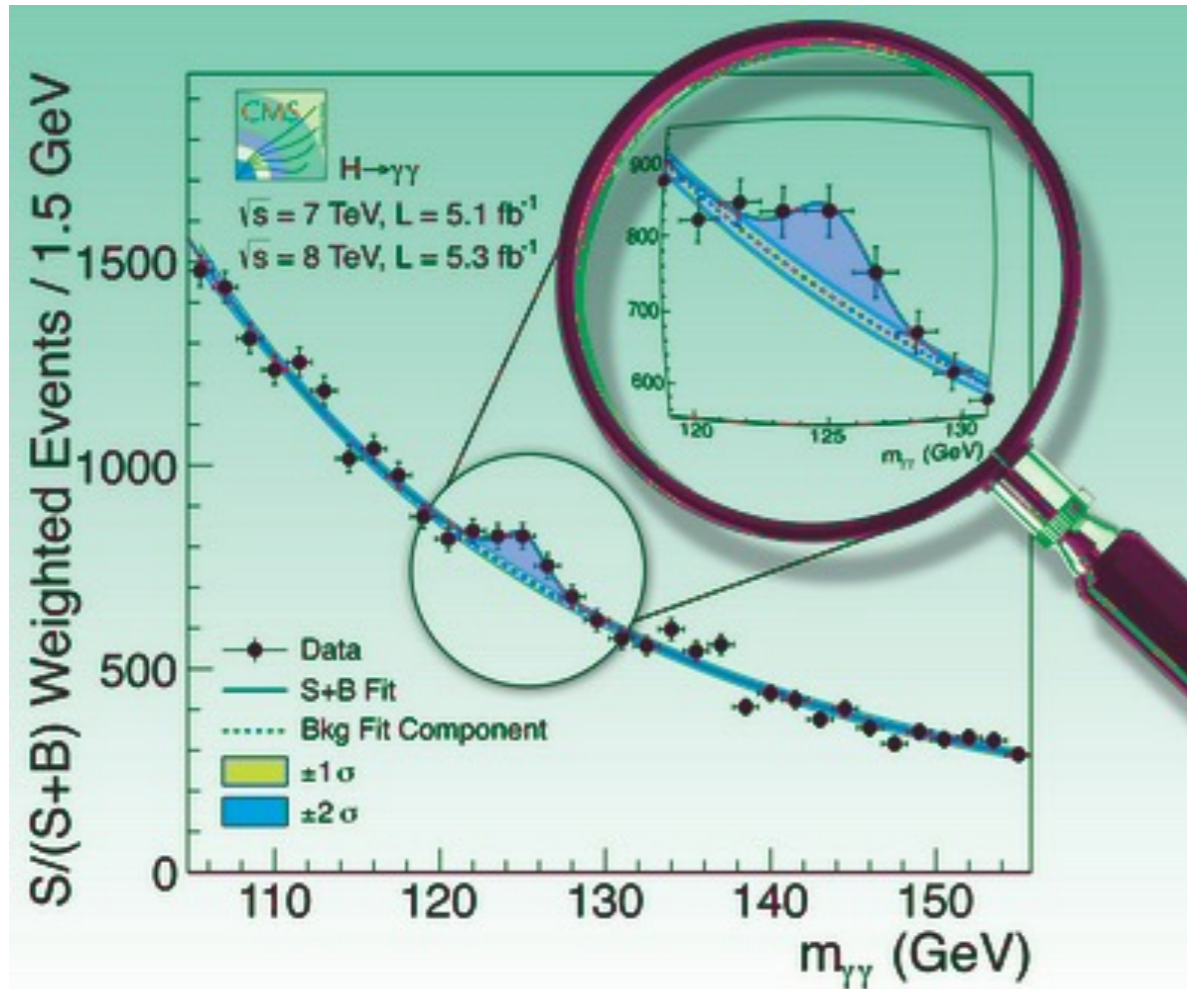
2012 : 125GeV Higgs boson



Higgs era of particle physics

2012 : 125GeV Higgs boson

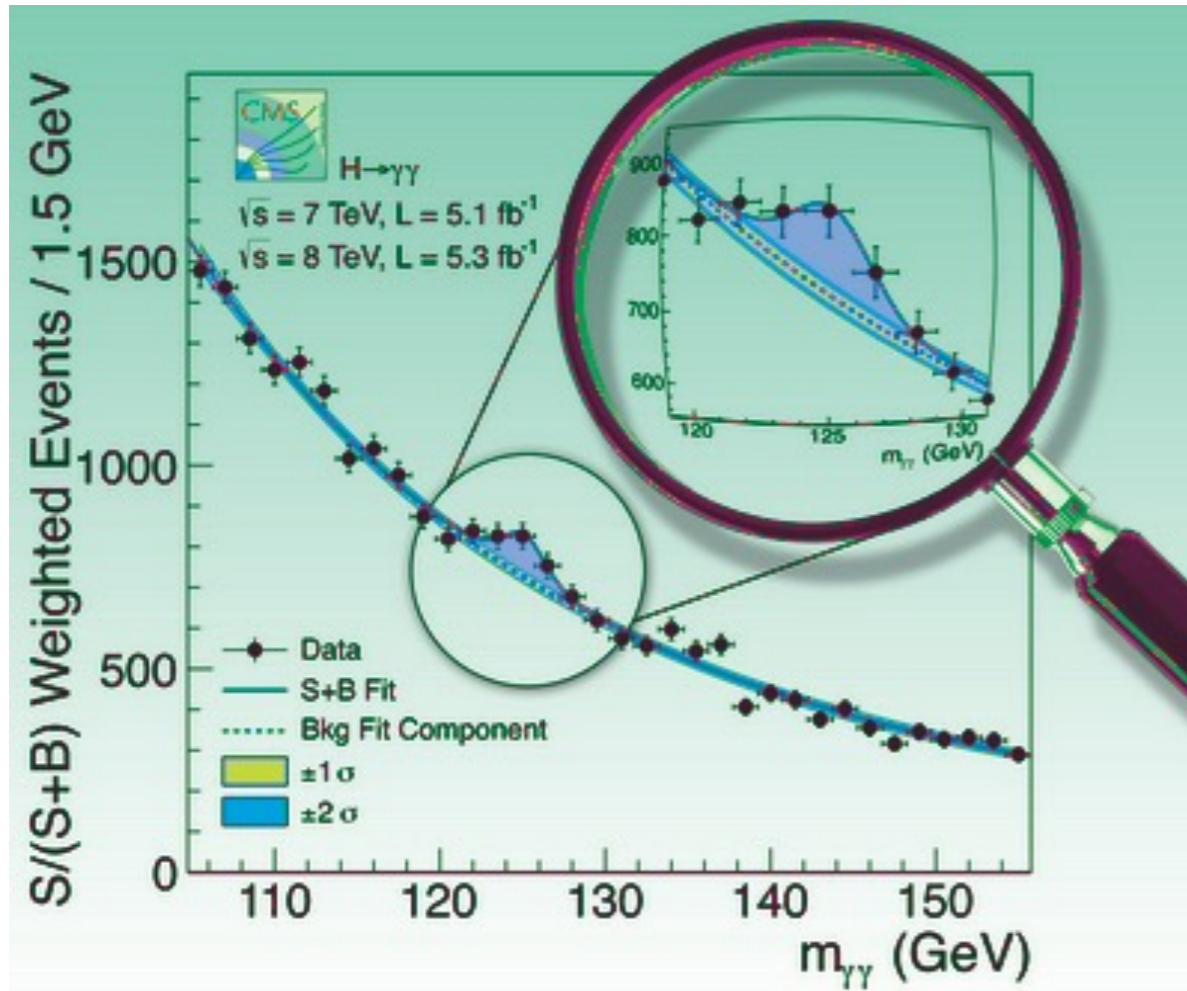
2015 : 3.4 σ excess in di-boson



Do not get too excited....

Higgs era of particle physics

2012 : 125GeV Higgs boson



Composite Higgs models

What is the mechanism that keeps the Higgs light?

Spontaneously broken symmetry \rightarrow massless Goldstone bosons

- **Flavor symmetry**: SSB leads to massless “pions”
- **Scale symmetry**: SSB leads to dilaton: **near-conformal models**

Composite Higgs models

What is the mechanism that keeps the Higgs light?

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- **Scale symmetry**: SSB leads to dilaton: **near-conformal models**

What is the mechanism that generates SM fermion masses

- **Bilinear coupling** - conformal TC : $y_\psi \bar{\psi}_L \psi_R \mathcal{O}_H$
- **Linear coupling** - Partial composite models: $\lambda_\psi \bar{\psi} \mathcal{O}_\psi$

Composite Higgs models

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- Linear coupling - Partial composite models: $\lambda_\psi \bar{\psi} \mathcal{O}_\psi$

Golterman, Shamir :

calculate the couplings of the low energy effective theory in terms of baryonic correlation functions ; in preparation to lattice simulations

arXiv:1502.00390

Composite Higgs models

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Composite Higgs models

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- **Scale symmetry:** SSB leads to dilaton: **near-conformal models**

Lattice simulations investigate strongly coupled but simple models

Couple them to Standard Model \rightarrow things will change

Challenge #1 : Is there a light dilaton?

Is the 0^{++} scalar of a near-conformal system light*?

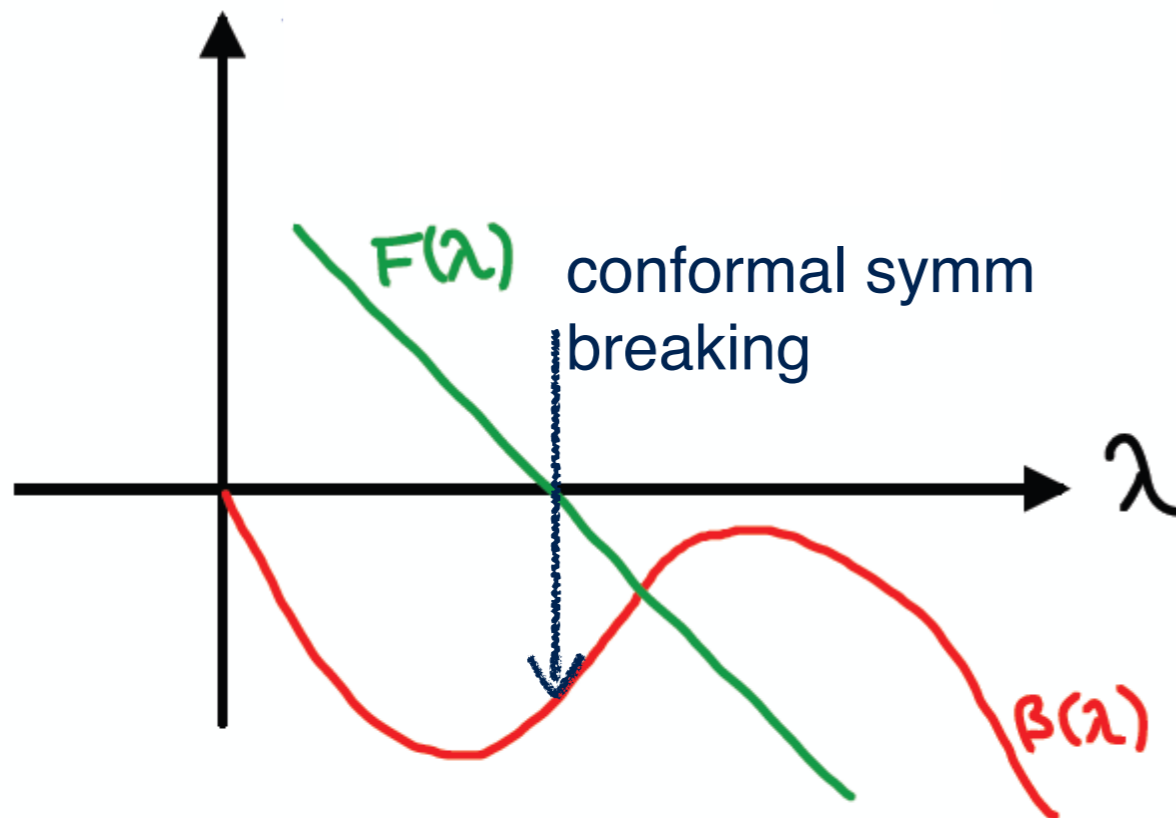
*How light is light enough?

Light dilaton?

Is the 0^{++} scalar of a near-conformal system light?

dilaton emerges from spontaneously conformal breaking
dilaton mass \approx value of the β function at breaking of conformal symmetry

(Csaki et al, arXiv:1305.3919)

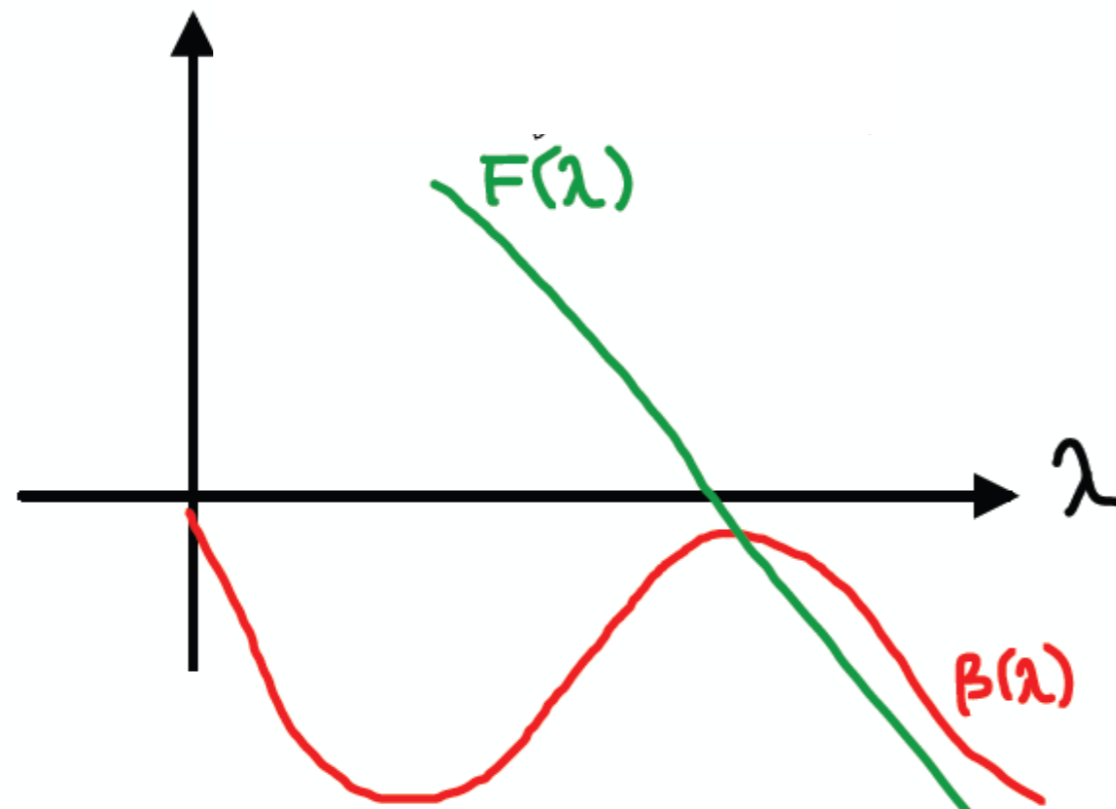


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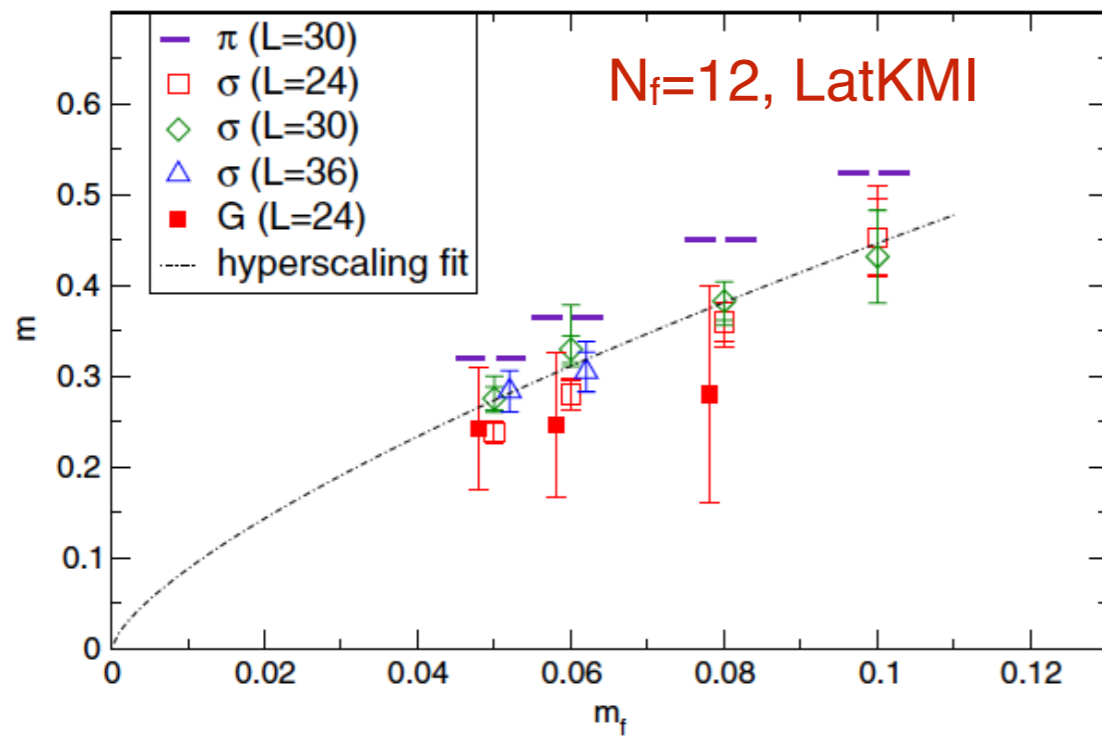
(Csaki et al, arXiv:1305.3919)



tuning is required
for a light dilaton

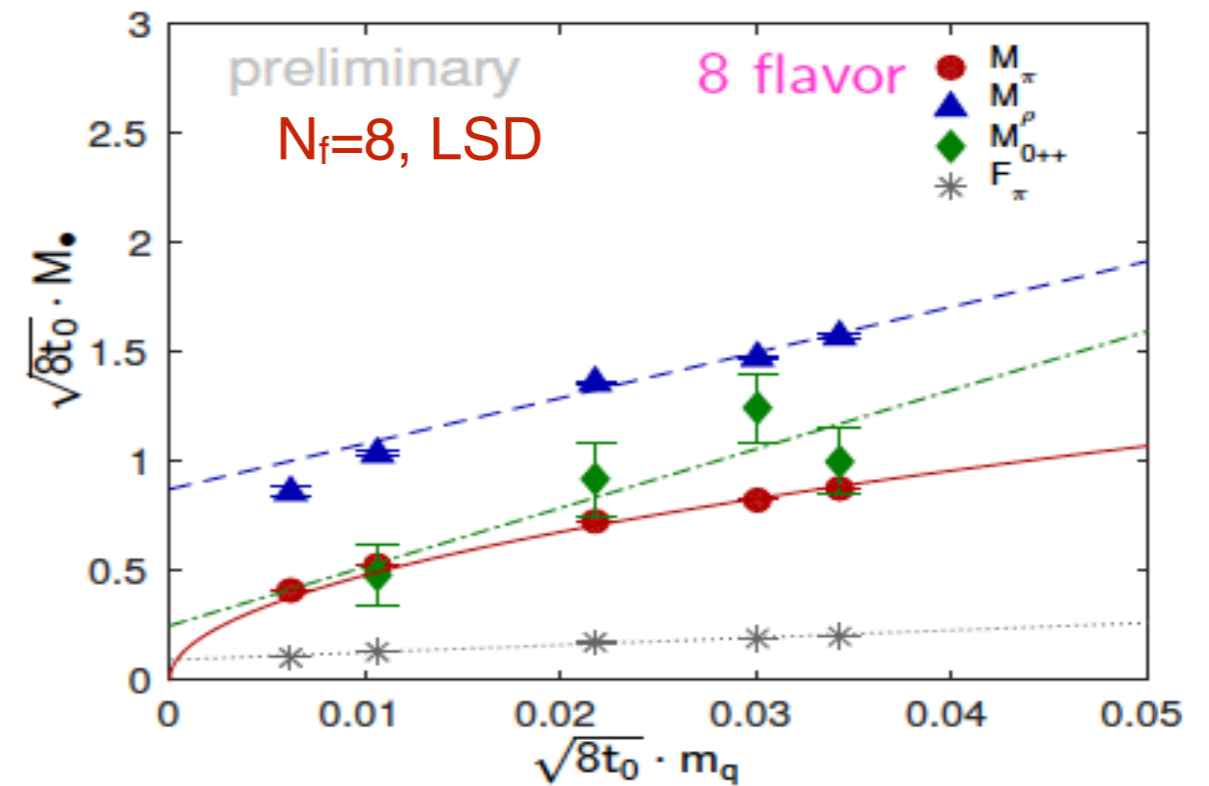
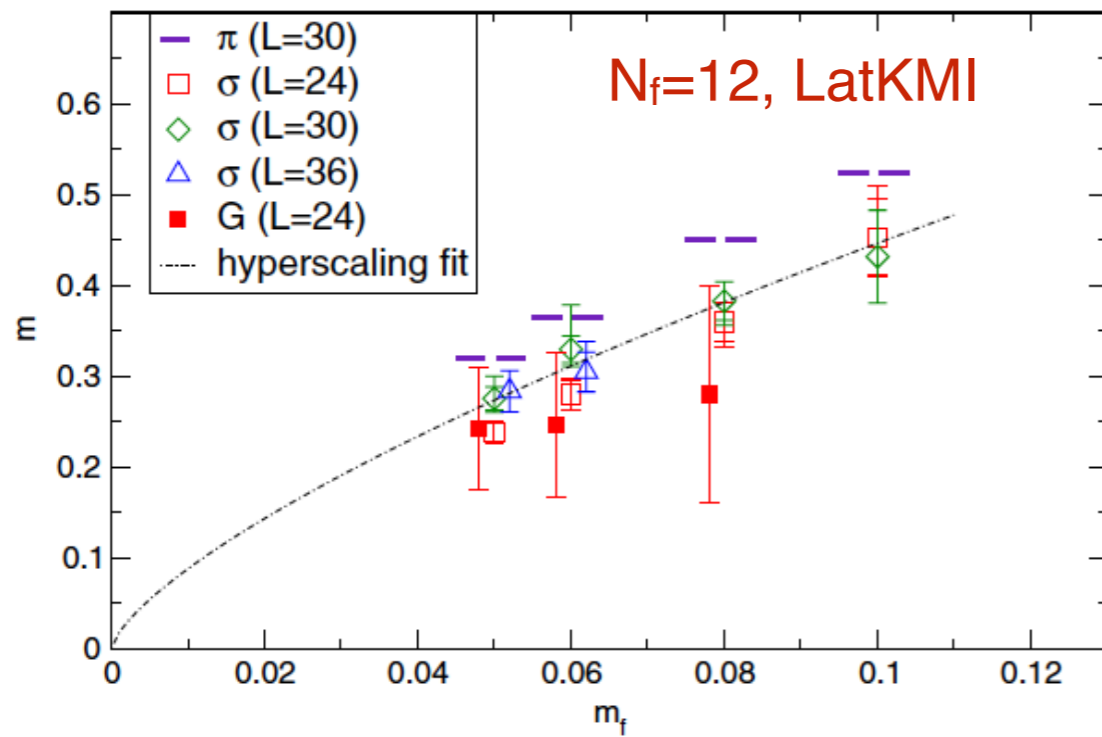
Light dilaton on the lattice

Lattice studies identified light scalar in many systems



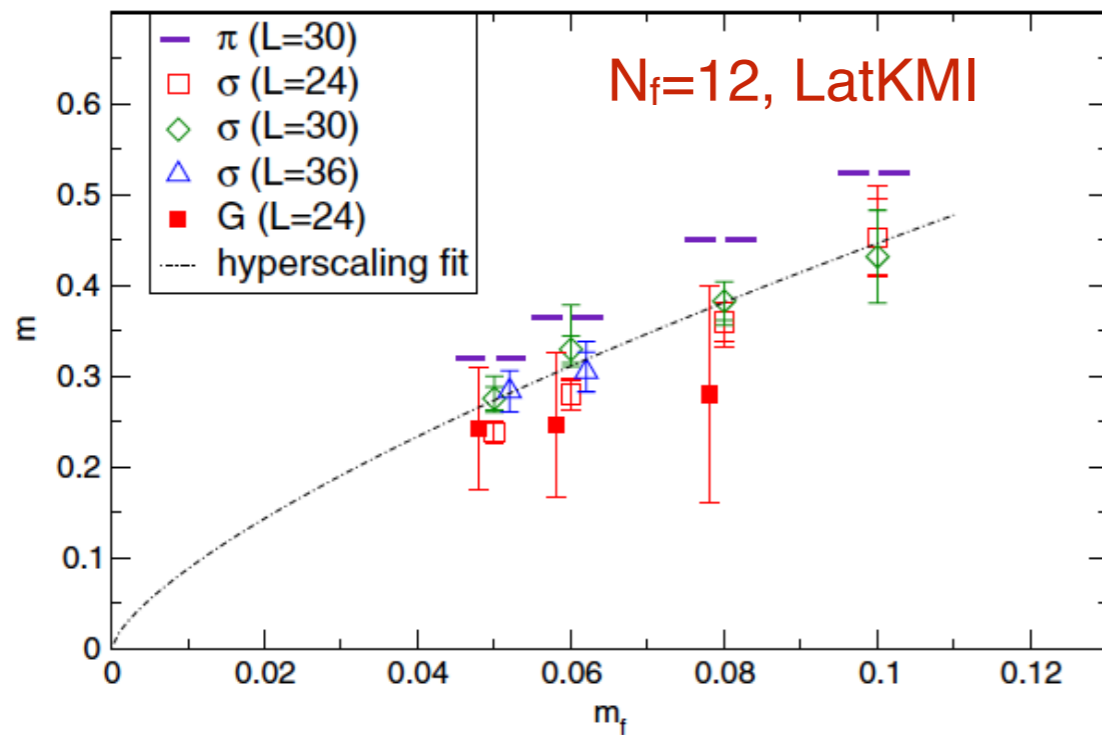
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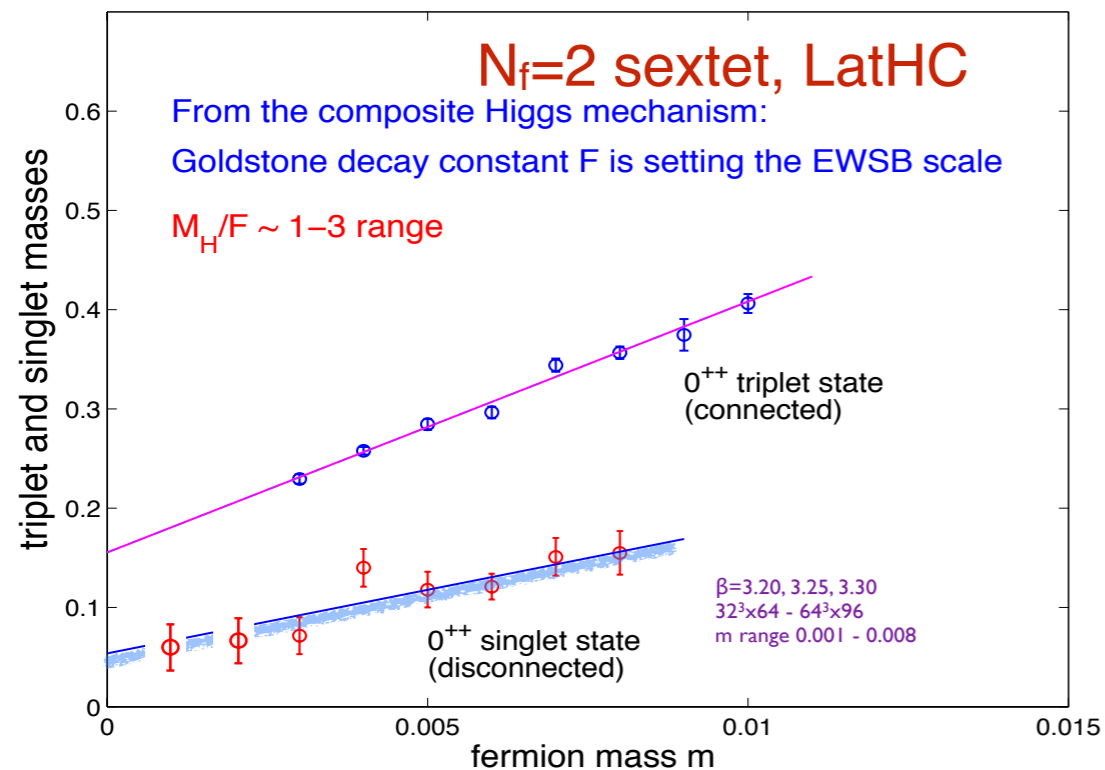
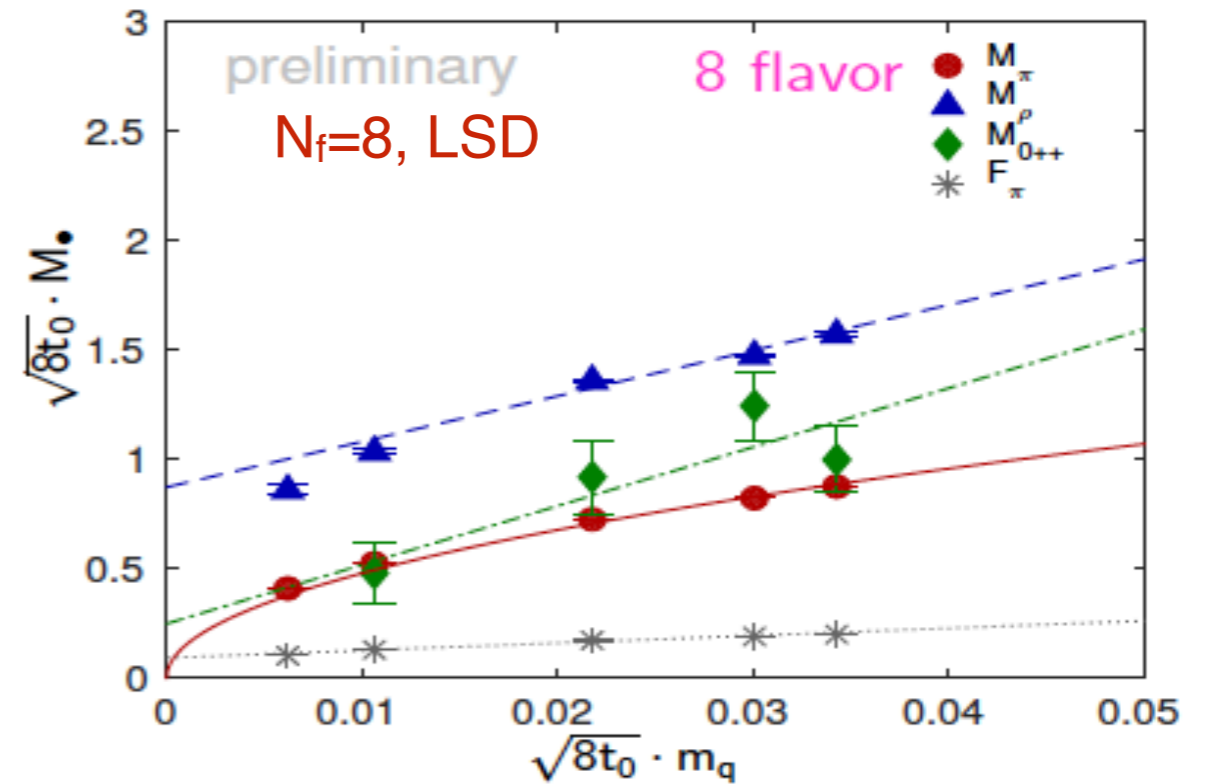


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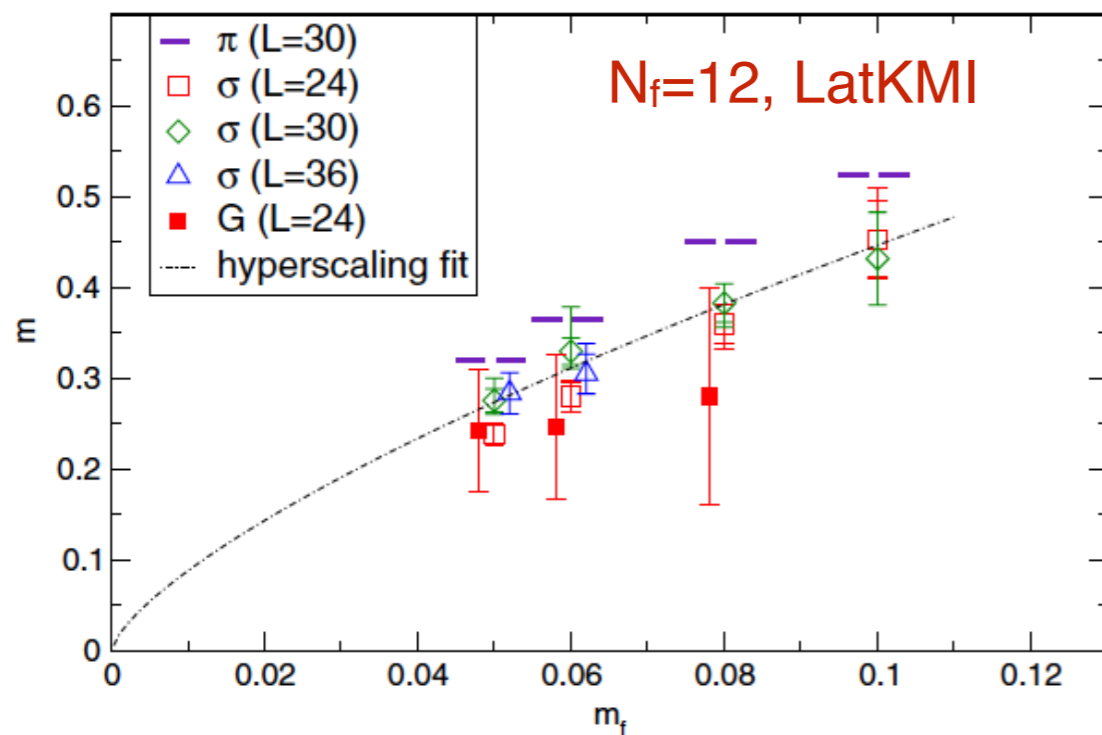


Triplet and singlet masses from 0^{++} correlators

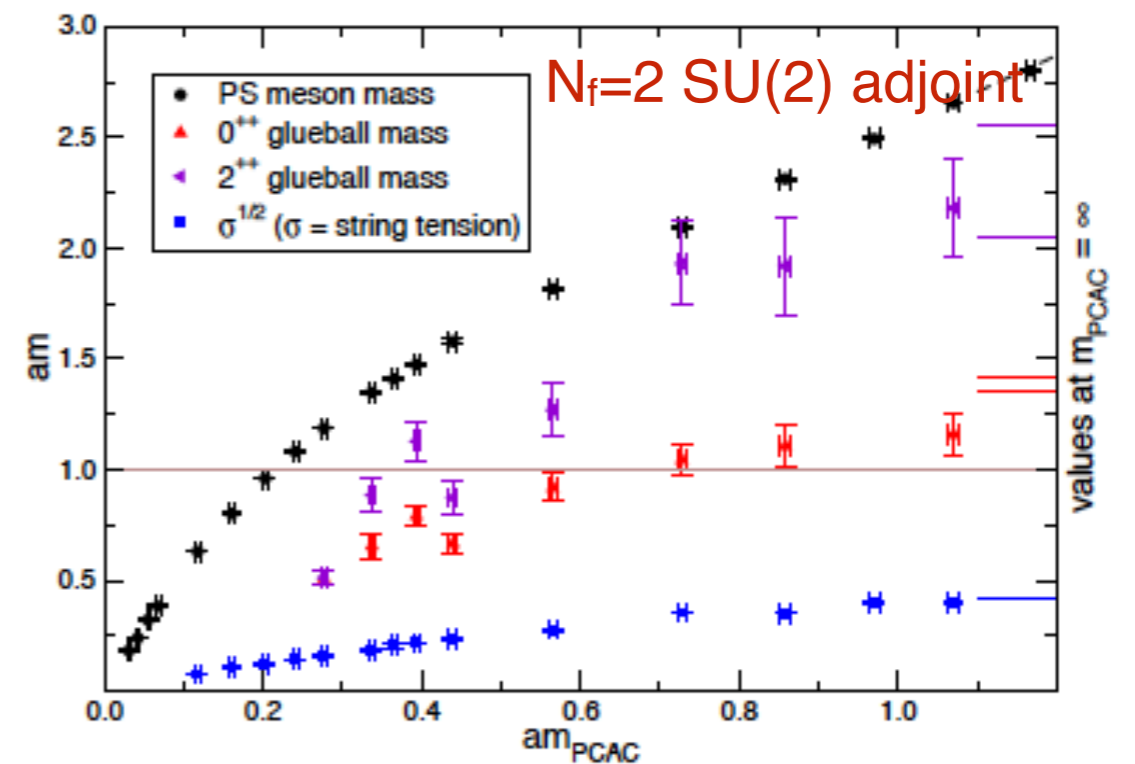
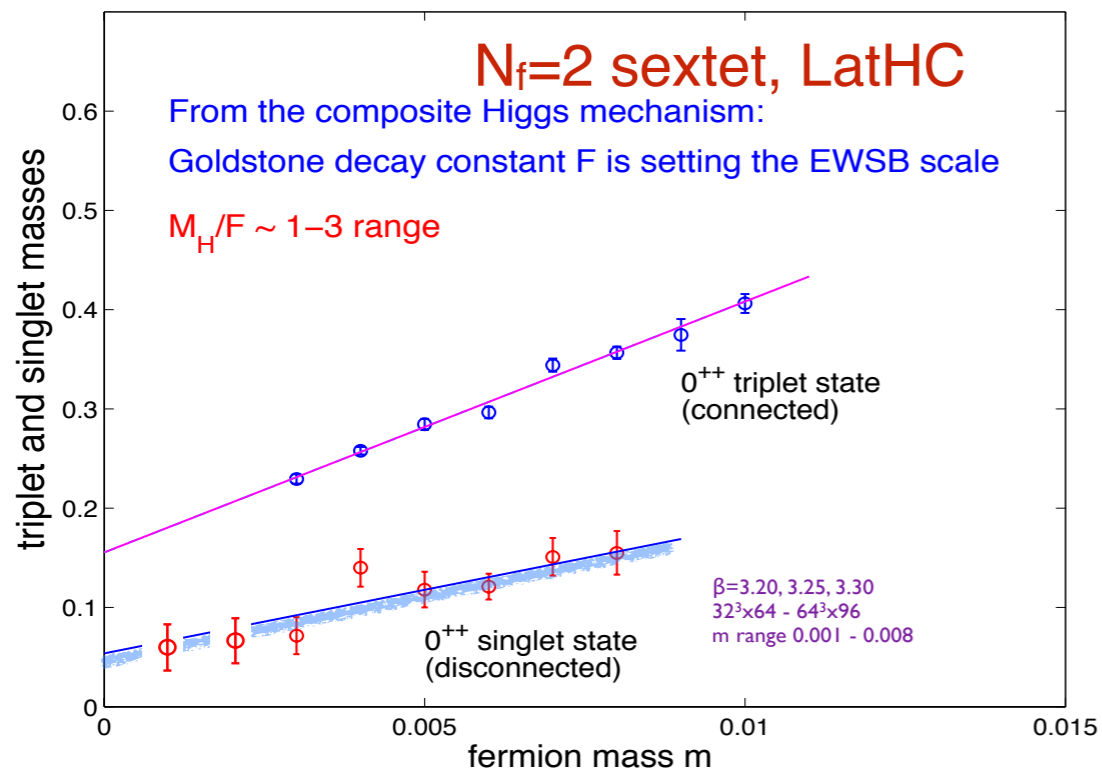
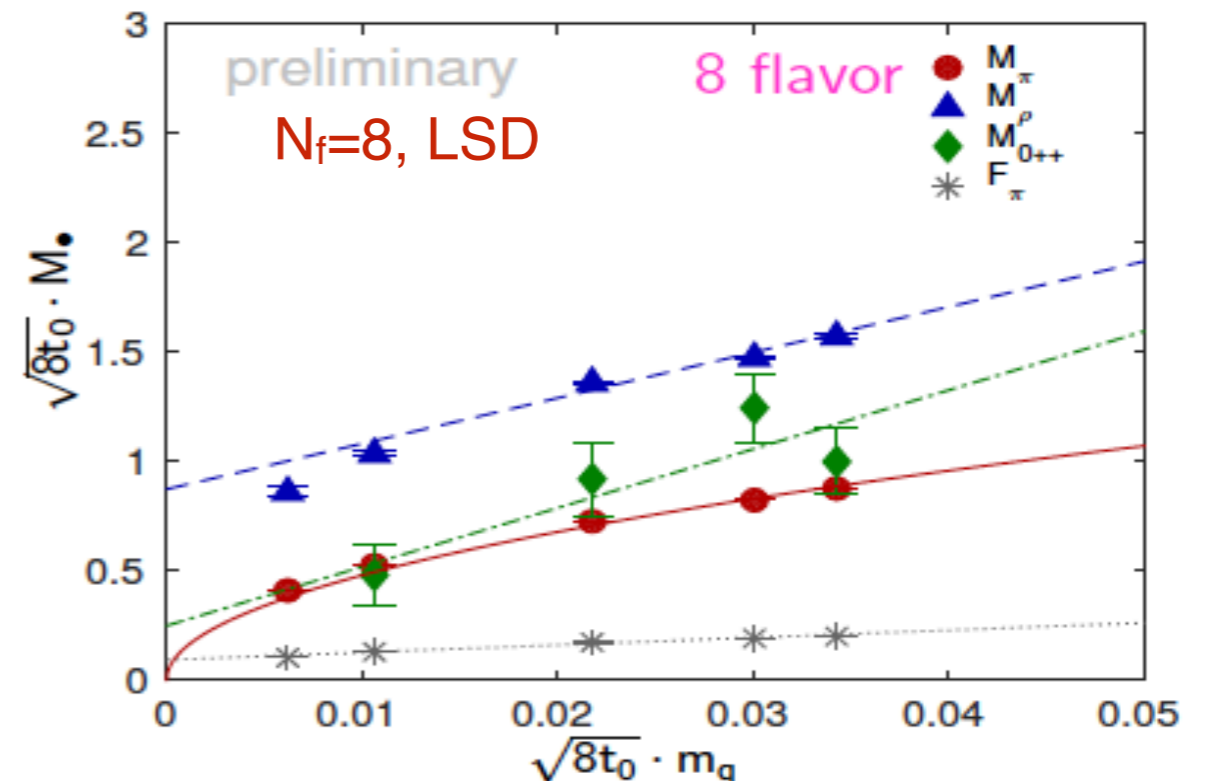


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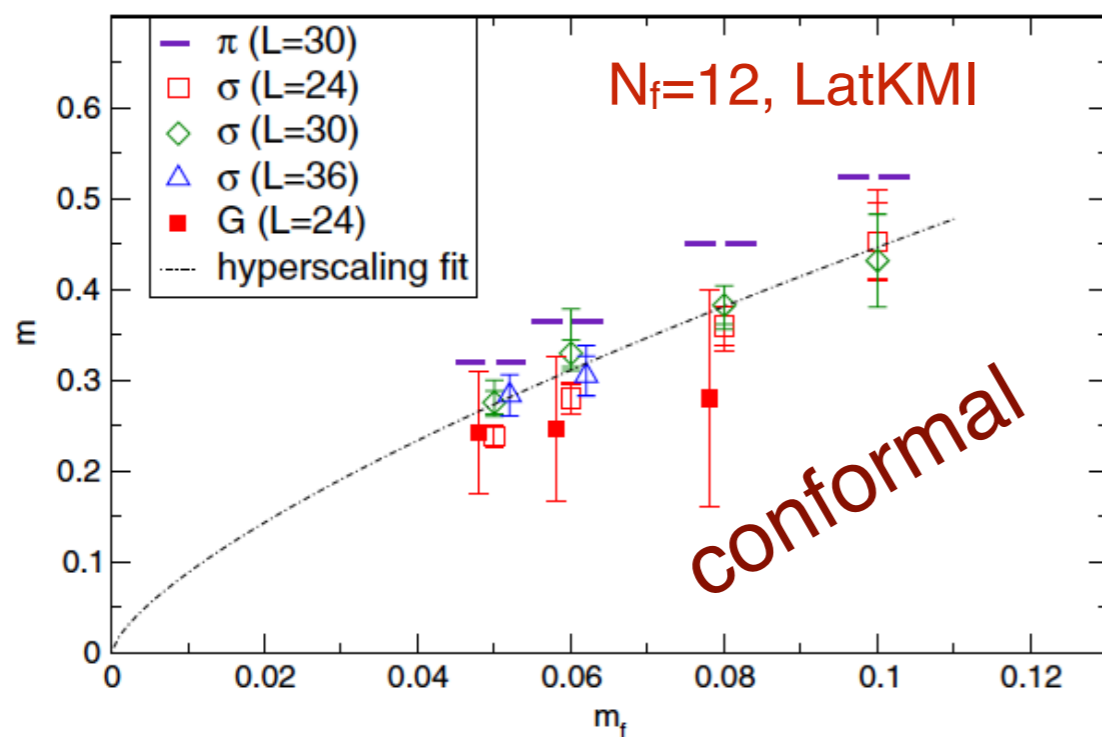


Triplet and singlet masses from 0^{++} correlators

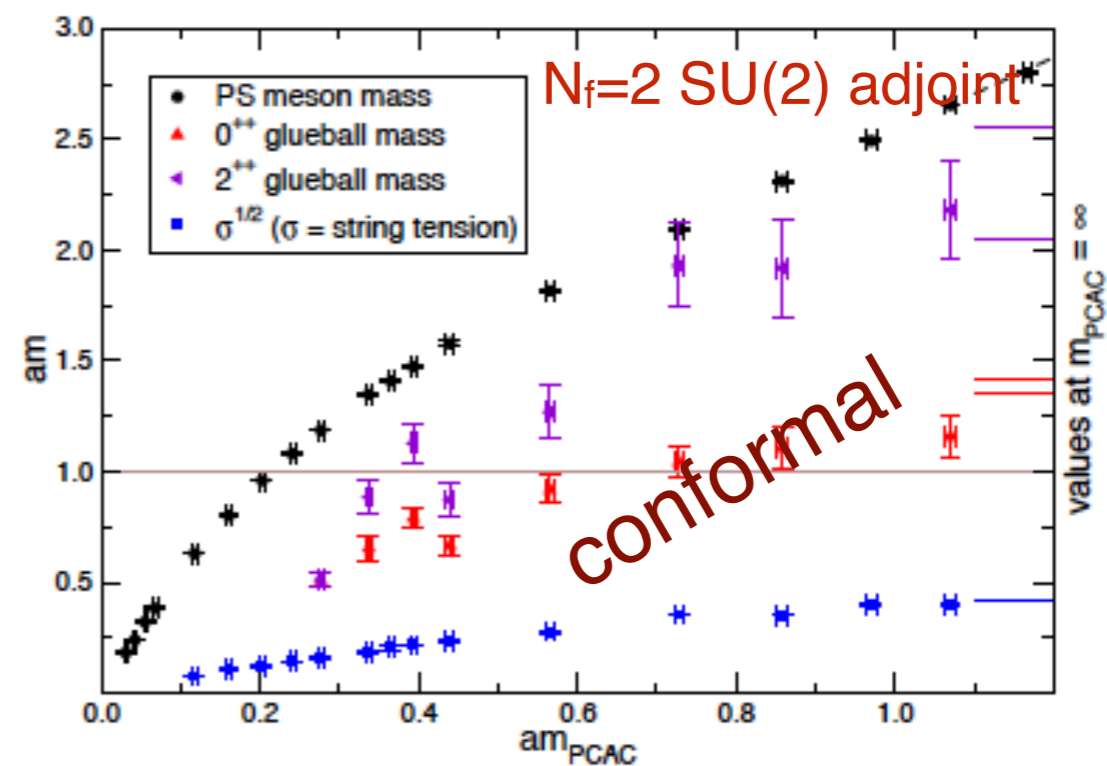
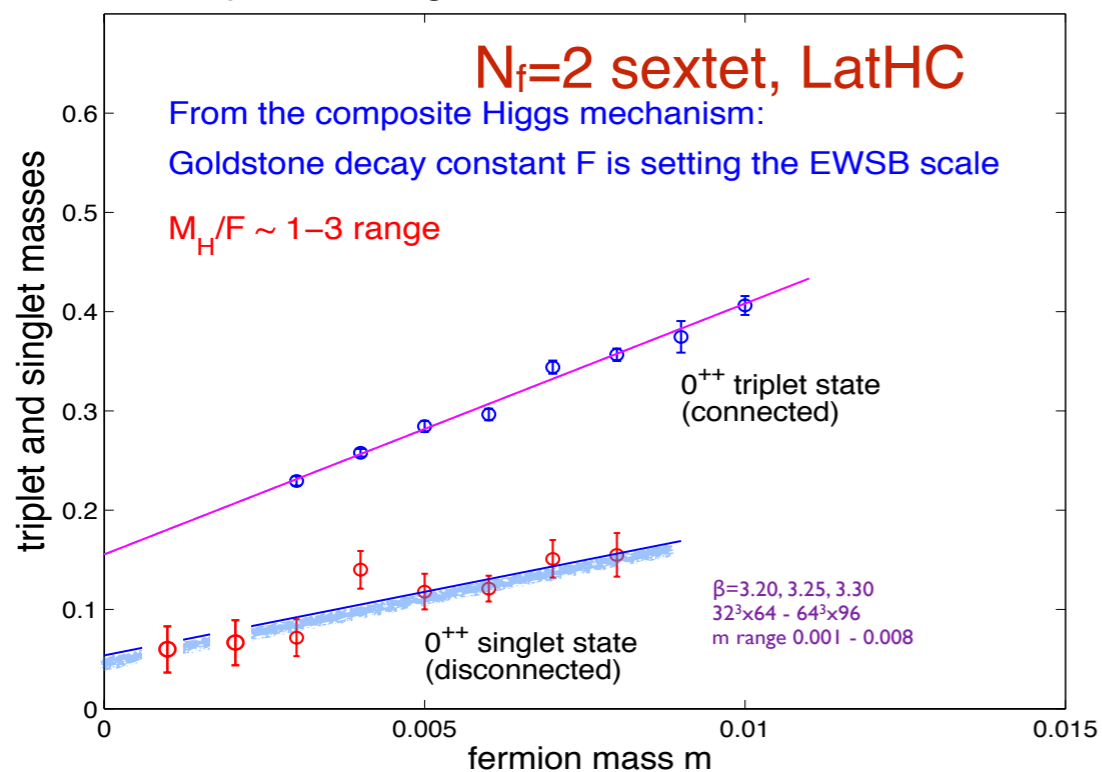
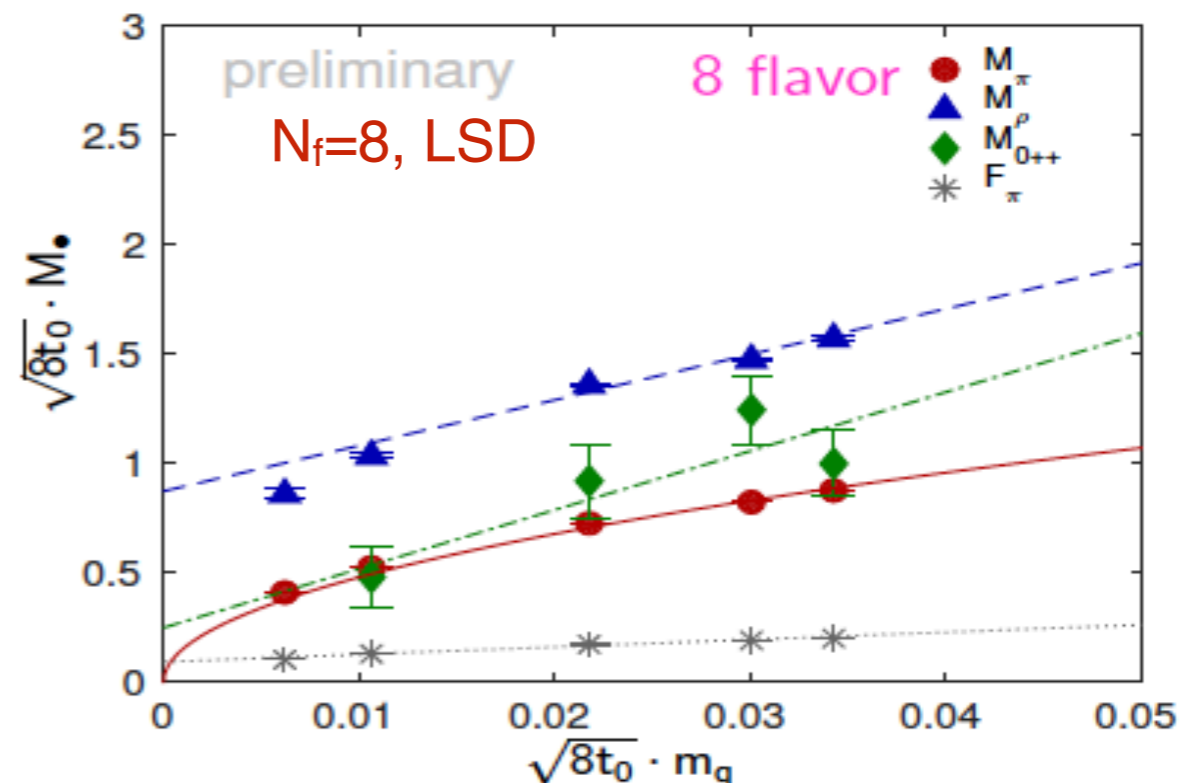


Light dilaton on the lattice

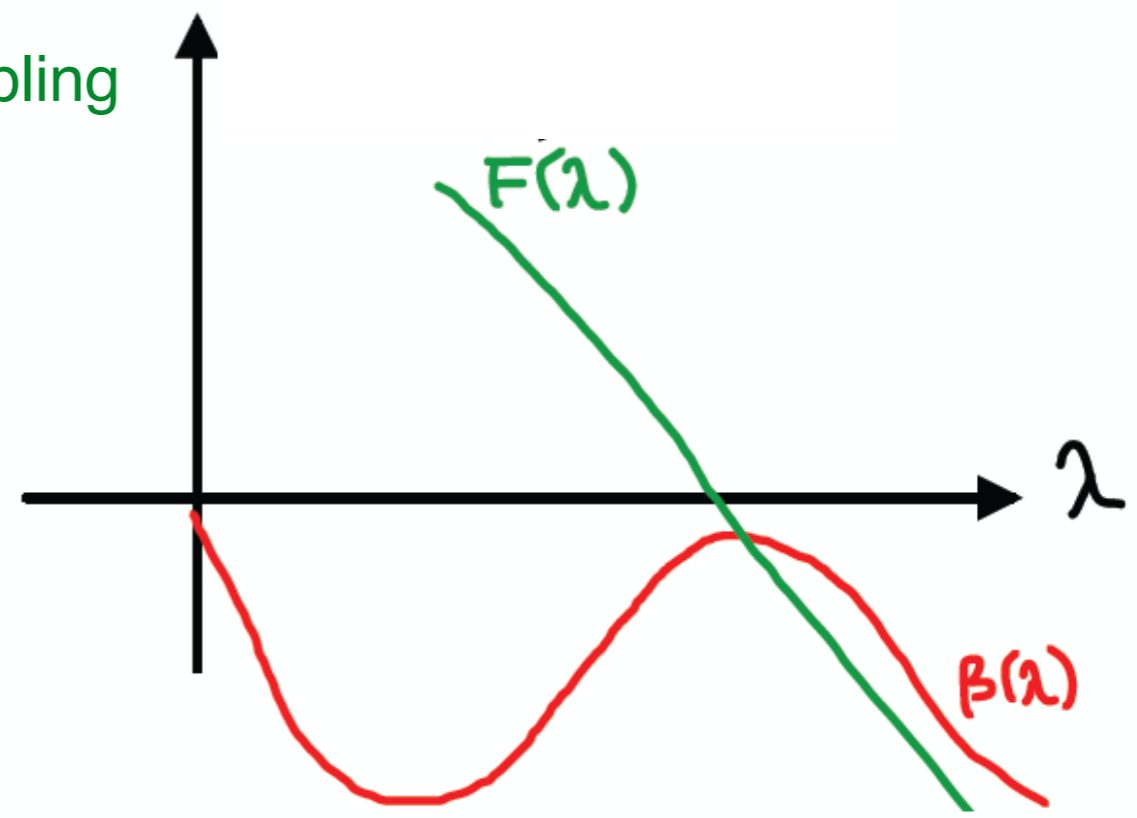
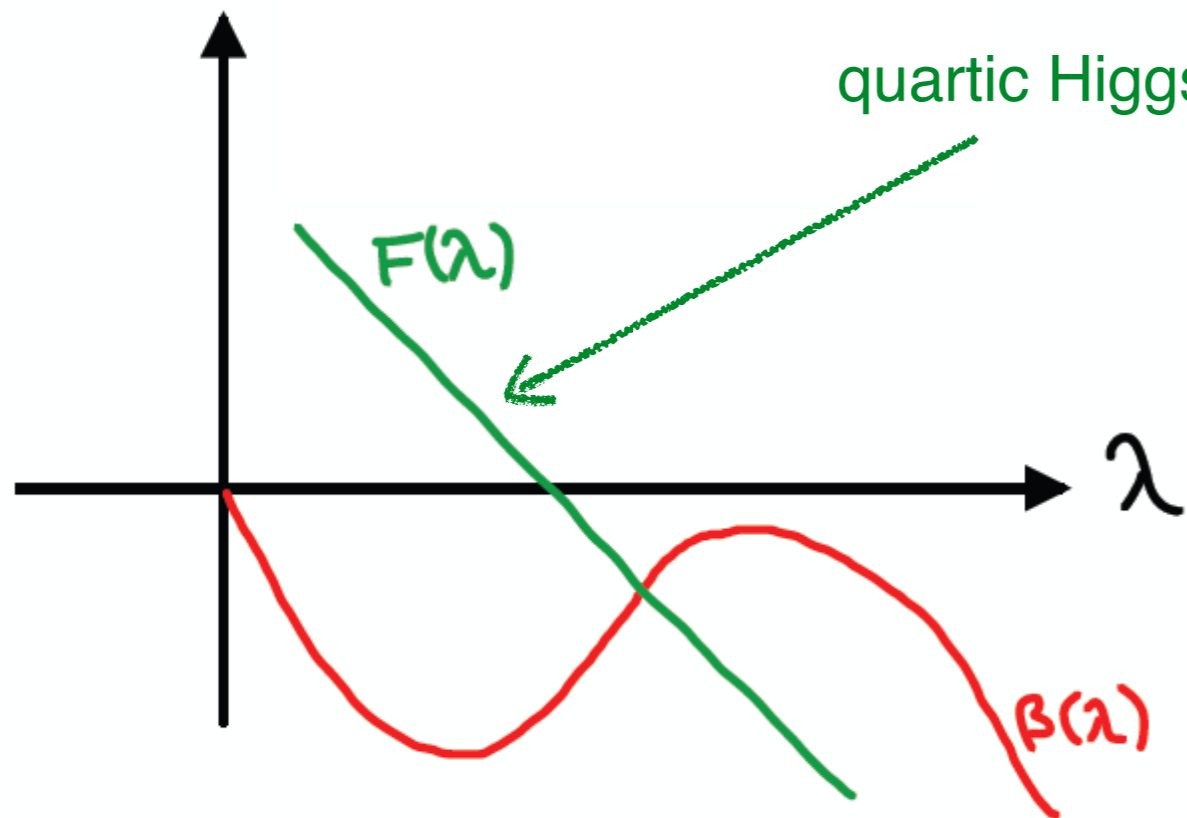
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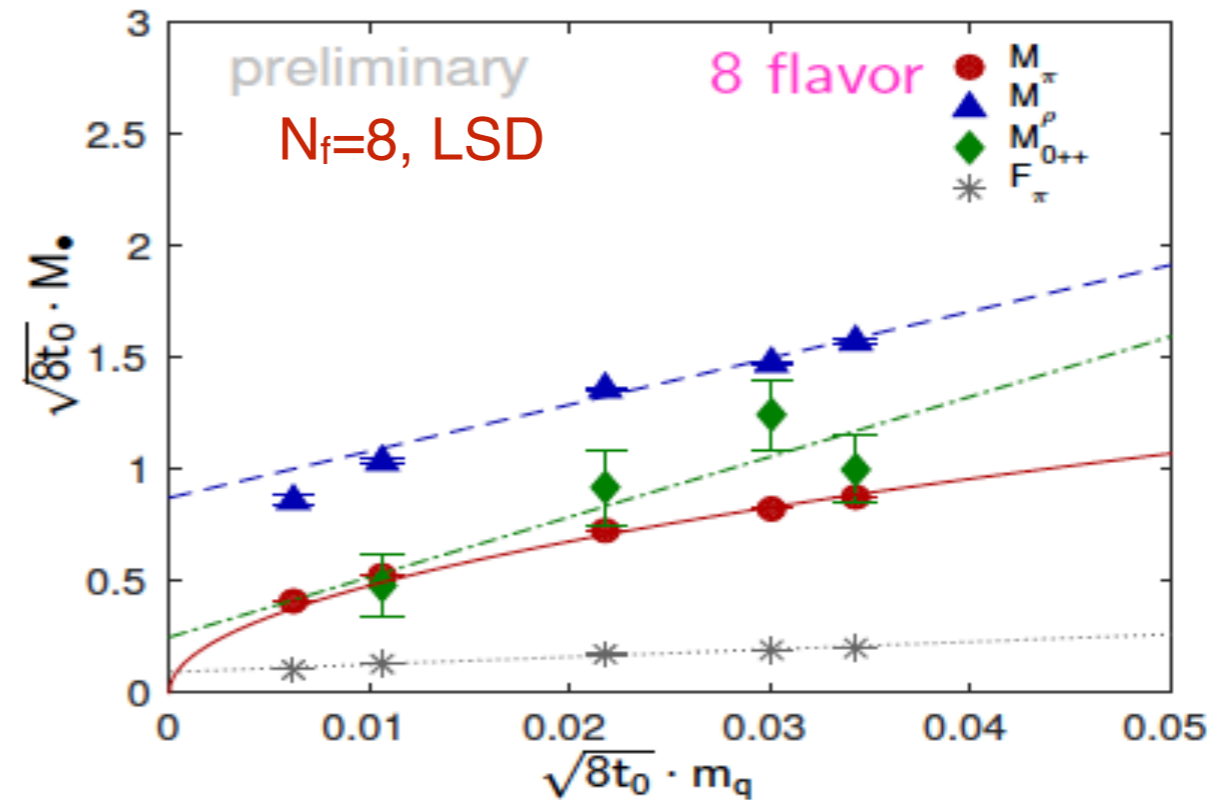
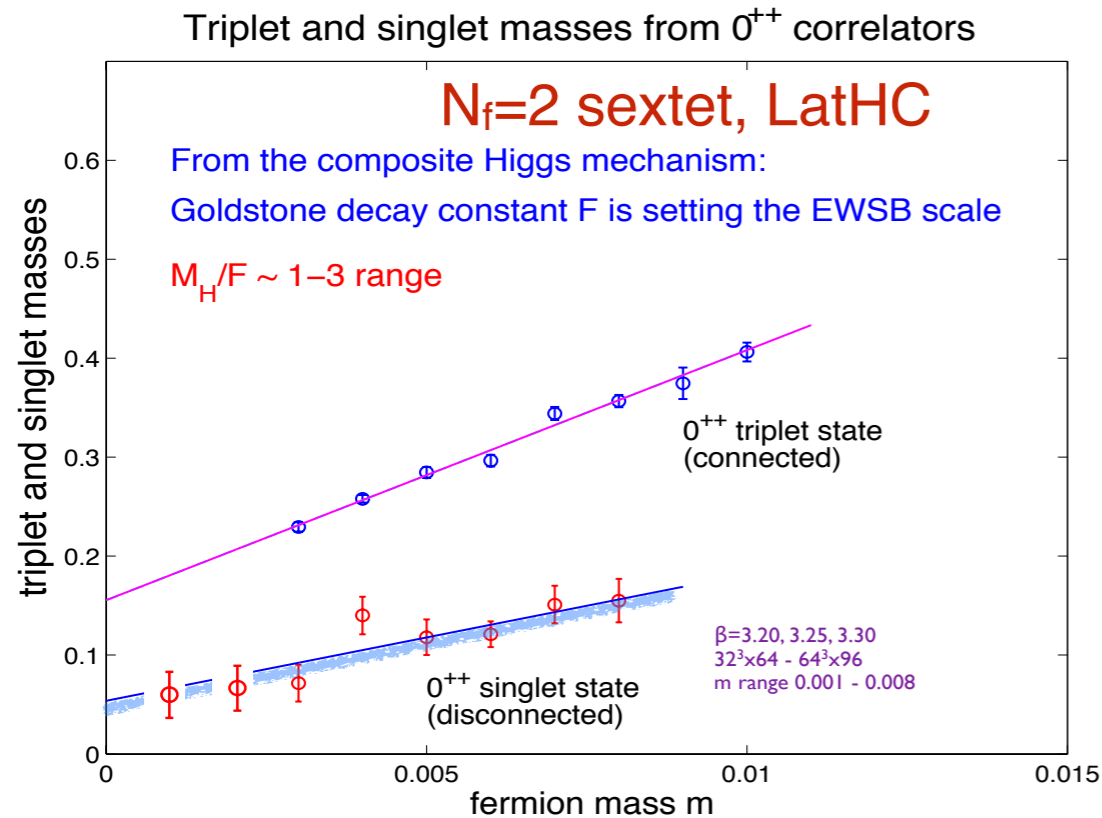


Light dilaton?



maybe we are lucky

Challenge #2 : Chiral extrapolation



Simulations are in a regime where the 0^{++} state is degenerate/lighter than the pion

Chiral extrapolations need to take this into account:

- some models, no solid approach yet

talk by J. Kuti

Challenge #3: What is the (right?) model

Guiding principles

- chirally broken*
- walking* with large anomalous dimension
- light 0^{++}
-

* or not??

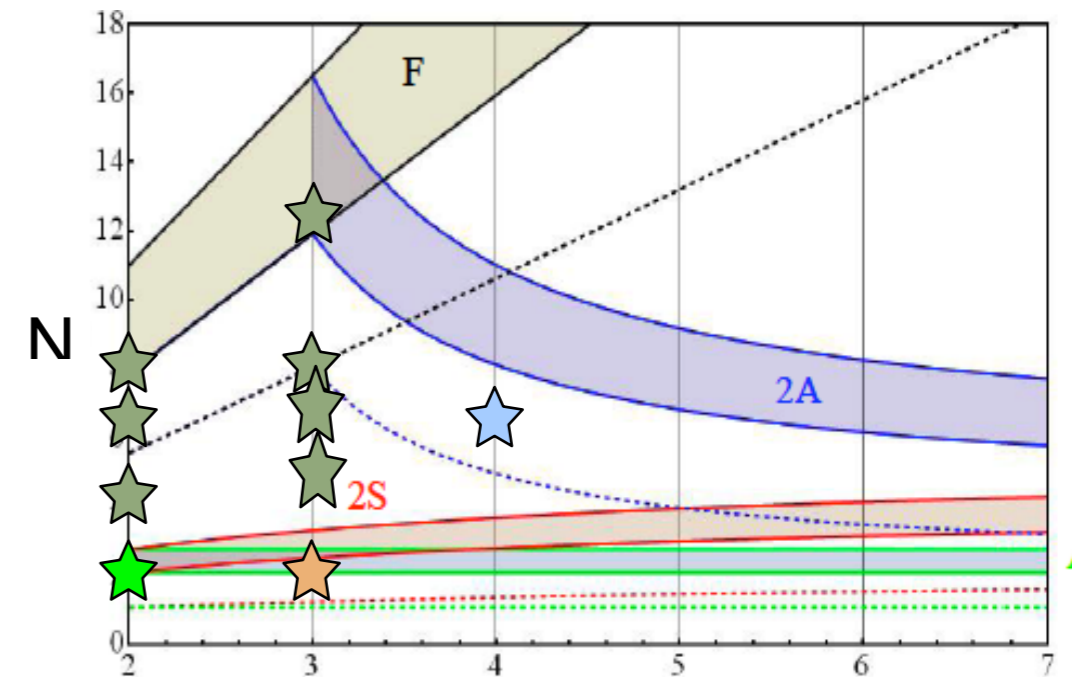
Challenge #3: What is the (right?) model

Guiding principles

- chirally broken*
- walking* with large anomalous dimension
- light 0^{++}
-

Two approaches :

- Look at many models and find generic features, identify pitfalls, etc (and then do large scale studies)
- Pick a likely model and do a large-scale study



Models & methods (this conference)

	N_f	Step Scaling	Finite Size S	Finite T	Eigen modes	Runnin coupli	Spectr. conn	Spectr. scalar
SU(2)	1A				X		X	X
	2A	X			X		X	X
	2F						X	
	4F						X	
	6F	X					X	
	8F	X					X	
SU(3)	4F	X		X	X		X	X
	8F	X	X	X	X	X	X	X
	4+8				X	X	X	X
	12F	X	X	X	X		X	X
	2S	X	X		X	X	X	X

Large scale (+ exploratory) studies

	N_f	Step Scaling	Finite Size S	Finite T	Eigen modes	Runnin coupli	Spectr. conn	Spectr. scalar
SU(2)	1A				X		X	X
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	2F						X	
	4F						X	
	6F	X					X	
	8F	X					X	
SU(3)	4F	X		X	X		X	X
	8F	X	X	X	X	X	X	X
	4+8				X	X	X	X
	12F	X	X	X	X		X	X
	2S	X	X		X	X	X	X

SU(3) 8F : LatKMI, LSD, LatHC;

talks by Ohki, Bennett,Rebbi, Weinberg, Nakayama

Large scale (+ exploratory) studies

	N_f	Step Scaling	Finite Size S	Finite T	Eigen modes	Runnin coupli	Spectr. conn	Spectr. scalar
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	8F	X	X	X	X	X	X	X
	4+8				X	X	X	X
	12F	X	X	X	X		X	X
	2S	X	X		X	X	X	X

SU(3) 2S: LatHC (+ CP^3 , Boulder)

talks by Kuti, Wong, Santanu, Nogradi, Hansen

posters by Holland,AH

Exploratory studies

	N_f	Step Scaling	Finite Size S	Finite T	Eigen modes	Runnin coupli	Spectr. conn	Spectr. scalar	talks
SU(2)	1A				X		X	X	Bergner
	2A	X			X		X	X	Bergner
	2F						X		Tahtinen
	4F						X		Tahtinen
	6F	X					X		Tahtinen
	8F	X					X		Leino
SU(3)	4F	X		X	X		X	X	Masafumi
	8F	X	X	X	X	X	X	X	
	4+8				X	X	X	X	Rebbi, Weinberg
	12F	X	X	X	X		X	X	D. Lin
	2S	X	X		X	X	X	X	

In this talk

	N_f	Step Scaling	Finite Size S	Finite T	Eigen modes	Runnin coupli	Spectr. conn	Spectr. scalar
SU(2)	1A				X		X	X
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	8F	X	X	X	X	X	X	X
	4+8				X	X	X	X
	12F	X	X	X	X		X	X
	2S	X	X		X	X	X	X

An example: $SU(3)$ $N_f=8$ fundamental

Perturbative prediction:

2 loop : just below conformal window

3 & 4 loop MS-bar : strongly coupled IRFP

Questions for lattice studies:

- is the model chirally broken?
- is it walking?
- what is the anomalous dimension in the walking range?
- is there a light scalar?
- π - π scattering
- etc

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Perturbative prediction:

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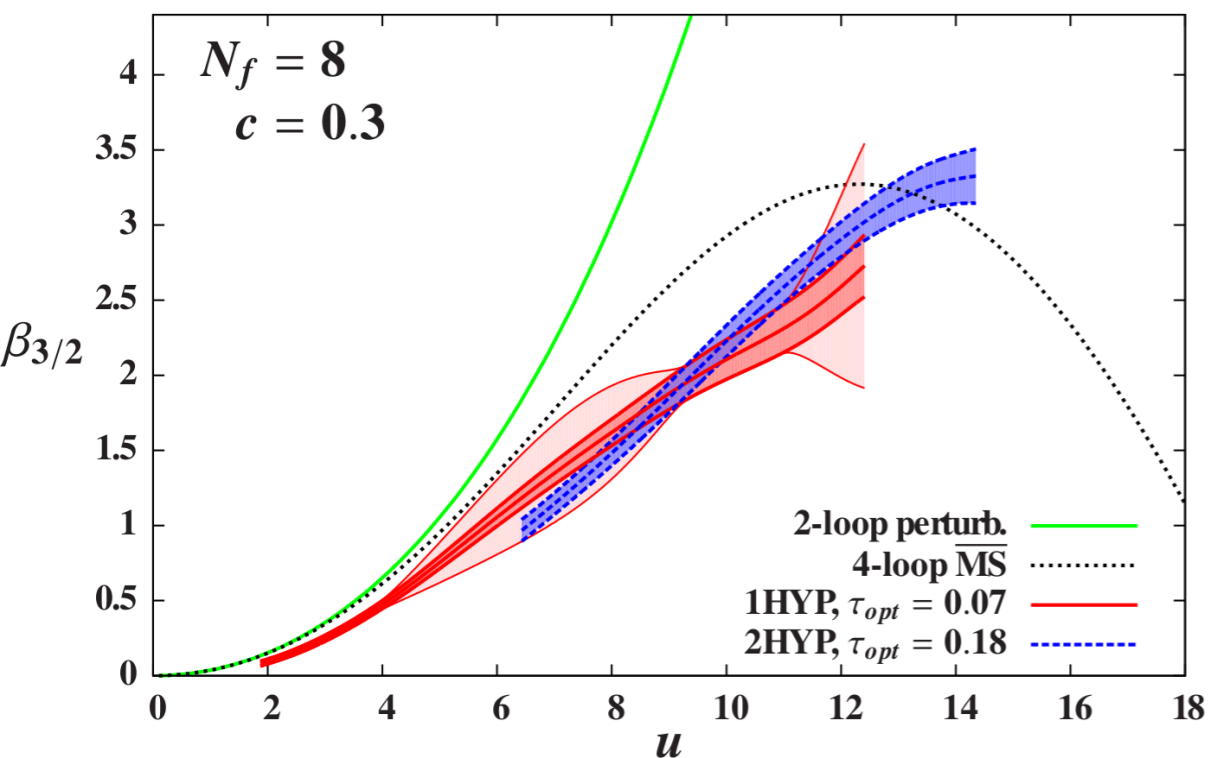
- is the model chirally broken?
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- what is the anomalous dimension in the walking range?
- is there a light scalar?
- $\pi - \pi$ scattering
- etc

Possibly the most frustrating BSM model

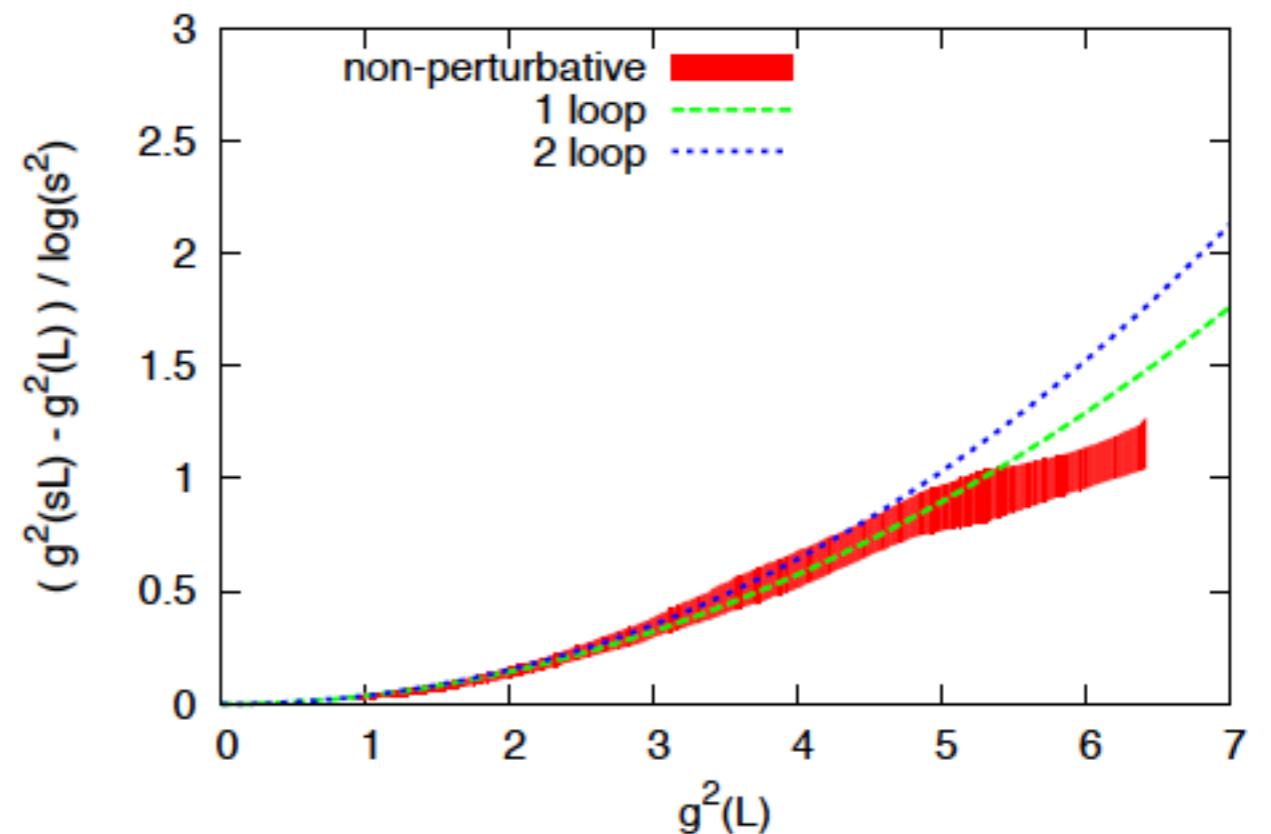
SU(3) $N_f=8$ fundamental - Step scaling

Two step scaling studies:

A.H., Schaich, Veernala, arXiv:1410:5886)



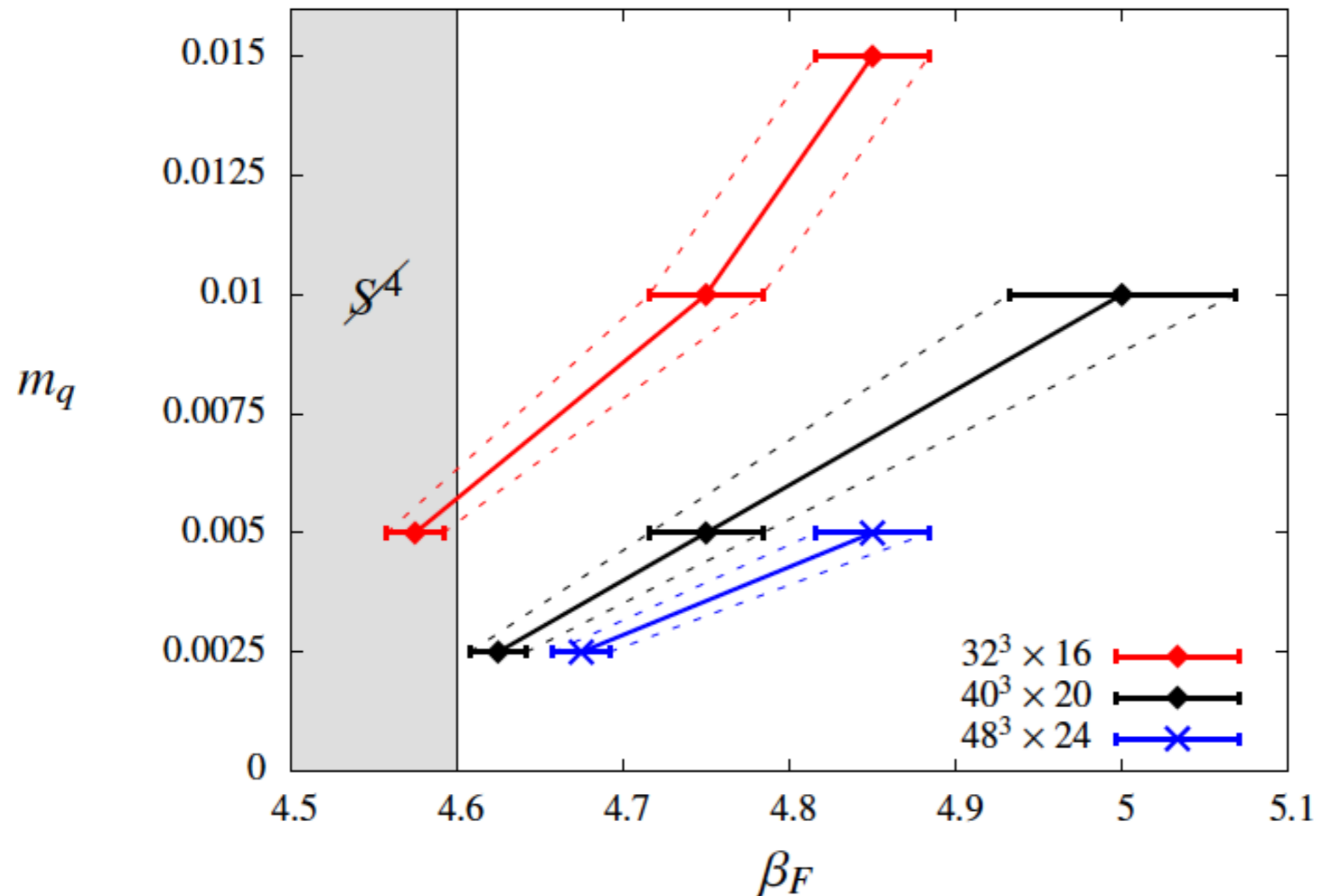
Lattice Higgs Coll. arXiv:1503.01132



- Step scaling function is monotonic up to $g^2 = 15$,
- Consistent with 4-loop $\overline{\text{MS}}$ -bar
- Range limited by bulk transition

SU(3) $N_f=8$ fundamental - finite T

No spontaneous chiral symmetry breaking in the chiral limit yet



SU(3) $N_f=8$ fundamental - finite size scaling

Finite size scaling: (in the chiral limit)

arXiv:1503.02359, talk by Nakayama

The propagator $G(t, N)$ scales as (up to $1/N$ corrections)

$$G(t; g, m_q, N, \mu) = \left(\frac{N'}{N} \right)^{3-2\gamma} G(t'; g', m'_q, N', \mu)$$

The quantity

$$\mathfrak{m}(t, N) = N \ln \frac{G(t, N)}{G(t+1, N)}. \quad \tau = t/N_t$$

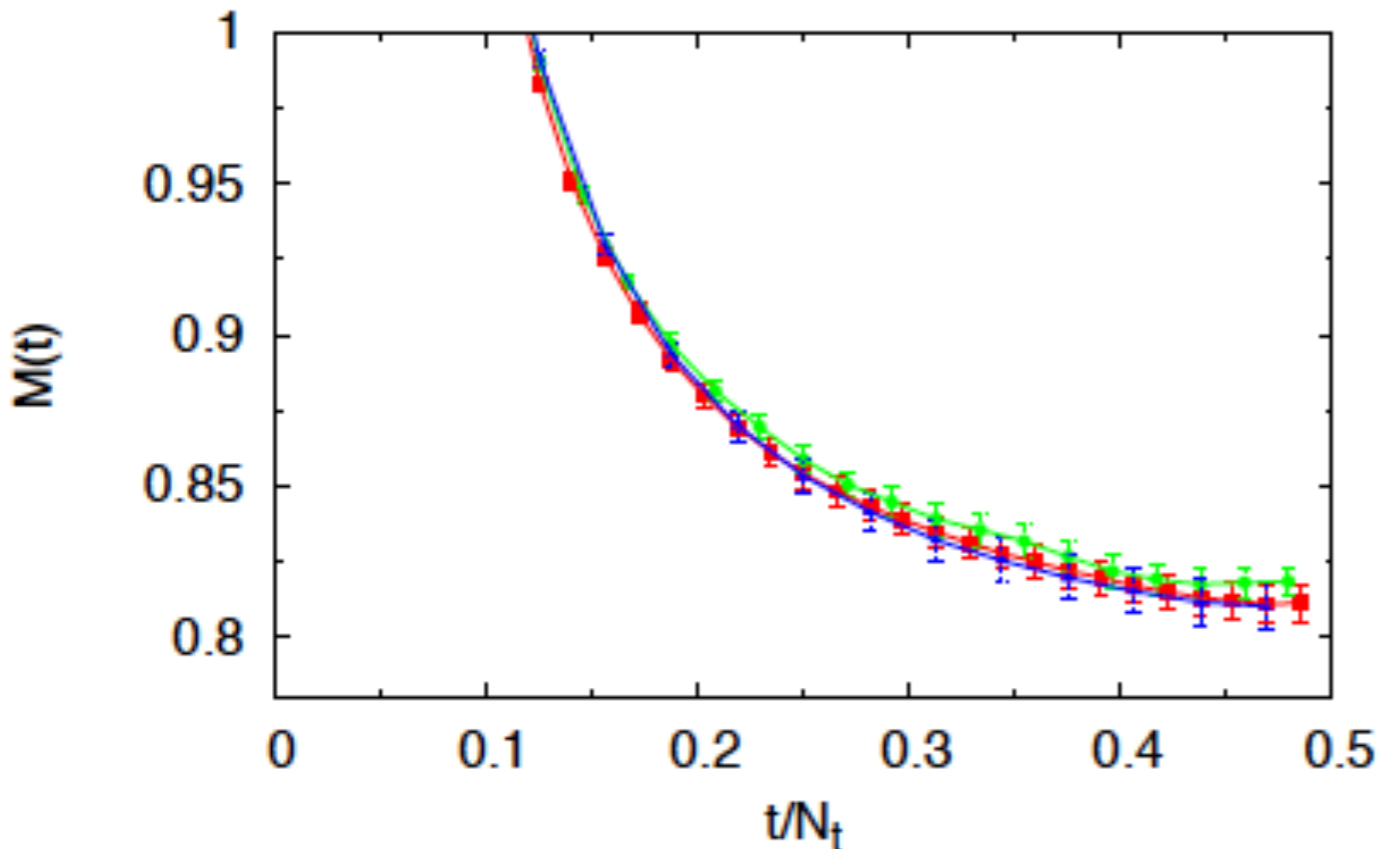
is scale invariant at the FP. Leading corrections from g^2 :

$$\begin{aligned} \mathfrak{m}(\tau, g, N) &= \mathfrak{m}(\tau, g, N') + \mathcal{B}(g) \ln \left(\frac{N}{N'} \right) \partial_g \mathfrak{m}(\tau, g, N') + \dots \\ &= \mathfrak{m}(\tau, g, N') \end{aligned}$$

→ tune β to find “curve collapse”: IRFP

SU(3) $N_f=8$ fundamental - finite size scaling

Effective mass: $N_f=08$; $\beta=2.4$, $K=0.147$



[arXiv:1503.02359](#), talk by Nakayama

Numerical tests with Wilson fermions suggest that $N_f=8$ is conformal

De Silva et al, based on different arguments and staggered fermions reach the same conclusion

[arXiv:1506.06396](#)

Caveats:

- do other operators predict the same β_{IRFP} ?
- does a volume squeezed chirally broken system look different?

An example: $SU(3)$ $N_f=8$ fundamental

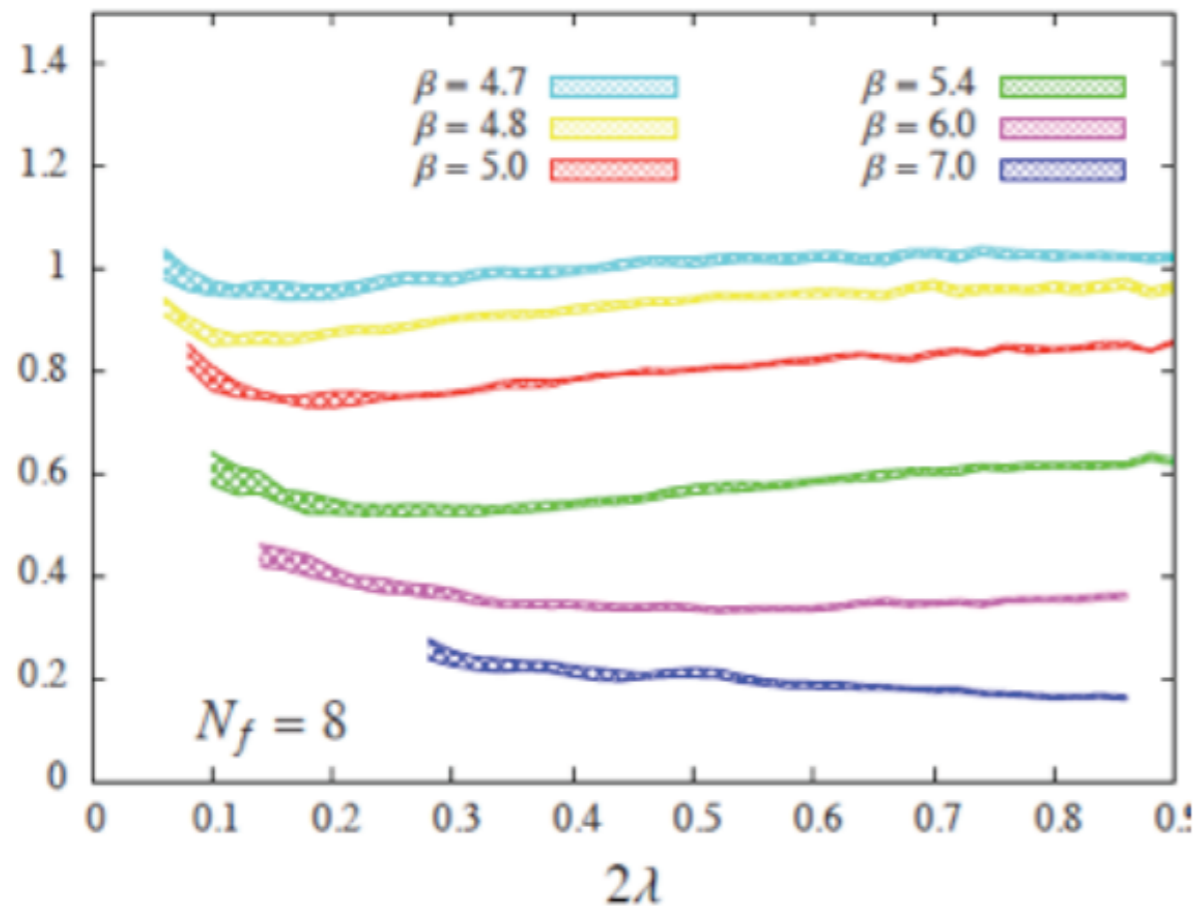
Questions for lattice studies:

- is the model chirally broken?
- is it walking?
- what is the anomalous dimension ?
- is there a light scalar?

SU(3) $N_f=8$ fundamental - mass anomalous dimension

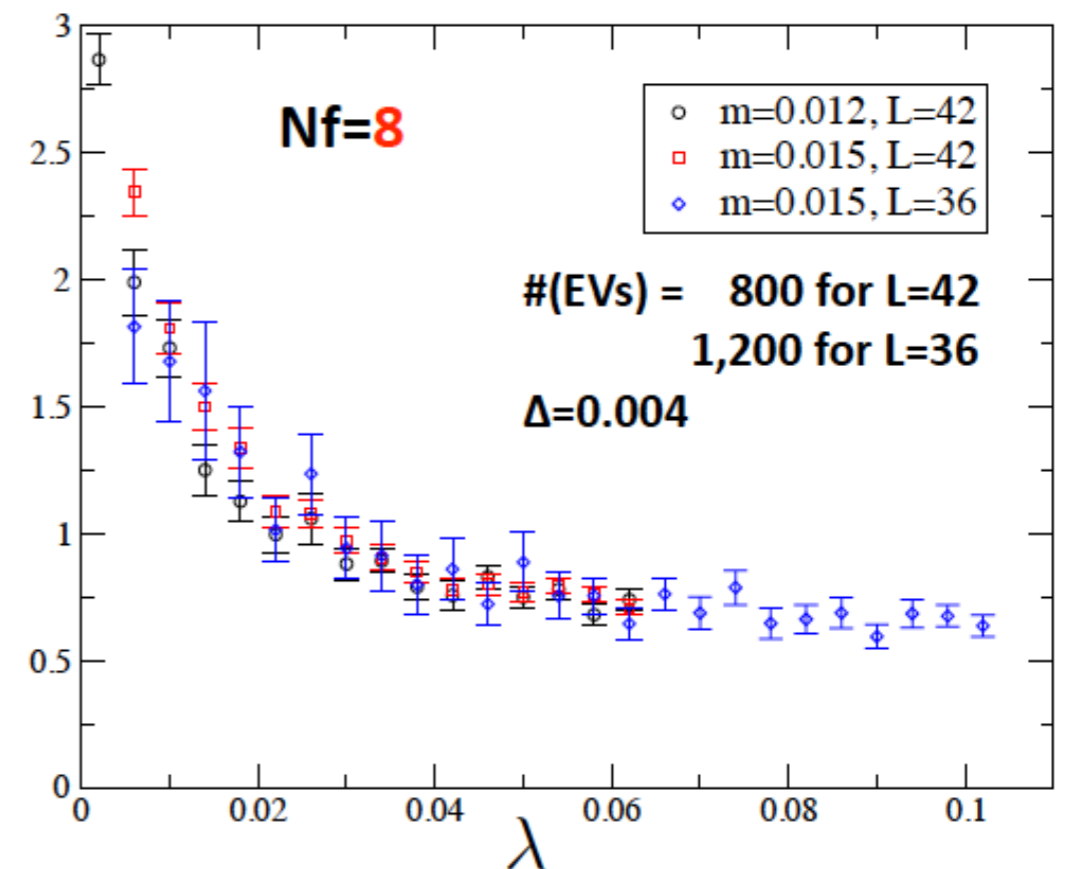
Anomalous dimension from Dirac operator mode number

Boulder



$\gamma_{\text{eff}}(\lambda)$

LatKMI



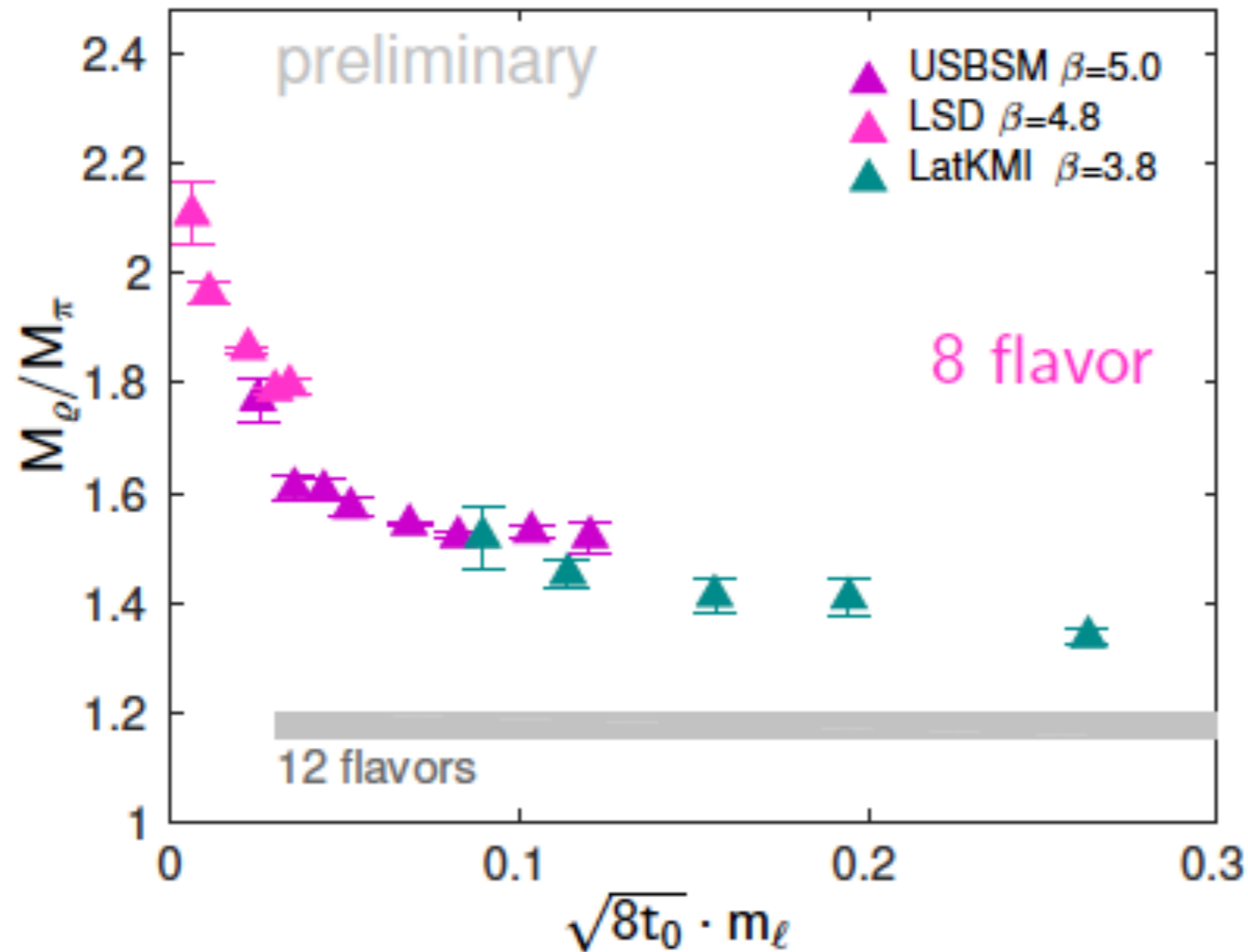
Strong β dependence
slow running, $\gamma \approx 1$ ($m=0$)

one β value, volume and
mass dependence

Anomalous dimension $\gamma \approx 1$ makes this model exciting

SU(3) $N_f=8$ fundamental - Spectrum

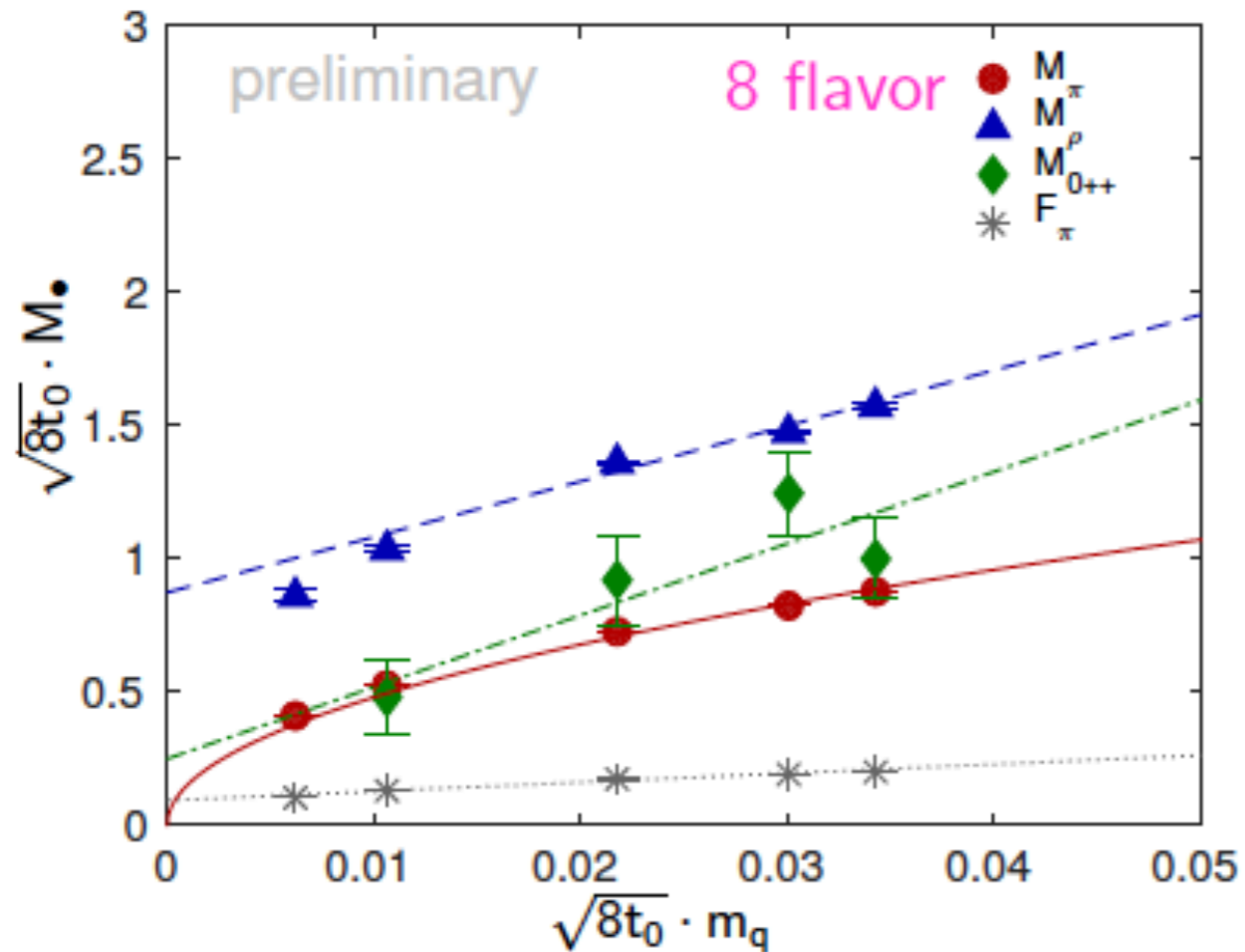
Both LatKMI and LSD collaborations have extensive spectrum data



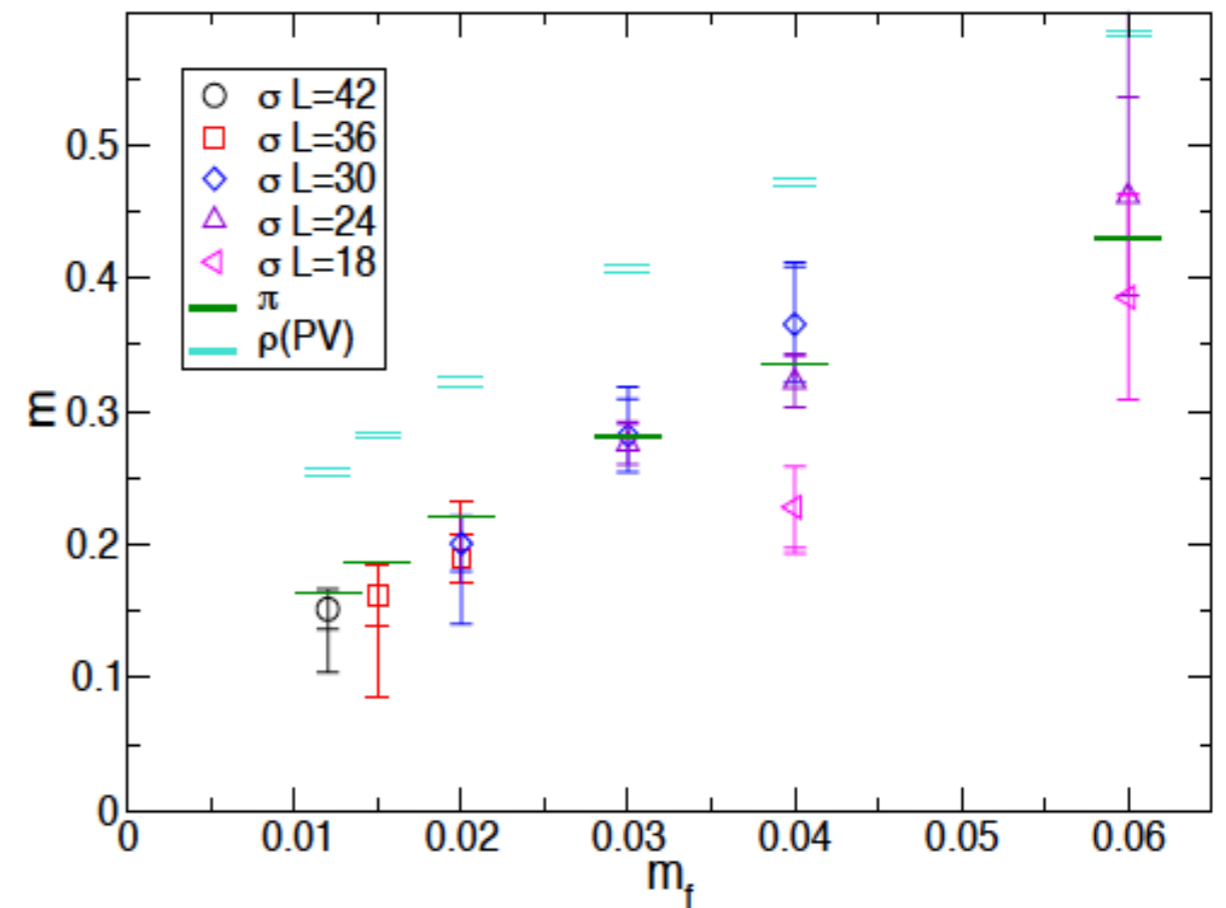
SU(3) $N_f=8$ fundamental - 0^{++} Spectrum

0^{++} is degenerate with the pion,
at lightest pion $m_{0^{++}} / m_\rho \lesssim 0.5$

LSD



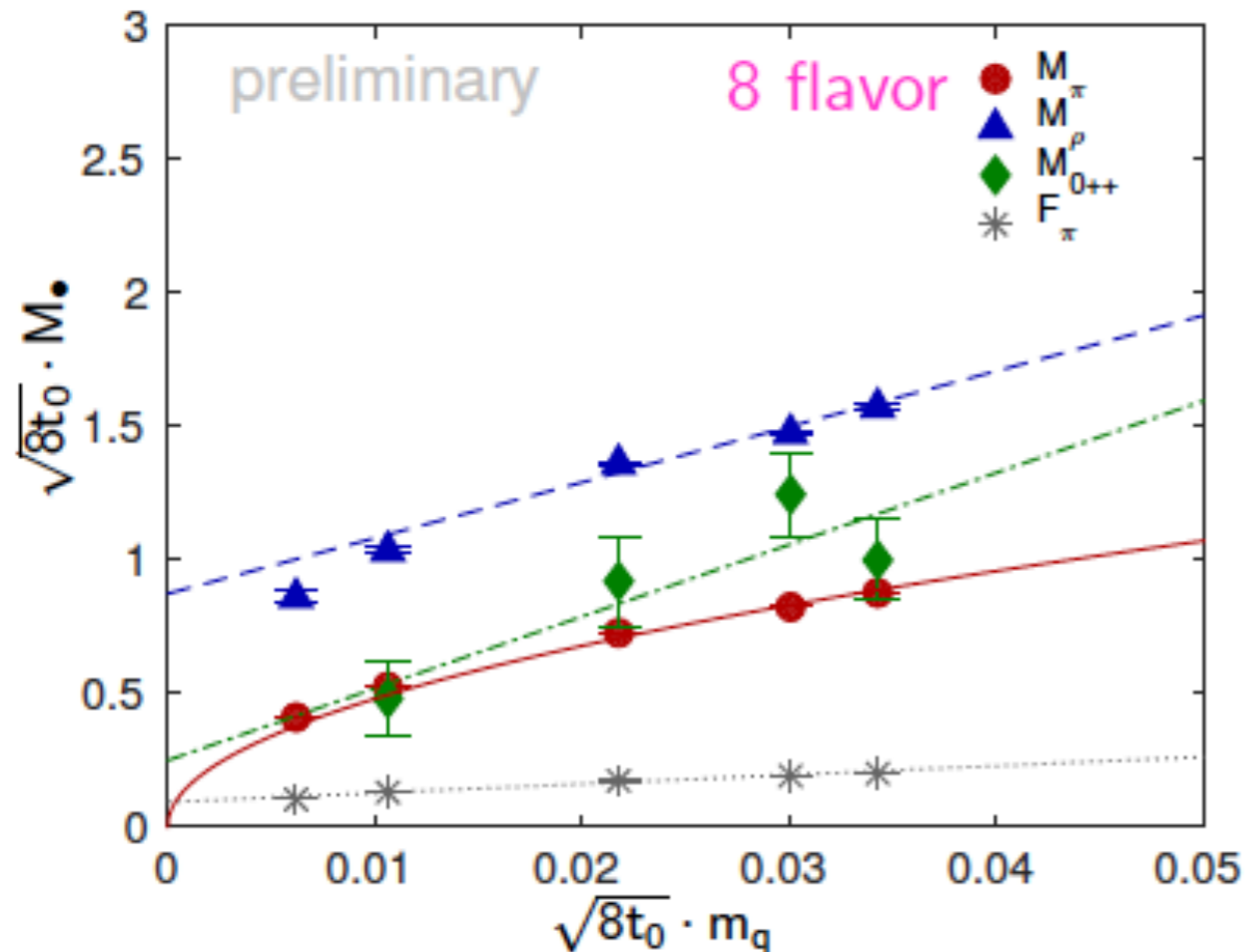
LatKMI



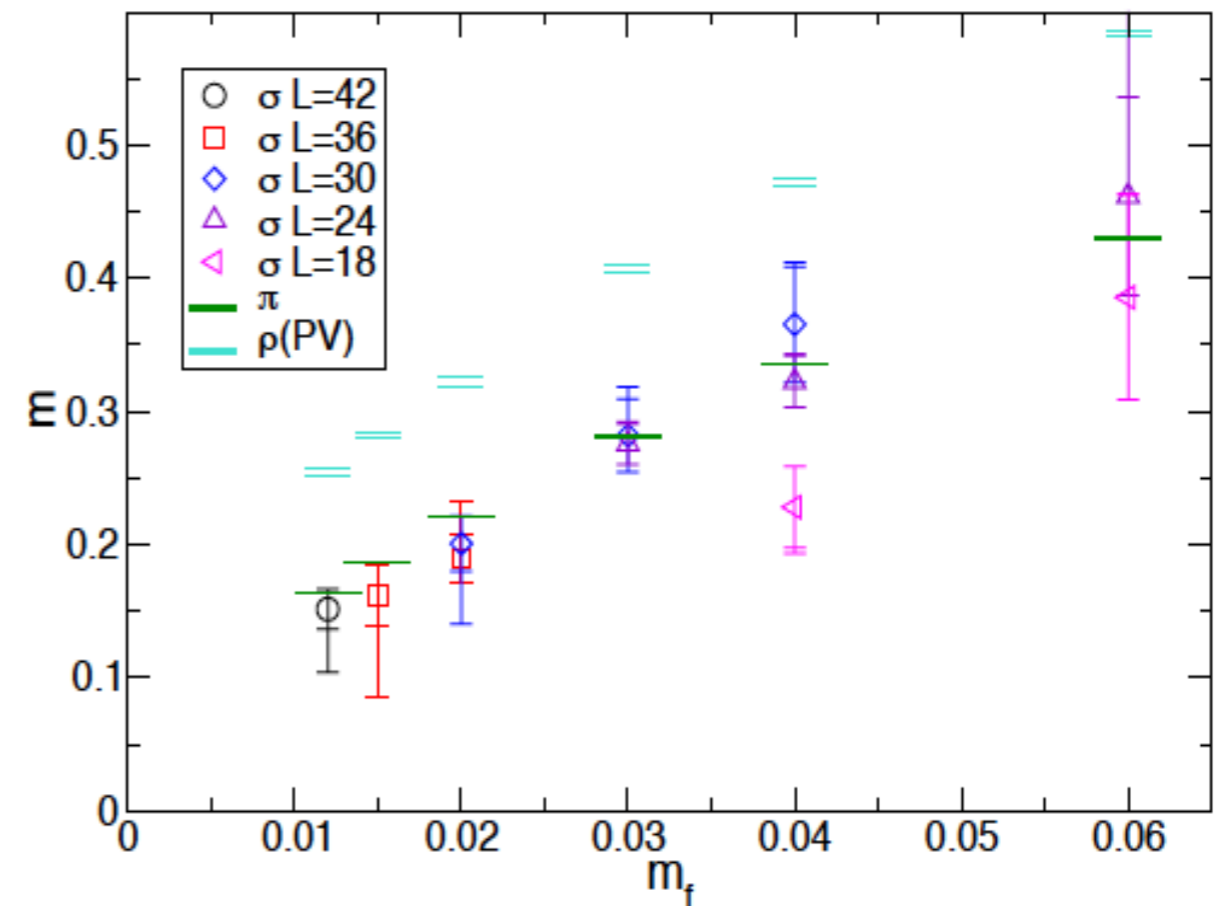
SU(3) $N_f=8$ fundamental - 0^{++} Spectrum

0^{++} is degenerate with the pion,
at lightest pion $m_{0^{++}} / m_\rho \lesssim 0.5$

LSD



LatKMI



So it is chirally broken with a light scalar, right?

Challenge # 4:

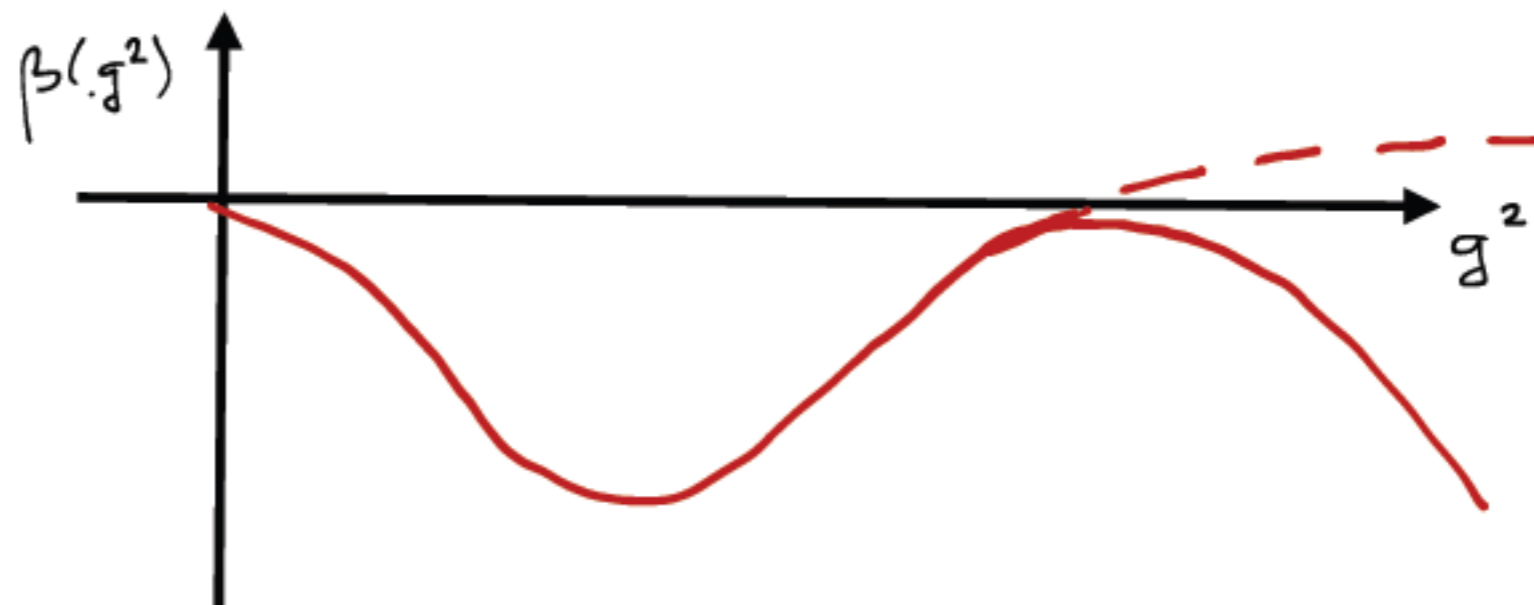
Conformal or chirally broken ?

	Step	Finite	Finite	Eigen	Runnin	Spectr.	Spectr.	Ref.
8F	?	?	?	?	?	?	?	

No definite signal of chiral symmetry breaking (or conformality)

Does it matter?

YES



Challenge # 4:

Conformal or chirally broken ?

	Step	Finite	Finite	Eigen	Runnin	Spectr.	Spectr.	Ref.
8F	?	?	?	?	?	?	?	

No definite signal of chiral symmetry breaking or conformality

Does conformality kill the model?

NO

Challenge # 4:

Conformal or chirally broken ?

	Step	Finite	Finite	Eigen	Runnin	Spectr.	Spectr.	Ref.
8F	?	?	?	?	?	?	?	

No definite signal of chiral symmetry breaking or conformality

Does conformality kill the model?

NO

We can move a conformal system into the chirally broken regime
(add a 4-fermion operator)

Even more exciting!

SU(3) with 4+8 flavors

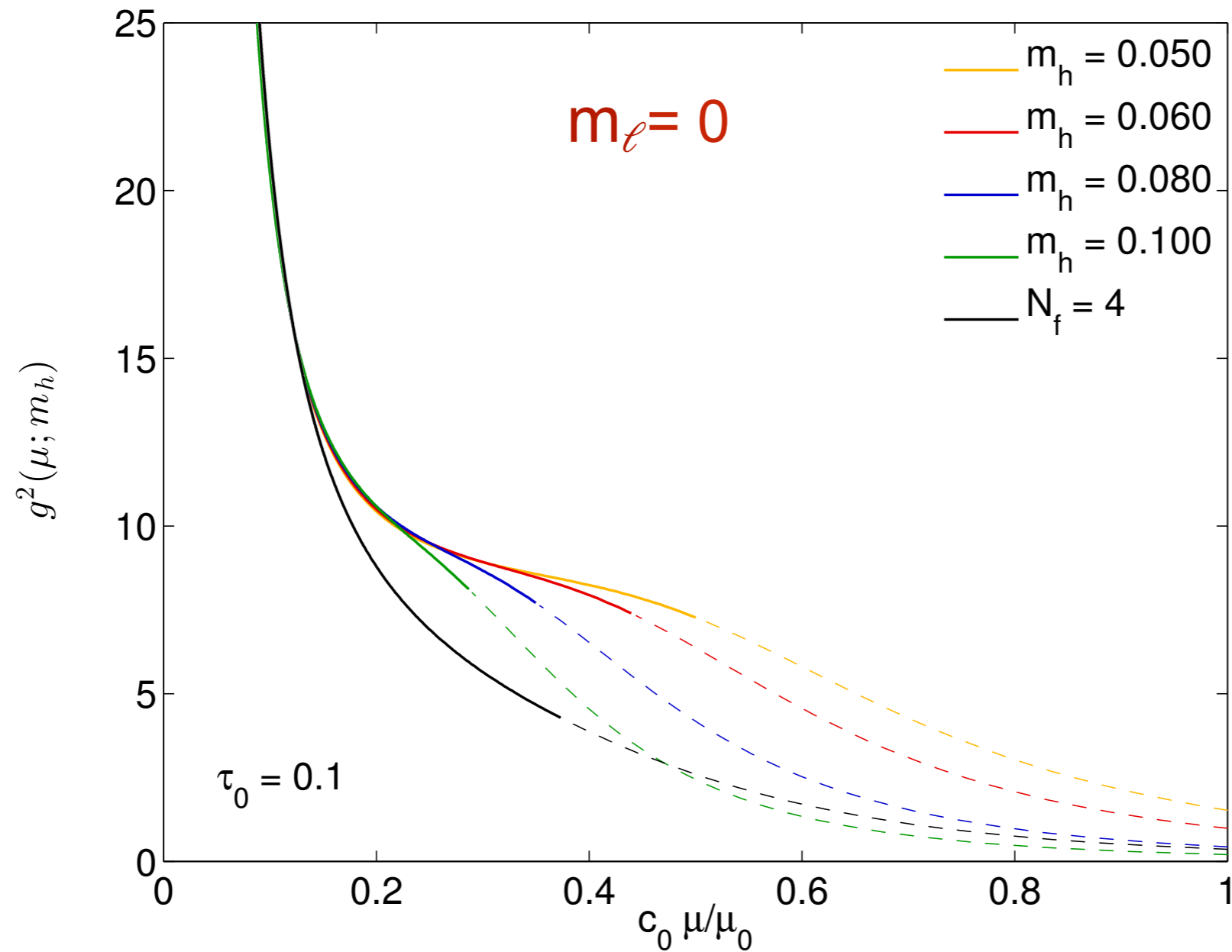
A system with 4 massless (light) and 8 heavier flavors

- IR : 4 flavor, chirally broken
- UV : 12 flavor conformal

Tune the mass of the 8 “heavy” flavors to interpolate between conformal and chirally broken dynamics

Boston-Boulder coll.
talks by Rebbi, Weinberg

4+8 flavors : “walking” running coupling



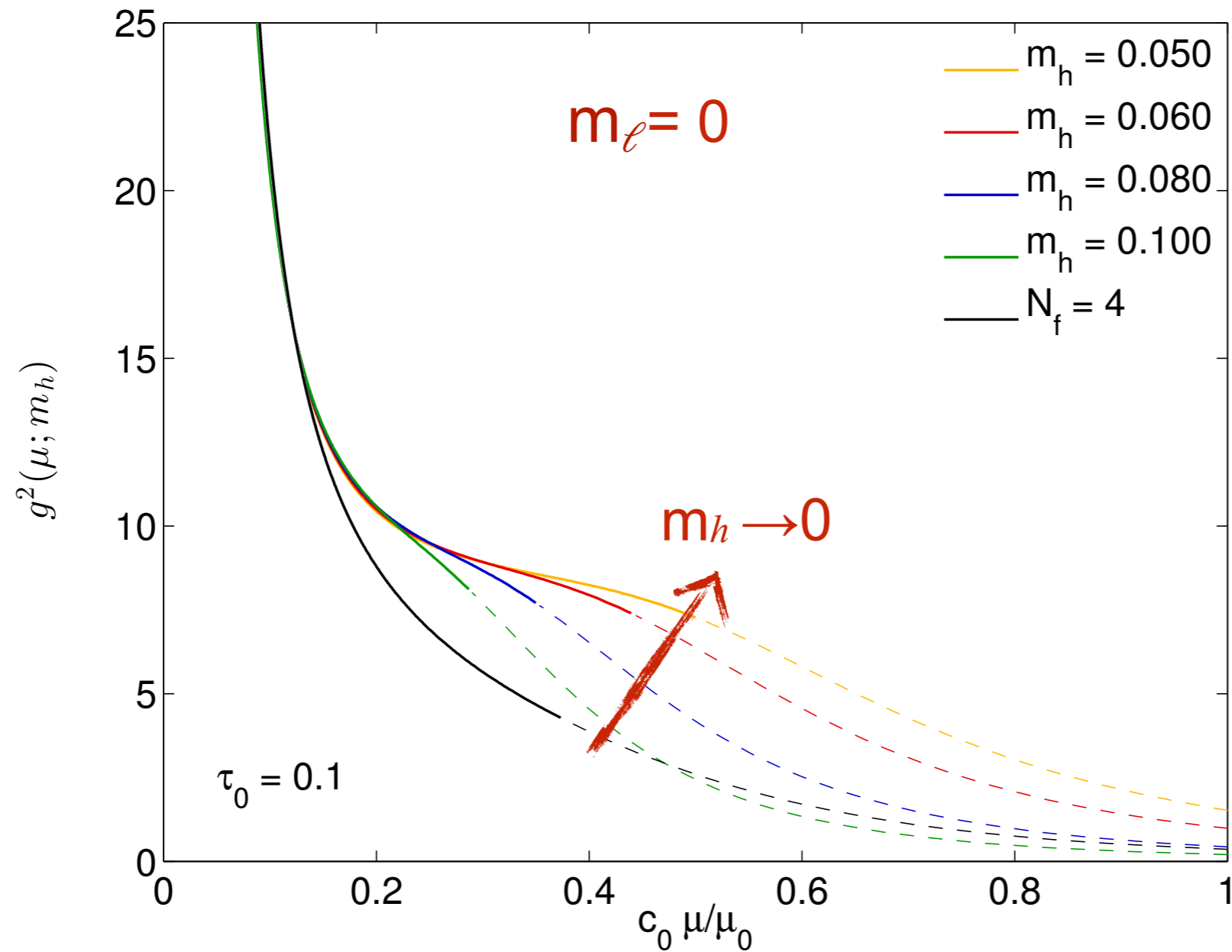
There are error bars on this plot!

$N_f=4$: running fast

$g_{GF}^2(\mu)$ develops a “shoulder” as $m_h \rightarrow 0$: this is walking !

Walking range can be tuned arbitrarily with m_h

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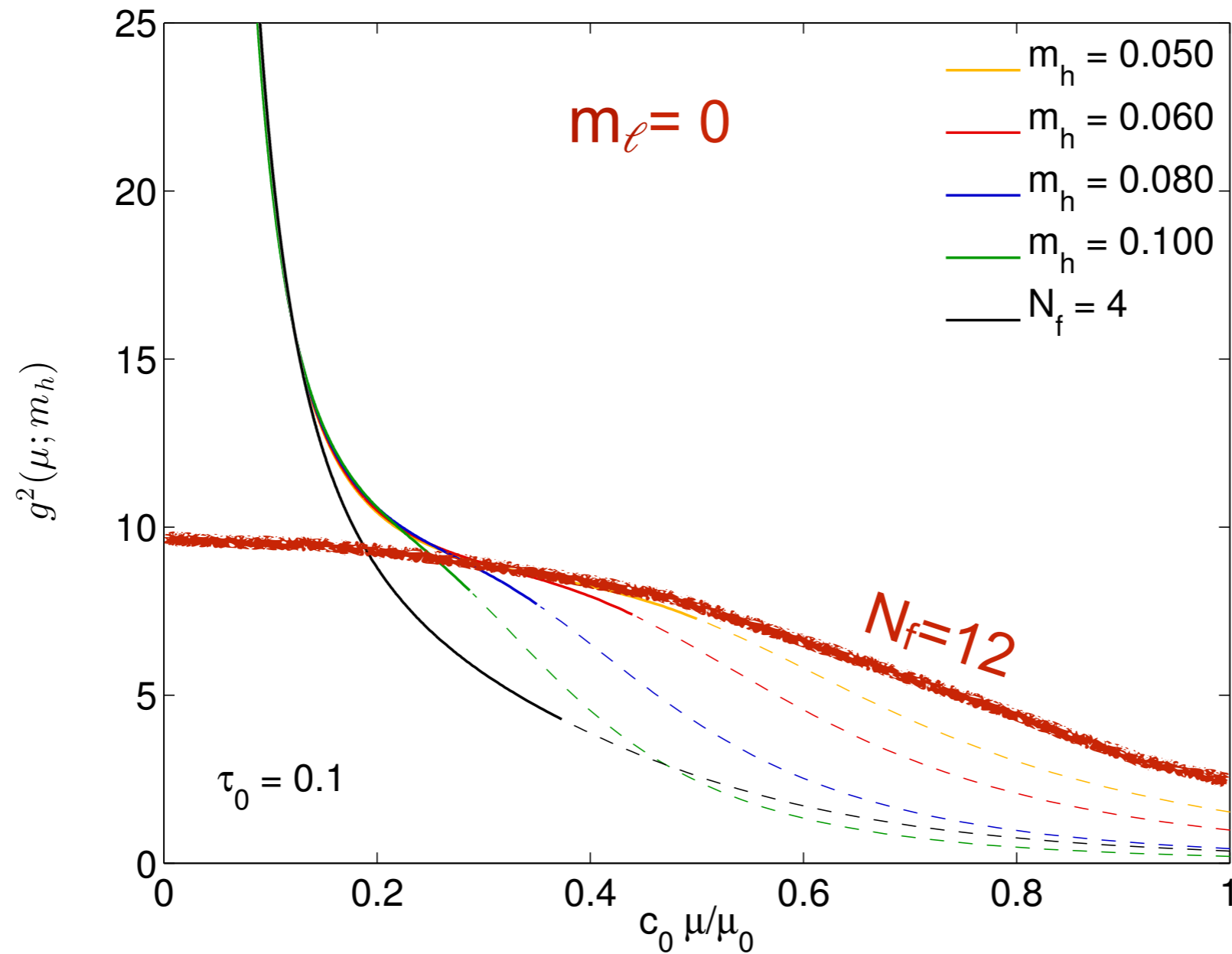
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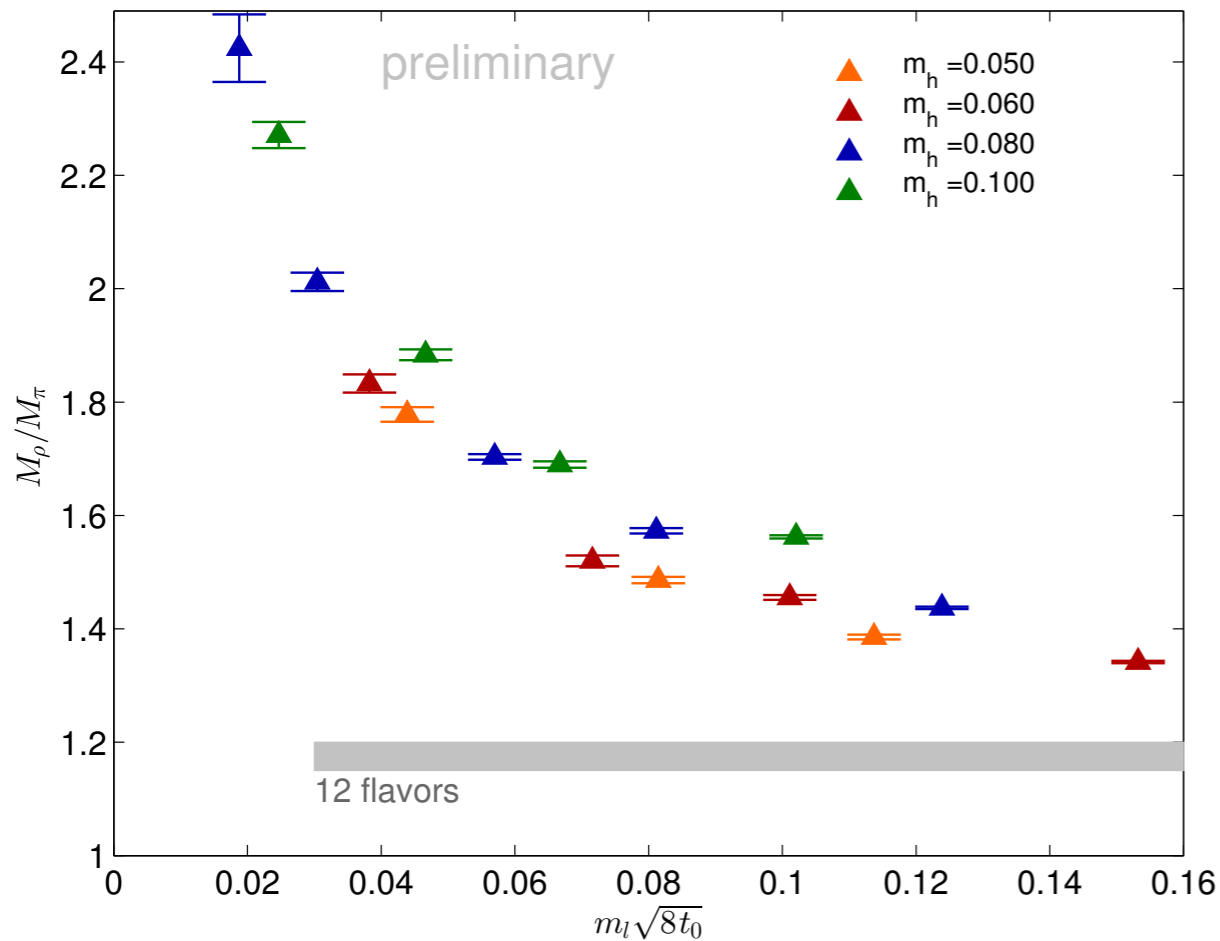
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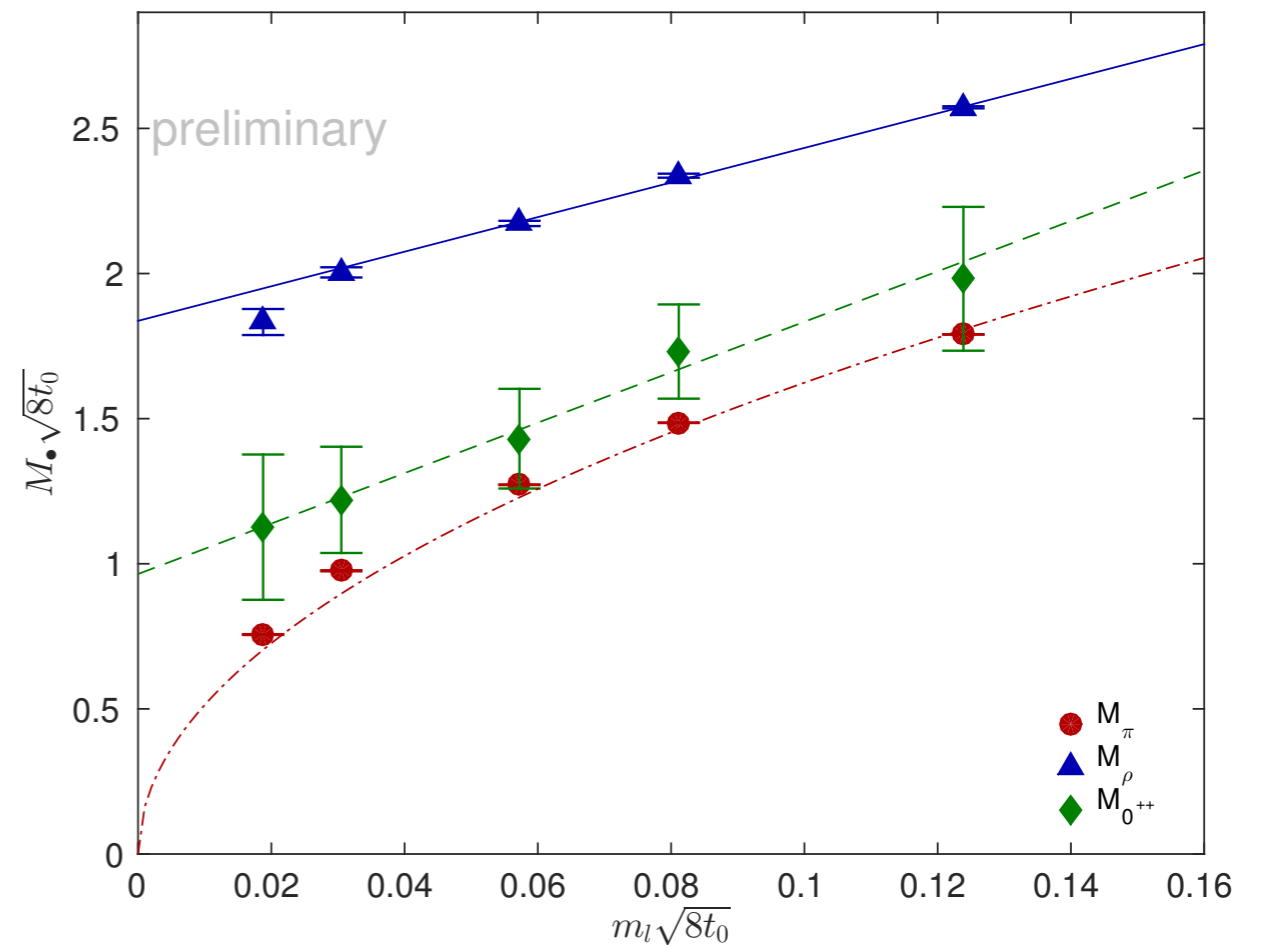
Walking range can be tuned arbitrarily with m_h

4+8 flavors : spectrum (at $m_h=0.080$)

M_ρ/M_π shows the approach to the chiral regime



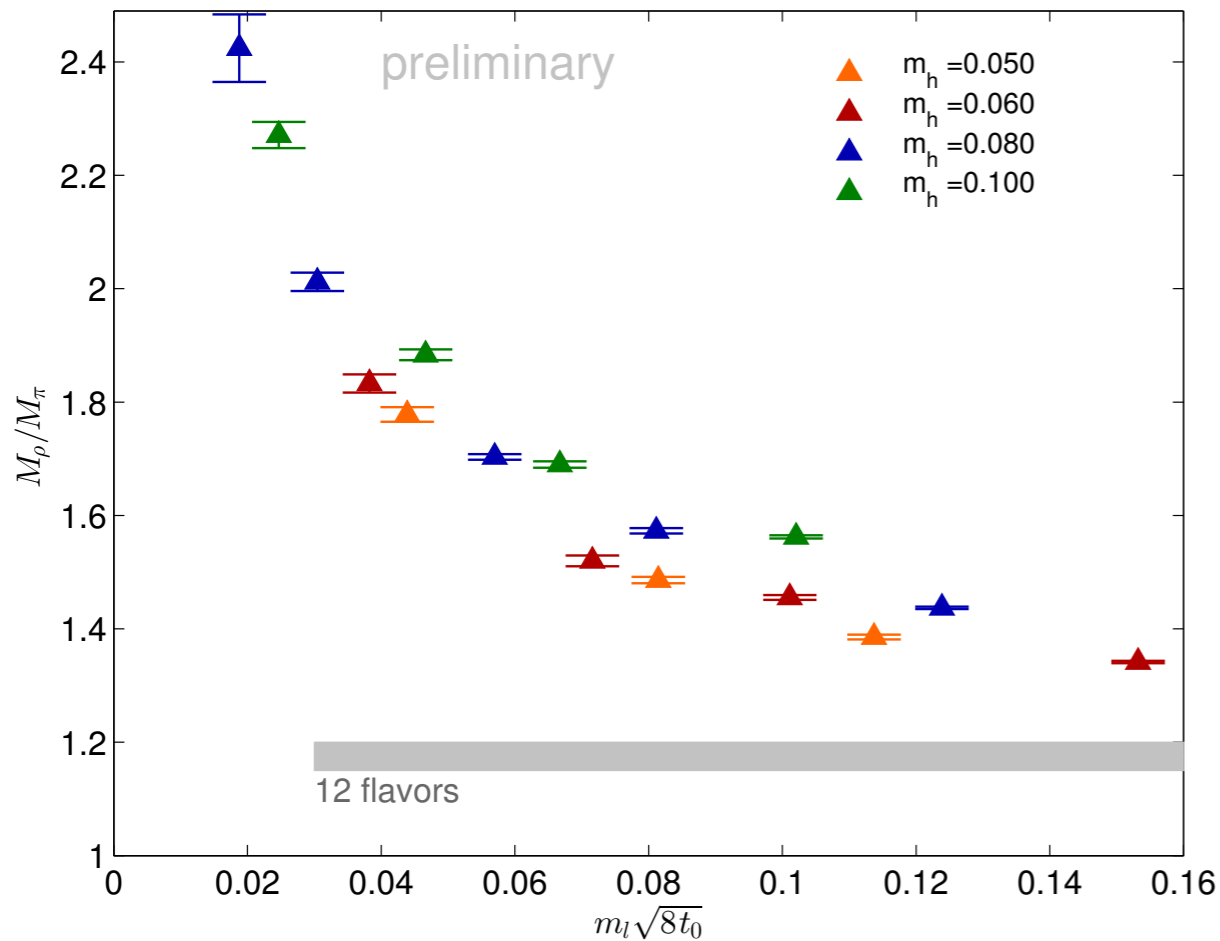
0^{++} is just above M_π
very different from $N_f=4$!



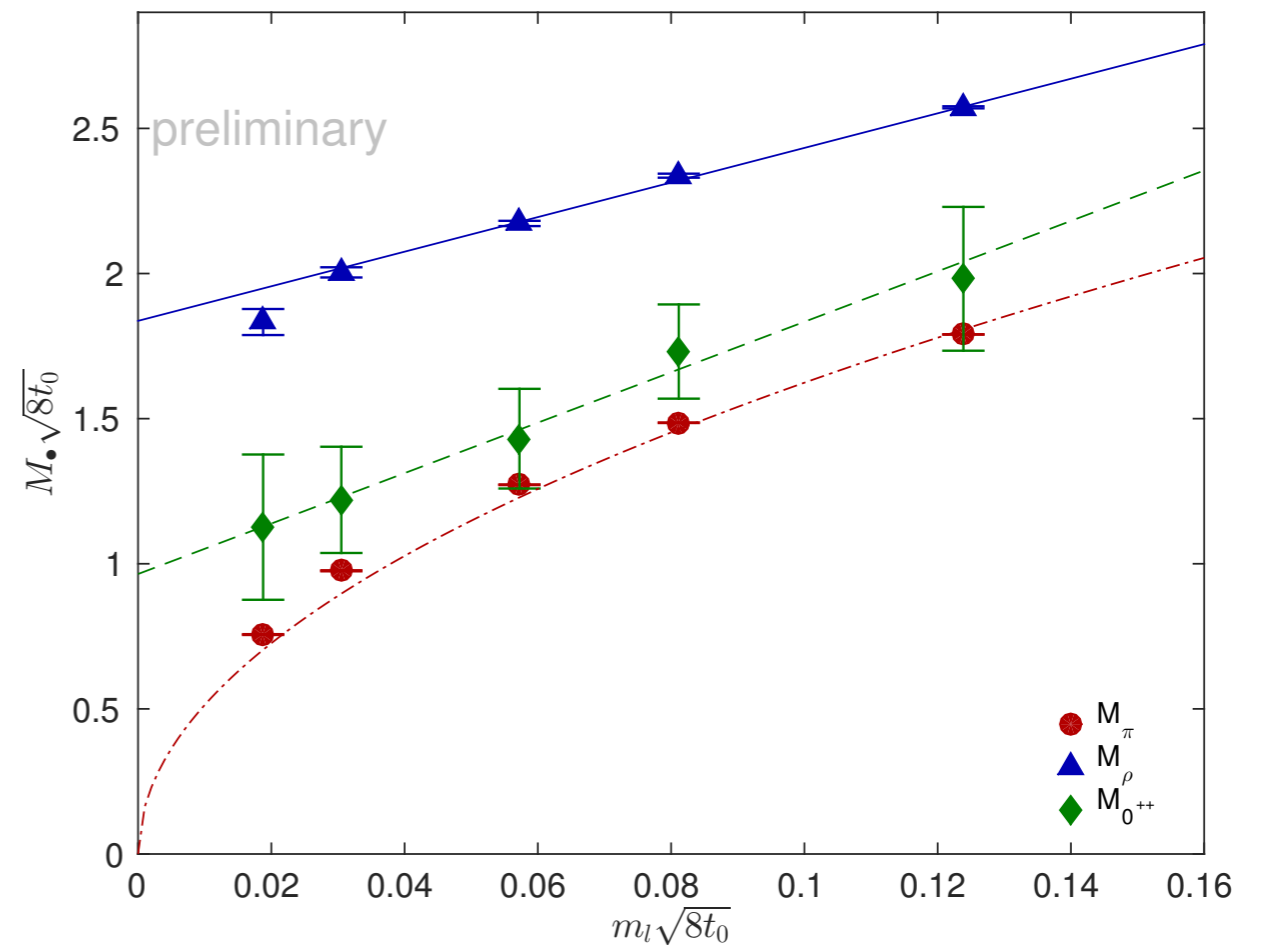
If tuning $m_h \rightarrow 0$ does not give a light enough 0^{++} , we can try 2+8 or a cascade of masses 2+ (6+1+1+1+1....)

4+8 flavors : spectrum (at $m_h=0.080$)

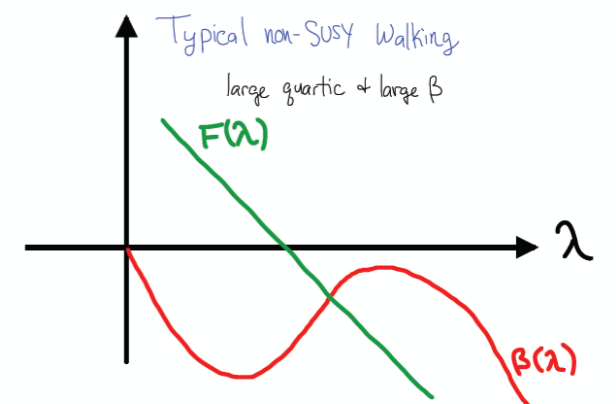
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SU(3) $N_f=2$ sextet

2-loop perturbation theory predicts an IRFP at $g^2 \sim 10$

3, 4 loop $\overline{\text{MS}}$ $g^2 \sim 6$

High statistics, extensive studies with staggered fermions favor chiral symmetry breaking with light scalar

(LatHC, talks by Kuti, Wong, Santano, Nogradi, Holland)

Emerging Wilson fermion study is consistent both with conformal hyper scaling and chiral symmetry breaking

(CP³, Hansen)

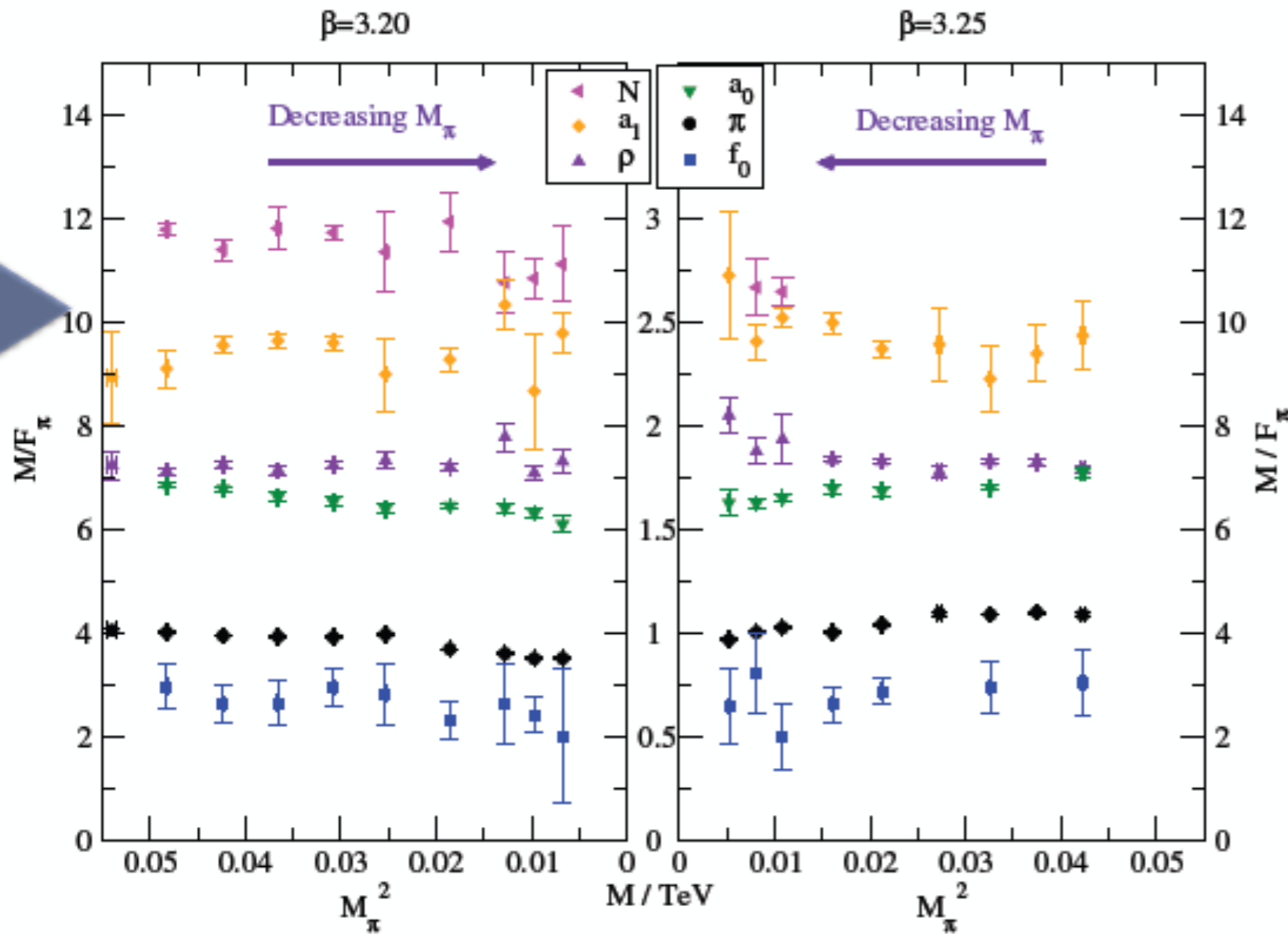
Step scaling studies with Wilson fermions are in tension with staggered

(Boulder coll, AH et al)

SU(3) $N_f=2$ sextet

Impressive spectrum from LatHC:

(talks by Kuti, Wong,
Santano, Nogradi, Holland)



Ratio of M_H/F_π vs M_π^2 is nearly constant

However M_π does not look like a Goldstone boson

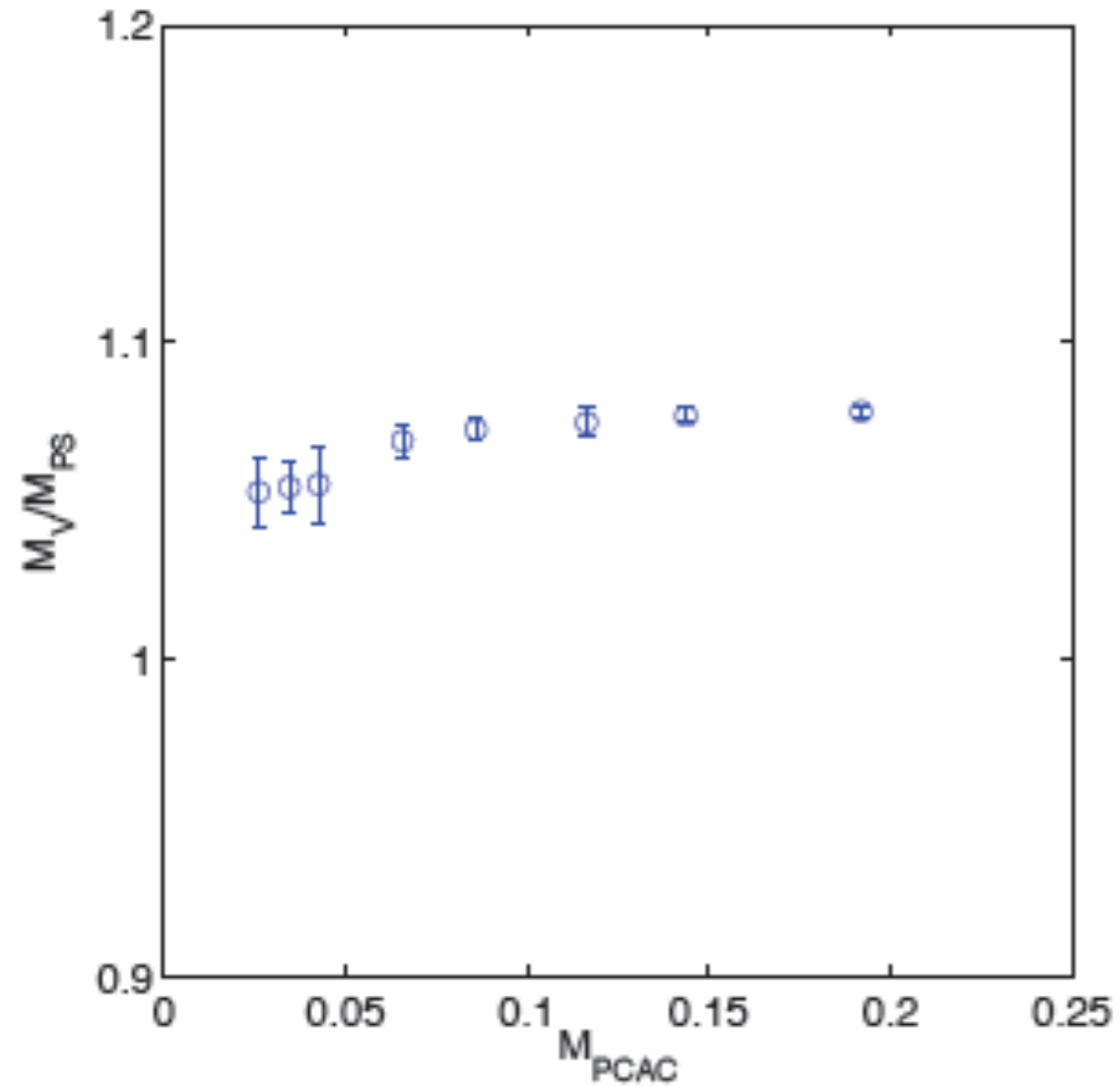
LatHC :

- the 3 lightest mass points might not be reliable
- the spectrum is not a good way to differentiate conformal vs chirally broken systems

SU(3) $N_f=2$ sextet

Preliminary spectrum from CP³

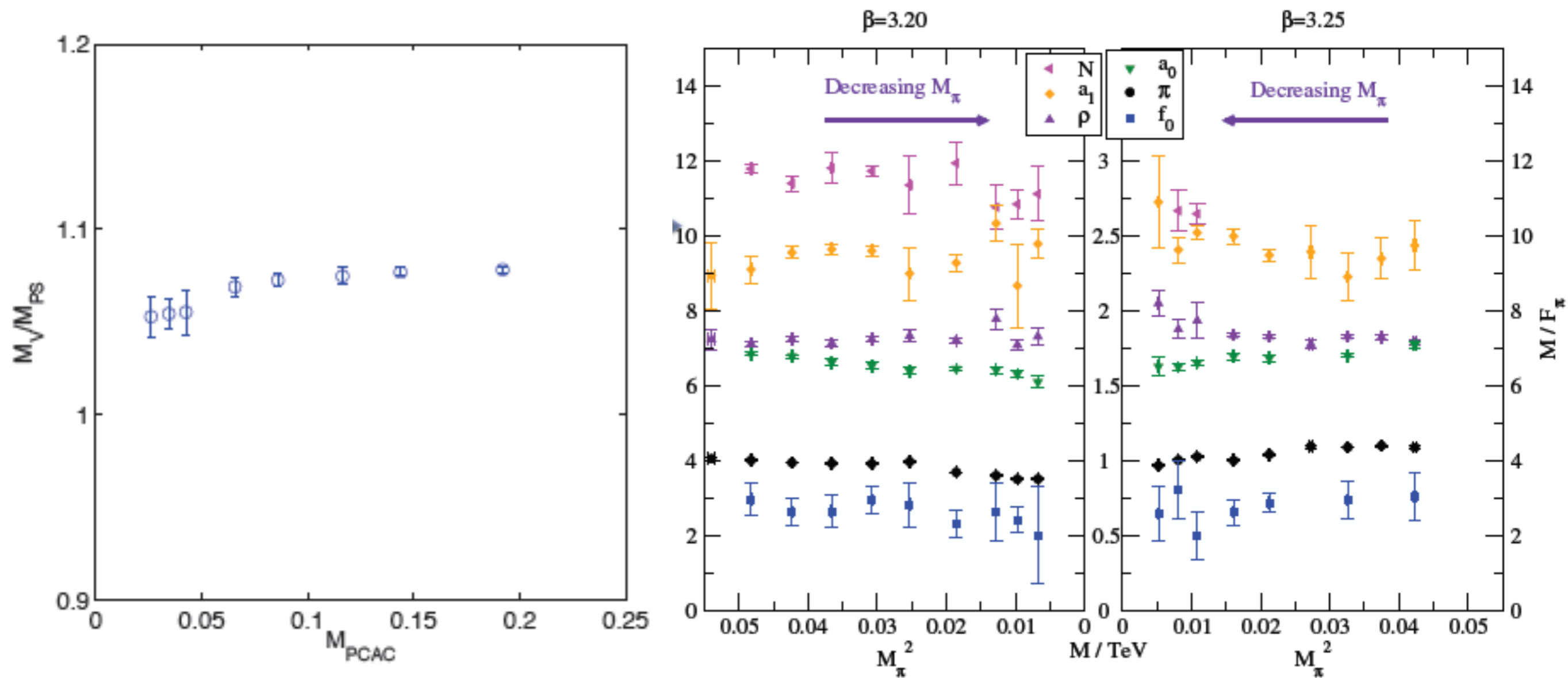
talk by Hansen



SU(3) $N_f=2$ sextet

Preliminary spectrum from CP³

talk by Hansen



The two collaborations probe quite different parameter spaces

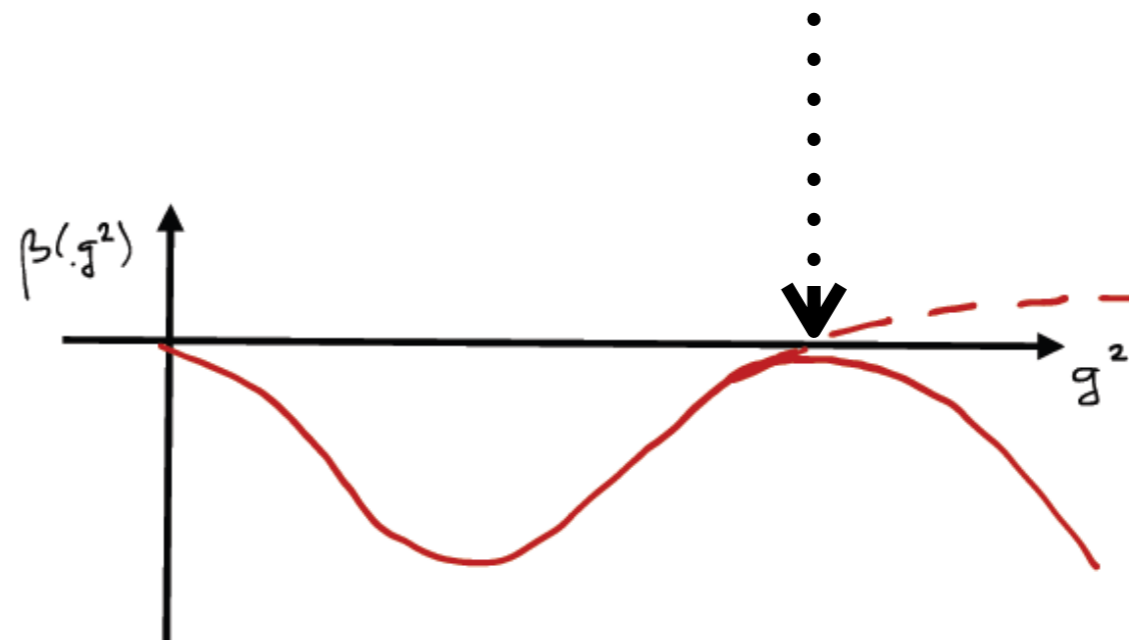
Challenge #(n+1): Universality

Even in QCD we want to verify that Wilson / staggered/overlap fermions predict the same IR physics

In a walking theory that is much harder.

In a conformal theory it might not even be true

(Wilson Renormalization Group - universality requires identical symmetries!)



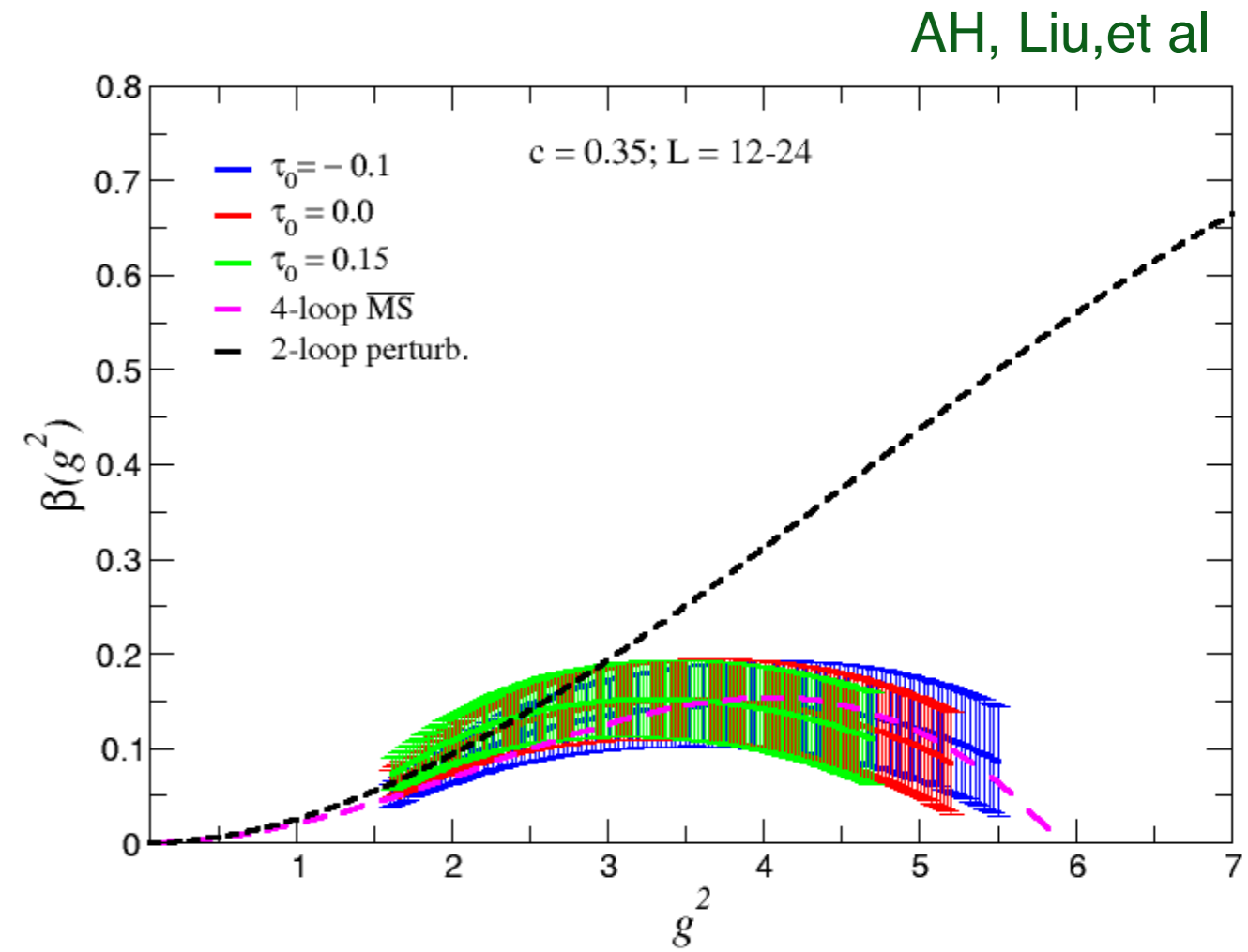
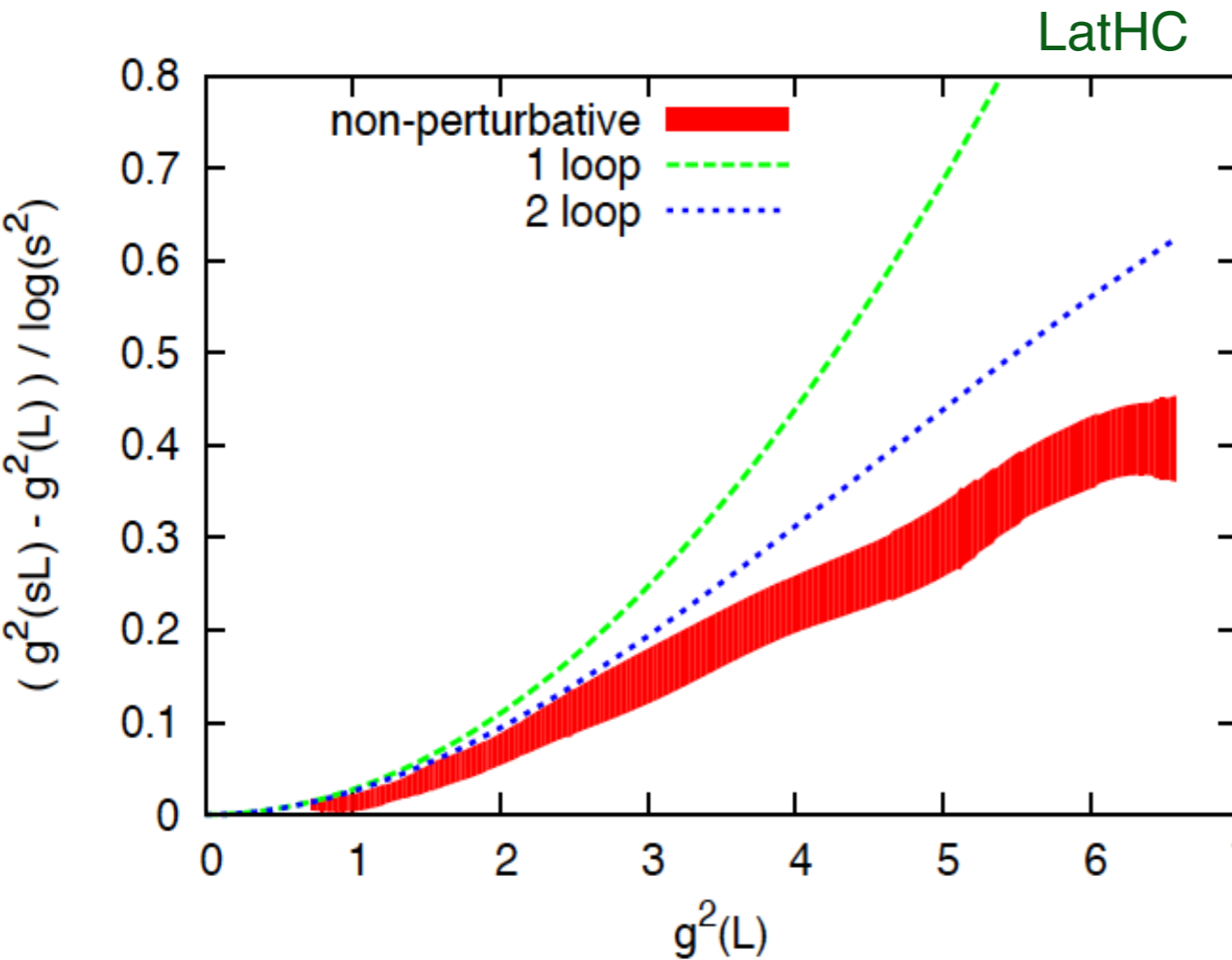
Step scaling of $N_f=2$ sextet

Step scaling

with staggered

vs

Wilson fermions



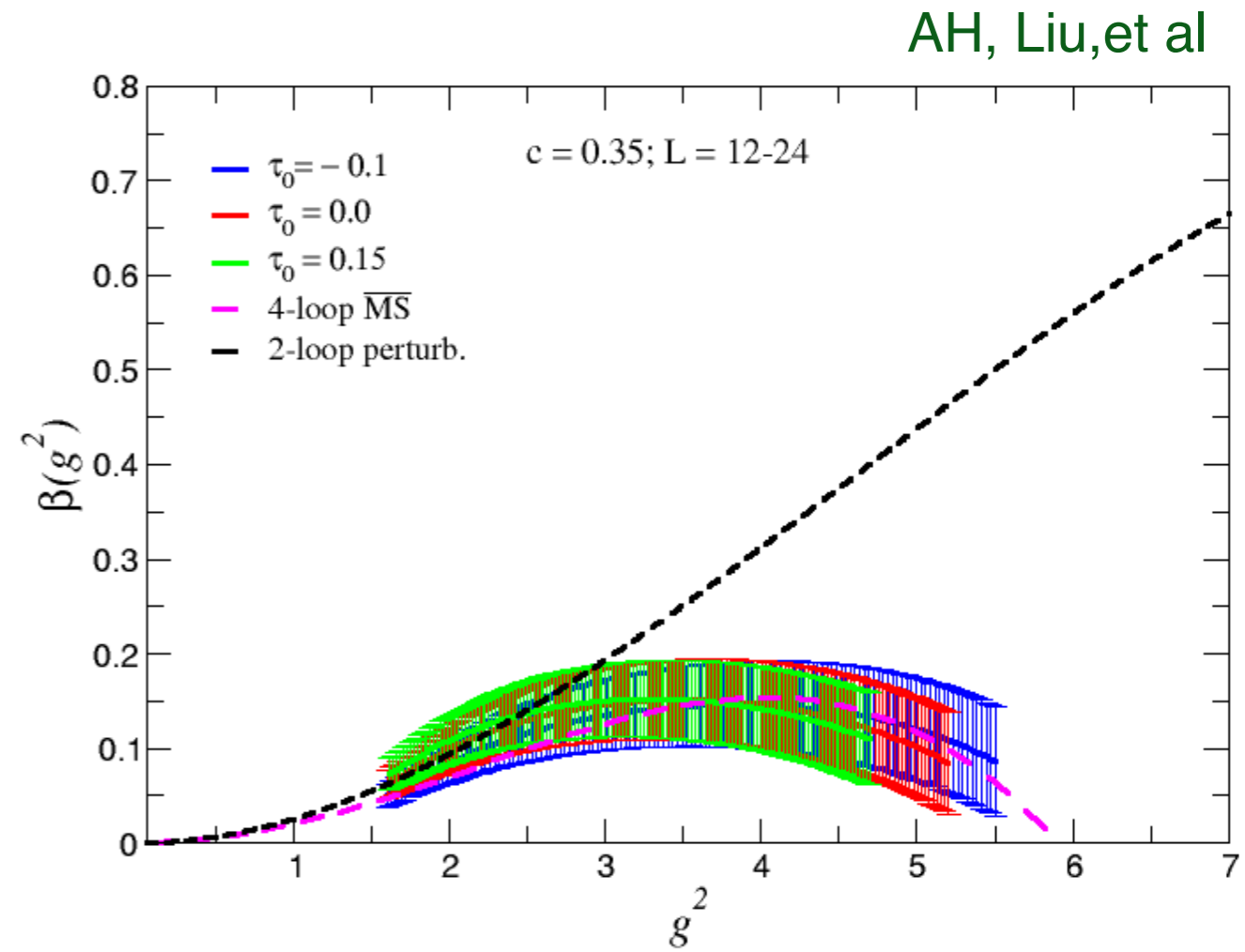
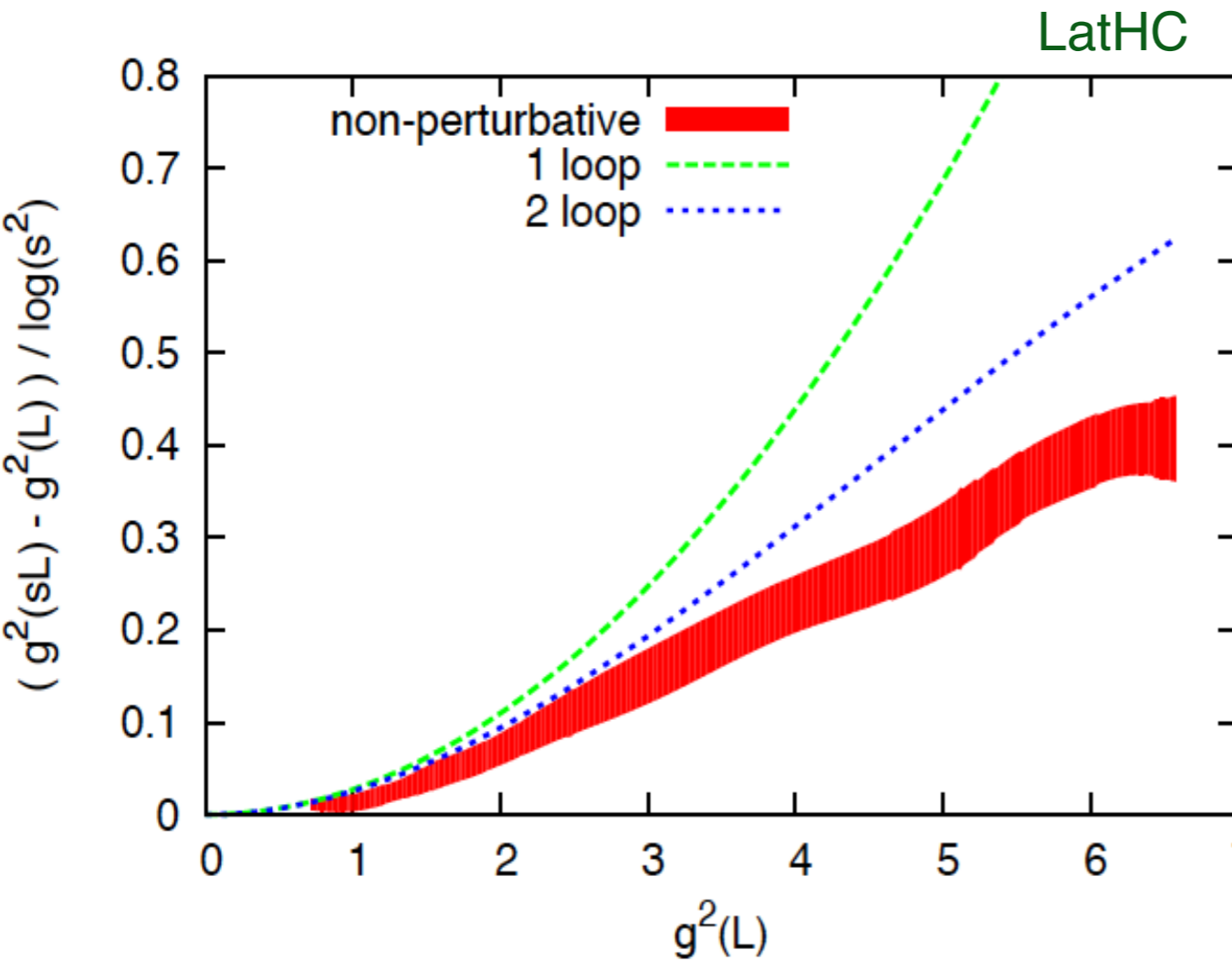
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Step scaling

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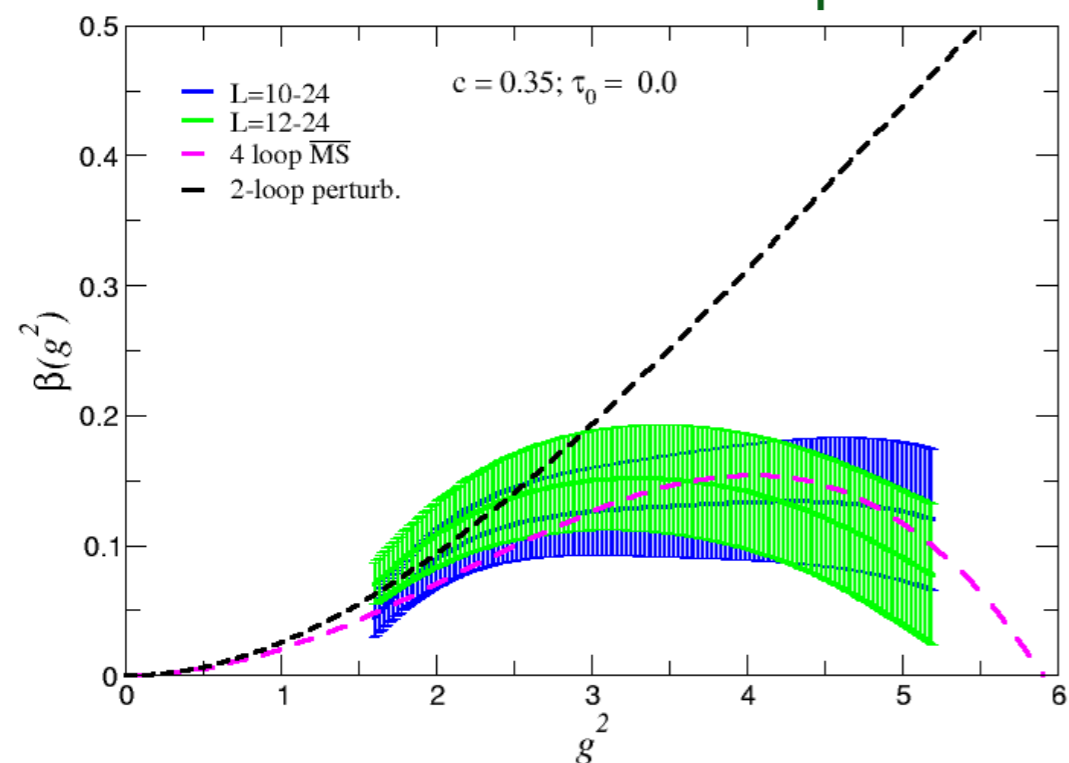


Is the tension due to universality?
The question has to be settled

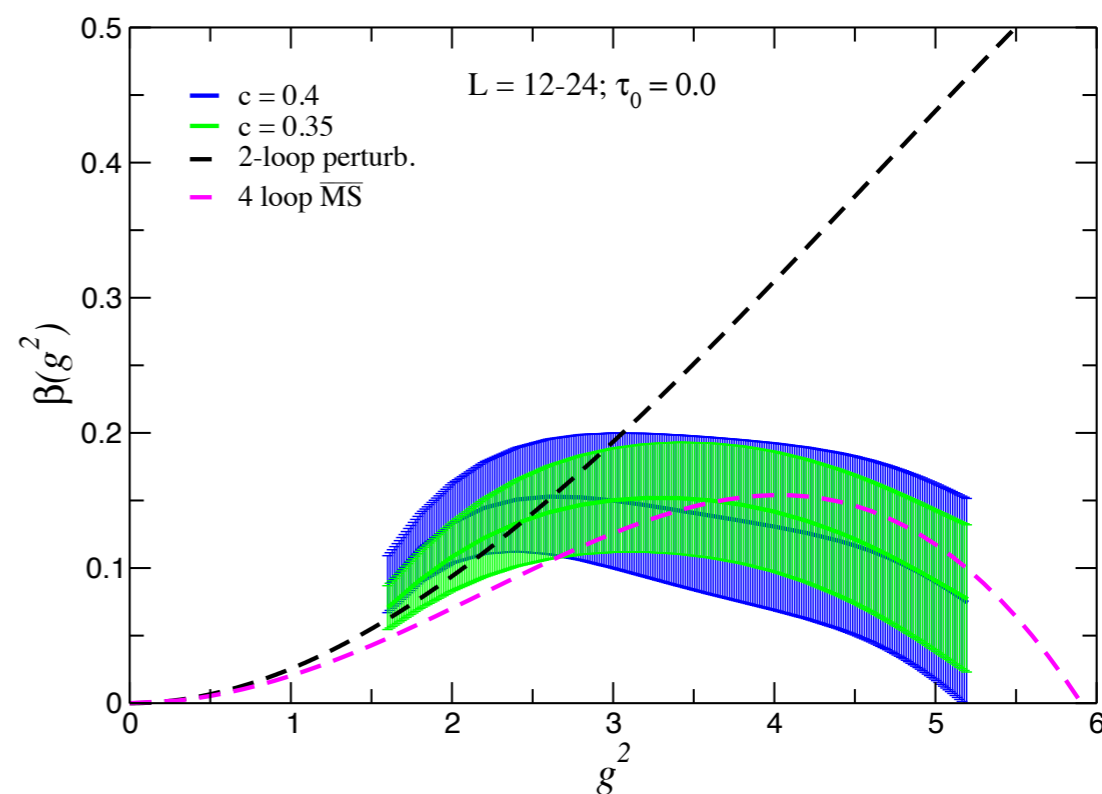
SU(3) $N_f=2$ sextet, Wilson fermions

Consistency checks:

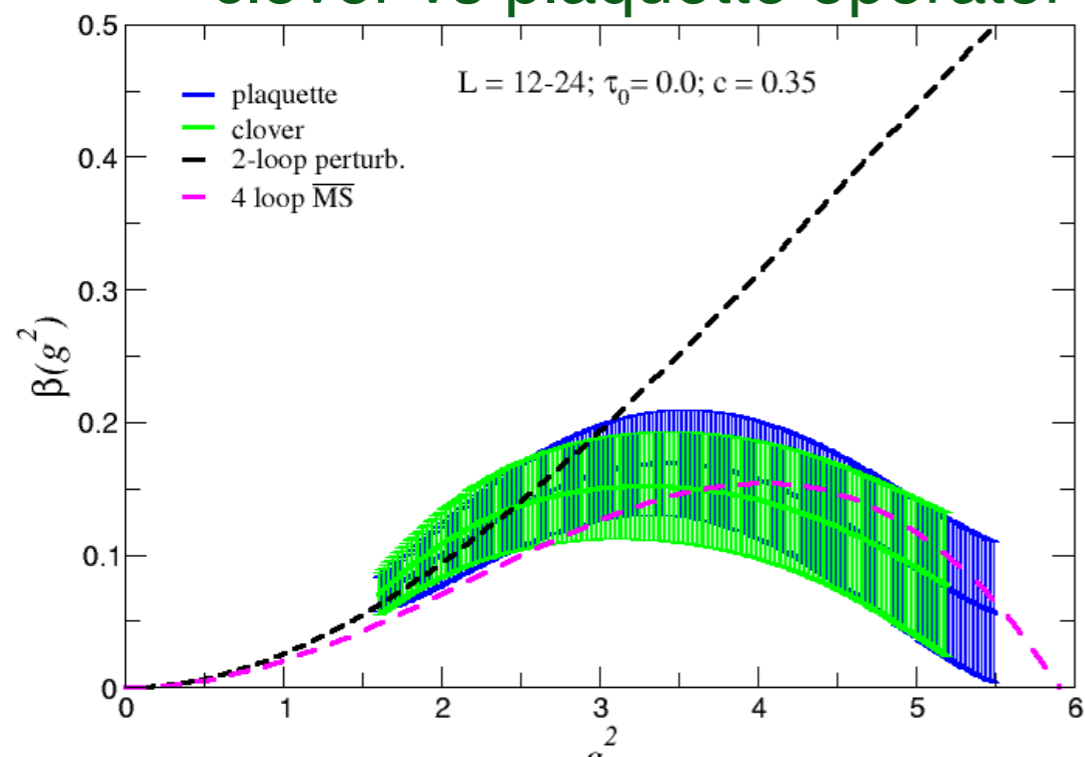
volume extrapolation



$c=0.35, c=0.4$



clover vs plaquette operator



Conclusion

BSM lattice studies:

Lattice calculations complement phenomenology :

we could have real impact !

- These calculations are difficult, even when not high precision
- Plenty of challenges, also solutions

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Future :

I put my bet on systems that interpolate between conformality and chiral symmetry breaking with a tunable parameter

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BSM lattice studies:

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Future :

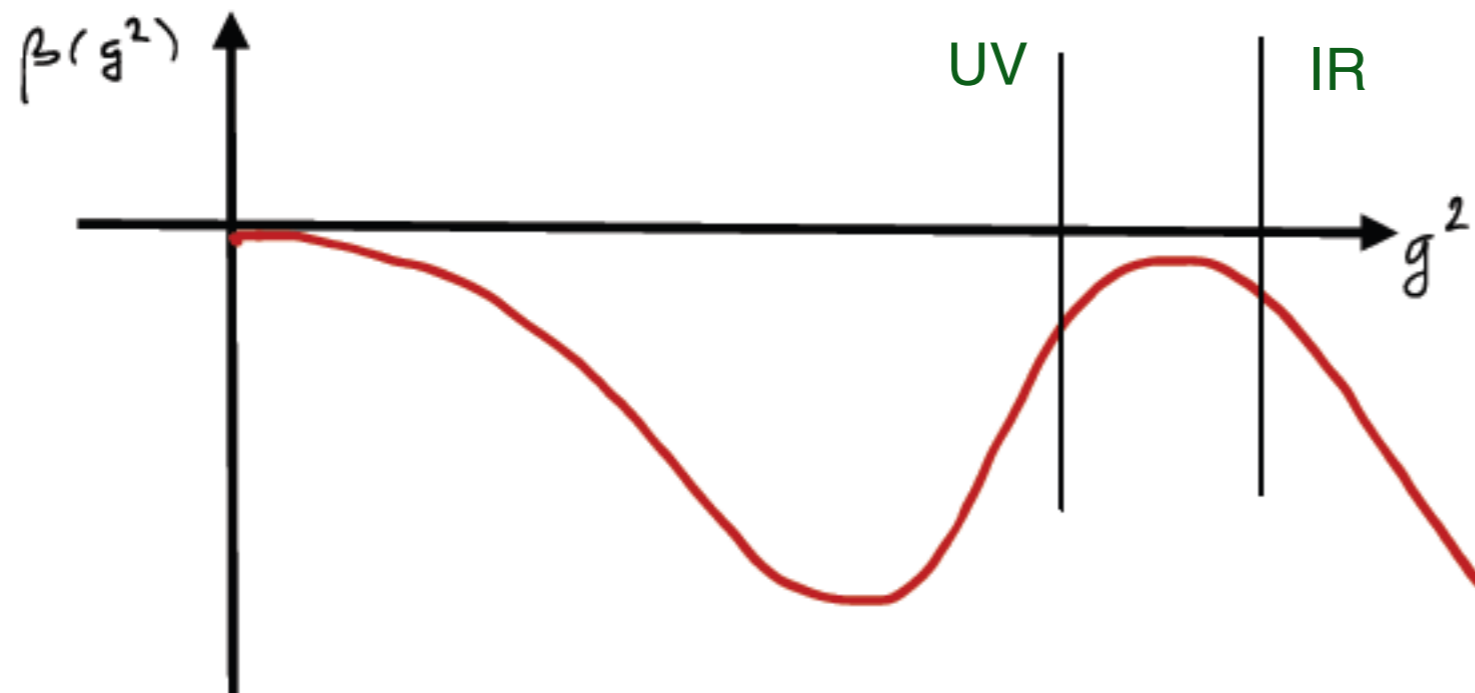
I put my bet on systems that interpolate between conformality and chiral symmetry breaking with a tunable parameter

Thank you

Challenge #5: Control of walking

Can we control the amount of walking?

need $(F^2L^2) < N_f$ — usually not (or barely) satisfied



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