Perturbative versus non-perturbative decoupling of heavy quarks

Francesco Knechtli

with M. Bruno, J. Finkenrath, B. Leder and R. Sommer

Bergische Universität Wuppertal

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Motivation

Including a dynamical charm quark

At which level of precision do effects of a dynamical quarks become visible? sub-percent effects in low energy $E \ll M_\mathrm{c}$ quantitites [Bruno, Finkenrath, FK, Leder and Sommer, 2015]
Here: compare renormalisation effects of charm to perturbation theory

Decoupling as an effective theory

QCD with N_q quarks, Λ_q (mass-independent scheme)
N_l light (massless) quarks; $N_\mathrm{q} - N_\mathrm{l}$ quarks of RGI mass M
Description in terms of a theory with only N_l light quarks

$$\mathcal{L}_\mathrm{dec} = \mathcal{L}_{\mathrm{QCD}_{N_\mathrm{l}}} + (\Lambda_\mathrm{q}/M)^2 \sum_i \omega_i \Phi_i + \mathrm{O}((\Lambda_\mathrm{q}/M)^4)$$

Heavy quarks: coupling $\overline{g}_\mathrm{dec}(\mu/\Lambda_\mathrm{dec})$ and power corrections

Factorisation formula

Ratio of Λ parameters

In the effective theory $\Lambda_{\rm l} = \Lambda_{\rm dec}(M, \Lambda_{\rm q})$

$$P_{\rm l,q}(M/\Lambda_{\rm q}) \;=\; \Lambda_{\rm dec}(M,\Lambda_{\rm q})/\Lambda_{\rm q}$$

P can be computed in perturbation theory

Factorisation formula

$m^{\rm had}$ is a hadron mass or an hadronic scale like r_0^{-1}, $t_0^{-1/2}$, w_0^{-1}
At leading order: $m_{\rm q}^{\rm had} = m_{\rm l}^{\rm had} + {\rm O}((\Lambda_{\rm q}/M)^2)$

$$\frac{m_{\rm q}^{\rm had}(M)}{m_{\rm q}^{\rm had}(0)} \;=\; Q_{\rm l,q}^{\rm had} \times P_{\rm l,q}(M/\Lambda_{\rm q}) + {\rm O}((\Lambda_{\rm q}/M)^2)$$

$Q_{\rm l,q}^{\rm had} = \frac{m_{\rm l}^{\rm had}/\Lambda_{\rm l}}{m_{\rm q}^{\rm had}(0)/\Lambda_{\rm q}}$ is independent of M

Matching of couplings

Leading order

Relation between the $\overline{\mathrm{MS}}$-couplings $\bar{g}_{\mathrm{dec}}(\mu/\Lambda_{\mathrm{dec}}) = \bar{g}_{\mathrm{l}}(\mu/\Lambda_{\mathrm{l}})$ and $\bar{g}(\mu/\Lambda) \equiv \bar{g}_{\mathrm{q}}(\mu/\Lambda_{\mathrm{q}})$

$$\bar{g}^2_{\mathrm{dec}}(\mu/\Lambda_{\mathrm{dec}}) = \bar{g}^2(\mu/\Lambda) + \mathrm{O}(\bar{g}^4(\mu/\Lambda)) \,.$$

At $\mu = m_*$ defined through $\overline{m}(m_*) = m_*$ [Weinberg, 1980; Bernreuther and Wetzel, 1982] the relation is known up to four loops [Grozin, Hoeschele, Hoff and Steinhauser, 2011; Chetyrkin, Kühn and Sturm, 2006]

$$\begin{aligned} \bar{g}^2_{\mathrm{dec}}(m_*/\Lambda_{\mathrm{dec}}) &= \bar{g}^2(m_*/\Lambda)\, C(\bar{g}(m_*/\Lambda)) \\ & \quad C(g) = 1 + c_2 g^4 + c_3 g^6 + \ldots \quad (c_1 = 0) \end{aligned}$$

Ratio of Λ parameters

Ratio $P = \Lambda_{\mathrm{dec}}(M,\Lambda)/\Lambda$

Using $\frac{\Lambda}{\mu} = \exp\left\{-\int^{\bar{g}} \mathrm{d}x \frac{1}{\beta(x)}\right\} = \exp(I_g(\bar{g}))$:

$$P_{\mathrm{l,q}}(M/\Lambda) \;=\; \exp\left\{I_g^{\mathrm{l}}(g_* \sqrt{C(g_*)}) - I_g^{\mathrm{q}}(g_*)\right\}$$

where $\exp(I_g^i(\bar{g}))=\left(b_0(N_i)\bar{g}^2\right)^{-b_1(N_i)/(2b_0(N_i)^2)}\mathrm{e}^{-1/(2b_0(N_i)\bar{g}^2)}\times[1+O(\bar{g}^2)]$

Using $\frac{M}{\overline{m}} = \exp\left\{-\int^{\bar{g}} \mathrm{d}x \frac{\tau(x)}{\beta(x)}\right\}$, the coupling $g_* = \bar{g}(\mu = m_*)$ is computed by inverting (M corresponds to $\overline{m} = m_*$)

$$\frac{\Lambda}{M} \;=\; \exp\left\{-\int^{g_*(M/\Lambda)} \mathrm{d}x \, \frac{1-\tau_{\mathrm{q}}(x)}{\beta_{\mathrm{q}}(x)}\right\}$$

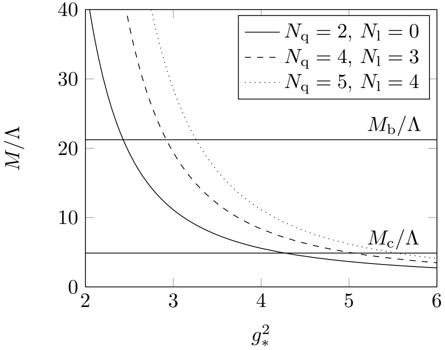
The coupling g_*

Loop expansion

$\beta_{\rm q}(\bar g) \overset{\bar g\to 0}{\sim} -\bar g^3\left\{b_0+\bar g^2 b_1+\ldots\right\};\ b_0=\frac{1}{(4\pi)^2}\left(11-\frac{2}{3}N_{\rm q}\right),\ b_1=\frac{1}{(4\pi)^4}\left(102-\frac{38}{3}N_{\rm q}\right)$

$\tau_{\rm q}(\bar g) \overset{\bar g\to 0}{\sim} -\bar g^2\left\{d_0+\bar g^2 d_1+\ldots\right\};\ d_0=8/(4\pi)^2$

e.g. two-loop: $\frac{1-\tau(x)}{\beta(x)} = -\frac{1}{b_0 x^3}+\frac{b_1}{b_0^2 x}-\frac{d_0}{b_0 x}+{\rm O}(x)$



4-loop relation

$\frac{\Lambda}{M}=$

$\exp\left\{-\int^{g_*(M/\Lambda)} {\rm d}x\ \frac{1-\tau_{\rm q}(x)}{\beta_{\rm q}(x)}\right\}$

The factor $P_{\mathrm{l,q}}$

$$P_{\mathrm{l,q}} = \frac{1}{k} \underbrace{\left(\frac{M}{\Lambda}\right)^{\eta_0}}_{\text{1-loop ``approx''}} \ell^{\eta_1^{\mathrm{M}}/2b_0(N_\mathrm{q})} \times \left[1 + \mathrm{O}\left(\frac{\log \ell}{\ell}\right)\right]$$

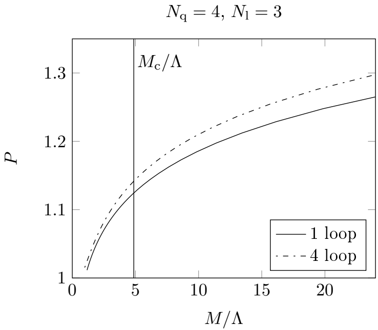
where $\ell = \log(M/\Lambda)$ and $(\tilde{b}_i(n) = b_i(n)/b_0(n))$

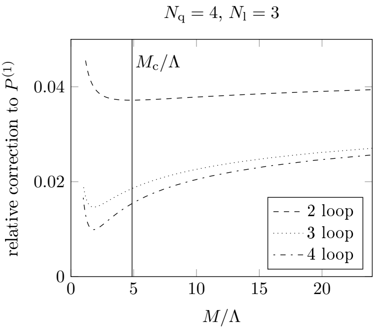
$$\log k = \frac{\tilde{b}_1(N_\mathrm{q})}{2b_0(N_\mathrm{q})} \log 2 - \frac{\tilde{b}_1(N_\mathrm{l})}{2b_0(N_\mathrm{l})} \log(2b_0(N_\mathrm{q})/b_0(N_\mathrm{l}))$$

N_q	N_l	η_0	$\eta_1^{\mathrm{M}}/2b_0(N_\mathrm{q})$	$\log(k)$
2	0	0.121212	0.010829	0.046655
4	3	0.074074	0.017284	0.012756
5	4	0.080000	0.025252	0.002622

Mass dependence of $P_{3,4}$

Decoupling of charm quark



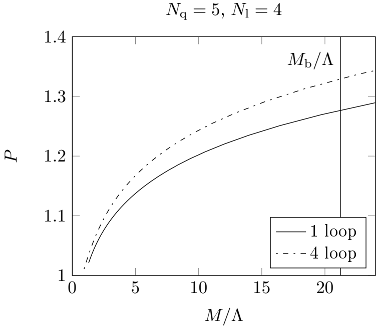


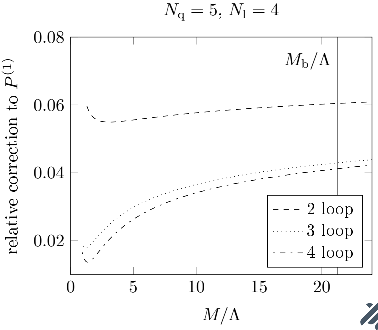
One-loop "approximation" $P^{(1)}_{l,q} = (M/\Lambda)^{\eta_0}$,

$\eta_0 = 1 - \frac{b_0(N_q)}{b_0(N_l)} > 0$ very close to P (accidentally)

Mass dependence of $P_{4,5}$

Decoupling of bottom quark





At $M = M_b$, perturbation theory can be used to compute $P_{4,5}$

Simulations

The model

We study a theory with $N_\mathrm{q} = 2$ heavy quarks and compare it to Yang-Mills theory ($N_\mathrm{l} = 0$)

We simulate $N_\mathrm{q} = 2$ O(a) improved Wilson quarks with plaquette gauge action

Ensembles

β	a [fm]	BC	$T \times L^3$	$M/\Lambda_{\overline{\mathrm{MS}}}$	kMDU	τ_{exp}
5.3	$0.0658(10)$	p	64×32^3	0.638(46)	1	0.04
		p	64×32^3	1.308(95)	2	0.08
		p	64×32^3	2.60(19)	2	0.02
5.5	$0.0486(\ 7)$	o	120×32^3	0.630(46)	8	0.3
		o	120×32^3	1.282(93)	8	0.1
		p	96×48^3	2.45(18)	4	0.1
5.7	$0.0341(\ 5)$	o	192×48^3	0.587(43)	4	0.3
		o	192×48^3	1.277(94)	4	0.3
		o	192×48^3	2.50(18)	8	0.2

Observables

Scale t_0

Wilson flow: scale t_0 [Lüscher, 2010]

$$\mathcal{E}(t_0) = 0.3\,, \quad \mathcal{E}(t) = t^2 \langle E(x,t) \rangle$$

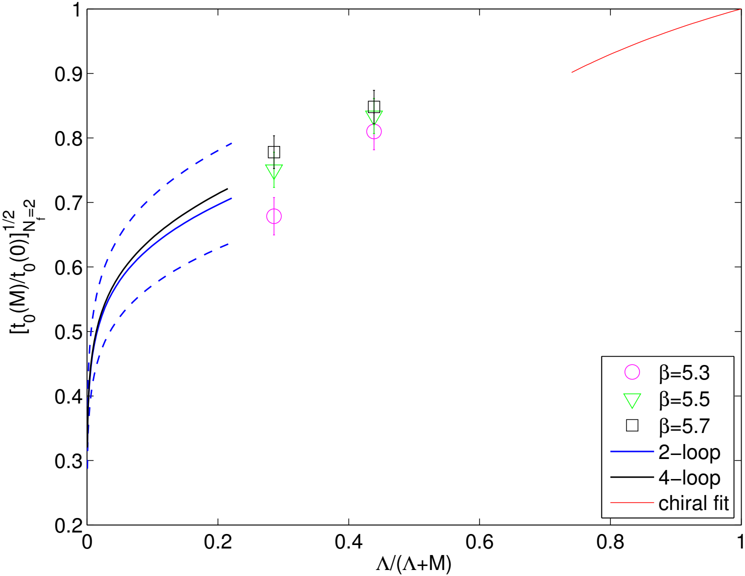
Factorisation formula for the mass-dependence

$$\sqrt{t_0(M)/t_0(0)} \;=\; 1/(P_{0,2}Q_{0,2}) + \mathrm{O}((\Lambda/M)^2)$$

We use $Q_{0,2} = [\sqrt{t_0(0)}\Lambda]_{N_\mathrm{q}=2}/[\sqrt{t_0}\Lambda]_{N_\mathrm{q}=0} \simeq 1.19(13)$ [ALPHA; Sommer, Lattice 2013]

We compute from simulations at lattice spacing a
$\sqrt{t_0(M)/t_0(0)} = \sqrt{t_0(M)/t_0(M_\mathrm{ref})} \times \lim_{a\to 0} \sqrt{t_0(M_\mathrm{ref})/t_0(0)}$
($M_\mathrm{ref}/\Lambda = 0.59$; continuum limit from $\beta = 5.3$, 5.5)

Results



1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0
0.2
0.4
0.6
0.8
1
$[t_0(M)/t_0(0)]^{1/2}_{N_f=2}$
Λ/(Λ+M)
β=5.3
β=5.5
β=5.7
2–loop
4–loop
chiral fit

Conclusions

Summary

- decoupling of heavy quarks leaves traces through renormalization and power corrections
- perturbation theory "converges" fast for decoupling of heavy quarks at leading order in $1/M$
- factorisation formula for the mass dependence of hadronic scales is confirmed by simulations of $N_\mathrm{q} = 2$ $\mathrm{O}(a)$ improved Wilson quarks at $a = 0.034\,\mathrm{fm}$ and $M \approx M_\mathrm{c}/2$

Outlook

- combined continuum limit with twisted mass simulations
- simulations at M_c using $a = 0.034 \ldots 0.018\,\mathrm{fm}$
- estimates of the effects of sea charm quark on observables like f_{D_s}, charmonium, charmonium-like exotics