Clover fermions in Numerical Stochastic Perturbation Theory

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Goal of this project

In recent years we enlarged applications of Numerical Stochastic Perturbation Theory [Di Renzo *et al.* 1993]

- Schrödinger functional formalism [Hesse *et al.* Lattice 2013]

current project

- clover fermions

In this talk I will

- briefly review:
  - clover fermions
  - Numerical Stochastic Perturbation Theory
- present two approaches to compute c_{SW} to higher orders:
  - quark-gluon vertex improvement
  - PCAC mass improvement
- show an example of viable computations

Clover fermions: recap

As well known in presence of Wilson fermions lattice QCD action suffers of $\mathcal{O}(a)$ artifacts due to mixing with operators of dimension 5.

According to Symanzik improvement program one finds that $\mathcal{O}(a)$ artifacts can be canceled adding to the action the term

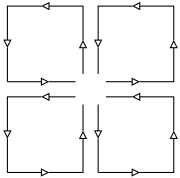
$$\mathcal{O} = a^5 \sum_x c_{SW} \bar{\psi}(x) \frac{i}{4} \sigma_{\mu\nu} \hat{F}_{\mu\nu}(x) \psi(x)$$

- $\hat{F}_{\mu\nu} = \frac{1}{8} \left\{ \hat{Q}_{\mu\nu}(x) - \hat{Q}_{\nu\mu}(x) \right\}$ lattice representation of gluon field tensor
- $\hat{Q}_{\mu\nu}(x)$ is the “clover” product of link variables
- $c_{SW} = c_{SW}(g_0)$ is the Sheikholeslami and Wholert (*clover*) coefficient

- in PT c_{SW} can be Taylor-expanded in powers of the bare coupling g_0 as

$$c_{SW}(g_0) = c_{SW}^{(0)} + c_{SW}^{(1)} g_0^2 + c_{SW}^{(2)} g_0^4 + \dots$$

- the presence of clover give rise to a growing number of diagrams when PT expanded



$\Rightarrow$ computing clover improved observables in ordinary diagrammatic can be (as usual) a little bit cumbersome.

NSPT

has proven to be a viable tool to perform PT computation on the lattice: why not use it for clover improved computations?

A SKETCH OF NSPT

- Let the system evolve according Langevin dynamic in a *"fictitious"* time t

$$\partial_t U(x,t) = \{-i\nabla S[U(x,t)] - i\eta(x,t)\}\, U(x,t)$$

where $\langle \eta(x,t) \rangle = 0 \quad \langle \eta(x,t)\eta(x',t') \rangle = 2\delta(x-x')\delta(t-t')$

- by expanding the link in a power series one gets a system of equations to be truncated at a given order (Stochastic PT)
- the differential equations can be traded for integral ones (in this way one would get diagrams); in our approach the integration is performed numerically on a computer

Correlation functions are introduced as averages over η:

$$\langle \phi(x_1,t_1)\ldots\phi(x_n,t_n)\rangle_\eta = \int \mathcal{D}[\eta] e^{-\frac{1}{4}\int dx' dt' \eta^2(x',t')} \phi(x_1,t_1)\ldots\phi(x_n,t_n)$$

The main assert of Stochastic Quantization is

$$\lim_{t\to\infty} \langle \phi(x_1,t)\ldots\phi(x_n,t)\rangle_\eta = \langle \phi(x_1)\ldots\phi(x_n)\rangle$$

Quark contributions can be included by adding a term to the action

$$S_{\text{eff}} = -\text{Tr} \ln M[U]$$

The Langevin equation receives a contribution

$$\nabla S_{\text{eff}} = -\text{Tr}\Big[(\nabla M[U])\, M[U]^{-1} \Big]$$

- the trace can be stochastically evaluated inserting a gaussian source ξ s.t. $\langle \xi_i \xi_j \rangle = \delta_{ij}$:

$$\text{Tr}\Big[(\nabla M)\, M^{-1} \Big] = \text{Re}\langle \xi^\dagger\, (\nabla M)\, M^{-1} \xi \rangle_\xi$$

- the Lie derivative ∇M can be analytically computed
- the expansion of U induces an expansion of the Dirac operator

$$M[U] = \sum_{n=0} \beta^{-n/2} M[U]^{(n)}$$

The inversion $\psi = M[U]^{-1}\xi$ is iteratively constructed:

- compute $\psi^{(0)} = {M^{(0)}}^{-1}\xi$
- iteratively construct $\psi^{(n)} = {M^{(0)}}^{-1}\left[\sum_{m=0}^{n-1} M^{(n-m)}\psi^{(m)}\right]$

One has to face only the inversion of $M^{(0)}$, but this can (usually) be computed analytically!
e.g. using Wilson fermions the inversion is trivial in momentum space, where the operator is diagonal.

Clover case

Since the clover term does not act at tree-level the inversion is not affected

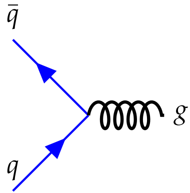
FINAL GOAL OF THE PROJECT: c_{SW} TO HIGHER LOOP

We are considering two approaches:

1. quark-gluon vertex improvement
2. PCAC mass measurement

Disclaimer: this is work in progress, we are trying to understand which will be the better approach.

QUARK-GLUON VERTEX IMPROVEMENT



The 1-PI vertex function $\Lambda^a_\mu(p,q)^{ij}_{\alpha\beta}$ can be obtained by Fourier transform and amputation of the quark-gluon vertex function

$$V^a_\mu(x,y,z)^{ij}_{\alpha\beta} = \langle \psi^i_\alpha(x)\bar{\psi}^j_\beta(z) A^a_\mu(y)\rangle$$

Sandwich $\Lambda^a_\mu(p,q)$ by the Dirac spinor (at tree-level, $m_q = 0$)

$$\bar{u}(q)\Lambda^{a,(0)}_\mu(p,q)u(p) = -gT^a\bar{u}(q)\left[i\gamma_\mu + (1 - c^{(0)}_{SW})\frac{a}{2}(p+q)_\mu\right]u(p)$$

at higher orders one has to consider all the possible symmetry-conserving terms, and obtains

$$\bar{u}(q)\Lambda^{a,(1)}_\mu(p,q)u(p) = -g^3T^a\bar{u}(q)\left[\gamma_\mu F_1 + a(p+q)_\mu G_1 + a(p-q)_\mu H_1\right]u(p)$$

Alternatively one can work off-shell, but has to consider a further term [Horsley *et al.* PRD78 (2008)]

PCAC MASS IMPROVEMENT

- PCAC relation allows to define an unrenormalized quark mass m_q by

$$m_q = \frac{\frac{1}{2}\left[\frac{1}{2}(\partial_0^L + \partial_0^R) f_A + c_A \partial_0^L \partial_0^R f_P\right]}{f_p}$$

where

$$f_A = \frac{1}{3}\sum_x \langle A_0^a(x) \bar{\psi}(\mathbf{y}) \gamma_5 \frac{1}{2}\tau^a \psi(\mathbf{z})\rangle$$

$$f_P = \frac{1}{3}\sum_x \langle P^a(x) \bar{\psi}(\mathbf{y}) \gamma_5 \frac{1}{2}\tau^a \psi(\mathbf{z})\rangle$$

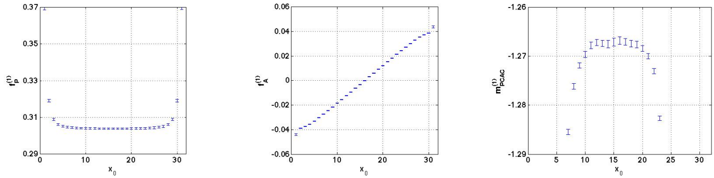
- in an improved theory (both c_{SW} and c_A) the PCAC mass should suffer $\mathcal{O}(a^2)$ effects – not $\mathcal{O}(a)$ –
- in the framework of the Schrödinger functional we can measure m_q with different background field and impose the difference to be order a^2

A similar strategy has already been used by other group

[Torrero and Bali, Lattice 2009]

PCAC MASS WITH PBC

We tried to explore the approach on existing configurations. Using PBC (and ABC) we computed f_A, f_P and m_{PCAC}



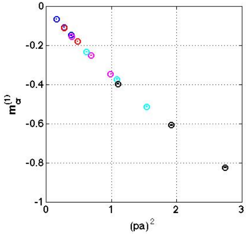
- measurement have been performed on 42 configurations $L = 32^4$
- fixed τ (extrapolation towards $\tau = 0$ required)
- errorbar are expected to be larger, still the signal looks stable

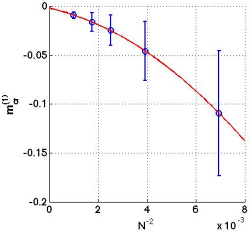
PROOF OF CONCEPT: QUARK CRITICAL MASS

The quark critical mass can be extracted from the propagator

$$\hat{S}(\hat{p}, \hat{m}_{cr}, \beta^{-1})^{-1} = i\hat{\not{p}} + \hat{m}_W(\hat{p}) - \hat{\Sigma}(\hat{p}, \hat{m}_{cr}, \beta^{-1})$$
$$\hat{\Sigma}(0, \hat{m}_{cr}, \beta^{-1}) = \hat{m}_{cr}$$

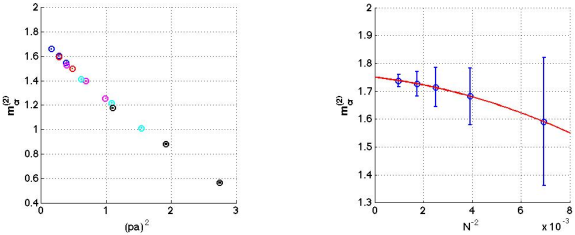
We performed the measurement on Iwasaki quenched configurations at different lattice volumes, and extrapolated towards the infinite volume limit





Since quark critical mass is known at 1-loop we plugged the counterterm into our measurement: as a result we can stay massless.

Two loop critical mass is a first (preliminar) result.



For quenched Iwasaki gauge action we find

$$m_{cr}^{(1)} = -0.001 \pm 0.003 \qquad m_{cr}^{(2)} = 1.75 \pm 0.01$$

Precision can be improved considering more momenta and performing hypercubic lattice expansion.

M.B., Di Renzo EPJC 73 (2013)

M.B., Hasegawa, Di Renzo EPJC 74 (2014)

Results and conclusions

Improved clover fermions are (partially) available in NSPT:

- we implemented the clover term in NSPT
- 2-loop computations are feasible, since 1-loop c_{SW} is known
- unquenched dynamic is almost ready

Final goal of the project is the computation of c_{SW} to higher orders via:

- quark-gluon vertex
- PCAC relations

Thanks for your attention