Calculation of the decay width of decuplet baryons

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based on *[Phys.Rev. D88 (2013) 3, 031501]* and *[arXiv:1507.02724 [hep-lat]]*

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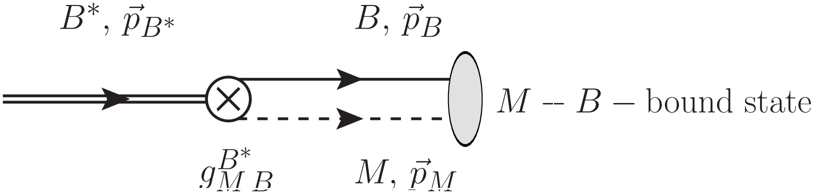
Outline

- Introduction
  - Motivation
  - Transfer matrix method
- Lattice calculation
  - transition amplitude from the lattice
  - extraction of coupling and width
  - results for two ensembles
- Conclusions and outlook

Characterization of baryonic resonance states on the lattice (E_R, Γ)



- strong decays, transitions from baryonic initial to final states of bound hadrons, $B^* \to M\,B$
- well known example: $\Delta \to \pi + N$
- volume method: phase shifts via finite volume energy spectrum *[Commun.Math.Phys. 104, 177 (1986), Commun.Math.Phys. 105, 153 (1986)]*
- transfer matrix method: attempt to estimate $\mathcal{M} \sim \langle f \,|\, H \,|\, i \rangle$ from lattice QCD *[Phys.Rev. D65, 094505 (2002)]*
- first test for a baryonic, strong transition $\Delta \to \pi N$ *[Phys.Rev. D88 (2013) 3, 031501]*

Introduction - transfer matrix method (I)

$$i\mathcal{M}(B^* \to MB)\,(2\pi)^4\,\delta\,(P_{\rm out} - P_{\rm in}) = \lim_{t_f - t_i \to \infty} \langle MB, t_f, {\rm out}\,|\,B^*, t_i, {\rm in}\rangle$$

- no physical transition on a Euclidean lattice in finite volume
- overlap of finite volume lattice states $\leftrightarrow$ if energies of levels are *sufficiently close*
- estimates from lattices with finite time extent $\leftrightarrow$ if transition amplitude *sufficiently small*
- final states at non-zero momentum with fine resolution relative momentum $\leftrightarrow$ lattice volume *sufficiently large*
- 3-momentum conservation on the lattice, but no energy conservation from initial to final state

$$E_f - E_i = E_{MB} - E_{B^*} \ll (t_f - t_i)^{-1}$$
$$E^* - E_f,\, E_i \gg (t_f - t_i)^{-1}$$

- Fermi's Golden Rule using $x = \langle MB\,|\,B^*\rangle$ from lattice QCD

$$\Gamma_{B^* \to MB} = 2\pi\,\langle |x|^2\rangle\,\rho\,(E)$$

Introduction - transfer matrix method (II)

- start from transfer matrix for system of 2 states

$$\mathrm{T} = \mathrm{e}^{-a\bar{E}} \begin{pmatrix} \mathrm{e}^{-a\delta/2} & ax & \cdots \\ ax & \mathrm{e}^{+a\delta/2} & \\ \vdots & & \ddots \end{pmatrix},$$

- transition amplitude $x = \langle B^* \mid M\,B \rangle$ parametrized by transfer matrix T
- $\bar{E} = (E_{B^*} + E_{M\,B})/2$ and $\delta = E_{M\,B} - E_{B^*}$
- restrict to 2-state sub-space for the spectrum, $\mathrm{span}\,\{|B^*\rangle,\, |M\,B\rangle\}$
- T with eigenstates of energy

$$E_{\pm} \approx \bar{E} \pm \sqrt{\delta^2/4 + x^2}$$

- summation over all possibilities of one transition $B^* \to M\,B$ (leading order)

$$\begin{aligned}\langle B^*,\, t_f \,|\, M\,B,\, t_i\rangle &= \langle B^* \,|\, \mathrm{e}^{-H(t_f - t_i)} \,|\, M\,B\rangle = \langle B^* \,|\, \mathrm{T}^{n_{fi}} \,|\, M\,B\rangle \\ &= ax\,\frac{\sinh(\delta\,\Delta t_{fi}/2)}{\sinh(a\delta/2)}\,\mathrm{e}^{-\bar{E}\Delta t_{fi}}\,,\end{aligned}$$

where $\Delta t_{fi} = t_f - t_i = an_{fi}$

- approximation of δ-functional in Euclidean time

$$a\sinh(\delta\,\Delta t_{fi}/2)/\sinh(a\delta/2) \xrightarrow[a\to 0]{\Delta t_{fi}\to\infty} 2\pi\,\delta(p^0_{M\,B} - p^0_{B^*})$$

- for sufficiently small δ linear expansion

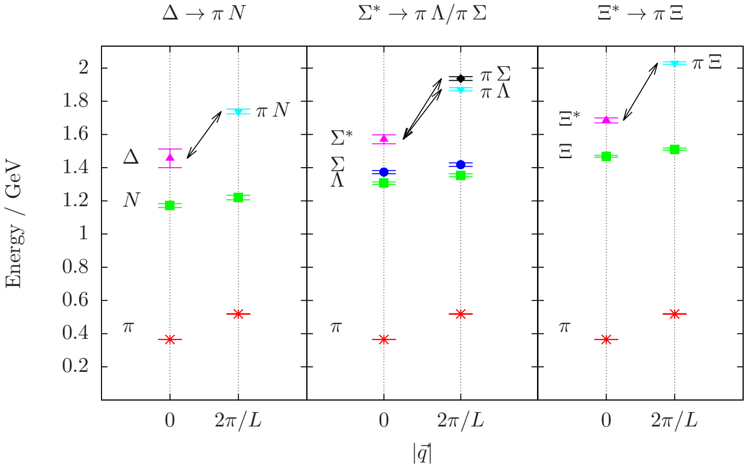
$$\langle B^*,\, t_f \,|\, M\,B,\, t_i\rangle = \left[ax\,\Delta t_{fi} + \mathcal{O}\left(\delta^2\,\Delta t_{fi}^3\right)\right]\,\mathrm{e}^{-\bar{E}\Delta t_{fi}} + \ldots$$

- ellipsis for higher order contributions, contributions from excited states (no asymptotic B^* and $M\,B$ states), mixing with other states

Lattice calculation - $B^* \to M\,B$ [Phys.Rev. D88 (2013) 3, 031501]

$$\Delta \to \pi\,N,\ \Sigma^* \to \pi\,\Lambda,\ \Sigma^* \to \pi\,\Sigma,\ \Xi^* \to \pi\,\Xi$$

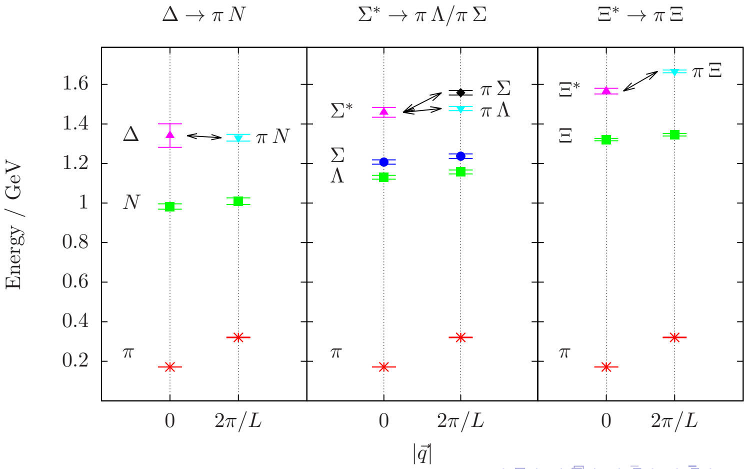
transitions for the hybrid action with $N_f = 2+1$ and $m_{PS} = 350\,\mathrm{MeV}$, $L = 3.4\,\mathrm{fm}$



Lattice calculation - $B^* \to M\, B$ [arXiv:1507.02724 [hep-lat]]

$$\Delta \to \pi\, N,\ \Sigma^* \to \pi\, \Lambda,\ \Sigma^* \to \pi\, \Sigma,\ \Xi^* \to \pi\, \Xi$$

transitions for the unitary domain wall fermion action with $N_f = 2 + 1$ and $m_{PS} = 180\,\mathrm{MeV}$, $L = 4.5\,\mathrm{fm}$



Lattice calculation - transition amplitude

- suitable ratio of 3-point and 2-point functions

$$R_{MB}^{B^*}(\Delta t_{fi}, \vec{Q}, \vec{q}) = \frac{C_\mu^{B^* \to M\,B}(\Delta t_{fi},\, \vec{Q}, \vec{q})}{\sqrt{C_\mu^{B^*}(\Delta t_{fi},\, \vec{Q})\, C^{M\,B}(\Delta t_{fi},\, \vec{Q},\, \vec{q})}}$$

- isospin channels

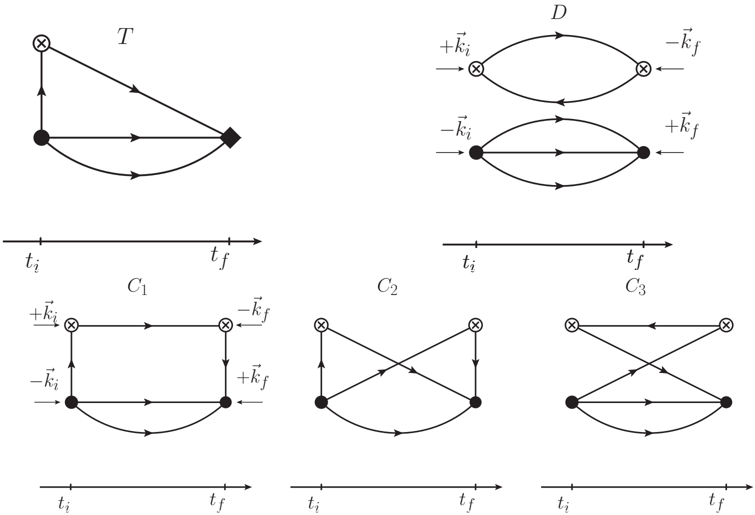
$$\Delta^{++} \to \pi^+\, N^+\,, \quad \Sigma^{*+} \to \pi^+\, \Lambda$$
$$\Sigma^{*+} \to \pi^+\, \Sigma^0\,, \quad \Xi^{*-} \to \pi^-\, \Xi$$

- standard interpolating operators for B^*, $M = \pi$, B; represent $M\,B$ by

$$J_{M\,B}^{\alpha}(t, \vec{q}, \vec{x}) = \sum_{\vec{y}} J_M(t, \vec{y} + \vec{x})\, J_B^{\alpha}(t, \vec{x})\, \mathrm{e}^{-i\vec{q}\vec{y}}$$

- $M - B$ system with relative momentum to generate overlap with $l = 1$ state; dominant contribution from coupling $s_B \oplus l \to J_{B^*} = 3/2$
- approximate $C^{M\,B} \approx C^M \times C^B$

Lattice calculation - transition amplitude



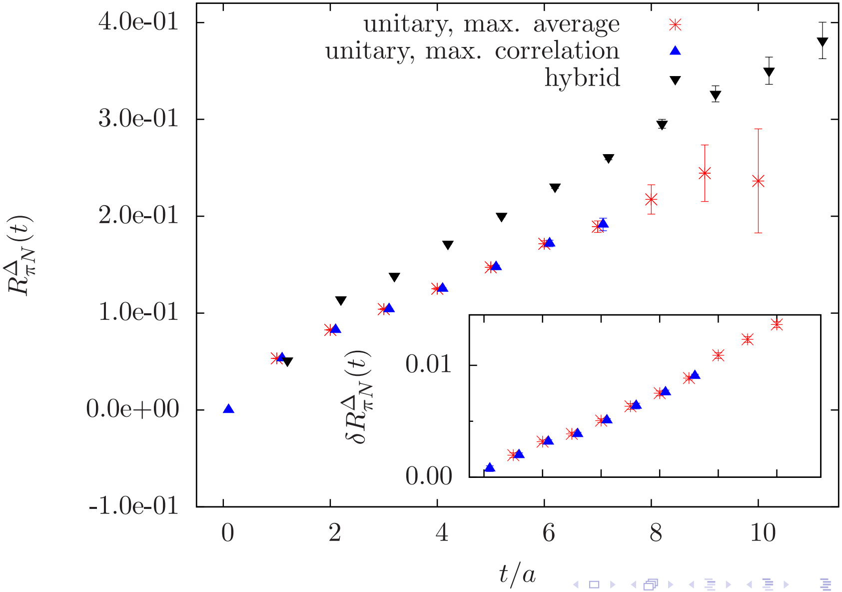
Lattice calculation - ensemble data

- hybrid action: domain wall valence quarks on $N_f = 2 + 1$ staggered sea @ $m_{PS} = 350\ \mathrm{MeV}$
  MILC ensemble *2864f21b676m010m050 [Phys.Rev. D64, 054506 (2001)]*
- unitary action with $N_f = 2 + 1$ domain wall fermions @ $m_{PS} = 180\ \mathrm{MeV}$ *[Phys.Rev. D79, 054502 (2009)]*

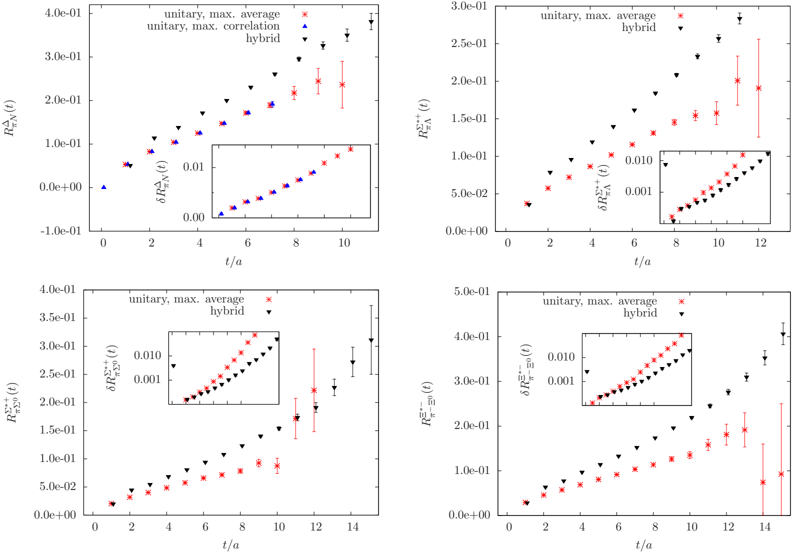
action	$L^3 \times T$	$m_{PS}/\,\mathrm{MeV}$	$a/\,\mathrm{fm}$	$L/\,\mathrm{fm}$	L_5/a	N_{conf}	N_{src}
hybrid	$28^3 \times 64$	350	0.124	3.4	16	210	4 indep.
unitary	$32^3 \times 64$	180	0.143	4.5	32	254	4 coh.

- Gaussian source and sink smearing with APE-smeared gauge links
- relative momentum: $\vec{q} = 2\pi/L \cdot \vec{e}_i$,
- averaging over $i = \pm 1, \pm 2, \pm 3$, forward and backward propagation

Lattice calculation - ratio signal

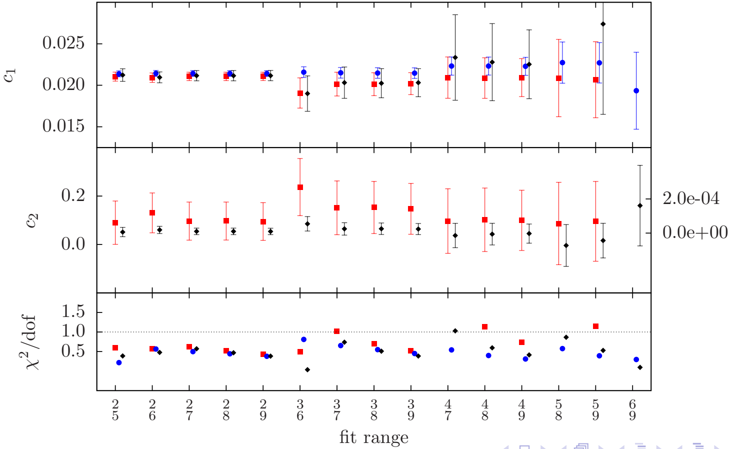


Lattice calculation - ratio signal

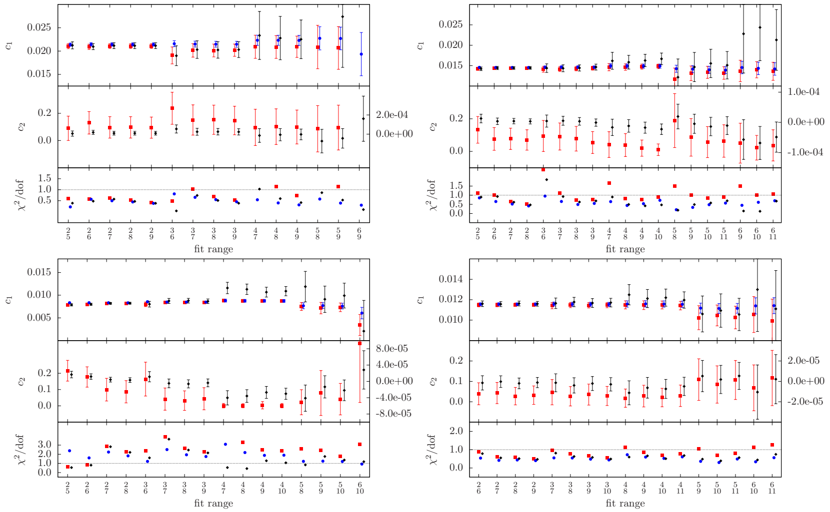


Lattice calculation - ratio fits

$$f_1(t) = c_0 + c_1\, a\, \frac{\sinh(c_2\, t/2)}{\sin(a c_2/2)}\,, \quad f_2(t) = c_0 + c_1\, \frac{t}{a} + c_2\, \left(\frac{t}{a}\right)^3 + \ldots$$



Lattice calculation - ratio fits



left to right, top to bottom: $\Delta \to \pi\, N$, $\Sigma^* \to \pi\, \Lambda$, $\Sigma^* \to \pi\, \Sigma$ and $\Xi^* \to \pi\, \Xi$

Lattice calculation - extraction of the coupling and width

- definition of the matrix element from c_1

$$c_1 = \sum_{\sigma_3,\,\tau_3} \frac{\mathcal{M}(\vec{Q},\,\vec{q},\,\sigma_3,\,\tau_3)}{\sqrt{N_{B^*}\,N_{M\,B}}}\,V\,\delta_{\vec{Q}\vec{Q}} \times \text{spin sum factor}$$

- normalizations of finite volume B^* and $M\,B$ states

$$\begin{aligned} N_{B^*} &= V\,\frac{E_{B^*}}{m_{B^*}} \\ N_{M\,B} &= N_M \times N_B = 2V\,E_M \times V\,\frac{E_B}{m_B}\,. \end{aligned}$$

- decomposition of $\mathcal{M}$ by connecting to LO effective field theory with coupling $g_{B\,M}^{B^*}$

$$\mathcal{M}(\vec{Q},\,\vec{q},\,\sigma_3,\,\tau_3) = \frac{g_{M\,B}^{B^*}}{2m_B}\,\bar{u}_{B^*}^{\mu\,\alpha}(\vec{Q},\sigma_3)\,q_\mu\,u_B^{\alpha}(\vec{Q}+\vec{q},\tau_3)\,C_{\mathrm{CG}}$$

Lattice calculation - results for the coupling $g_{MB}^{B^*}$ and width $\Gamma_{MB}^{B^*}$

$$\text{coupling} \quad g_{MB}^{B^*} = c_1 \, \frac{\sqrt{N_{B^*}\, N_{MB}}}{V\, C_{\mathrm{CG}}} \, \frac{2m_B}{|\vec{q}|} \, \left(\frac{1}{3} \, \frac{E_B(\vec{q}^{\,2}) + m_B}{m_B} \right)^{-1/2} .$$

process	unitary	hybrid	PDG
$\Delta^{++} \leftrightarrow \pi^+ N^+$	23.7 (0.7) (1.1)	26.7 (0.6) (1.4)	29.4 (0.3)
$\Sigma^{*+} \leftrightarrow \pi^+ \Lambda$	18.5 (0.3) (0.5)	23.2 (0.6) (0.8)	20.4 (0.3)
$\Sigma^{*+} \leftrightarrow \pi^+ \Sigma^0$	16.1 (0.3) (1.9)	19.0 (0.7) (2.9)	17.3 (1.1)
$\Xi^{*-} \leftrightarrow \pi^- \Xi^0$	21.0 (0.3) (0.3)	25.6 (0.6) (4.3)	19.4 (1.9)

$$\text{width} \quad \Gamma_{M\,B}^{B^*} = 2\pi \left[\frac{2\,c_1^2}{2s_{B^*} + 1} \right] \rho(E_{MB}) \quad \text{in GeV}$$

process	unitary	hybrid	PDG
$\Delta^{++} \leftrightarrow \pi^+ N^+$	119.4 (7.9) (4.5)	238.5 (12.2) (16.2)	118 (2)
$\Sigma^{*+} \leftrightarrow \pi^+ \Lambda$	54.5 (2.1) (1.3)	143.9 (7.4) (6.1)	31.3 (8)
$\Sigma^{*+} \leftrightarrow \pi^+ \Sigma^0$	17.6 (0.8) (2.1)	58.3 (3.4) (6.8)	4.2 (5)
$\Xi^{*-} \leftrightarrow \pi^- \Xi^0$	35.1 (1.1) (0.4)	126.0 (5.6) (18.5)	9.9 (1.9)

Conclusions and outlook

- problem: "rigid" kinematical setup given by the lattice parameters
- study of systematics (a, m_{PS}, L)
- may require a fine-tuned, combined extrapolation in lattice spacing, pion mass and spatial volume to keep the method applicable
- interesting cases of impact for this method: resonances of higher spin, negative parity
- apply more known techniques (enlarged operator basis, moving frames) ... with growing computational cost

Thank you very much for your attention.