Strong coupling expansion of the t-V model

Marcin Szyniszewski*[,1,2], Evgeni Burovski[1]

[1] *Physics Department, Lancaster University, Lancaster, LA1 4YB, UK*

[2] *NoWNano DTC, University of Manchester, Manchester, M13 9PL, UK*

* *E-mail for corresponding author: mszynisz@gmail.com*

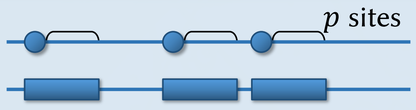
1 Introduction

The generalised t-V model [1] of fermions distributed on a chain of L sites with p.b.c.:

$$H = -t\sum_{i=1}^{L}\left(\phi_i^\dagger\phi_{i+1} + \text{h.c.}\right) + \sum_{i=1}^{L}\sum_{m=1}^{p} U_m \phi_i^\dagger \phi_i \phi_{i+m}^\dagger \phi_{i+m}$$

p – interaction range

Away from critical density



- Luttinger liquid
- Highly degenerate ground state
- Interacting "hard rods"

Critical density $Q_C = \frac{q}{p+1}$; $q = 1, \ldots, p$

- Mott insulator
- Simple ground state
- "Rods" filling the lattice

2 The Objective

Spacing: $p > 1$ Non-integrable

Solved only in the 1st order perturbation [1].

Try reaching higher orders and finite system sizes.
Describe critical behaviour.

Spacing: $p = 1$ Integrable

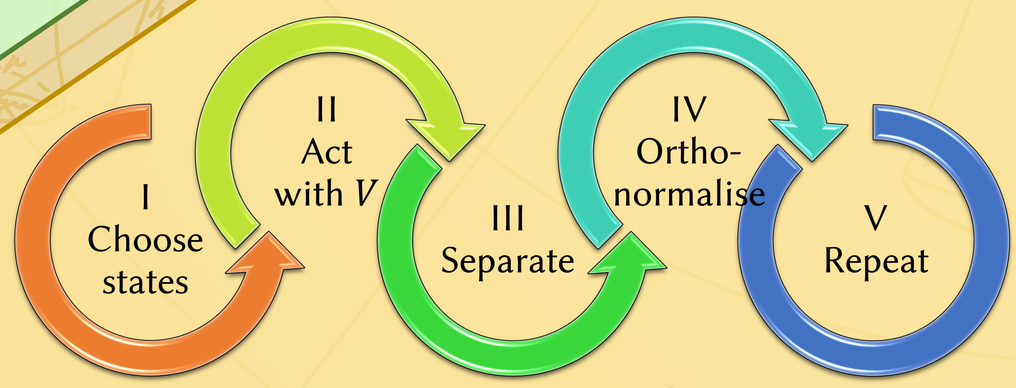
Bethe ansatz approach [2].

3 Strong Coupling Expansion

Assume:

$$H = H_0 + \lambda V$$

$\lambda \ll 1$, V can be treated as a perturbation [3].



E.g. ground state $|0\rangle$ → $V|0\rangle$ → $|1\rangle$, $|1'\rangle$ → $|\tilde{1}\rangle$, $|\tilde{1}'\rangle$ → Include in the new basis

The Hamiltonian matrix in new basis:

SCE step 0	0th and 1st perturbation orders	2nd	0			
step 1	2nd	3rd	4th	0	0	0
step 2	0	4th	5th	6th	0	
step 3	0	0	6th	7th	⋱	
…	0	0	0	⋱	⋱	
	0					Subspace not connected to initial states by V^n

Truncated Hamiltonian with complete information about initial subspace up to the 5th order.

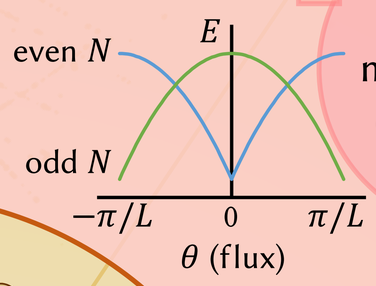
With every SCE step we are increasing the accuracy by two orders in λ.

4 Results and Criticality

Ground state energy formulae for near critical densities

- Critical density $1/(p+1)$
- Precision up to $\mathcal{O}(t^8)$
- General results including all values of p

Small magnetic flux dependence



- Kinetic term becomes: $\sum_i\left(e^{i\theta}\phi_i^\dagger\phi_{i+1} + \text{h.c.}\right)$
- K defines all critical exponents

Critical parameter K can be easily calculated

$$K = \frac{\pi \csc\left(\frac{\pi(p+1)}{2L+4p}\right)}{4L(p+1)} + \mathcal{O}(t)$$

5 Other Phases

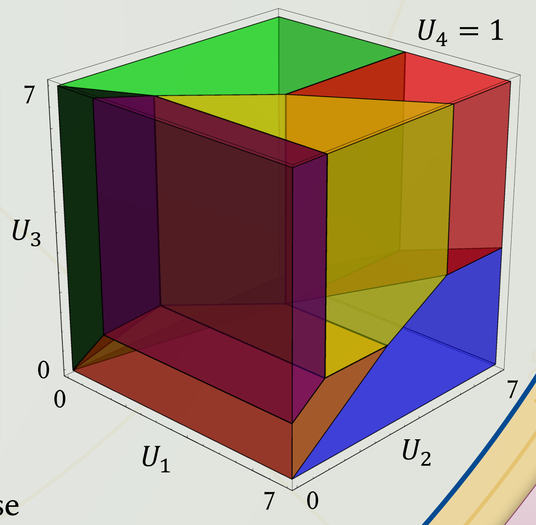
Other phases are present if

$$U_m < (U_{m-1} + U_{m+1})/2$$

is not satisfied and thus potential has an unusual behaviour.

Example for $p = 4, Q = 1/3$:

	Ground state unit cell	Energy
	●○○	$\frac{1}{3}U_3$
	●●○○○○	$\frac{1}{6}U_1$
	●○●○○○	$\frac{1}{6}(U_2 + U_4)$
	●●○○○●○○○	$\frac{1}{9}(U_1 + 2U_4)$
	●○●○●○○○○	$\frac{1}{9}(2U_2 + U_4)$
	●○●○○●○●○○○○	$\frac{1}{12}(2U_2 + U_3)$
	●●●○○○○●○●○○○○●○●○○○○	$\frac{1}{21}(2U_1 + 3U_2)$



- For $Q = \frac{1}{p+1}$ ⟶ one trivial phase
- For $Q = \frac{1}{p}$ ⟶ phases can be described in detail
- For $Q = \frac{1}{p-1}$, etc. ⟶ number of phases increases rapidly; no simple way of description

For a non-zero kinetic energy ($t \neq 0$) [4]:

- Phases with long range interactions (e.g. bond order)
- Luttinger liquid despite critical density

6 Conclusions & Outlook

Strong coupling expansion

- Simple way of reaching higher order perturbations **numerically & analytically**
- Critical parameter K easily obtained
- High precision results for both **integrable** and **non-integrable** models

Possible further work:

- Temperature dependence
- Time dependence

References

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The University of Manchester

Lancaster University