Extracting the η' meson mass from gluonic correlators in lattice QCD

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arXiv:150?.xxxxx

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[JLQCD collaboration]

What I talked at Lattice 2014

New method for topological susceptibility

1. Gluonic topology density after Wilson flow

$$q(x) = c\mathrm{Tr} F_{\mu\nu}\tilde{F}^{\mu\nu}(x)$$

-> smooth gauge field, good noise reduction.

2. Subvolume topology

$$\chi_t^{\mathrm{sub}} \equiv \int_{|x|<r_{cut}} d^4x \langle q(x)q(0)\rangle$$

-> shorter auto-correlation than global Q.

3. Subtract bias from global topology

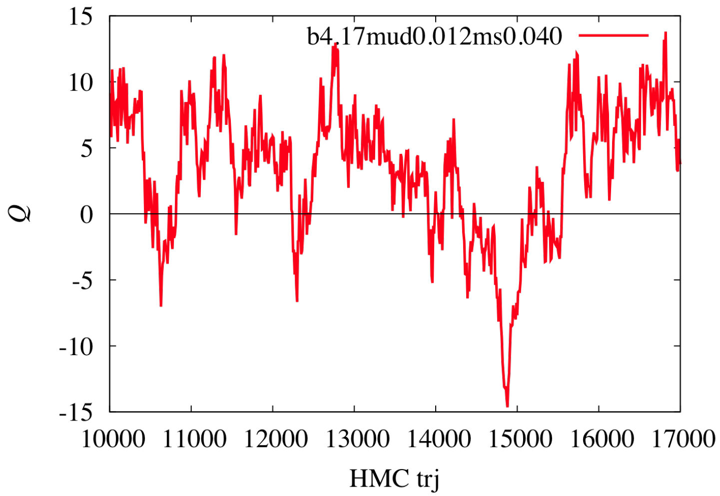
$$\langle q(x)q(0)\rangle_Q \rightarrow_{|x|\rightarrow \mathrm{large}} \frac{1}{V}\left[\frac{Q^2}{V} - \chi_t\right] + O(V^{-2}),$$

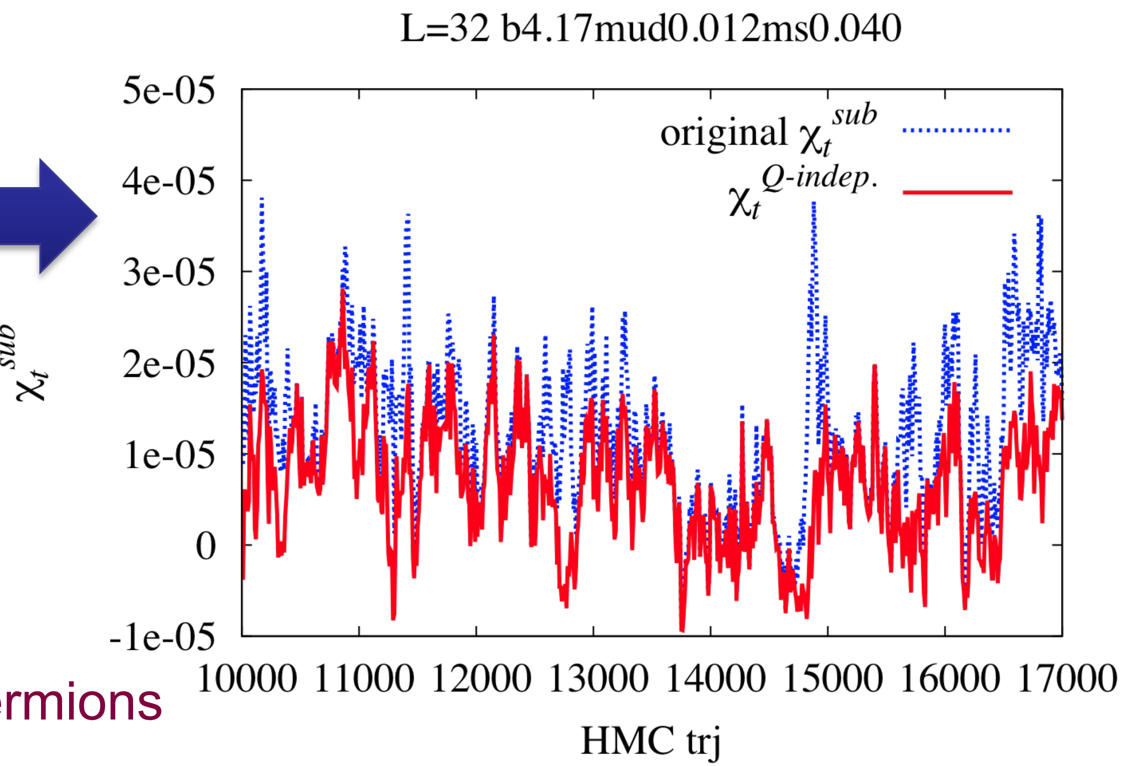
[Aoki, F, Hashimoto, Onogi 2007]

New χ_t has shorter auto-correlation.

$$\langle \chi_t^{Q-\text{indep.}} \rangle = \frac{V}{V - V_{sub}} \left(\chi_t^{sub} - \frac{V_{sub}}{V^2} Q^2 \right)$$

Global Q



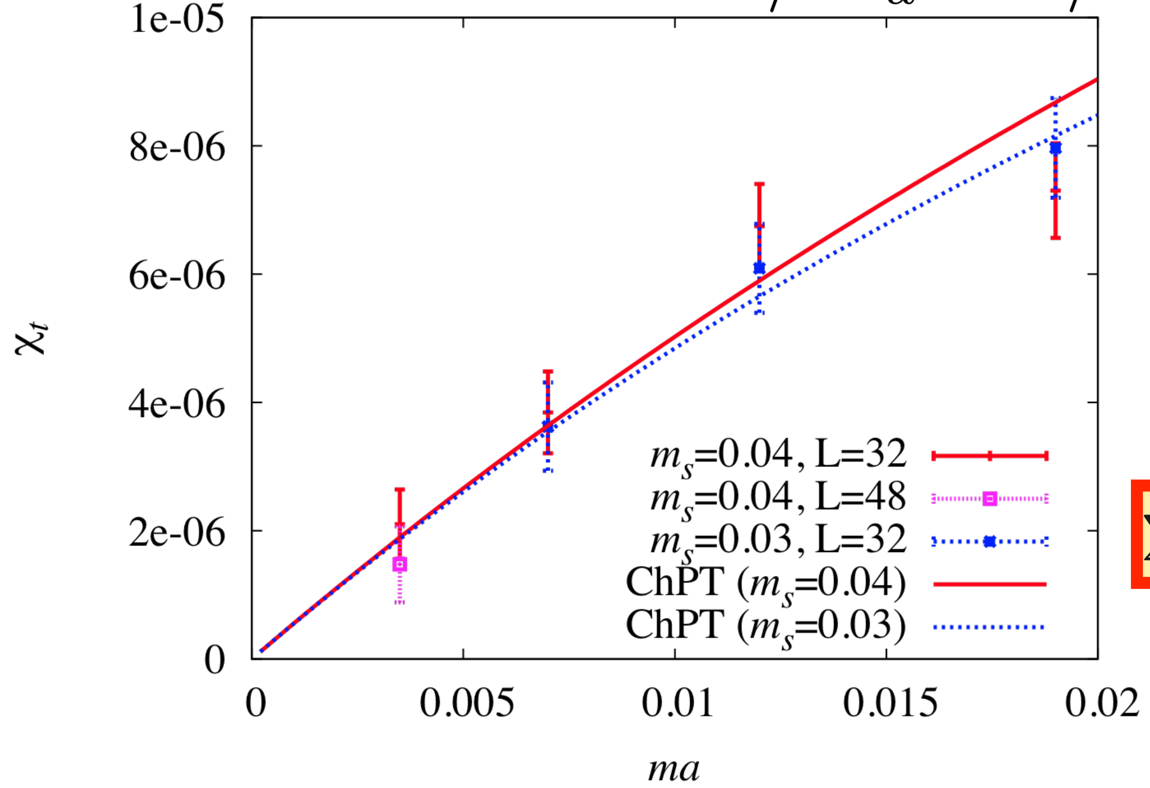


JLQCD 2014 with Mobius DW fermions

Topological susceptibility

$\chi_t^{Q\mathrm{-indep.}}$ agrees with ChPT prediction :

$$\chi_t^{\mathrm{ChPT}} = \frac{\Sigma}{1/m_u + 1/m_d + 1/m_s}$$



JLQCD 2014
with Mobius DW fermions

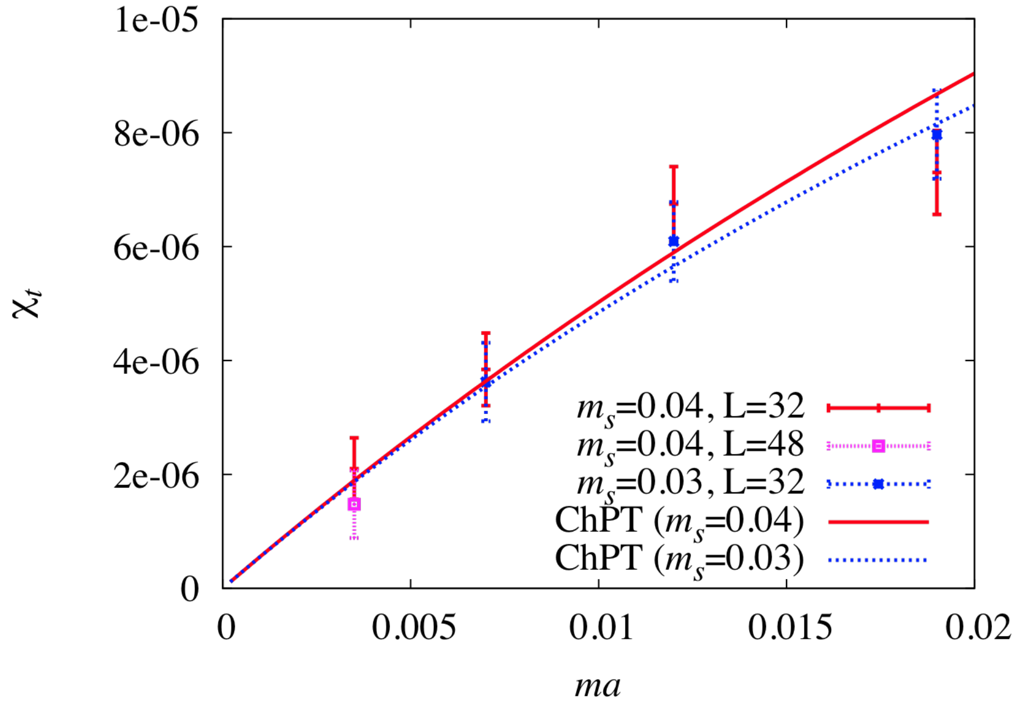
1-param fit
→

$\Sigma^{\overline{\mathrm{MS}}\ 1/3}(2\mathrm{GeV}) \sim 255\mathrm{MeV}.$

Topological susceptibility

$\chi_t^{Q-\text{indep.}}$ agrees with ChPT prediction.

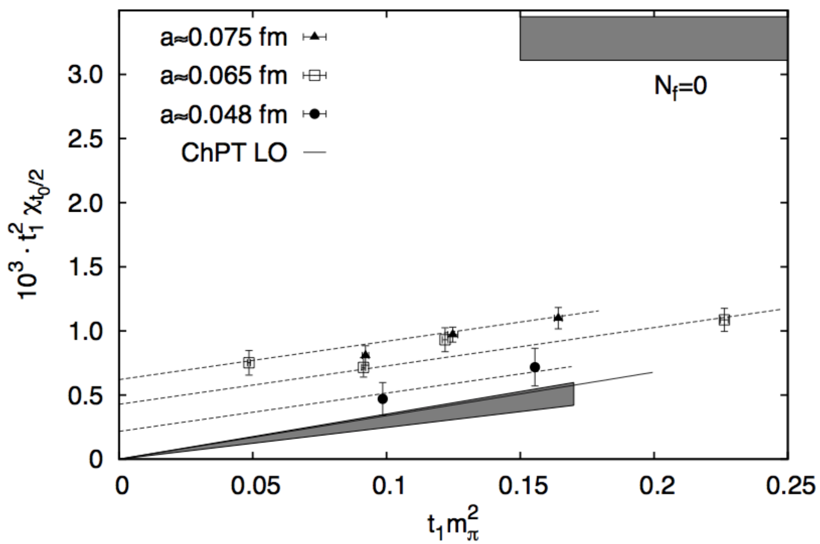
Our work w/ Mobius DW fermions



Cf. w/ Wilson fermions

[Bruno, Schaefer, Sommer 14]

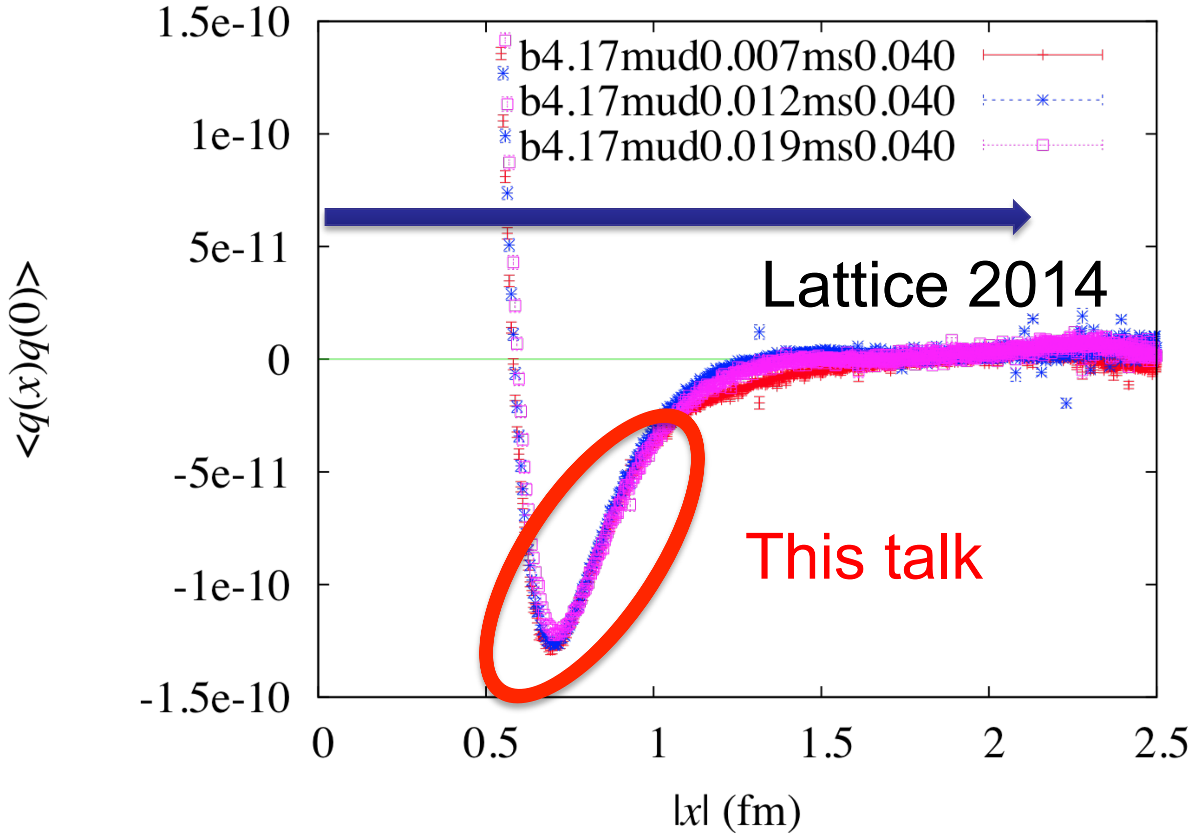
$N_f = 2$



Today's target

= Same correlator but shorter region.

$$\langle q(x)q(0)\rangle$$



1. Introduction

Eta-prime meson is interesting.

$$\eta' = \bar{\Psi}\gamma_5\Psi \quad (\Psi = (u,d,s)^T)$$

would-be a Nambu-Goldstone boson
for $U(1)_A$ symmetry but heavy
(due to anomaly) = $U(1)_A$ problem

[related talks by Tomiya, Cossu, Sharma Sat. morning]

Eta-prime meson is difficult.

$$\eta' = \bar{\Psi}\gamma_5\Psi \quad (\Psi = (u, d, s)^T)$$

difficult quantity to treat on a lattice because of "disconnected" part in the correlators :

$$\langle \eta'(x)\eta'(y)\rangle \simeq \langle \bar{\Psi}\gamma_5\Psi(x)\bar{\Psi}\gamma_5\Psi(y)\rangle + \langle \bar{\Psi}\gamma_5\Psi(x)\bar{\Psi}\gamma_5\Psi(y)\rangle$$

connected | disconnected

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Disconnected part is expensive.

$$\langle \bar{\Psi}\gamma_5\Psi(x)\bar{\Psi}\gamma_5\Psi(y)\rangle = \langle \mathrm{Tr} D^{-1}(x,y) D^{-1}(x,y)^* \rangle$$

➡ Enough to solve $D(x,z)s(z) = v(x), \quad s(z) = \delta_{y,z}$

$$\langle \bar{\Psi}\gamma_5\Psi(x)\bar{\Psi}\gamma_5\Psi(y)\rangle = \langle \mathrm{Tr}\gamma_5 D^{-1}(x,x) \mathrm{Tr}\gamma_5 D^{-1}(y,y)\rangle$$

➡ Requires $D(x,z)s(z) = v(x), \quad s(z) = \delta_{x,z}$

many times for different x and y.

Disconnected part is noisy

due to pion's contamination.

$$\langle \eta'(x)\eta'(y)\rangle \simeq \langle \bar{\Psi}\gamma_5\Psi(x)\bar{\Psi}\gamma_5\Psi(y)\rangle + \langle \bar{\Psi}\gamma_5\Psi(x)\bar{\Psi}\gamma_5\Psi(y)\rangle$$

$$= Ae^{-m_\pi|x-y|} - Ae^{-m_\pi|x-y|} + Be^{-m_{\eta'}|x-y|}$$

noisy　　noisy　　our target

In this work

We use

[Similar (quenched) works : Chowdhury et al. 2014, Ohki this conference 14th 14:00 BSM]

1. gluonic (topology density) operator :

$$\eta' = \frac{\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s}{\sqrt{3}} \rightarrow \frac{1}{32\pi^2} \mathrm{Tr}\epsilon_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma}$$

→ Free from pion's fluctuation, Numerically less expensive.

2. smeared links via Wilson flow

→ Noise reduction.

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2. Topological charge density and (short) Wilson flow
3. Lattice simulations
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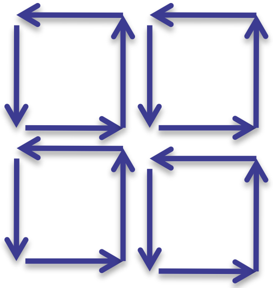
2. Topological charge density and (short) Wilson flow

Topological charge density

$$F_{\mu\nu}^{lat} =$$  $$q(x) = \frac{1}{32\pi^2}\mathrm{Tr}\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}^{lat}F_{\rho\sigma}^{lat}$$

1. $\langle q(x)q(y)\rangle$ is numerically cheap with FFT(fast Fourier transformation).
2. No (direct) pion's contamination.
3. Global topological charge

$$\sum_x q(x) = Q + O(a^2)$$

Wilson flow smearing

Wilson flow equation

$$\partial_t A_\mu(t,x) = -\frac{\partial S_{YM}}{\partial A_\mu}, \quad A_\mu(0,x) = A_\mu(x)$$

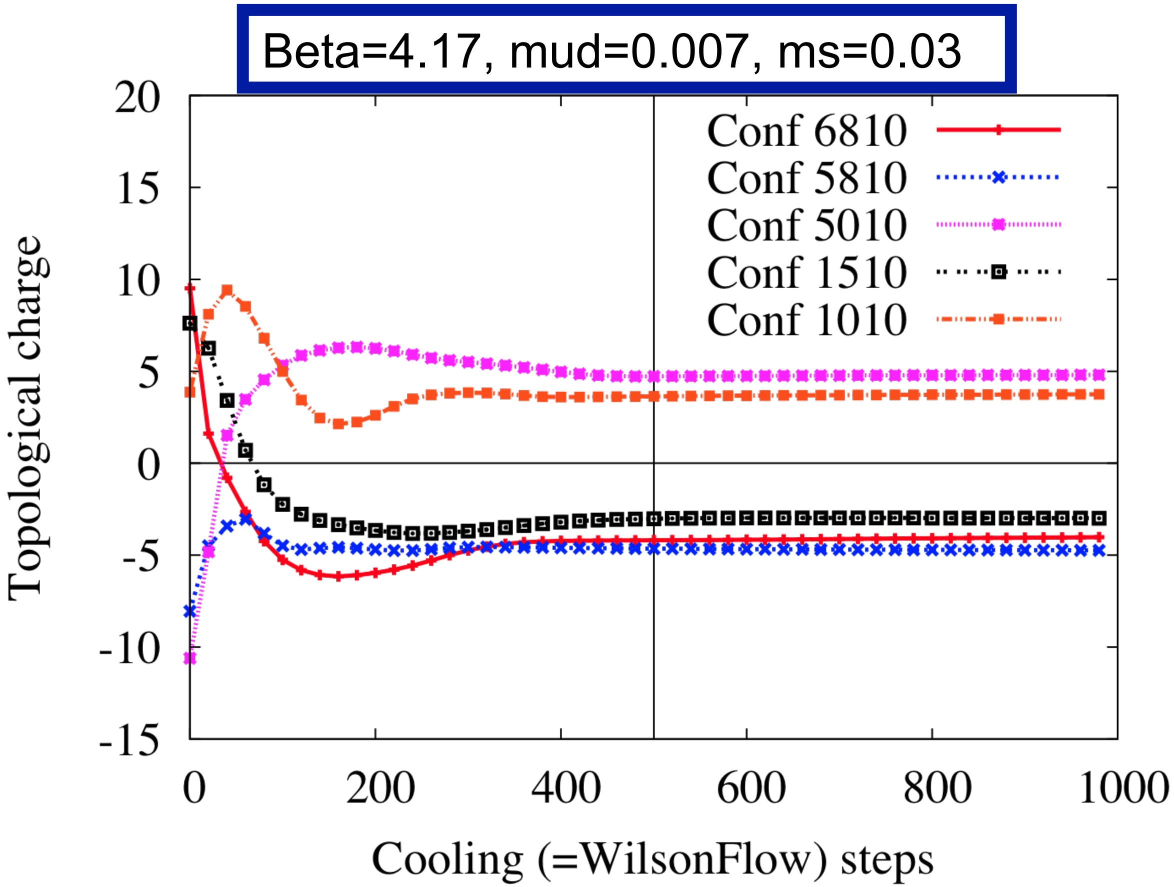
smoothed length $\quad d \sim \sqrt{8t}$

Gluonic correlators are UV finite.

[Luscher & Weisz 2011]

Toplogical charge

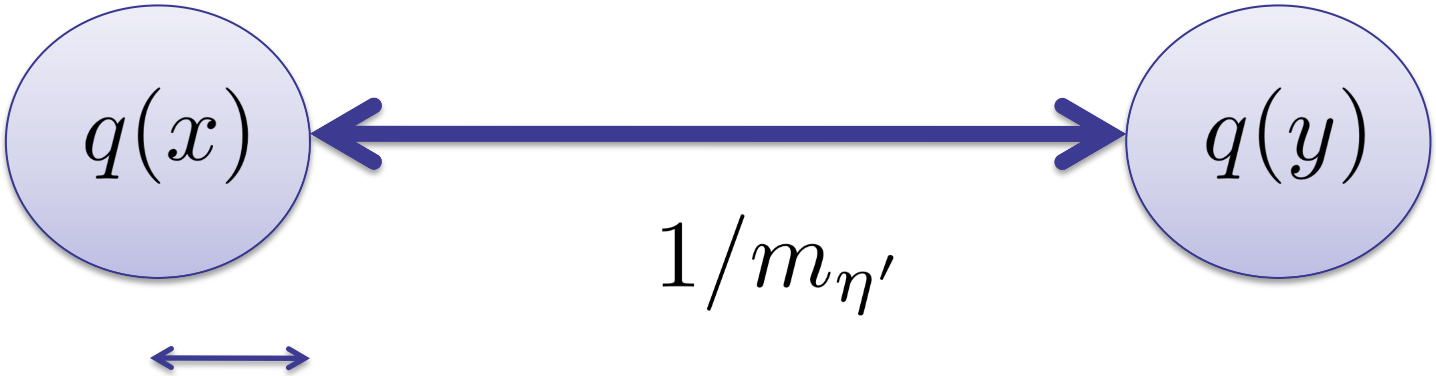
Flow time history of $\sum_x q(x) = Q + O(a^2)$



After Wilson flow around $\sqrt{8t} \sim 0.5\text{fm}$ topological charge does not change.

Wilson time flow has to be short…

Available window for flow time



$\sqrt{8t}$

$$\sqrt{8t} \ll (\text{fit range})/m_{\eta'}$$

Our choice : $\sqrt{8t} < 0.25\text{fm}, \quad (\text{fit range}) > 0.7\text{fm}.$

3. Lattice simulations

JLQCD's new project

Simulations on bigger & finer lattices started.

Computers @KEK: SR11000 (2 TFLOPS) + BG/L (57 TFLOPS)

→ SR16000 (55 TFLOPS) + BG/Q (1.2 PFLOPS)

Lattice cut-off : 1.8 GeV → 2.5, 3.6, 4.5 GeV

Lattice size : 16^3 x48 → 32^3 x64, 48^3x96, 64^3x128

(Physical size : 1.8 fm → 2.6 fm ~ 4 fm)

Fermion action : overlap fermion→ (improved) DomainWall

Pion mass : 200-400 MeV

Hitachi SR16000

IBM Blue Gene/Q

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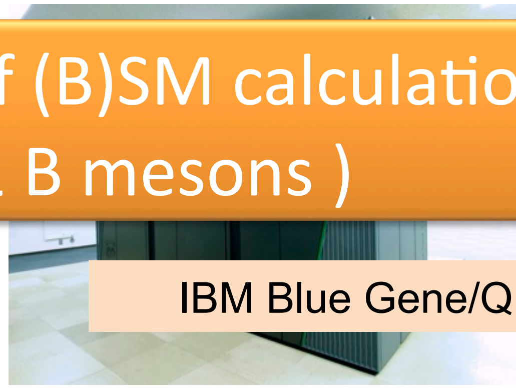
Our goal = 1% precision of (B)SM calculations
(in particular, D & B mesons)

Hitachi SR16000



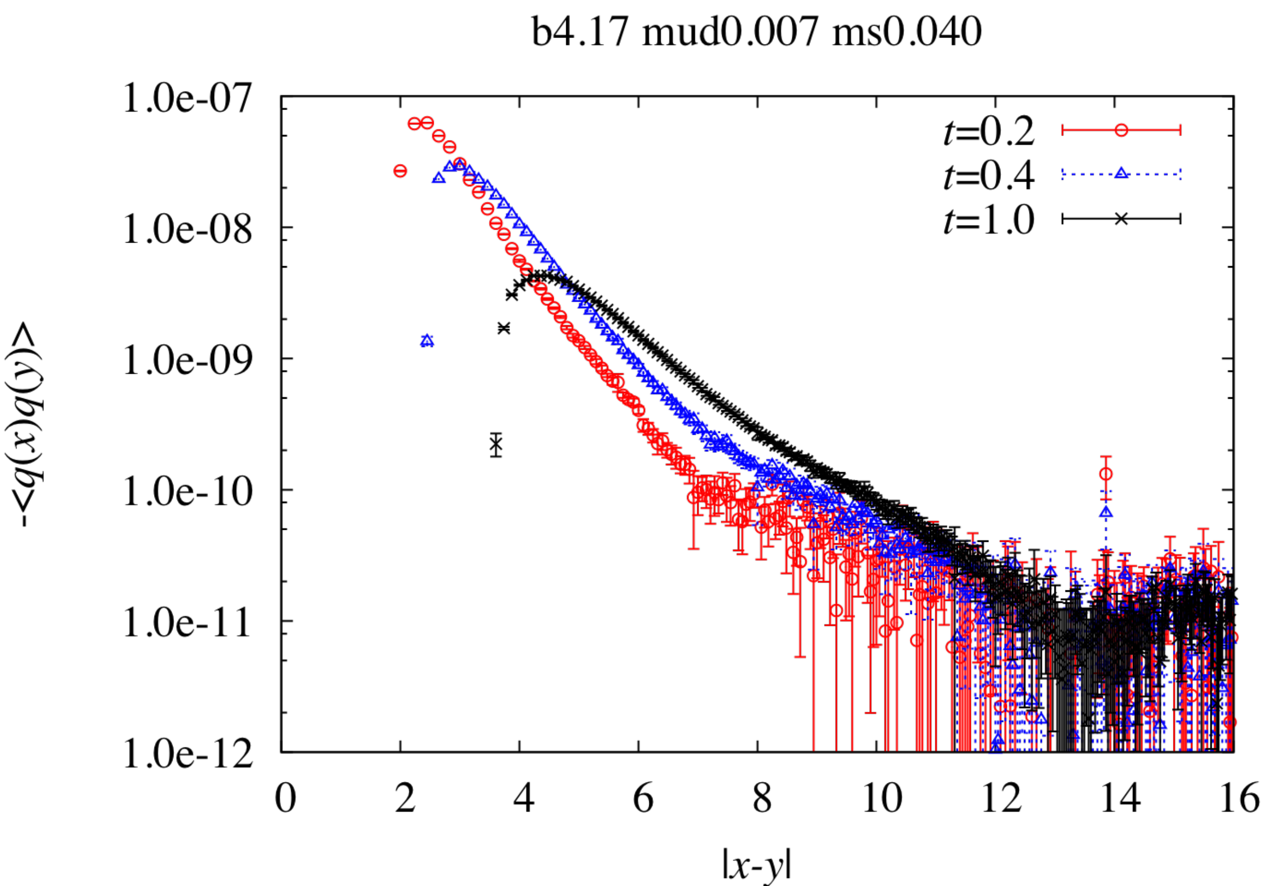
IBM Blue Gene/Q

Talks by JLQCD members

- B. Fahy, meson spectrum and decay const. 14th, 16:30
- K. Nakayama, Charmonium, 15th, 16:50
- T. Suzuki, Dmeson decays, 16th, 11:20
- N. Yamanaka, nucleon axial and tensor charges, 15th, 17:10
- G. Cossu, A. Tomiya, axial U(1) at finite T, 18th 9:00, 9:20
- T. Kaneko light meson form factors, 14th, 16:30
- S. Hashimoto, Dirac spectrum, 15th, poster
- M. Tomii, renormalization & OPE, 15th, 16:30

Topology density correlator

$$-\langle q(x)q(y)\rangle \sim AK_1(m_{\eta'}|x-y|)/|x-y|$$



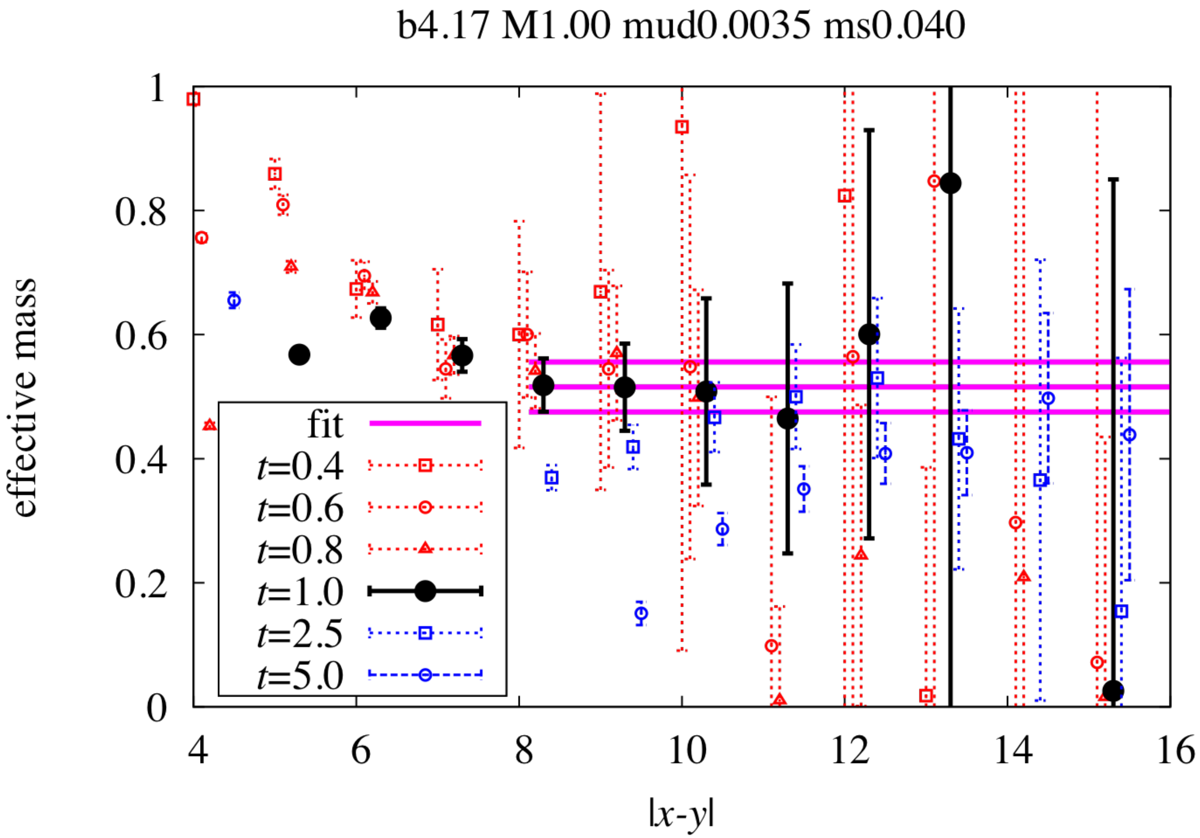
As the Wilson flow time t increases,

1. Better signals
2. Short $(\sim \sqrt{8t})$ correlation destroyed.

[similar observations w/ different coolings : de Forcrand et al. 1997, Horvath et al. 2003, 2005, Koma et al. 2005…]

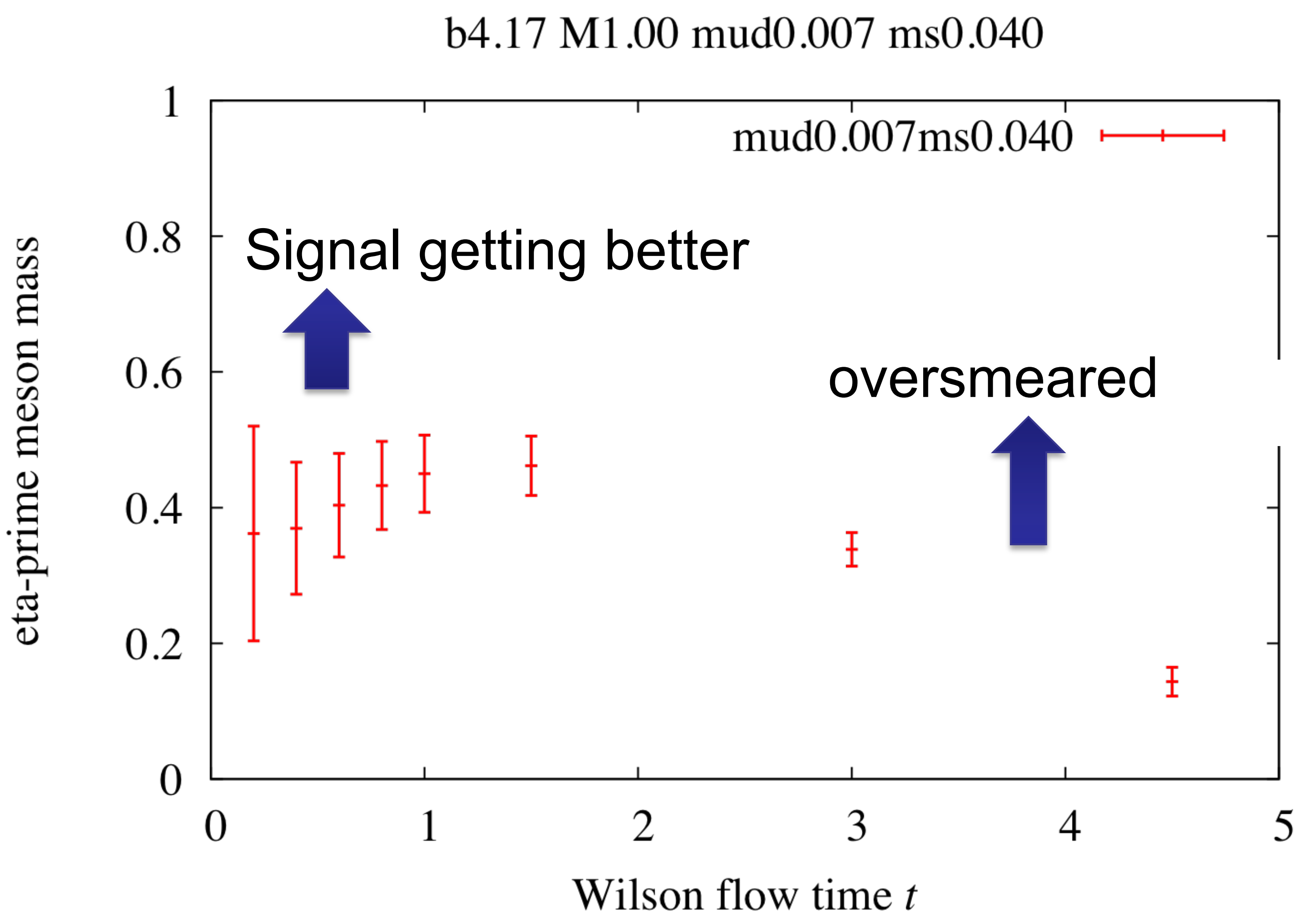
Effective mass plot

$$m_{\text{eff}}(|x-y|) \sim -\frac{1}{\Delta x} \ln \frac{\langle q(x+\Delta x) q(y) \rangle}{\langle q(x) q(y) \rangle}$$

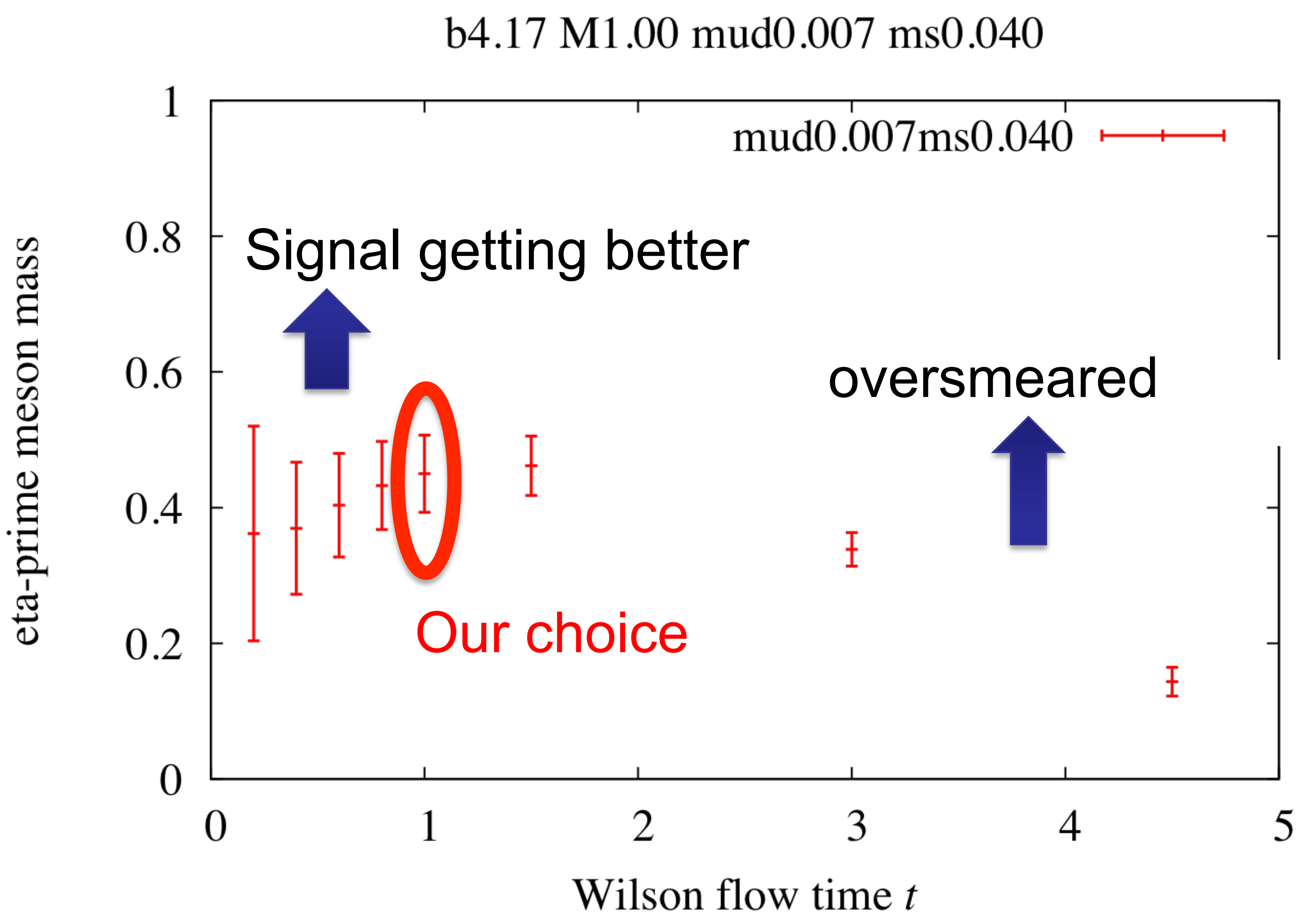


Fit range
|x-y| > 8 (~0.7 fm)

Flow time dependence of fit

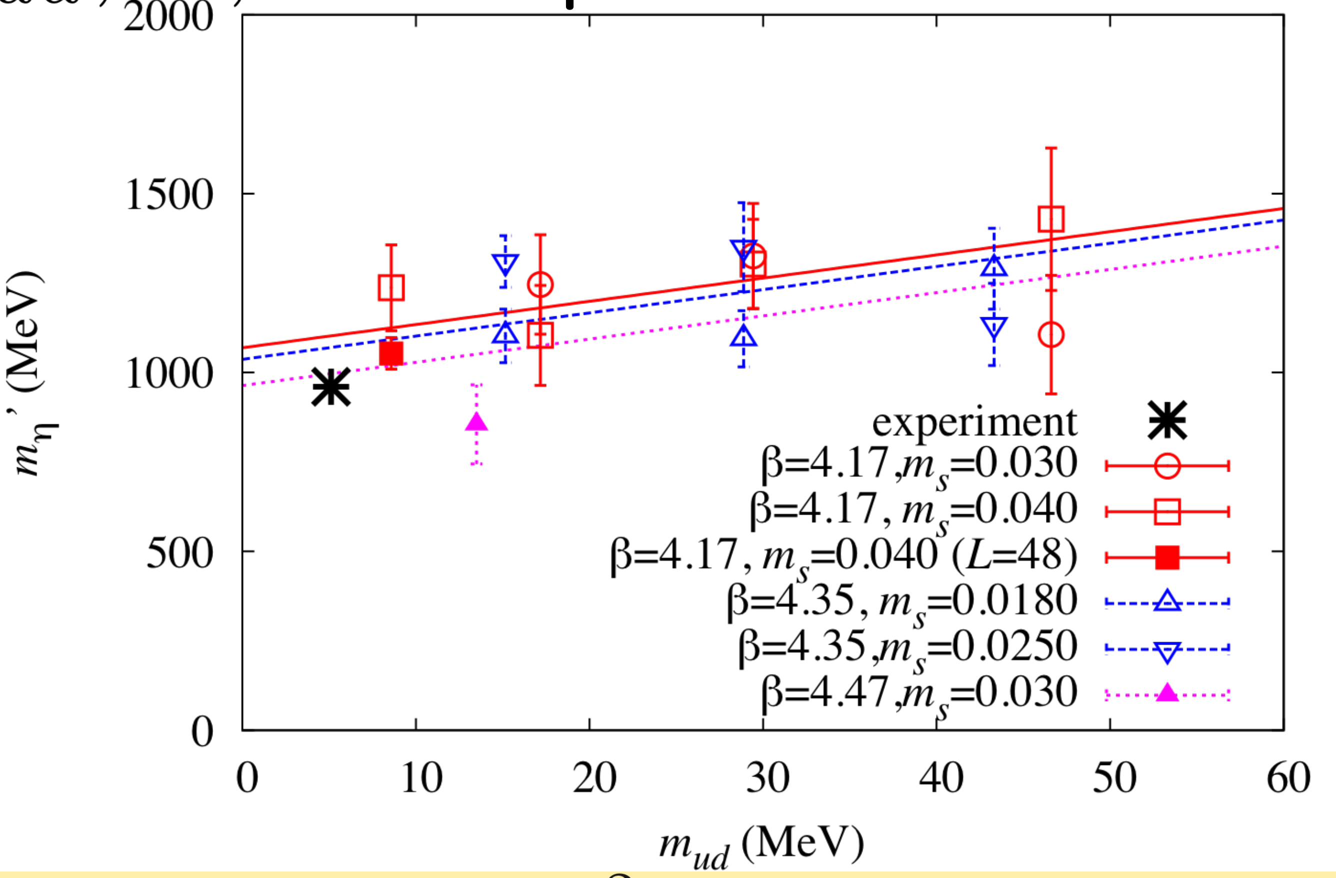


Flow time dependence of fit



Results

$m_{ud}, \ a, \ V$ dependences look mild.



linear fit in $m_{ud}, \quad a^2 \to m_{\eta'} = 1.00(11)$ GeV.

Mixing with eta meson

SU(3) breaking -> mixing with eta:

$$\eta_0 = \frac{\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s}{\sqrt{3}} = \eta' \cos\theta + \eta \sin\theta$$

So far we have neglected θ .

Cf. previous works

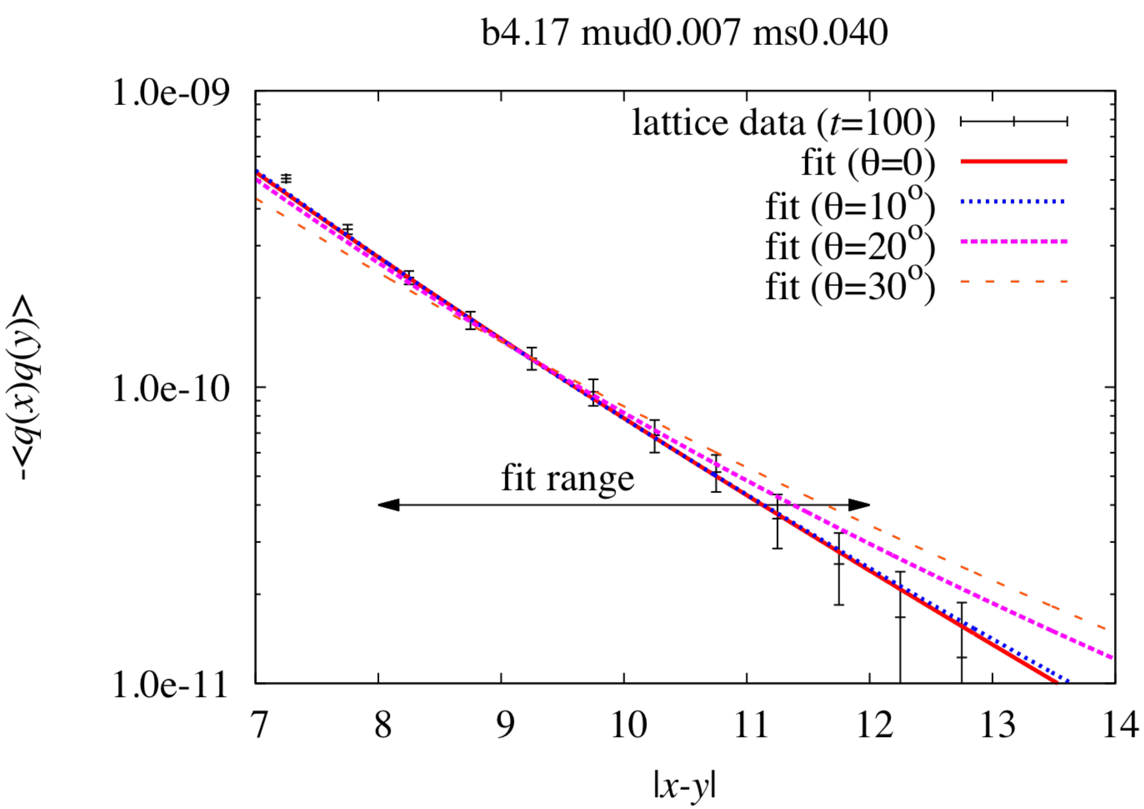
[RBC/UKQCD 2010, UKQCD 2012…]

fit combined with $\eta_8 = \eta' \sin\theta + \eta \cos\theta$

$$\to m_\eta \;\text{ and }\; m_{\eta'}$$

If | θ | < 20º, eta meson's contribution is negligible.

Modifying the fit curve w/ measured $m_{\eta'}$ and m_{η} (using GMOR relation)



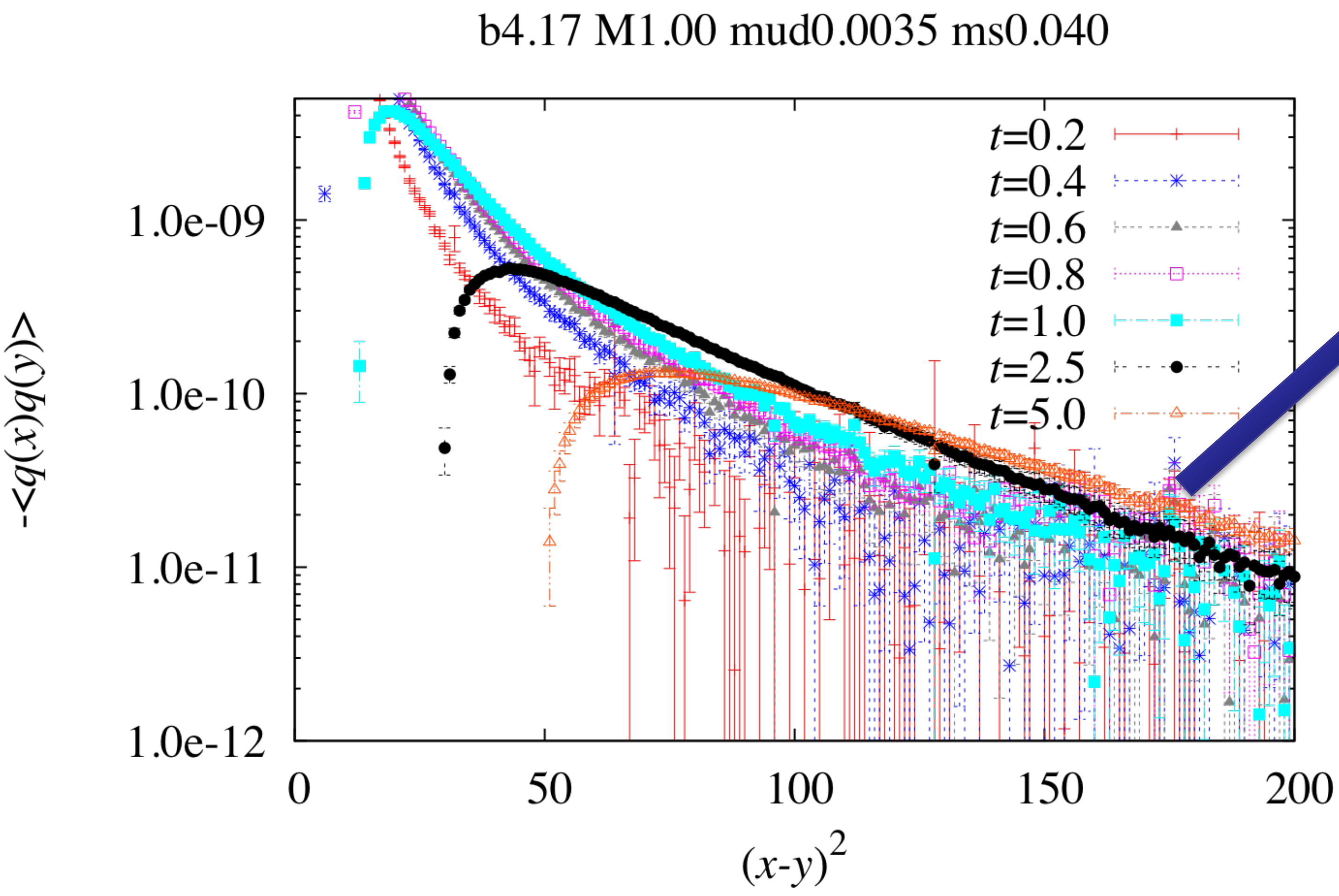
In our fit range, contribution from eta is small if $|\theta| < 20^o$

Cf. phenomenological value (at phys.point)

$$-\theta = 10^o\text{–}25^o$$

Still interesting to look for eta

Correlator at longer Wilson flow



m_η & θ ?

4. Summary

Summary

Gluonic operator (topology density)

+

Short Wilson flow

numerical cost is negligible,

free from pion's contamination,

good noise reduction.

→ eta-prime meson mass

(+ eta meson mass & mixing angle)

Numerical cost

<q(x)q(y)> + Wilson flow

= at most 1/10000

of disconnected fermon integrals,

(or essentially zero when you perform a Wilson flow for other measurements.)

Backup slide 1

Correlator w/o momentum insertion

$$\langle q(x)q(0)\rangle = \frac{A}{|x|}K_1(m_{\eta'}|x|)$$

* The data at same |x| are averaged.

Backup slide 3

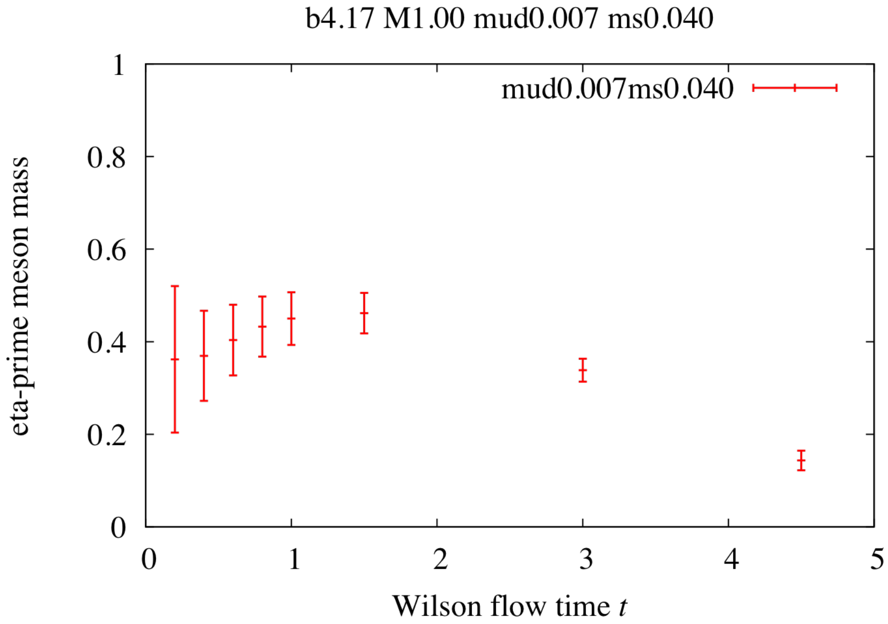
Non-trivial t dependence

Gaussian model [Alpha collaboration 2014]

-> correction $\sim e^{-(|x-y|/\sqrt{8t}-m\sqrt{8t})^2}\dfrac{m(8t)^{3/2}}{|x-y|^2}$

< 1% in our case.

In fact, t dependence looks mild.



Topological susceptibility

$\chi_t^{Q\text{–indep.}}$ agrees with ChPT prediction :

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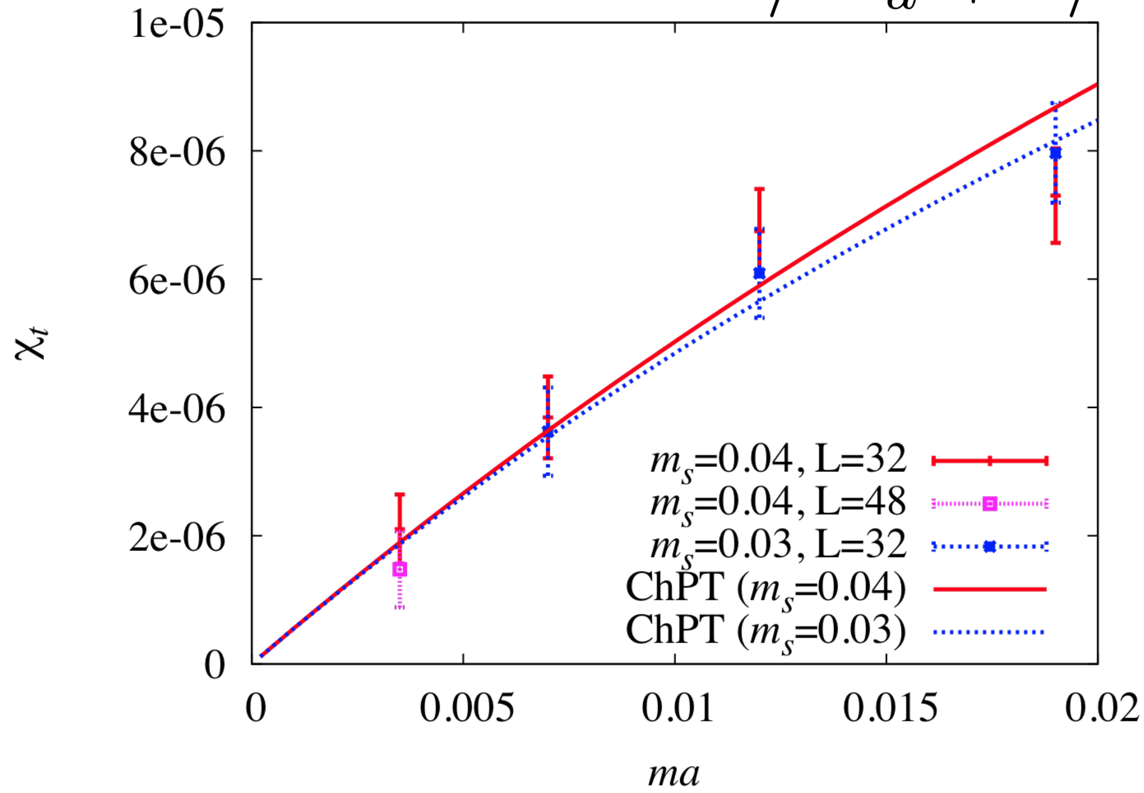


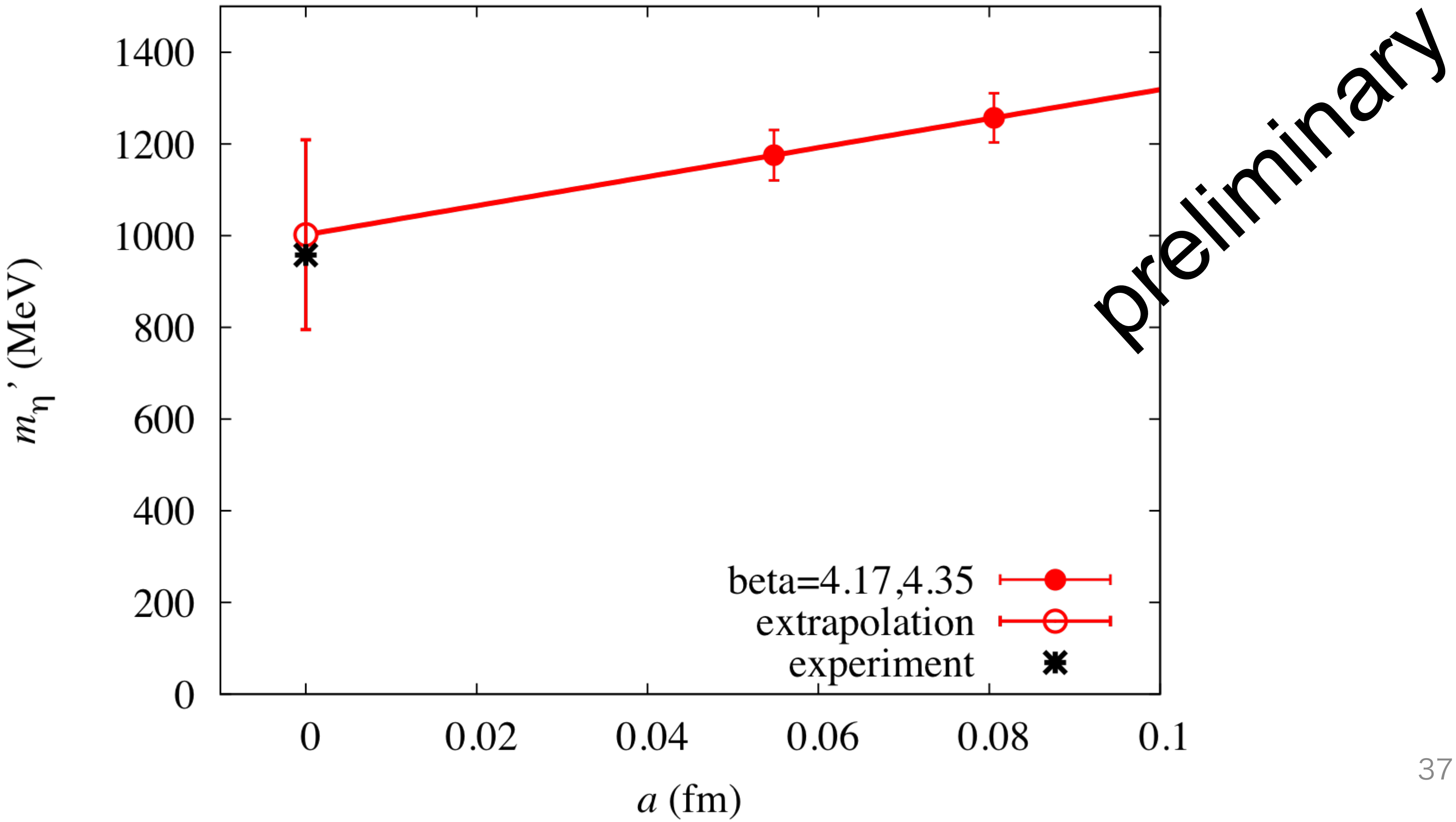
Fit with the chiral condensate

$$\Sigma^{1/3} = 250\mathrm{MeV}.$$

$\beta = 4.17\ (a^{-1} \sim 2.4\mathrm{GeV})$

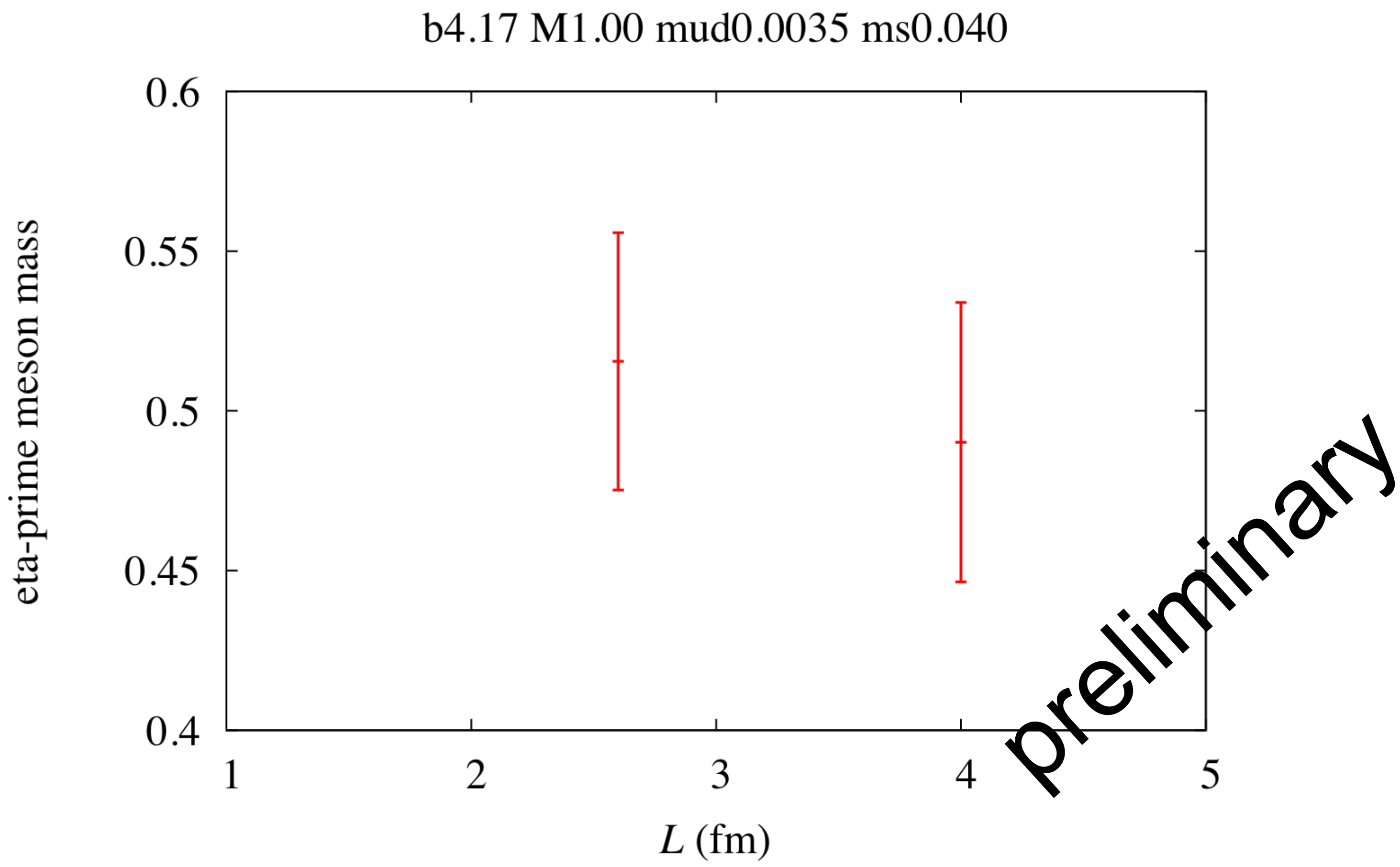
Other systematics

Continuum limit



Other systematics

Finite V scaling at the lightest mass



Other systematics

Chiral limit