Decay constants f_B and f_{B_s} from HISQ simulations

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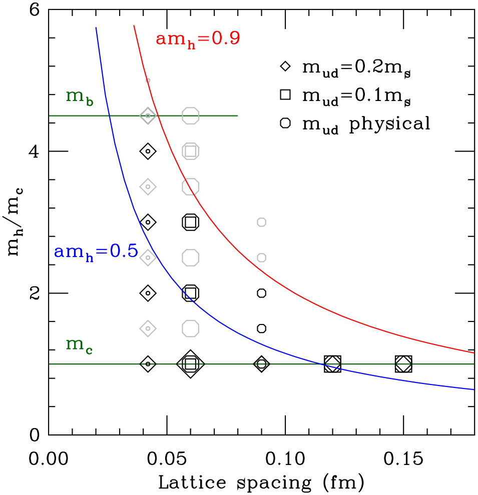
Need for precise values of f_B and f_{B_s}

- Provide accurate SM predictions for rare decays such as $B \to \tau\nu$, neutral B mixing, and $B_s \to \mu^+\mu^-$.
- Probe the $V - A$ structure of the Wub vertex.
- Resolve/sharpen tension between inclusive and exclusive $|V_{ub}|$.

What is new in this work?

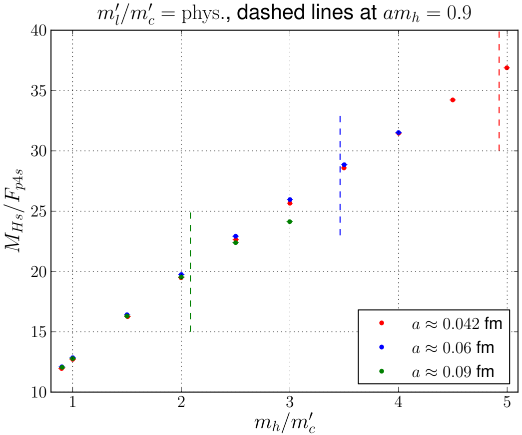
- Follows method for HISQ heavier than charm from HPQCD [1110.4510]
  - Incorporates HQET in the chiral/continuum analysis model
  - HISQ decay constants don't need renormalization
  - Compared with HPQCD, we use HISQ ensembles (they used asqtad) and physical-mass light quarks. Statistics.
- Extend our earlier calculation of f_D, etc using HISQ light and charm quarks. [PRD **90**, 074509]
  - Now we have data with HISQ heavier than charm
- Use five lattice spacings as small as 0.045 fm.
- Results preliminary. Will present methodology and error projection.

Ensembles and heavy quark masses



- Heavy quark masses *vs* lattice spacing
- Symbol size is proportional to data sample size
- Drop some m_h/m_c' points for data thinning and others to avoid large discretization errors ($am_h > 0.9$)

Heavy quark discretization errors



- Intermediate scale parameter $F_{p4s}(a \to 0) \approx 154$ MeV.
- We stop at $am_c' = 0.9$ and model heavy-quark effects below that.

Decay constant methodology

- Pseudoscalar density-density correlator

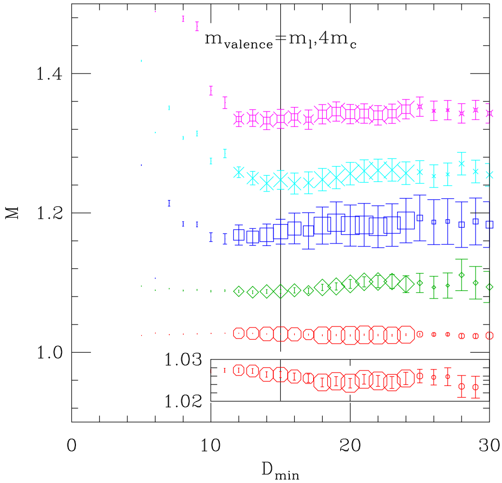
$$C_{\mathrm{pt-pt}}(t) \to A_{\mathrm{pt-pt}} \exp(-M_{H_q} t)$$

- Decay constant

$$f_{H_q} = (m_h + m_l)\sqrt{\frac{3VA_{pt-pt}}{2M_{H_q}^3}}$$

- Several heavy valence masses, so we use "H" instead of "B".
- We use $q = \{u, d, s\}$
- NOTE: No matching factors for HISQ pseudoscalar density.
- Calculate correlators with
  - Sources: random and Coulomb-gauge wall
  - Sinks: point
- Fit to 3+2 states and 2+1 states to check effect of excited states.

Example: two-point (3+2)-state spectrum *vs* $D_{\min}$



- Spectrum for a heavy-light meson with valence masses $4m_c$, m_l vs $D_{\min}$. ($m_l = m_u = m_d$)
- 0.045 fm physical mass ensemble
- Ground and excited state masses of both parities for a (3+2)-state fit.
- Inset magnifies ground state
- Priors constrain the excited state mass splittings.
- Vertical line shows our choice for $D_{\min}$

HQET

- Heavy quark effective theory (HQET) models the heavy-quark mass dependence of f_{H_q}

$$J^{\mathrm{QCD}} \equiv C_i(m_Q, \mu) J_i^{\mathrm{HQET}}(\mu) + \mathcal{O}(1/m_Q)$$

- Wilson coefficient

$$C_i(m_Q, \mu) = \left[\frac{\alpha_S(m_Q)}{\alpha_S(\mu)}\right]^{-6/25} \left[1 + \mathcal{O}\left(\alpha_S(m_Q), \alpha_S(\mu)\right)\right] + \mathcal{O}\left(1/m_Q\right)$$

- So (using M_{H_s} at physical sea quark masses and physical s quark mass)

$$f_{H_q}\sqrt{M_{H_q}} = \left[\alpha_S(M_{H_s})\right]^{-6/25} \left[1 + \mathcal{O}\left(\alpha_S(M_{H_s})\right)\right] \; \Phi_{H_q}$$

- where

$$\Phi_{H_q} = \tilde{\Phi}_0 \left(1 + k_1 \frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}} + k_2 \frac{\Lambda^2_{\mathrm{HQET}}}{M^2_{H_s}}\right) (1 + \mathrm{chiral\ log/analytic\ terms})$$

- Discretization correction for LEC. [HPQCD, PRD 75, 054502].

$$\tilde{\Phi}_0 = \Phi_0[1 + c_1\alpha_S(a\Lambda)^2 + c_2(a\Lambda)^4 + c_3\alpha_S(am_h)^2 + c_4(am_h)^4 + \dots]$$

Chiral expression

- Heavy-Meson Rooted All-Staggered χPT [Bernard and Komijani, PRD **88**, 094017(2013)]

$$
\begin{aligned}
(1 + \text{chiral log/analytic terms}) &= 1 + \text{NLO staggered logs} \\
&+ \text{NLO analytic terms} \\
&+ \text{NNLO} + \text{NNNLO analytic terms} \\
\text{NLO analytic terms} &= \left(L_s + L_{s,M} \frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}} \right) (2m_l + m_s) \\
&+ \left(L_{\mathrm{v}} + L_{\mathrm{v},M} \frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}} \right) (m_{\mathrm{v}}) + c_{a,\Xi} a^2 \ .
\end{aligned}
$$

- Note chiral expression includes the $M^*_{H_q} - M_{H_q}$ hyperfine splitting, which also depends on the heavy quark mass.
- Central fit: NLO, NNLO, NNNLO has 26 parameters

Adjustment for sea charm mass

- Integrating out the charm quark yields an effective three-quark theory

$$\Lambda^{(3)}_{\rm QCD}(m_c) = \Lambda^{(4)}_{\rm QCD}\left[\frac{m_c}{\Lambda^{(4)}_{\rm QCD}}\right]^{2/27}$$

- Starting from a fixed value of strong coupling at very high energy

$$\frac{\Lambda^{(3)}_{\rm QCD}(m_c')}{\Lambda^{(3)}_{\rm QCD}(m_c)} = \left[\frac{m_c'}{m_c}\right]^{2/27}$$

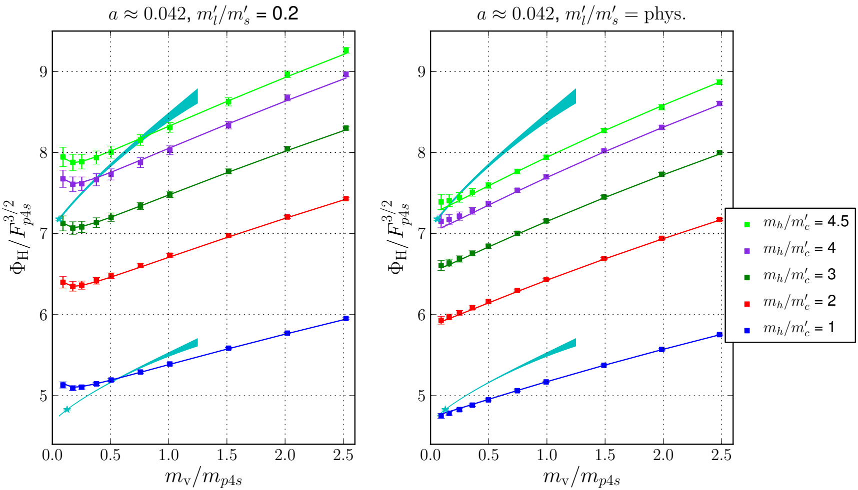
- Any hadronic quantity like f is proportional to $\Lambda^{(3)}_{\rm QCD}$ at leading order
- For Φ_{H_q} in HQET/chiral limit we have

$$\Phi_0 \propto \left[\Lambda^{(3)}_{\rm QCD}\right]^{3/2} \quad \Rightarrow \quad \frac{\Phi_0(m_c')}{\Phi_0(m_c)} = \left[\frac{m_c'}{m_c}\right]^{3/27}$$

- Sea charm effects can be taken into account (m_c = physical, m_c' = simulation):

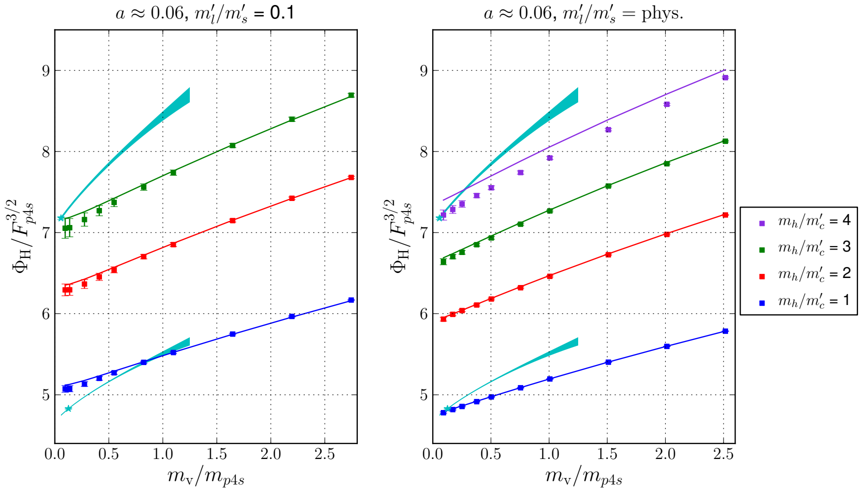
$$\Phi_{H_q} = \Phi_0\left[\frac{m_c'}{m_c}\right]^{3/27}\left(1 + k_1\frac{\Lambda_{\rm HQET}}{M_{H_s}} + k_2\frac{\Lambda^2_{\rm HQET}}{M^2_{H_s}} + k_1'\frac{\Lambda_{\rm HQET}}{m_c'}\right)(1 + \text{chiral/analytic terms})$$

Sample chiral fit result: 0.045 fm data



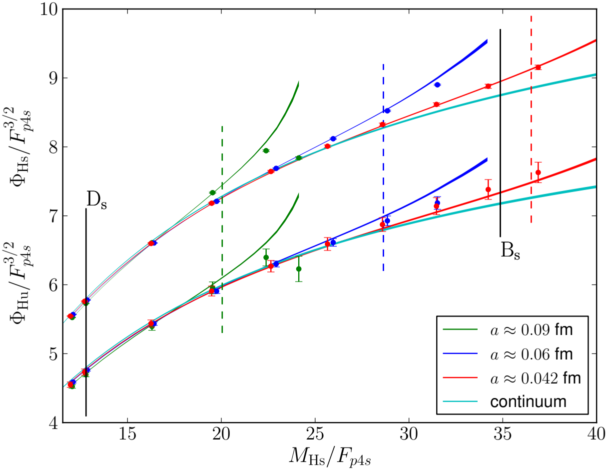
- Continuum extrapolation shown in cyan.

Sample chiral Fit result: 0.06 fm data



- Continuum extrapolation shown in cyan.

Chiral Fits

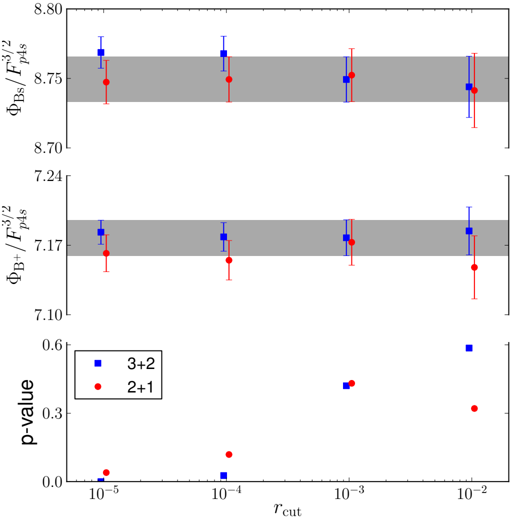


- Illustration of growing heavy-quark discretization error. (First reported by HPQCD.)

Aside: Thinning the data

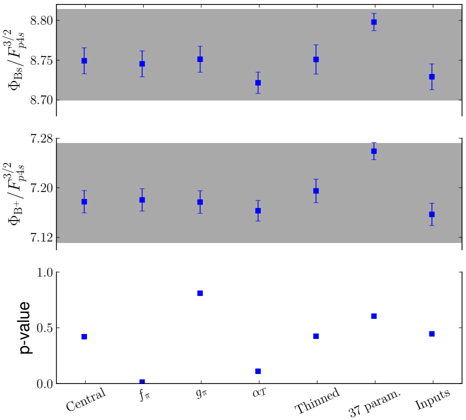
- Common problem: strong correlations in data lead to very small eigenvalues of the chiral fit correlation matrix and a nearly singular inverse.
- Eigenvectors for tiny eigenvalues characterize the correlations.
- SVD thinning procedure: drop some of the small eigenmodes. Specifically,
  - Introduce $r_{\rm cut}$ (say 10^{-3})
  - For each ensemble determine the largest eigenvalue $\lambda_{\rm max}$
  - Drop those modes with eigenvalue smaller than $r_{\rm cut}\lambda_{\rm max}$
  - Reduce d.o.f. by the number of dropped eigenvectors.

Stability under SVD cuts



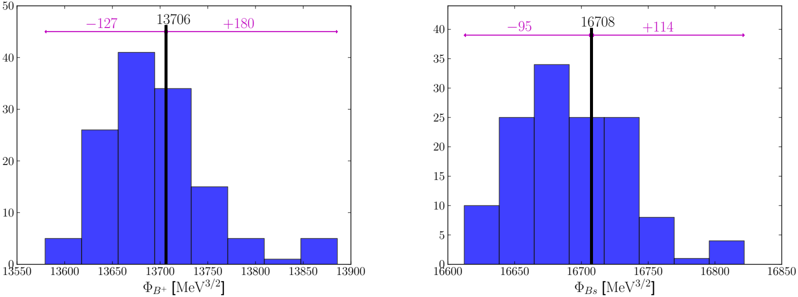
- As $r_{\rm cut}$ increases the errors increase along with the "*p*-value".
- Choose 10^{-3}

Systematics: sampling of fit alternatives



- Error band estimates the full systematic error
- f_π: replace f_K with f_π in chiral terms
- g_π: let g_π float with prior 0.53(8)
- α_T Use α from measured taste splitting
- Thinned: Drop half the light quark masses.
- 37 param. Add 11 higher-order HQ terms + no SVD cut.
- Inputs: Alternative F_{4ps} scale determination.

Histogram



- Includes 132 plausible analysis variations.
- Use extrema and symmetrize to estimate the systematic error.

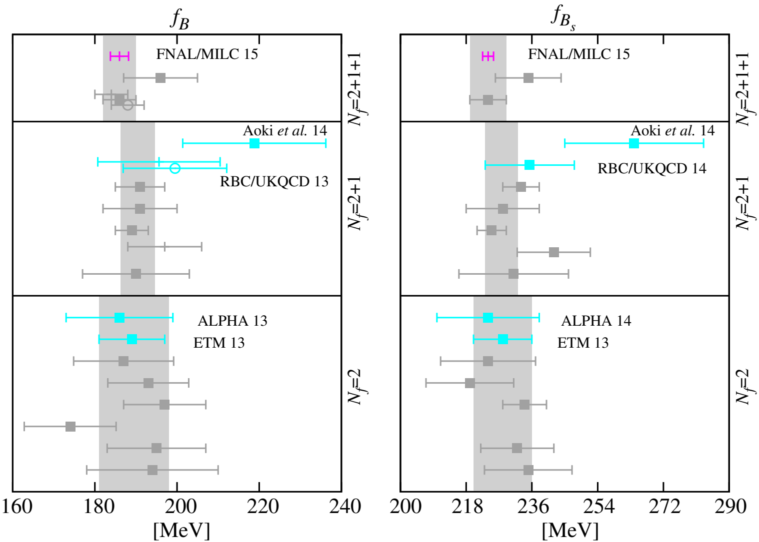
Projected error budget

	f_{B^+}	f_{B_s}
Histogram*	2.1	1.4 MeV
Statistics	0.6	0.5
Charm sea-quark mass tuning	—	—
Finite volume	0.2	0.3
E. M.	0.1	0.1
Scale (f_π)	0.3	0.4
Total	2.2	1.5 MeV

* Includes chiral fit, light- and heavy-quark discretization, excited state contamination.

Comparison with other results

[Updates Chris Bouchard, Lattice 14]



Shows only our projected error. Take the FLAG 2013 central value for now.

Conclusion and Outlook

- We have presented the status of our analysis of f_B and f_{B_s} from a calculation with all HISQ quarks à la HPQCD.
- The errors are much reduced compared with Fermilab quarks (next talke) because HISQ needs no matching factors.
- The errors are reduced compared with the HPQCD HISQ study in [1110.4510].
  - Physical mass ensembles, especially at 0.045 fm help anchor the chiral analysis
  - The data set is larger, and we do a chiral extrapolation.
  - HISQ sea quarks have smaller discretization errors than asqtad.
- We are generating a 0.03 fm ($m_{ud}/m_s = 0.2$) ensemble ($am_b = 0.6$). Analysis is in progress.

Backup Slides

Heavy-Meson Rooted All-Staggered χPT

[Bernard and Komijani, Phys. Rev. D88 (2013) 9, 094017]

$$
\begin{aligned}
&(1 + \text{chirallog/analytic terms}) \\
&= \quad 1 + \frac{1}{16\pi^2 f^2}\frac{1}{2}\left\{ -\frac{1}{16}\sum_{\mathcal{S},\Xi'} \ell(m^2_{\mathcal{S}x,\Xi'}) - \frac{1}{3}\sum_{j\in\mathcal{M}_I^{(3,x)}} \frac{\partial}{\partial m^2_{X,I}}\left[R_j^{[3,3]}(\mathcal{M}_I^{(3,x)};\mu_I^{(3)})\ell(m_j^2)\right]\right. \\
&\quad - \left(a^2\delta'_V \sum_{j\in\mathcal{M}_V^{(4,x)}} \frac{\partial}{\partial m^2_{X,V}}\left[R_j^{[4,3]}(\mathcal{M}_V^{(4,x)};\mu_V^{(3)})\ell(m_j^2)\right] + [V\to A]\right) \\
&\quad - 3g_\pi^2 \frac{1}{16}\sum_{\mathcal{S},\Xi'} J(m_{\mathcal{S}x,\Xi'}, \Delta^* + \delta_{\mathcal{S}x}) \\
&\quad - g_\pi^2 \sum_{j\in\mathcal{M}_I^{(3,x)}} \frac{\partial}{\partial m^2_{X,I}}\left[R_j^{[3,3]}(\mathcal{M}_I^{(3,x)};\mu_I^{(3)})J(m_j,\Delta^*)\right] \\
&\quad \left. - 3g_\pi^2\left(a^2\delta'_V \sum_{j\in\mathcal{M}_V^{(4,x)}} \frac{\partial}{\partial m^2_{X,V}}\left[R_j^{[4,3]}(\mathcal{M}_V^{(4,x)};\mu_V^{(3)})J(m_j,\Delta^*)\right] + [V\to A]\right)\right\} \\
&\quad + \left(L_s + L_{s,M}\frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}}\right)(2m_l + m_s) + \left(L_{\mathrm{v}} + L_{\mathrm{v},M}\frac{\Lambda_{\mathrm{HQET}}}{M_{H_s}}\right)(m_{\mathrm{v}}) + c_{a,\Xi}a^2 \ .
\end{aligned}
$$

Topological charge history