Implementation of a non-perturbative matching strategy between heavy–light currents in HQET and QCD

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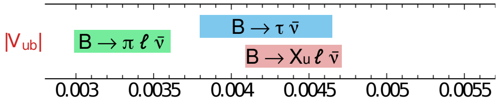
Motivation: Determination of V_{ub}

Cabibbo–Kobayashi–Maskawa matrix $V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$

can be determined from various $b \to u$ processes

- inclusive semi-leptonic $B \to X_u \ell \bar{\nu}$
- exclusive semi-leptonic $B \to \pi \ell \bar{\nu}$
  from lattice QCD: hadronic form factor $f_+(q^2)$
- leptonic $B \to \tau \bar{\nu}$
  from lattice QCD: hadronic decay constant f_B



[PDG, 2014]

- 3σ tension, V_{ub} puzzle
- new physics?
- need reliable lattice input

V_{ub} via $\mathrm{B} \to \pi \ell \bar{\nu}$ in the Standard Model

- $f_+(q^2)$ needed to determine $|V_{\mathrm{ub}}|$

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} = \frac{G_F^2 |V_{\mathrm{ub}}|^2}{192\pi^3 m_{\mathrm{B}}^3} \left(\lambda(q^2)\right)^{3/2} \left|f_+(q^2)\right|^2 \qquad q = p_{\mathrm{B}} - p_\pi$$

- $f_+(q^2)$ can be determined from the matrix element $\langle \pi(p_\pi) | V^\mu | \mathrm{B}(p_{\mathrm{B}}) \rangle$,

$$\langle \pi(p_\pi) | V^\mu | \mathrm{B}(p_{\mathrm{B}}) \rangle = f_+(q^2) \left[p_{\mathrm{B}} + p_\pi - \frac{m_{\mathrm{B}}^2 - M_\pi^2}{q^2} q^\mu \right] + f_0(q^2) \frac{m_{\mathrm{B}}^2 - M_\pi^2}{q^2} q^\mu$$

- which we want to compute on the lattice.

HQET

- effective theory for systems with one heavy quark
- action and operators are expanded in an asymptotic power series of $1/m_{\rm b}$ [Eichten 1988, Eichten & Hill 1990]

$$\text{action:} \quad \mathcal{L}^{\rm HQET} = \underbrace{\bar\psi_{\rm b} D_0 \psi_{\rm b}}_{\mathcal{O}(1)} \underbrace{- \omega_{\rm kin}\mathcal{O}_{\rm kin} - \omega_{\rm spin}\mathcal{O}_{\rm spin}}_{\mathcal{O}(1/m_{\rm b})} + \ldots$$

$$\mathcal{O}_{\rm kin} = \bar\psi_{\rm b} \mathbf{D}^2 \psi_{\rm b} \qquad \mathcal{O}_{\rm spin} = \bar\psi_{\rm b} \boldsymbol{\sigma} \cdot \mathbf{B} \psi_{\rm b}$$

$$\text{operators:} \quad \mathcal{O}_{\rm R}^{\rm HQET} = Z_{\mathcal{O}}^{\rm HQET} \left[\mathcal{O}^{\rm stat} + \sum_i c_{\mathcal{O}_i} \mathcal{O}_i \right]$$

- parameters: $(m_{\rm bare}, \omega_{\rm kin}, \omega_{\rm spin}, Z_{\mathcal{O}}^{\rm HQET}, c_{\mathcal{O}_1}, \ldots)$
- all $\mathcal{O}_i$ with same quantum numbers and correct dimension must be taken into account

Considered Operators

Previously: A_0

- to compute decay constant f_B

$$A_{0,\mathrm{R}}^{\mathrm{HQET}} = Z_{A_0}^{\mathrm{HQET}} \left[A_0^{\mathrm{stat}} + \sum_{i=1}^{2} c_{A_{0,i}} A_{0,i} \right]$$

$$A_{0,1} = \bar{\psi}_\mathrm{l} \gamma_5 \gamma_i \frac{1}{2} \left(\nabla_i - \overleftarrow{\nabla}_i \right) \psi_\mathrm{b} \quad A_{0,2} = \bar{\psi}_\mathrm{l} \gamma_5 \gamma_i \frac{1}{2} \left(\nabla_i + \overleftarrow{\nabla}_i \right) \psi_\mathrm{b}$$

- $A_{0,2}$ vanishes due to sum over **x** if BC are periodic
- 5 parameters left: $(m_{\mathrm{bare}}, \omega_{\mathrm{kin}}, \omega_{\mathrm{spin}}, Z_{A_0}^{\mathrm{HQET}}, c_{A_{0,1}})$

[ALPHA Collaboration, arxiv:1203.6516]

Considered Operators

Now: A_0, A_k, V_0, V_k

- application: form factor f_+
- 14 new parameters, e.g. from

$$V_{k,\mathrm{R}}^{\mathrm{HQET}} = Z_{V_k}^{\mathrm{HQET}} \left[V_k^{\mathrm{stat}} + \sum_{i=1}^{4} c_{V_{k,i}} V_{k,i} \right]$$

$$V_{k,1} = \bar{\psi}_{\mathrm{l}} \gamma_k \gamma_i \frac{1}{2} \left(\nabla_i - \overleftarrow{\nabla}_i \right) \psi_{\mathrm{b}} \quad V_{k,2} = \bar{\psi}_{\mathrm{l}} \frac{1}{2} \left(\nabla_k - \overleftarrow{\nabla}_k \right) \psi_{\mathrm{b}}$$

$$V_{k,3} = \bar{\psi}_{\mathrm{l}} \gamma_k \gamma_i \frac{1}{2} \left(\nabla_i + \overleftarrow{\nabla}_i \right) \psi_{\mathrm{b}} \quad V_{k,4} = \bar{\psi}_{\mathrm{l}} \frac{1}{2} \left(\nabla_k + \overleftarrow{\nabla}_k \right) \psi_{\mathrm{b}}$$

- complete set of 19 parameters:

$$\begin{aligned}(&m_{\mathrm{bare}}, \omega_{\mathrm{kin}}, \omega_{\mathrm{spin}},\\ &Z_{A_0}^{\mathrm{HQET}}, c_{A_{0,1}}, c_{A_{0,2}}, Z_{A_k}^{\mathrm{HQET}}, c_{A_{k,1}}, c_{A_{k,2}}, c_{A_{k,3}}, c_{A_{k,4}},\\ &Z_{V_0}^{\mathrm{HQET}}, c_{V_{0,1}}, c_{V_{0,2}}, Z_{V_k}^{\mathrm{HQET}}, c_{V_{k,1}}, c_{V_{k,2}}, c_{V_{k,3}}, c_{V_{k,4}})\end{aligned}$$

Matching

$$\boldsymbol{\omega} = (m_{\text{bare}}, \omega_{\text{kin}}, \omega_{\text{spin}},\\ Z_{A_0}^{\text{HQET}}, c_{A_{0,1}}, c_{A_{0,2}}, Z_{A_k}^{\text{HQET}}, c_{A_{k,1}}, c_{A_{k,2}}, c_{A_{k,3}}, c_{A_{k,4}},\\ Z_{V_0}^{\text{HQET}}, c_{V_{0,1}}, c_{V_{0,2}}, Z_{V_k}^{\text{HQET}}, c_{V_{k,1}}, c_{V_{k,2}}, c_{V_{k,3}}, c_{V_{k,4}})$$

- parameters can be determined via *matching* HQET and QCD
- matching condition:

$$\Phi^{\text{QCD}}(L, M, 0) \stackrel{!}{=} \Phi^{\text{HQET}}(L, M, a)$$

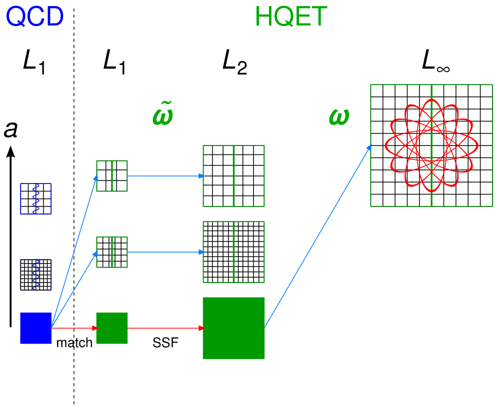
with 19 suitable observables Φ

Why non-perturbative matching?

- operator mixing in effective theory induces power divergences $\propto g^{2l}/a^n,\ n \in \{1, 2\}$
- truncated terms in pert. series can spoil continuum limit
- $\Rightarrow$ *non*-perturbative matching needed

[ALPHA Collaboration, Heitger & Sommer, JHEP0402(2004)022]

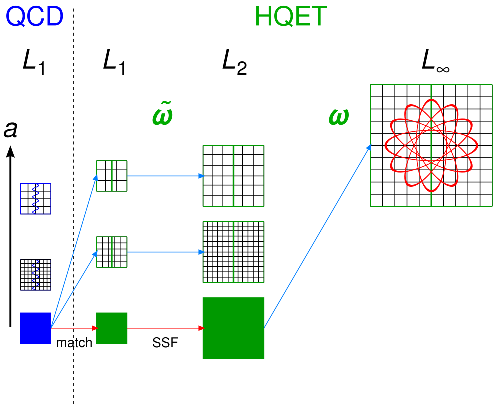
Matching



- choose $n_{\rm par}$ observables $\mathbf{\Phi} = (\Phi_1, \ldots, \Phi_{n_{\rm par}})$, sensitive to the HQET parameters, with a linear expansion

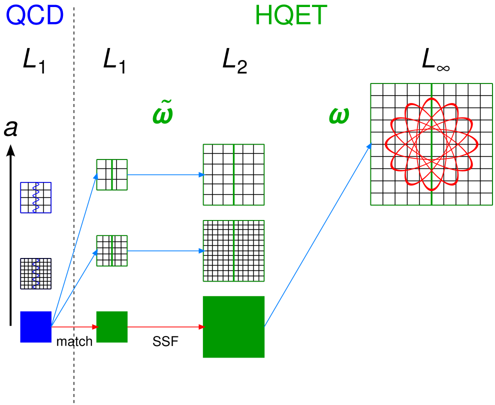
$$\mathbf{\Phi}^{\rm HQET} = \boldsymbol{\eta} + \varphi \cdot \tilde{\boldsymbol{\omega}} \qquad \text{(matrix–vector notation)}$$

Matching



- small volume (L_1): compute observables in relativistic QCD $\mathbf{\Phi}^{\mathrm{QCD}}(L_1, M, a)$ and extrapolate to continuum limit $\mathbf{\Phi}^{\mathrm{QCD}}(L_1, M, 0)$

Matching

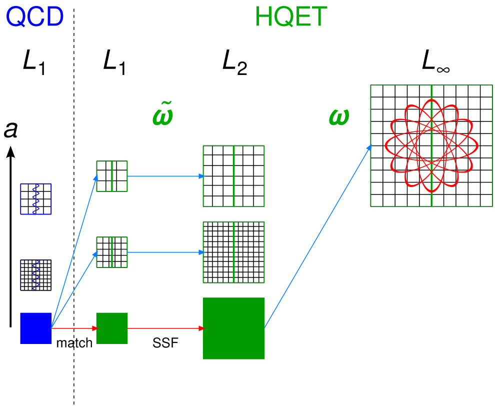


- matching with HQET ($a \lesssim 0.05\,\mathrm{fm}$):

$$\mathbf{\Phi}^{\mathrm{HQET}}(L_1, M, a) = \boldsymbol{\eta}(L_1, a) + \varphi(L_1, a) \cdot \tilde{\omega}(M, a) \stackrel{!}{=} \mathbf{\Phi}^{\mathrm{QCD}}(L_1, M, 0)$$

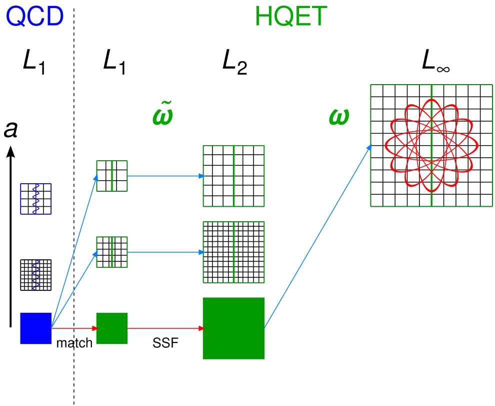
$$\Rightarrow \tilde{\omega}(M, a) = \varphi^{-1}\left[\mathbf{\Phi}^{\mathrm{QCD}} - \boldsymbol{\eta}\right]$$

Matching



- step scaling: use $\tilde{\omega}(M, a)$ to compute observables $\mathbf{\Phi}^{\mathrm{HQET}}(L_2, M, a)$ in larger volume, extrapolate to continuum

Matching

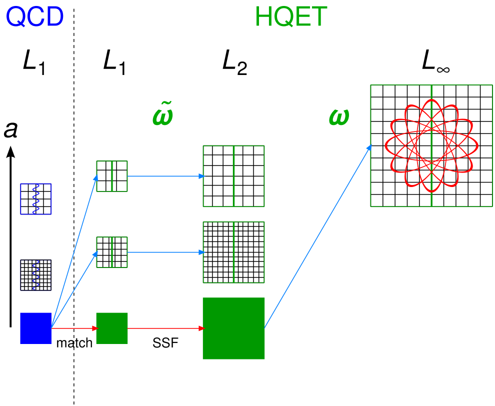


- determine parameters $\omega(M,a)$ for larger a

$$\omega(M,a) = \varphi(L_2,a)^{-1}\left[\Phi^{\mathrm{HQET}}(L_2,M,0) - \eta(L_2,a)\right]$$

[ALPHA Collaboration, Blossier et al., arXiv:1203.6516]

Matching



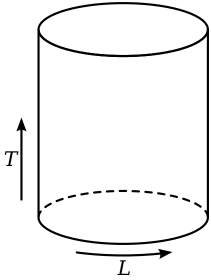
- use $\omega(M, a)$ to compute desired large-volume observables

Schrödinger Functional

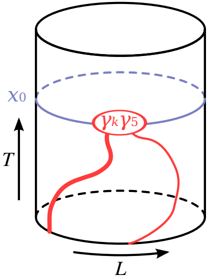
- observables defined in Schrödinger-functional setup
- finite volume $T \times L^3$
- Dirichlet BC in time, periodic BC in space
- additional phase $\boldsymbol{\theta}$ for fermions:

$$\psi(x_0, \mathbf{x} + \mathbf{n} \cdot L) = \mathrm{e}^{\mathrm{i}\boldsymbol{\theta}\cdot\mathbf{n}} \cdot \psi(x)$$

- boundary fields ζ used to build CF
- well known renormalization properties



Schrödinger Functional



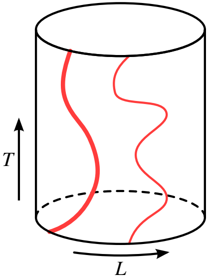
Example (Boundary–Bulk CF)

$$k_{V_k}(x_0,\boldsymbol{\theta}_\mathrm{l},\boldsymbol{\theta}_\mathrm{b}) = -\frac{a^6}{6L^3}\sum_{\mathbf{x},\mathbf{y},\mathbf{z},k}\Big\langle\,\underbrace{\bar\psi_\mathrm{l}(x)\gamma_k\psi_\mathrm{b}(x)}_{V_k(x)}\times$$

$$\times\,\bar\zeta_\mathrm{b}(\mathbf{y})\gamma_k\zeta_\mathrm{l}(\mathbf{z})\Big\rangle$$

Example (Boundary–Boundary CF)

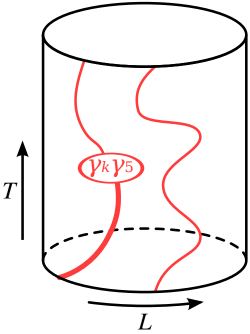
$$F_1(\boldsymbol{\theta}_\mathrm{l},\boldsymbol{\theta}_\mathrm{b}) = -\frac{a^{12}}{2L^6}\sum_{\mathbf{u},\mathbf{v},\mathbf{y},\mathbf{z}}\Big\langle\bar\zeta_\mathrm{l}'(\mathbf{u})\gamma_5\zeta_\mathrm{b}'(\mathbf{v})\bar\zeta_\mathrm{b}(\mathbf{y})\gamma_5\zeta_\mathrm{l}(\mathbf{z})\Big\rangle$$



3-point correlators

- also 3-point correlators added
- perturbative studies favor obs. built from 3-pt. correlators
- to be checked non-perturbatively

$$J^1_{A_1}(x_0, \boldsymbol{\theta}_\mathrm{l}, \boldsymbol{\theta}_\mathrm{b}) = -\frac{a^{15}}{2L^6} \sum_{\mathbf{uvyzx}} \langle \bar{\zeta}'_{\mathrm{l}'}(\mathbf{u}) \gamma_1 \zeta'_{\mathrm{l}}(\mathbf{v}) \times$$
$$\times\, \bar{\psi}_\mathrm{l}(x) \gamma_1 \gamma_5 \psi_\mathrm{b}(x) \bar{\zeta}_\mathrm{b}(\mathbf{z}) \gamma_5 \zeta_{\mathrm{l}'}(\mathbf{y}) \rangle$$



Choice of Observables

- 19 observables for 19 free parameters
- observables $\mathbf{\Phi}$ shall...
  - be sensitive to the parameters $\boldsymbol{\omega}$
  - yield small $1/m_\mathrm{b}$ corrections
- θ most important free parameter
- investigated in tree-level study by M. Della Morte, S. Dooling et al. [ALPHA Collaboration, arxiv:1312.1566]
- four additional, similar sets of observables will be tested

Example

- Φ_{14} sensitive to $c_{V_{k,1}}$ and $c_{V_{k,2}}$

$$\Phi_{14}^{\mathrm{QCD}} = \ln\left(\frac{k_{V_k}(T/2,\theta_1,\theta_1)}{k_{V_k}(T/2,\theta_2,\theta_2)}\right)$$

$$\Phi_{14}^{\mathrm{HQET}} = \underbrace{\Phi_{14}^{\mathrm{stat}}}_{\boldsymbol{\eta}} + \underbrace{\omega_{\mathrm{kin}}\Phi_{14}^{\mathrm{kin}} + \omega_{\mathrm{spin}}\Phi_{14}^{\mathrm{spin}} + c_{V_{k,1}}\Phi_{14,1} + c_{V_{k,2}}\Phi_{14,2}}_{\varphi\cdot\boldsymbol{\omega}}$$

Choice of Observables

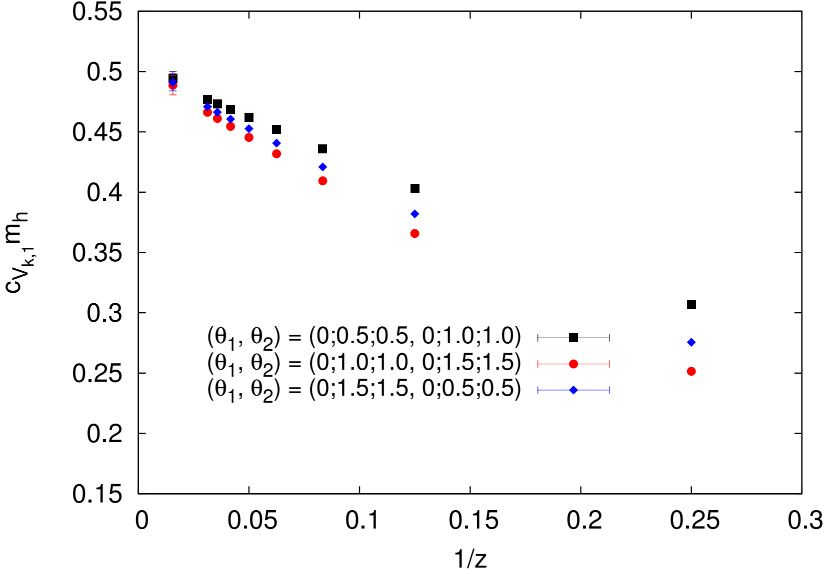


Figure: $c_{V_{k,1}}$ at tree-level determined via different θ versus inverse mass

Ensembles and Measurements

- re-using ensembles from matching of A_0
- $N_f = 2$, $\theta = (0.5, 0.5, 0.5)$

	L	T	β	L/a	meas. status
QCD	L_1	L_1	$6.16 \dots 6.64$	$20 \dots 40$	DONE
QCD	L_1	$L_1/2$	$6.16 \dots 6.64$	$20 \dots 40$	DONE
HQET	L_1	L_1	$5.26 \dots 5.96$	$6 \dots 16$	DONE
HQET	L_1	$L_1/2$	$5.26 \dots 5.96$	$6 \dots 16$	DONE
HQET	L_2	L_2	$5.26 \dots 5.96$	$12 \dots 32$	TO DO
HQET	L_2	$L_2/2$	$5.26 \dots 5.96$	$12 \dots 32$	TO DO

- measurements at seven different renormalized masses
- various combinations of θ of light and heavy quarks to support several matching strategies

Summary

- HQET: effective theory for heavy–light systems
- parameters via non-pert. matching to QCD
- want to determine parameters for all components of axial and vector current
- matching observables investigated in tree-level study
- created SFCF program for measurement
- completing measurements
- large-volume simulations to obtain $f_+(q^2)$ being done in parallel

Results from a Test Analysis

- very preliminary, only to demonstrate that you can get numbers out of the proposed matching strategy
- at $\beta = 5.7580$

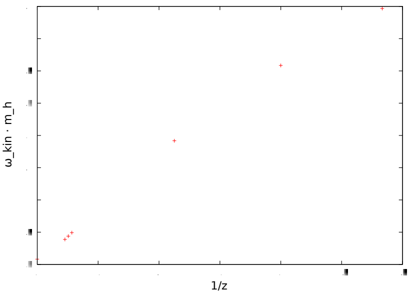


Figure: ω_{kin} vs. inverse mass

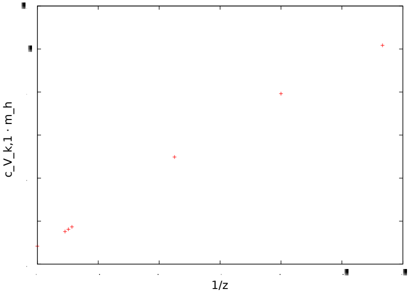


Figure: $c_{V_{k,1}}$ vs. inverse mass

[P. Korcyl]