An application of the hybrid Monte Carlo algorithm for realized stochastic volatility model

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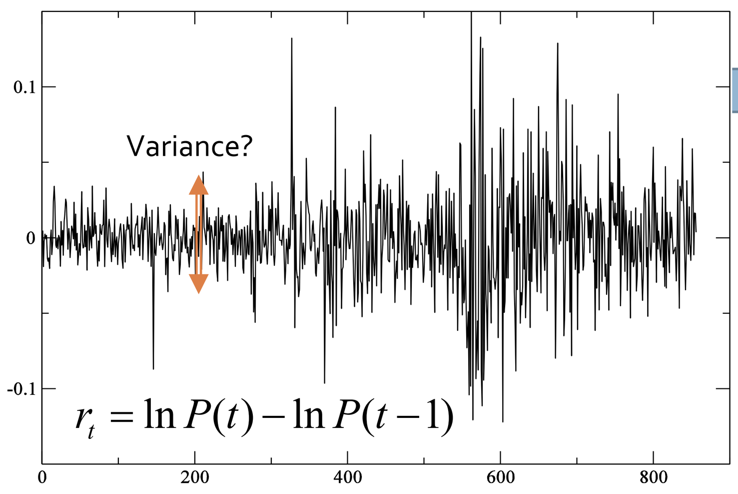
Introduction

Daily Return Time Series of Nissan Motor Co.



From 2006 July 4 to 2009 Dec 30

How can we estimate time-varying volatility (variance) from return data?

Volatility plays a crucial role in finance, such as in risk management.

We use the realized stochastic volatility (RSV) model which combines the stochastic volatility model and the realized volatility (RV).

Realized Stochastic Volatility Model

Return time series by RSV model

$$r_t = \sigma_t \varepsilon_t, \qquad \varepsilon_t \sim N(0,1)$$

$$h_t = \mu + \phi(h_{t-1} - \mu) + \eta_t \qquad h_t = \ln \sigma_t^2$$

$$\ln RV_t = \xi + h_{t-1} + u_t \qquad \eta_t \sim N(0, \sigma_\eta^2)$$

$$u_t \sim N(0, \sigma_u^2)$$

Parameters of the model

$$\mu, \phi, \sigma_\eta, \xi, \sigma_u$$

Bayesian Inference

We employ the Bayesian inference for the parameter estimation of the model.

$$\theta = \omega, \phi, \sigma_\eta$$

Defined from the likelihood function

$$\langle \theta \rangle = \int \theta f(r_t \mid \theta, h) dh_1 \cdots dh_T d\theta$$

We integrate this integral by the Markov Chain Monte Carlo.

The volatility variables are updated by the HMC method that can be paralleled.

Hybrid Monte Carlo Algorithm

$$H(p,h) = \frac{1}{2}\sum_{i=1}^{T} p_i^2 - \ln f(r_t \mid \theta, h)$$

Volatility candidates are generated by solving the Hamilton equations of motion.

$$\begin{cases} \dfrac{dh_i}{d\tau} = \dfrac{\partial H(p,h)}{\partial p_i} \\ \dfrac{dp_i}{d\tau} = -\dfrac{\partial H(p,h)}{\partial h_i} \end{cases}$$

Numerical integration by the leapfrog method.

Volatility candidates are accepted according to the Metropolis test.

GPU coding

The HMC algorithm can be paralleled. We perform the HMC by the GPU and use OpenACC for GPU coding. The OpenACC program with appropriate directive based coding can achieve the similar speedup with CUDA Fortran.

GPU coding environment

CPU: Intel i7-4770 3.4GHz

GPU: GeForce GTX 760

Compiler：PGI Fortran（PGI 14.6）

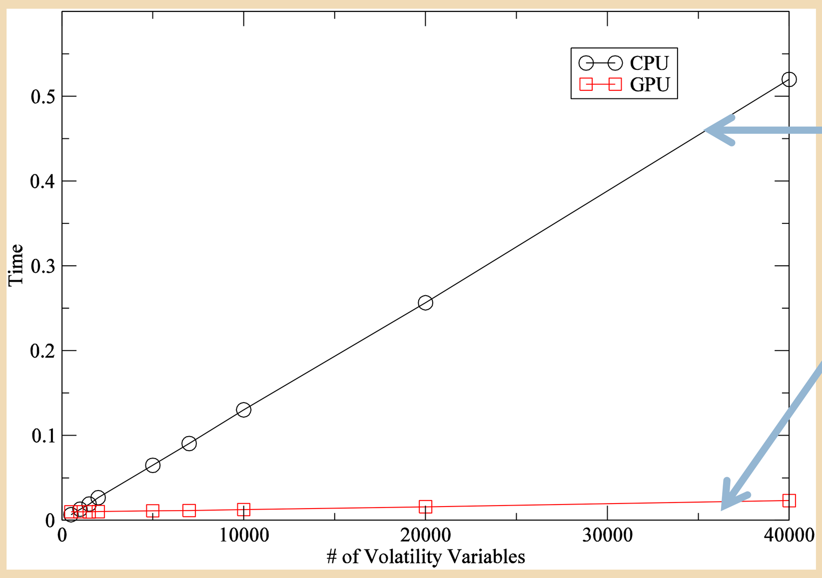
CUDA6.0

HMC by OpenACC

```
!$acc data copy(h,p)
do j=1, n                                      Repeat n times
  !$acc kernels
  h_i(τ+Δτ/2) = h_i(t+) + p_i(τ)Δτ/2            i=1,T
  p_i(τ+Δτ) = p_i(τ) - ∂H/∂h_i Δτ               i=1,T
  h_i(τ+Δτ) = h_i(t+) + p_i(τ+Δτ)Δτ/2          i=1,T
  !$acc end kernels
enddo
!$acc end data
```

Comparison of Computational time

900 trajectories with n=200 step-times

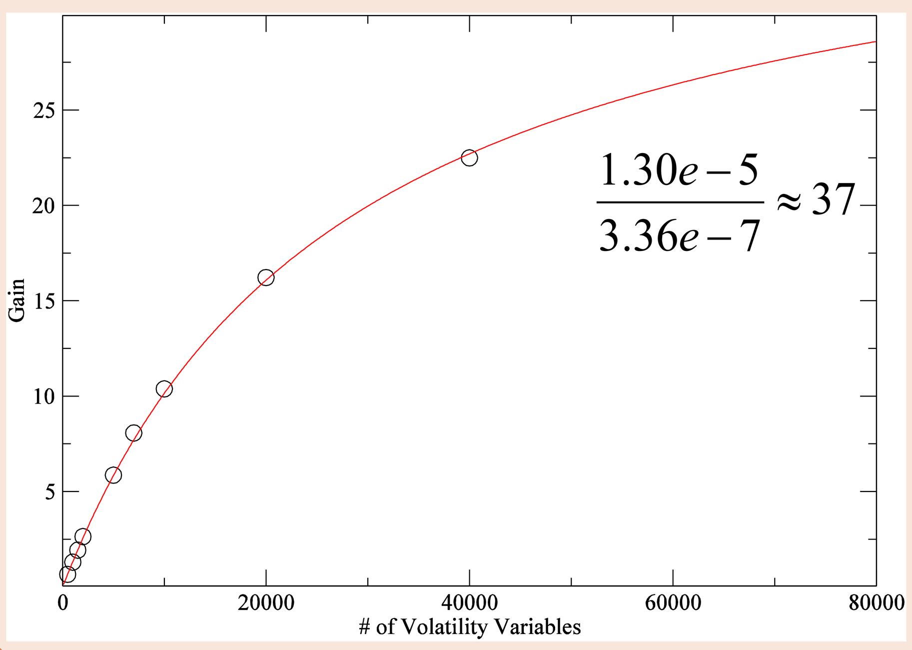


Fitting to f(T) =a+c*T

	a	c
CPU	-1.85e-4	1.30e-5
GPU(OpenACC)	9.41e-3	3.36e-7

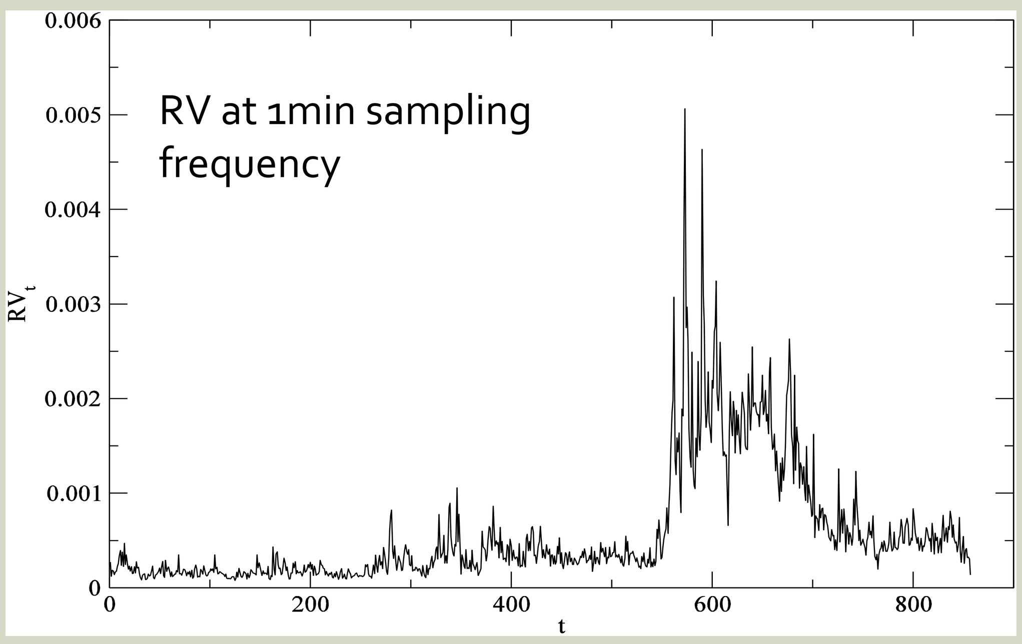
Gain of GPU over CPU

Gain=CPUTIME(CPU)/CPUTIME(GPU)



Empirical result

Data used: Daily Return and Realized volatility at 1min sampling frequency of Nissan Motor Co.



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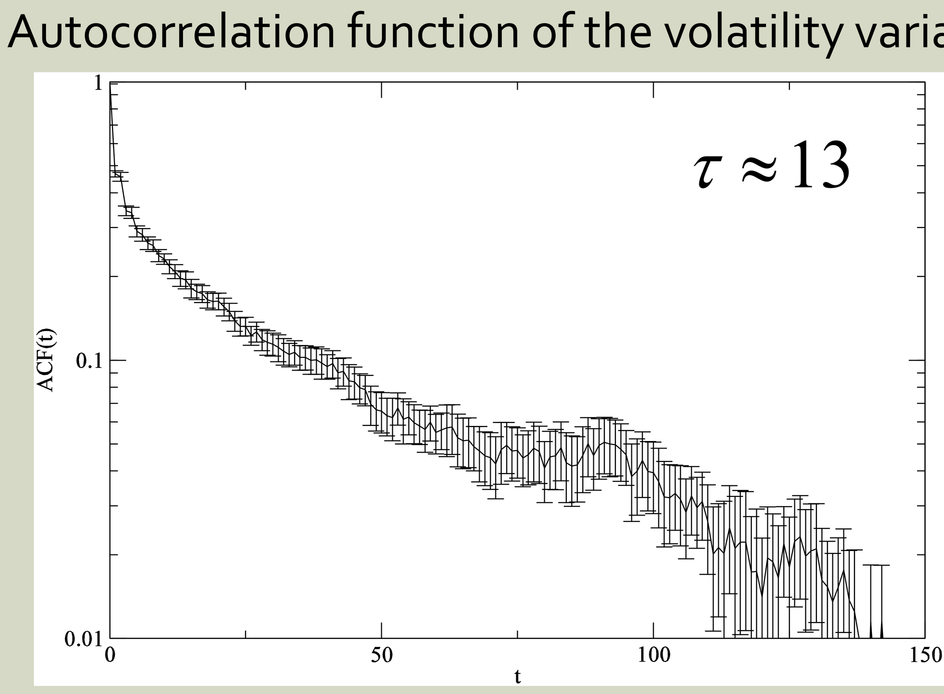
RV = sum of squared high-frequency returns

Autocorrelation function of the volatility variable (h_{100})



$\tau \approx 100$(Metropolis)

Parameters obtained by the Markov Chain Monte Carlo.

μ	ϕ	σ_η	ξ	σ_u
0.9849(1)	-7.617(5)	0.0237(2)	-0.370(3)	0.0465(1)

Conclusions

- We have performed the Bayesian estimation of the realized stochastic volatility model.
- The hybrid Monte Carlo method was used for the Bayesian estimation and performed on GPU. We used the OpenACC for GPU coding.
- It is found that the GPU (GTX760) can be faster than the CPU (Intel i7-4770 3.4GHz) when the size of time series is big.
- It might be interesting to employ a multivariate stochastic volatility model that has the large number of volatility variables.

References

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