Study of the hadronic contribution to the running of the QED coupling

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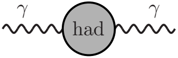
$\Delta\alpha_{\rm QED}^{\rm had}$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\rm QED}(Q^2)}$$

- vacuum polarisation: charge screening $\rightsquigarrow$ running of QED coupling
- Standard Model (SM) precision tests and sensitivity to new physics requires precise knowledge of $\Delta\alpha_{\rm QED}(Q^2)$: input parameter of SM
- $\alpha =$ 1/137.035999074(44) [0.3 ppb] [PDG, 2013]
- $\alpha(M_Z^2) = 1/128.952(14)$ $[10^{-4}]$ $\rightsquigarrow$ 10^5 less accurate . . .

[M. Davier et al., 1010.4180]

- uncertainty in $\alpha(M_Z^2)$ is significantly larger than that of M_Z
- hadronic effects : $\alpha(Q^2)$ depends strongly on Q^2 at low energies hadronic uncertainties propagate . . .



$\Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\mathrm{QED}}(Q^2)}$$

- leading order (LO) contribution

$$\int \mathrm{d}^4x\, e^{iQx}\, \langle J_\mu(x)\, J_\nu(0)\rangle \;=\; (Q_\mu Q_\nu - Q^2\,\delta_{\mu\nu})\, \Pi(Q^2)$$

$$J_\mu(x) \;=\; \sum_{f=1}^{N_{\mathrm{f}}} Q_f\, \overline{\psi}_f(x)\gamma_\mu\psi_f(x)$$

$Q_f \in \{-1/3, 2/3\}$

- $\Pi(Q^2)$: photon vacuum polarisation function (VPF)

$\Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\mathrm{QED}}(Q^2)}$$

- leading order (LO) contribution

$$\int \mathrm{d}^4x \, e^{iQx} \, \langle J_\mu(x) \, J_\nu(0) \rangle \;=\; (Q_\mu Q_\nu - Q^2 \, \delta_{\mu\nu}) \, \Pi(Q^2)$$

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$Q_f \in \{-1/3, 2/3\}$

- $\Pi(Q^2)$: photon vacuum polarisation function (VPF)

$$\Delta\alpha_{\mathrm{QED}}(Q^2) = 4\pi\alpha \left(\Pi(Q^2) - \Pi(0) \right)$$

- Adler function $D(Q^2)$:

$$\frac{D(Q^2)}{Q^2} = 12\pi^2 \, \frac{d\,\Pi\,(q^2)}{dq^2} = -\frac{3\pi}{\alpha} \, \frac{d}{dq^2} \Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}(q^2)$$

$Q^2 = -q^2$

$\Delta\alpha_{\rm QED}^{\rm had}$

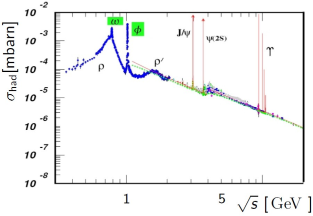
$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\rm QED}(Q^2)}$$

- the VPF $\Pi(Q^2)$ and the Adler function $D(Q^2)$ $\rightsquigarrow$ $\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$ and $a_\mu^{\rm had}[{\rm LO}]$

- phenomenological approach :

  dispersion relation + optical theorem + $(e^+e^- \to \text{hadrons})$ cross section

$$\Delta\alpha_{\rm QED}^{\rm had}(Q^2) = -\frac{\alpha\, Q^2}{3\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{R_{\rm had}(s')}{s'(s' - Q^2)}$$



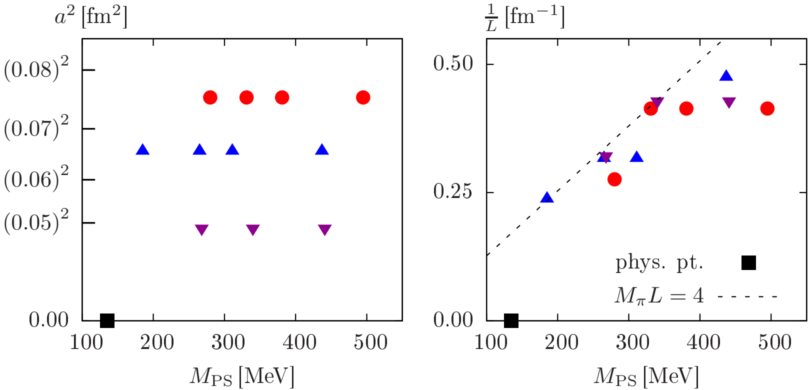
  compared to $a_\mu^{\rm had}[{\rm LO}]$, low-energy regions contribute less

  theoretical prediction that relies on experimental data

- lattice QCD

lattice setup

- [Mainz, 1112.2894] [talk by Hanno Horch]



$N_f = 2$ $\mathcal{O}(a)$ improved Wilson fermions [CLS]

strange and charm are quenched : s_Q , c_Q

only quark-connected contributions

increased statistics

1000 $\div$ 4000 meas. per ensemble

scale from f_K [ALPHA, 1205.5380]

lattice VPF

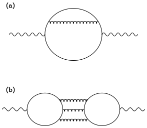
Local current

$$J_\mu^{(1,\,f)}(x) = Z_{\rm V}\,\overline{\psi}_f(x)\,\gamma_\mu\,\psi_f(x)$$

conserved-local correlator

$$a^6 \Big\langle \sum_{f=1}^{N_{\rm f}} \Big(Q_f\, J_\mu^{({\rm ps},\,f)}(x) \Big) \sum_{f'=1}^{N_{\rm f}} \Big(Q_{f'}\, J_\nu^{(1,\,f')}(0) \Big) \Big\rangle$$

$$\Pi_{\mu\nu}(\hat{Q}) = a^4 \sum_x e^{iQ(x+a\hat{\mu}/2)} \langle\, J_\mu^{({\rm ps})}(x)\, J_\nu^{(1)}(0)\,\rangle \quad \rightsquigarrow \quad \Pi(\hat{Q}^2)$$



$$\hat{Q}_\mu = \frac{2}{a}\sin\left(\frac{aQ_\mu}{2}\right)$$

Adler function

the Adler function $D(Q^2)$ is related to the vacuum polarization by

$$D(Q^2) = 12\,\pi^2\,Q^2\,\frac{d\Pi(Q^2)}{dQ^2}$$

compute the Adler function :

- analytic derivative:

  fit a function to $\Pi(Q^2)$ and compute its derivative

  Padé ansatz :

$$\Pi_{fit}(Q^2) = \Pi(0) + Q^2\left(\frac{p_1}{p_2 + Q^2} + \frac{p_3}{p_4 + Q^2}\right)$$

$$Q^2\,\frac{d}{dQ^2}\Pi_{fit}(Q^2) = Q^2\left(\frac{p_1 p_2}{(p_2 + Q^2)^2} + \frac{p_3 p_4}{(p_4 + Q^2)^2}\right)$$

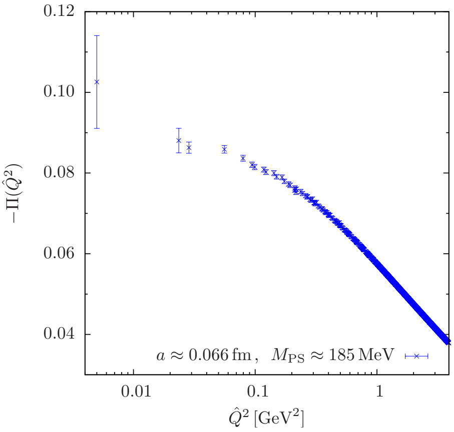
- numerical derivative:

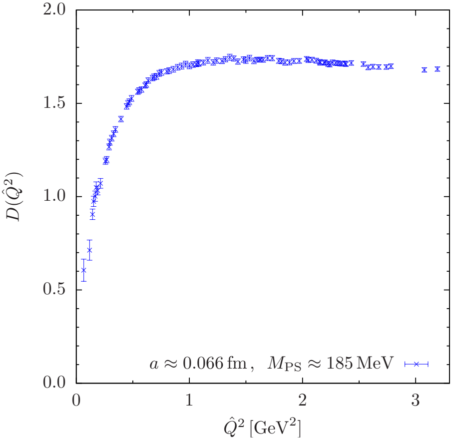
  apply linear or quadratic fits of varying ranges to determine the derivative of $\Pi(Q^2)$

Adler function : numerical derivative

$$D(Q^2) = 12\,\pi^2\ \frac{d\Pi(Q^2)}{d\log Q^2}$$

u, d





Adler function : combined fit

Adler function:

$$D(Q^2) = 12\,\pi^2\,Q^2\,\frac{d\Pi(Q^2)}{dQ^2}$$

- fit form :

$$D(Q^2) = \mathrm{Padé}(Q^2)\;[\,1 \;+\; \mathbf{discr.} \;+\; \mathbf{mass}\,]\;,$$

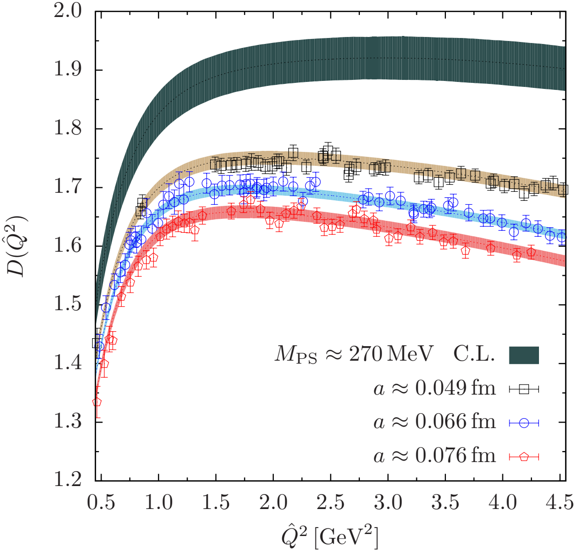
$$D(Q^2) = Q^2\left(\frac{p_1}{(p_2+Q^2)^2} + \frac{p_3}{(p_4+Q^2)^2}\right)\times$$
$$\left[1 + (d_1\,a + d_2\,|aQ|) + \left(\frac{c_1}{Q^2}\right)\left(M_{\mathrm{PS}}^2 - M_\pi^2\right) + \left(\frac{c_2}{Q^4}\right)\left(M_{\mathrm{PS}}^2 - M_\pi^2\right)^2\right]$$

- consider 11 ensembles with different a, M_{PS}
- consider also variations over these fit forms
- (u,d) , s_Q and c_Q

Adler function : combined fit $Q^2 \in [0.5, 4.5]\,\mathrm{GeV}^2$

$$D(Q^2) = \frac{3\pi}{\alpha}\frac{d}{d\log(Q^2)}\Delta\alpha^{\rm had}_{\rm QED}(Q^2)$$

u, d

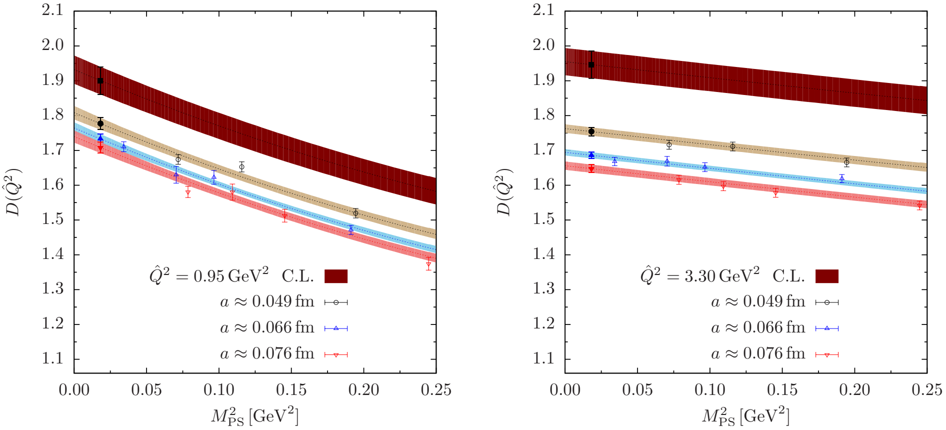


Padé [1, 2] with O(a) lattice artefacts and quadratic form in $M_{\rm PS}^2$ $\chi^2/\mathrm{d.o.f} = 0.93$

Adler function : M_{PS} dependence

$$D(Q^2) = \frac{3\pi}{\alpha}\,\frac{d}{d\log(Q^2)}\,\Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}(Q^2)$$

u, d

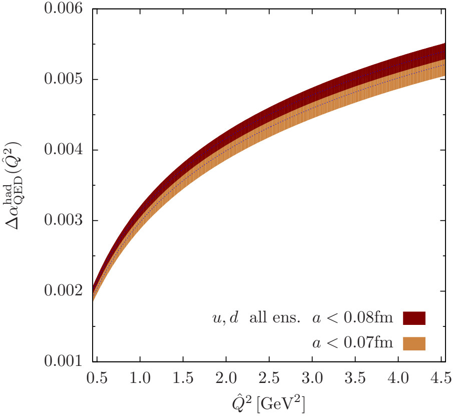


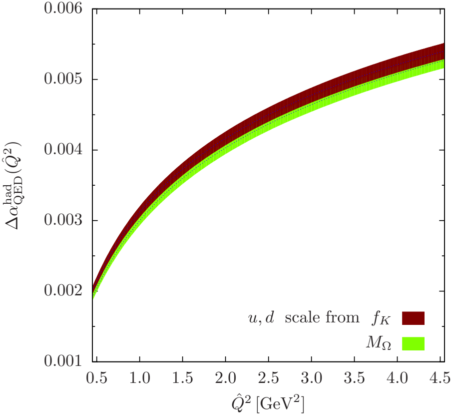
Padé [1, 2] with O(a) lattice artefacts and quadratic form in M^2_{PS} $\chi^2/\mathrm{d.o.f} = 0.93$

$\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$: systematic effects

$$D(Q^2) = \frac{3\pi}{\alpha}\,\frac{d}{d\log(Q^2)}\,\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$$

u, d; $a \to 0$; $M_\pi^{\rm phys}$



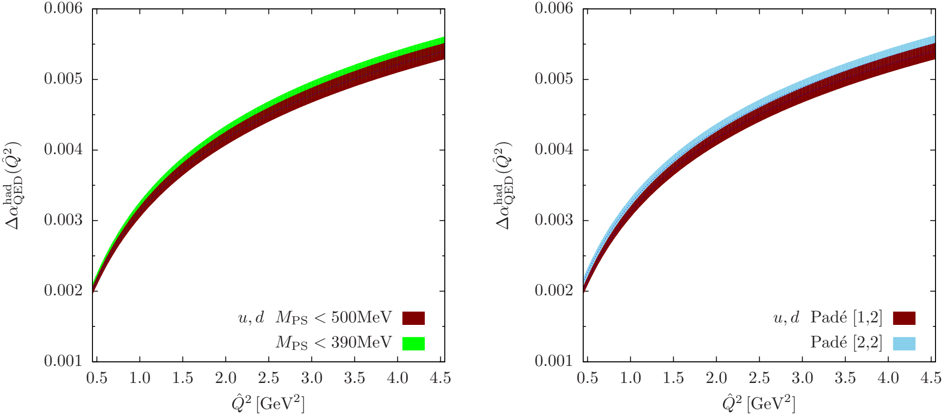


Padé [1, 2] with O(a) lattice artefacts and quadratic form in $M_{\rm PS}^2$

$\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$: systematic effects

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(Q^2)} \Delta\alpha_{\rm QED}^{\rm had}(Q^2)$$

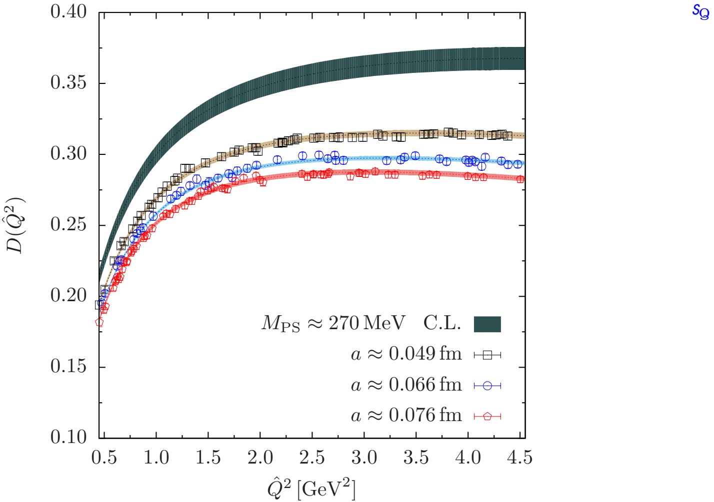
u, d; $a \to 0$; $M_\pi^{\rm phys}$



$O(a)$ lattice artefacts with quadratic form in $M_{\rm PS}^2$

Adler function : strange quark $Q^2 \in [0.5,\ 4.5]\,\mathrm{GeV}^2$

$$D(Q^2) = \frac{3\pi}{\alpha}\,\frac{d}{d\log(Q^2)}\,\Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}(Q^2)$$

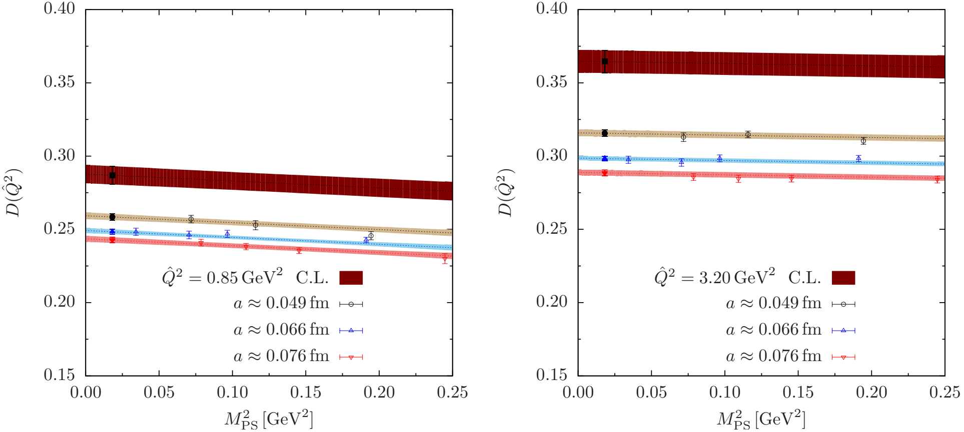


Padé [1, 2] with $O(a)$ lattice artefacts and linear form in M_{PS}^2 $\chi^2/\mathrm{d.o.f} = 0.87$

Adler function : strange quark

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(Q^2)} \Delta\alpha_{\rm QED}^{\rm had}(Q^2)$$

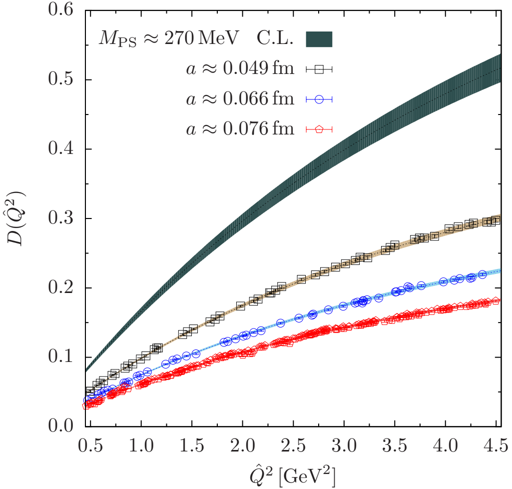
s_Q



Padé [1, 2] with O(a) lattice artefacts and linear form in $M_{\rm PS}^2$ $\chi^2/\mathbf{d.o.f} = 0.87$

Adler function : charm quark $Q^2 \in [0.5, 4.5]\,\mathrm{GeV}^2$

$$D(Q^2) = \frac{3\pi}{\alpha}\frac{d}{d\log(Q^2)}\Delta\alpha^{\mathrm{had}}_{\mathrm{QED}}(Q^2)$$



c_Q

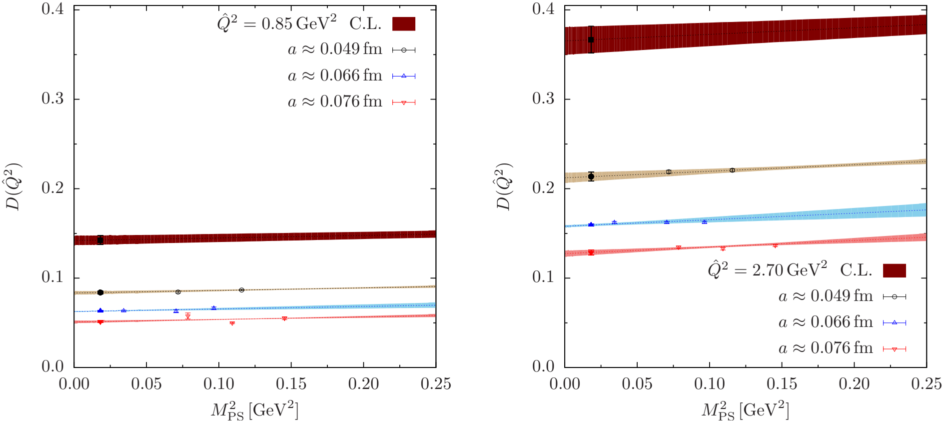
Padé [1, 1] with $O(a)$ lattice artefacts and linear form in M^2_{PS} $\chi^2/\mathrm{d.o.f} = 1.42$

$M_{\mathrm{PS}} < 390\,\mathrm{MeV}$

Adler function : charm quark

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(Q^2)} \Delta\alpha_{\rm QED}^{\rm had}(Q^2)$$

c_Q



Padé [1, 1] with $O(a)$ lattice artefacts and linear form in $M_{\rm PS}^2$ $\chi^2/\mathrm{d.o.f} = 1.42$

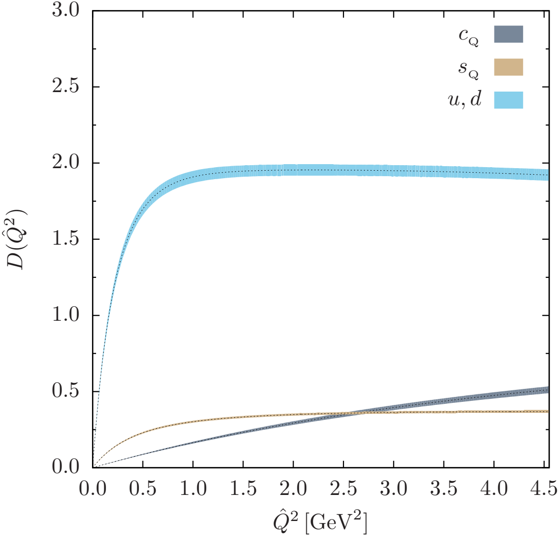
$M_{\rm PS} < 390\,\mathrm{MeV}$

Adler function : flavour contributions

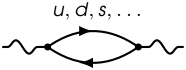
$$D(Q^2) = \frac{3\pi}{\alpha}\frac{d}{d\log(Q^2)}\Delta\alpha^{\text{had}}_{\text{QED}}(Q^2)$$

$a \to 0$

M_π^{phys}



u, d, s_Q, c_Q



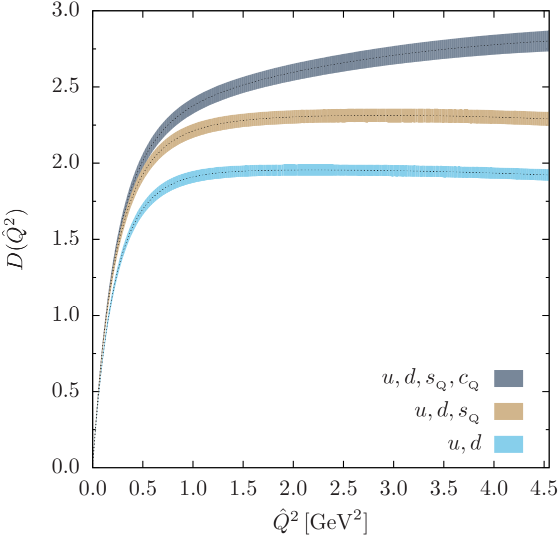
[PRELIMINARY]

Adler function : flavour contributions

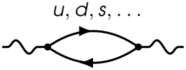
$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(Q^2)} \Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}(Q^2)$$

$a \to 0$

M_π^{phys}



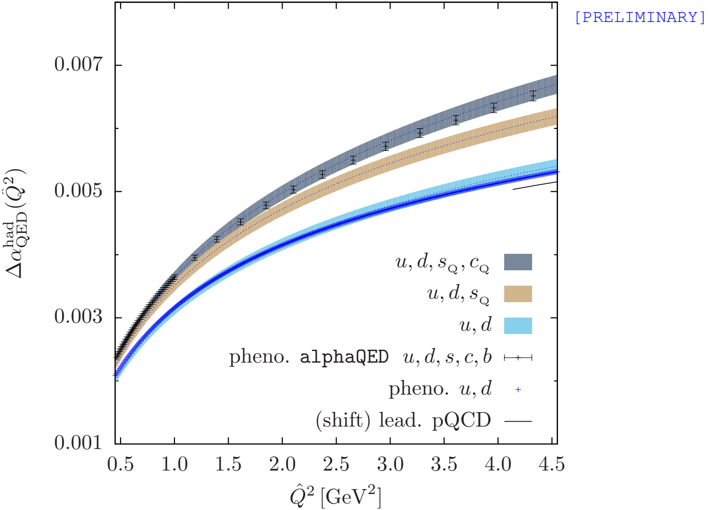
u, d, s_Q, c_Q



[PRELIMINARY]

running QED coupling: $\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\rm QED}(Q^2)}$$



pheno. u, d : [Bernecker & Meyer, 1107.4388]
pheno. u, d, s, c, b : [alphaQED package, F. Jegerlehner]
see also lattice u, d, s, c : [ETMC, 1505.03283]

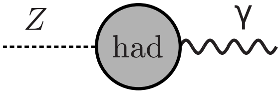
$\Delta^{\mathrm{had}} \sin^2\theta_W(Q^2)$

$$\sin^2\theta_W(Q^2) \;=\; \sin^2\theta_W\left(Q^2=0\right)\;\left(1 - \Delta\sin^2\theta_W(Q^2)\right)$$

$$\text{with}\quad \sin^2\theta_W\left(Q^2=0\right) \;=\; \alpha/\alpha_2 \;=\; 0.23871(9)$$

[Kumar et al., 1302.6293]

- LO hadronic contribution to the $SU(2)_L$ coupling α_2



- $\Delta^{\mathrm{had}}\sin^2\theta_W(Q^2) \;=\; \Delta\alpha^{\mathrm{had}}_{\mathrm{QED}}(Q^2) \;-\; \Delta\alpha_2^{\mathrm{had}}(Q^2)$

- for instance, (u,d) contribution :

$$\Delta^{\mathrm{had}}_{ud}\sin^2\theta_W(Q^2) \;=\; \Delta^{ud}\alpha^{\mathrm{had}}_{\mathrm{QED}}(Q^2)\;\left(1 - \frac{9}{20}\,\frac{\alpha_2}{\alpha}\right)$$

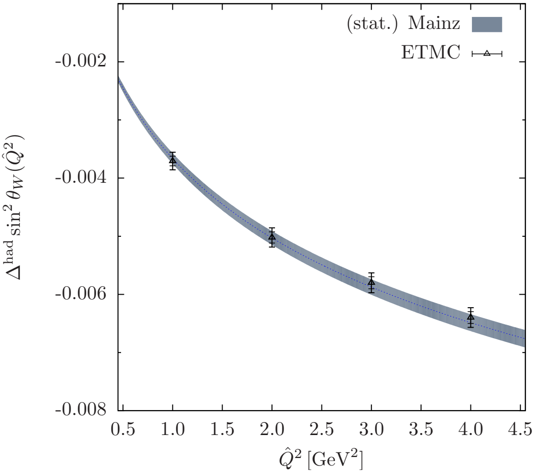
[ETMC, 1505.03283]

[talk by Vera Gülpers: Wed.,15h]

$\Delta^{\text{had}} \sin^2 \theta_W(Q^2)$

u, d, s_Q, c_Q

[PRELIMINARY]



u, d, s, c : [ETMC, 1505.03283]

conclusions

- lattice determination of the LO hadronic contribution to the running of the QED coupling and of $\sin^2\theta_W$
- Adler function : $\rightsquigarrow$ $\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$, $\Delta^{\rm had}\sin^2\theta_W(Q^2)$, α_s , $a_\mu^{\rm had}$
- combination of methods : Adler function, standard VPF and "mixed representation" methods $\rightsquigarrow$ $a_\mu^{\rm had}$ [following talk: Hanno Horch]
- $\Delta^{\rm had}\sin^2\theta_W(Q^2)$: quark-disconnected diagrams [talk by Vera Gülpers: Wed.,15h]

Lattice VPF

Conserved current

$$J_\mu^{(\mathrm{ps},\,f)}(x) = \frac{1}{2}\left(\overline{\psi}_f(x+a\hat{\mu})(1+\gamma_\mu)U_\mu^\dagger(x)\,\psi_f(x) \;-\; \overline{\psi}_f(x)(1-\gamma_\mu)U_\mu(x)\,\psi_f(x+a\hat{\mu})\right)$$

Local current

$$J_\mu^{(1,\,f)}(x) = Z_\mathrm{V}\,\overline{\psi}_f(x)\,\gamma_\mu\,\psi_f(x)$$

conserved-local correlator

$$a^6\left\langle \sum_{f=1}^{N_\mathrm{f}}\left(Q_f\,J_\mu^{(\mathrm{ps},\,f)}(x)\right)\,\sum_{f'=1}^{N_\mathrm{f}}\left(Q_{f'}\,J_\nu^{(1,\,f')}(0)\right)\right\rangle$$

$\Delta\alpha_{\mathrm{QED}}^{\mathrm{had}}$

$$\alpha(Q^2) = \frac{\alpha}{1 - \Delta\alpha_{\mathrm{QED}}(Q^2)}$$

- Combine experimental data and perturbation theory (PT)

$$\begin{aligned}\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) = {} & \Delta\alpha_{\mathrm{had}}^{(5)}(-M_0^2)^{\mathrm{exp}} \\ & + \left[\Delta\alpha_{\mathrm{had}}^{(5)}(-M_Z^2) - \Delta\alpha_{\mathrm{had}}^{(5)}(-M_0^2)\right]^{\mathrm{pQCD}} \\ & + \left[\Delta\alpha_{\mathrm{had}}^{(5)}(M_Z^2) - \Delta\alpha_{\mathrm{had}}^{(5)}(-M_Z^2)\right]^{\mathrm{pQCD}}\end{aligned}$$

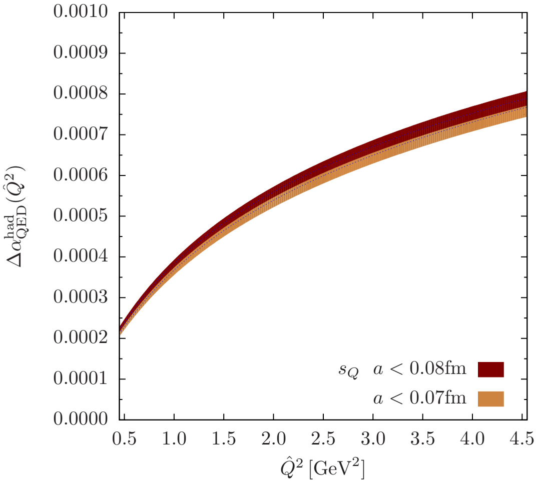
$$M_0^2 = (2.5\,\mathrm{GeV})^2 \approx 6\,\mathrm{GeV}^2$$

- lattice QCD

$\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$: strange quark syst. effects

$$D(Q^2) = \frac{3\pi}{\alpha}\frac{d}{d\log(Q^2)}\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$$

s_Q ; $a \to 0$; $M_\pi^{\rm phys}$

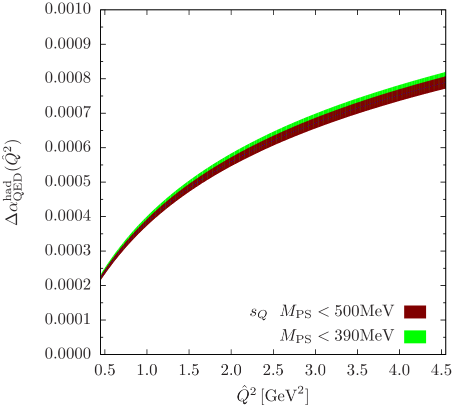


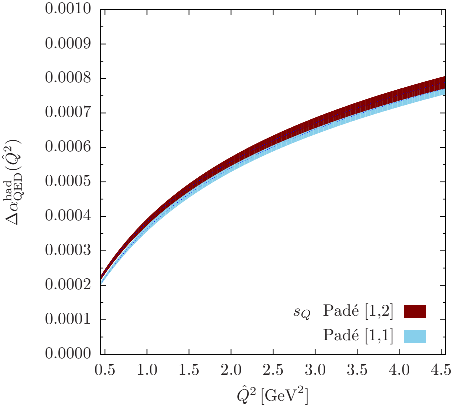
Padé [1, 2] with $O(a)$ lattice artefacts and linear form in $M_{\rm PS}^2$

$\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$: strange quark syst. effects

$$D(Q^2) = \frac{3\pi}{\alpha}\frac{d}{d\log(Q^2)}\Delta\alpha_{\rm QED}^{\rm had}(Q^2)$$

s_Q ; $a \to 0$; $M_\pi^{\rm phys}$



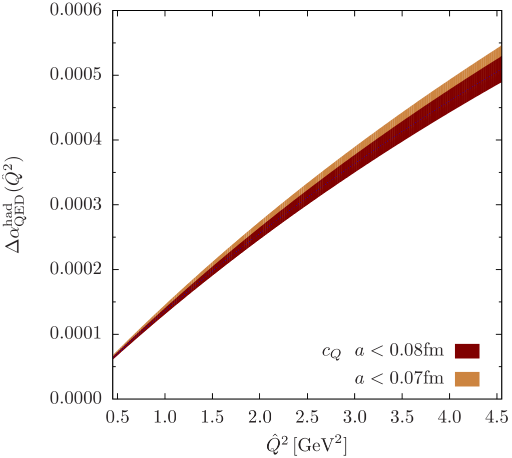


O(a) lattice artefacts and linear form in $M_{\rm PS}^2$

Adler function : charm quark

$$D(Q^2) = \frac{3\pi}{\alpha} \frac{d}{d\log(Q^2)} \Delta\alpha_{\rm QED}^{\rm had}(Q^2)$$

c_Q ; $a \to 0$; $M_\pi^{\rm phys}$



Padé [1, 1] with $O(a)$ lattice artefacts and linear form in $M_{\rm PS}^2$

$M_{\rm PS} < 390\,\rm MeV$