Combining ordinary and topological finite volume effects for fixed topology simulations

Arthur Dromard[1]
and
Wolfgang Bietenholz[2], Urs Geber[2], Héctor Mejía-Díaz[2], Marc Wagner[1]

[1]University of Frankfurt am Main
[2]Universidad Nacional Autónoma de México

JOHANN WOLFGANG GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Outline

1. Introduction
   - Motivation
2. Physical results from fixed topology simulation
   - Correlators and partition function at fixed topology
   - Extracting physical mass from fixed topology
   - Numerical results: pure SU(2)
   - Extracting topological susceptibility from fixed topology
3. Including ordinary finite volume effects
   - Ordinary finite volume effects
   - Combining ordinary and topological finite volume effects

Outline

Motivation

Problem 1

Topology freezes for a too small lattice spacing $a < 0.05fm$[a]

[a] Luscher, Martin JHEP 1008 (2010) 071

Problem 2

Topology fixed in purpose for some algorithms to avoid technical problems (Overlap)[a]

[a] H. Fukaya, *et al.* *Phys. Rev.* **D 73** (2006) 014503.

Problem 3

Mixed action: different near-zero modes between sea Wilson quarks and valences overlap quarks: use only trivial topology (Q=0) [a]

[a] K. Cichy, G. Herdoiza and K. Jansen, Nucl. Phys. B **847**, 179 (2011)

Outline

Topology and Correlators

To take into account topology in your action

- Action with topological term:

$$S_E(\theta) = S_E - i\theta \frac{g^2}{32\pi^2} \int F_{\mu\nu}\tilde{F}_{\mu\nu} = S_E - i\theta Q[A]$$

- θ-term does not modify EOM
- Experimental measure of θ_{QCD}: $\theta_{QCD} \approx 0$

Path integral at fixed Q does not correspond to a physical theory

- No Hamiltonian!
- We can still defined a partition function, correlators and masses at fixed topology.

$$Z_Q = \int Z(\theta) e^{(-i\theta Q)}$$

$$Z_Q C_Q = \int Z(\theta) C(\theta) e^{(-i\theta Q)}$$

Outline

Mass at fixed topological charge

- The mass at fixed topology of particles is given by the BCNW equation [1]:

$$\boxed{M_Q = M(0) + \frac{M''(0)}{2\chi_T V}\left(1 - \frac{Q^2}{\chi_T V}\right) + \mathcal{O}\left(\frac{1}{(\chi_T V)^2}\right)}$$

- Fixing the topology implies finite volume effects (TFV effect)
  - Expansion on $\frac{1}{\chi_T V}$ and $\frac{Q^2}{\chi_T V}$
  - $\chi_T V \in \{1, \ldots, 10\}$ for our studies

[1] Brower, R. et al. Phys.Lett. B560 (2003) 64-74, Aoki, Sinya et al. Phys.Rev. D76 (2007) 054508

Method

$$M_Q = M(0) + \frac{M''(0)}{2\chi_T V}\left(1 - \frac{Q^2}{\chi_T V}\right)$$

Method to extract physical mass:

1. Compute M_Q using only configurations in a single topological sector for different volumes and topological charges.
2. Fit $M(V,Q)$ with the BCNW-equation for $\chi_T V > max(|Q|,1)$.
   1. get the parameters: $M(\theta = 0), \chi_T,$ and $M''(0)$.

To test this fixed topology method:

1. Compute $M(0)$ using traditional method (unfixed and unfrozen topology)
2. Extract $M(0)$ using fixed topology simulations
3. Compare 1. and 2.

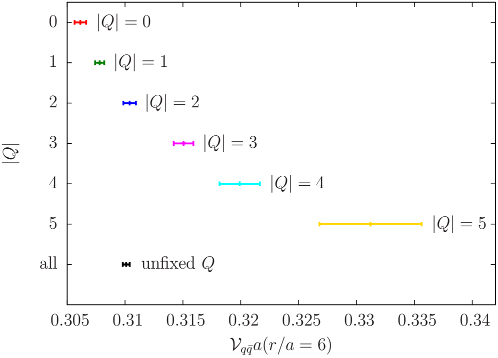
Outline

SU(2) Yang-Mills Theory

$$\mathscr{L} = \frac{1}{4} F_{\mu\nu} F_{\mu\nu}$$

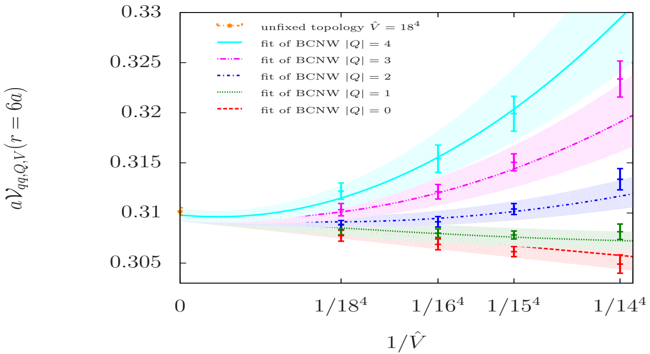
Set-up

- Observable: static potential $V_{qq}(R)$ for $R = 1$ to 6
- $\beta = 2.5$ which correspond to $a \approx 0.073 fm$
- Volumes: $14^4, 15^4, 16^4$ & 18^4
- Number of configurations: 4000 per volume



SU(2) Yang-Mills Theory

$$M_Q = M(0) + \frac{M''(0)}{2\chi_T V}\left(1 - \frac{Q^2}{\chi_T V}\right)$$



$V_{qq}(R = 6) = 0.3097(5)$ from fixed Q

$V_{qq}(R = 6) = 0.3101(3)$ unfixed topology simulation (ref)

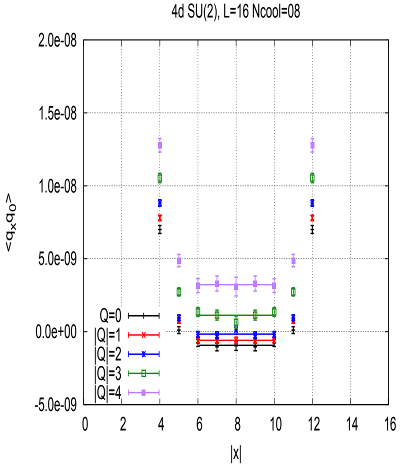
Outline

The AFHO[2] equation

- The AFHO[2]-equation

$$\langle q(x)q(0)\rangle_Q \underset{x\to\infty}{\approx} -\frac{\chi_T}{V}\left(1-\frac{Q^2}{\chi_T V}\right)$$

- The topological susceptibility can be extracted from the fit of a plateau for large x
- Need only one topological sector and one volume
- Results in agreement with unfixed and literature references



unfixed Q $\chi_T = 7.0(0.3)\times 10^{-5}$,
ex:$|Q| = 2$, $\chi_T = 7.1(0.8)\times 10^{-5}$

[2] S. Aoki, H. Fukaya, S. Hashimoto and T. Onogi, *Phys. Rev.* **D76** (2007) 054508.

Summary of these results

1. Fixing Q results in topological finite volume effects (TFV).
2. The method is working well to extract the mass under the condition that $\chi_T V > max(|Q|, 1)$
3. The method to extract topological susceptibility is also working well
4. Possibility to combine both:
   - to extract mass with a better precision
   - to extract mass from only one volume.

Additional difficulties: TFV effects are in competition with ordinary finite volume effects (OFV): short window to apply the method
$\Rightarrow$ Including OFV effects in the equation to increase the window

Outline

Ordinary finite volume effects

Ordinary finite volume effects are due to:

- simulation at finite volume
- periodic boundary conditions

$\Rightarrow$The particle can interact with an image of itself !

Ordinary finite volume effects (OFV) on a particle of mass M:

$$\text{SU(N) equation:} \quad M_L - M_{L=\infty} \propto \frac{1}{L} e^{-\frac{\sqrt{3}}{2} mL}$$

$$\text{QCD equation:} \quad m_{\pi,L} - m_{\pi,L=\infty} \propto \frac{1}{L} K_1(m_\pi L)$$

with m: mass of the lightest particle, L: length of the box

Ordinary finite volume effects in QCD:

- Extremely costly to generate configurations $\Rightarrow$small volumes
- m_π is small in QCD.

$\Rightarrow$Difficulties to get rid of ordinary finite volume effects

Outline

Combining ordinary and topological finite volume effects

- Working at fixed topology with ordinary finite volume effects: need to combine both kind of finite volume effects.
  - Need to calculate OFV in θ-vacuum using Lüscher method (for SU(N) and QCD)
  - Calculate the mass at fixed topology with OFV.
- Leading order (LO): BCNW-equation , OFV

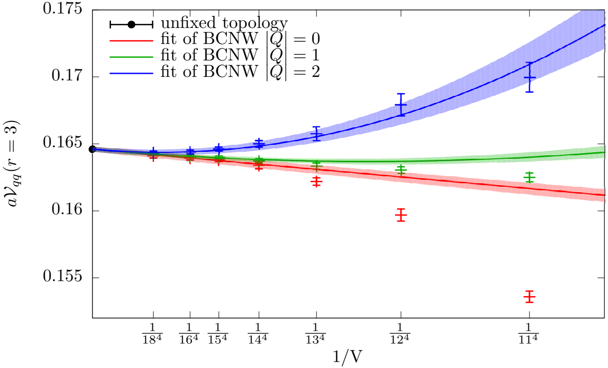
$$M_{Q,L}^{SU(N)} = M(0) + \frac{M''(0)}{2\chi_T V}\left(1 - \frac{Q^2}{\chi_T V}\right) - \frac{A}{m^2(0)L} e^{-\frac{\sqrt{3}}{2} m(0)L} + \mathcal{O}\left(\frac{e^{-\frac{\sqrt{3}}{2} m(0)L}}{(\chi_T V)}\right)$$

$$m_{\pi,Q,L} = m_\pi(0) + \frac{m_\pi^{(2)}(0)}{2\chi_T V}\left(1 - \frac{Q^2}{\chi_T V}\right) + \frac{B m_\pi(0)}{L} K_1(m_\pi(0)L) + \mathcal{O}\left(\frac{K_0(m_\pi(0)L)}{(\chi_T V)}\right)$$

- NLO: OFV effects will depend of Q.
- Our result is in agreement with result obtained for QCD with ChPT calculation in θ-vacuum [3]

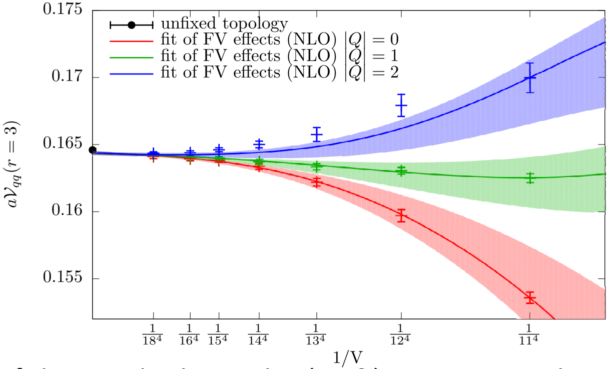
[3] S. Aoki and H. Fukaya, Phys. Rev. D **81**, 034022 (2010)

Ordinary finite volume effects at fixed topology



- Ordinary finite volume effects for $V < 14^4$ (discrepancy with the BCNW fit)
- Different OFV effects for different topological charges $\Rightarrow$Need to go to next leading order

Ordinary finite volume effects at fixed topology



- Fit of the next leading order (NLO) equation combining topological and ordinary finite volume effects

	$\hat{\mathcal{V}}_{q\bar{q},Q,V}(r=3a)$	$\hat{m}$	$\hat{\chi}_T \times 10^5$
fit results, NLO eq.	0.16437(15)	0.67(10)	9.5(2.0)
unfixed top. results	0.16455(7)	0.723(23)	7.0(0.9)

Summary

1. Study the possibility to work at fixed topology
    1. Show the efficiency of the method to extract mass from frozen topology simulation
    2. Precise results obtained on the mass
    3. Good results to obtained topological susceptibility
2. Combination of ordinary finite volume effects and topological finite volume effects
    1. Equation combining both finite volume effects
    2. Promising test on SU(2) Yang-Mills theory

- Outlook
    - More tests on the combination of ordinary finite volume effects and topological finite volume effects.
    - Full QCD (in-going)