# Dashen's theorem and electromagnetic contributions to pseudoscalar meson masses

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# Introduction

We are carrying out simulations in QCD + QED
Both gauge theories fully dynamical (charges of sea-quark loops included)
QED stronger than real world, so that we can see effects easily, and scale back to physical $\alpha_{EM}$

$$\beta_{EM} = 0.8,\ e^2 = 1.25,\ \alpha_{EM} \approx 0.1$$

Partially quenched, with $u, d, s$ and $n$ quarks ($n$ a fictional quark of charge 0).
No disconnected graphs, so $u\bar{u}, d\bar{d}, s\bar{s}, n\bar{n}$ do not mix. To a good approximation

$$M^2_{\pi^0} = [M^2(u\bar{u}) + M^2(d\bar{d})]/2$$

# Introduction

First look at pseudoscalar mesons.

Targets

Find $\pi^+$-$\pi^0$ splitting.

Find physical $\kappa$ for $u, d, s,$ needed to predict splittings in baryons. Delicate number - $u, d$ mass difference.

Dissect meson mass into a QCD part and a QED part

Theory discussion

Lattice results.

# Scheme dependence

In pure QCD, $m_u = m_d$ is unproblematic. equal bare mass implies equal renormalised mass, regardless of scale or scheme.
In QED+QCD, mass ratios between quarks of different charge are not invariant.
$\gamma_m = 6C_F g^2 + 6Q_f^2 e^2 + \cdots$
$u$ mass runs faster than $d$ mass.
If $m_u = m_d$ in one scheme, will not be so in another.
Also - no good way to compare masses at physical $e^2$ with masses at $e^2 = 0$.

# Scheme dependence

$M_\gamma^2 = M^2(g^2, e^2, m_{phys}) - M^2(g^2, 0, m_?)$
but we have to carefully specify exactly which pure QCD parameters correspond to the physical parameters.

Popular choice: same quark masses at $\overline{MS}$ with $\mu = 2$ GeV.

We will suggest an alternative, but also transform to get the $\overline{MS}$ result.

# Symmetric Point

We use same approach as for pure QCD.

Find a symmetric point, with all three quark masses equal. Write down allowed expansion terms. QCD + QED works very much like pure QCD. Electric charge is an octet, build up polynomials in charge and mass splitting.

Main difference - must have even powers of charge, so leading EM $\sim e^2$, leading QCD $\sim \delta m$.

# Symmetric Point

Symmetric point, all neutral mesons have the same mass

$a\delta m \equiv \frac{1}{2\kappa} - \frac{1}{2\kappa_{sym}}$

Use distance from symmetric point as measure of SU(3) breaking.

# Mass Formula

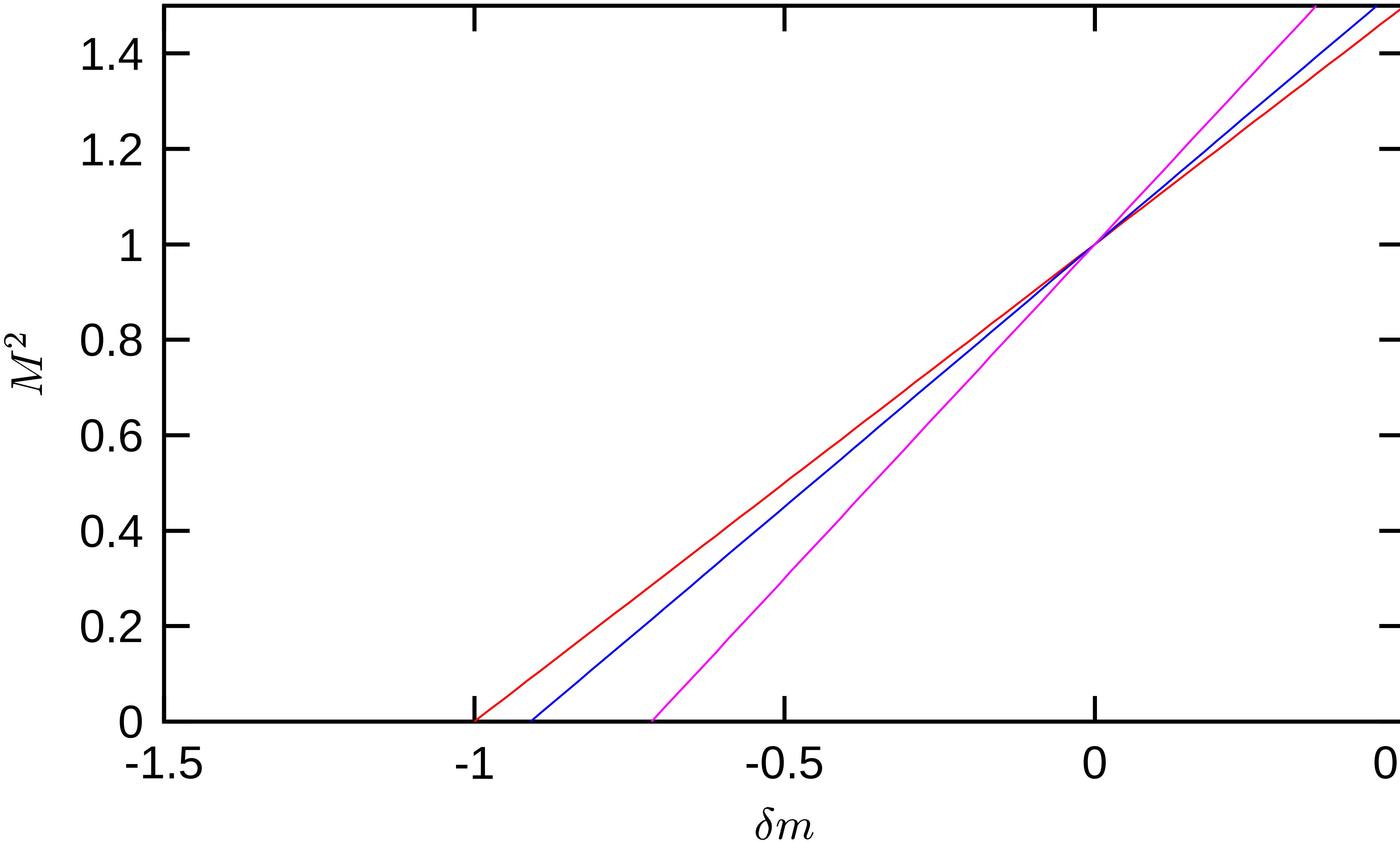
$$
\begin{aligned}
M^2(a\bar{b}) \quad = \quad & M^2 + \alpha(\delta\mu_a + \delta\mu_b) \\
& + \beta_0 \tfrac{1}{6}(\delta m_u^2 + \delta m_d^2 + \delta m_s^2) + \beta_1(\delta\mu_a^2 + \delta\mu_b^2) \\
& + \beta_2(\delta\mu_a - \delta\mu_b)^2 \\
& + \beta_0^{EM}(e_u^2 + e_d^2 + e_s^2) + \beta_1^{EM}(e_a^2 + e_b^2) + \beta_2^{EM}(e_a - e_b)^2 \\
& + \gamma_0^{EM}(e_u^2\delta m_u + e_d^2\delta m_d + e_s^2\delta m_s) + \gamma_1^{EM}(e_a^2\delta\mu_a + e_b^2\delta\mu_b) \\
& + \gamma_2^{EM}(e_a - e_b)^2(\delta\mu_a + \delta\mu_b) + \gamma_3^{EM}(e_a^2 - e_b^2)(\delta\mu_a - \delta\mu_b) \\
& + \gamma_4^{EM}(e_u^2 + e_d^2 + e_s^2)(\delta\mu_a + \delta\mu_b) \\
& + \gamma_5^{EM}(e_a + e_b)(e_u\delta m_u + e_d\delta m_d + e_s\delta m_s)
\end{aligned}
$$

# Mass Formula

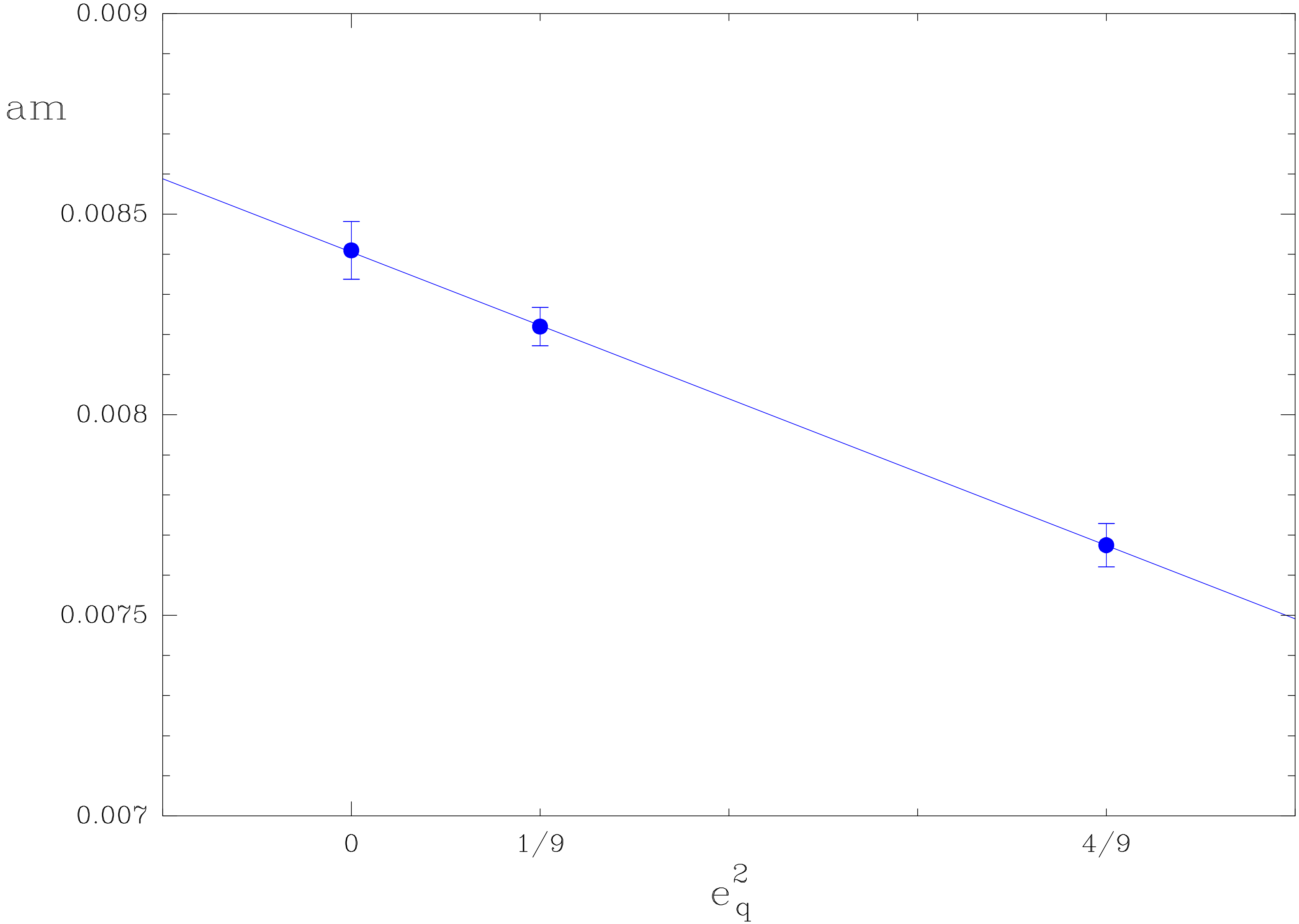
Lots of coefficients.
One worth remembering, leading QCD term

$$M^2(a\bar{b}) = \alpha(m_a + m_b) + \cdots$$

# Sketch



# Symmetric Point



# Symmetric Point

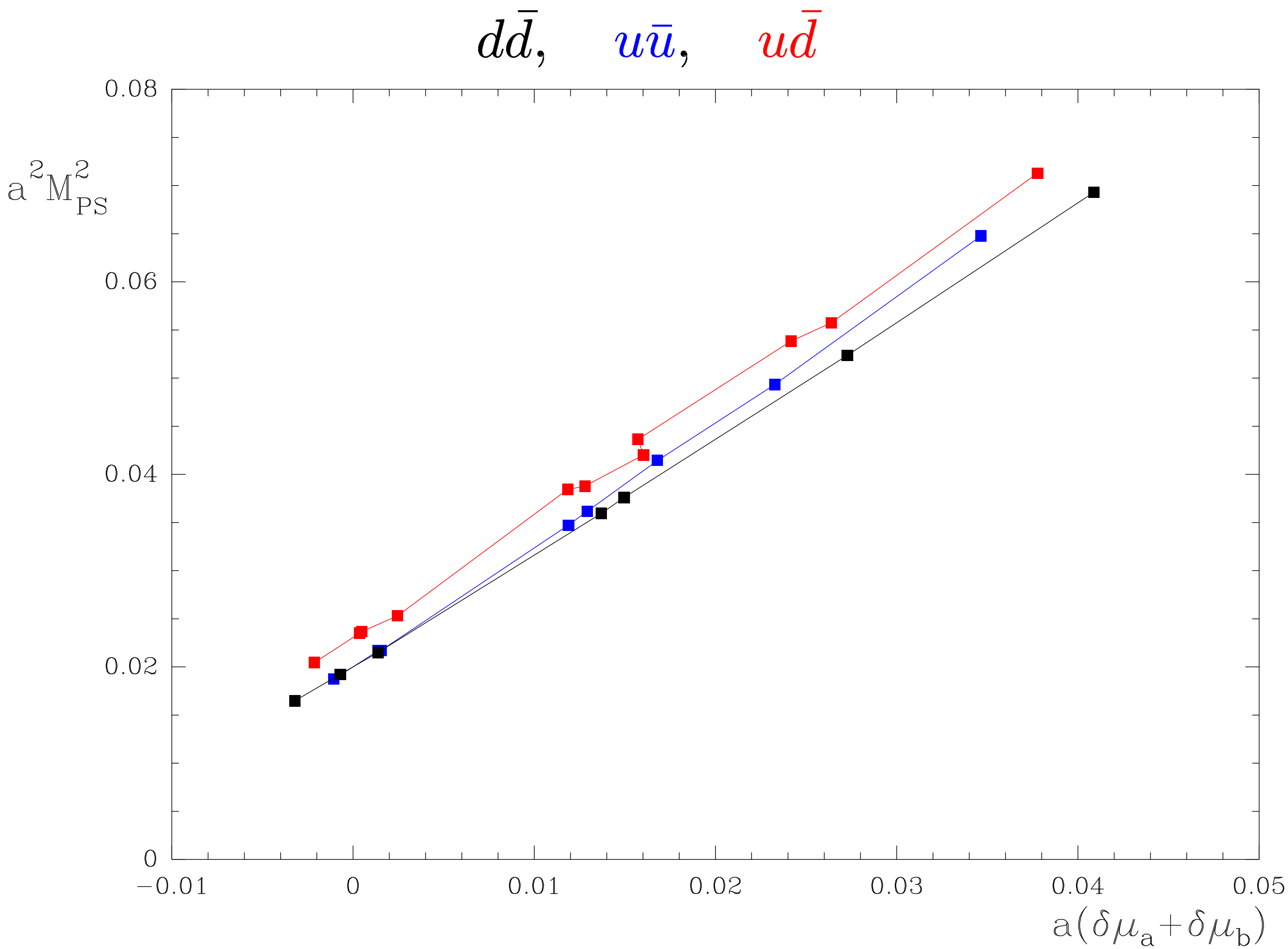
Unlike pure QCD, equal meson mass at symmetric point no longer means equal bare quark mass - gradient of $u\bar{u}, d\bar{d}, n\bar{n}$ differ.

We rescale (renormalise) to remove this effect, make the renormalised quark masses at the symmetric point equal.

$$\delta\mu_f^D = (1 + KQ_f^2 e^2)\delta\mu_f$$

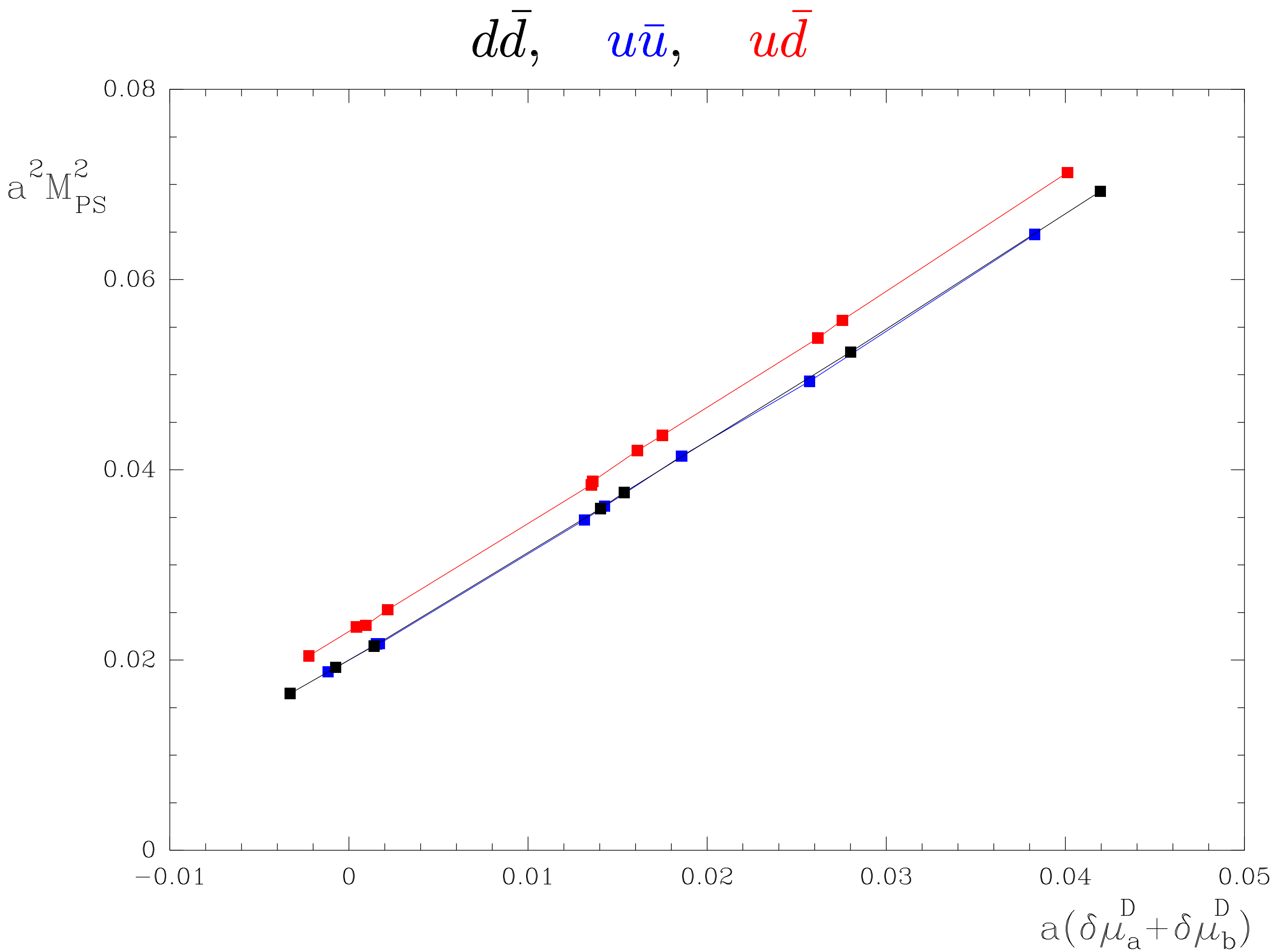
By construction, this simplifies the neutral mesons - what does it do to the charged?

# Bare Mass



$d\bar{d}$, $u\bar{u}$, $u\bar{d}$
$a^2M^2_{PS}$
0.08
0.06
0.04
0.02
0
−0.01
0
0.01
0.02
0.03
0.04
0.05
$a(\delta\mu_a+\delta\mu_b)$

# Dashen Mass



$d\bar{d}$, $u\bar{u}$, $u\bar{d}$
$a^2 M_{PS}^2$
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0
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0.02
0.03
0.04
0.05
$a(\delta\mu_a^D + \delta\mu_b^D)$

# Dashen scheme

$M_\gamma^2 = M^2(g^2, e^2, m_{phys}) - M^2(g^2, 0, m_?)$

One prescription for choosing the quark masses in the (unphysical) pure QCD case - use the neutral meson masses. Tune $m_u$ by requiring

$$M_{u\bar{u}}^2(g^2, e^2, m_{phys}) = M_{u\bar{u}}^2(g^2, 0, m_{QCD})$$

etc.

Since (QCD+QED) mass matches the QCD mass, this scheme has zero EM contribution to neutral pseudoscalars by definition.

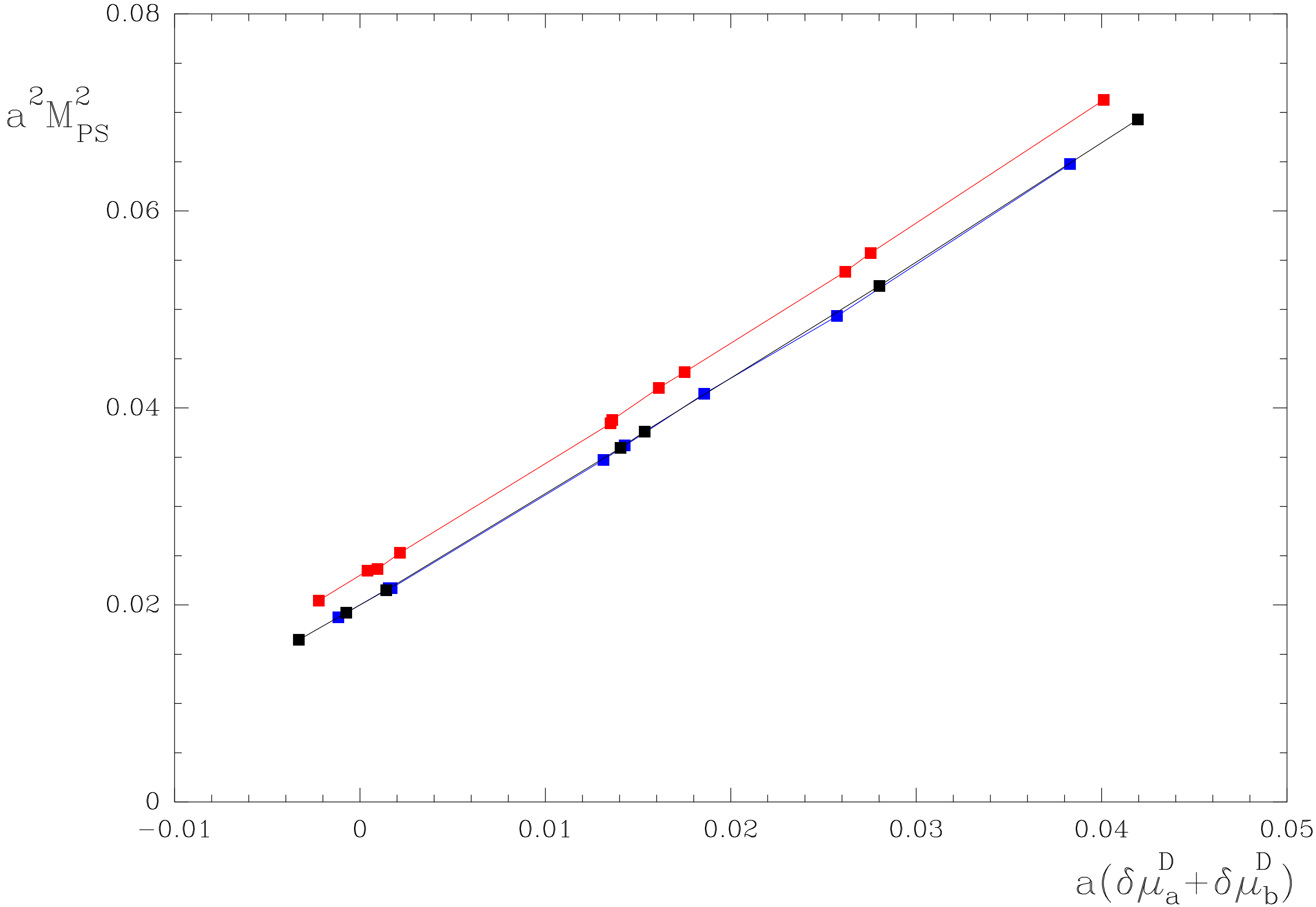
# Dashen's Theorem(s)

The QED contribution to the energy of the neutral pseudoscalar mesons is zero

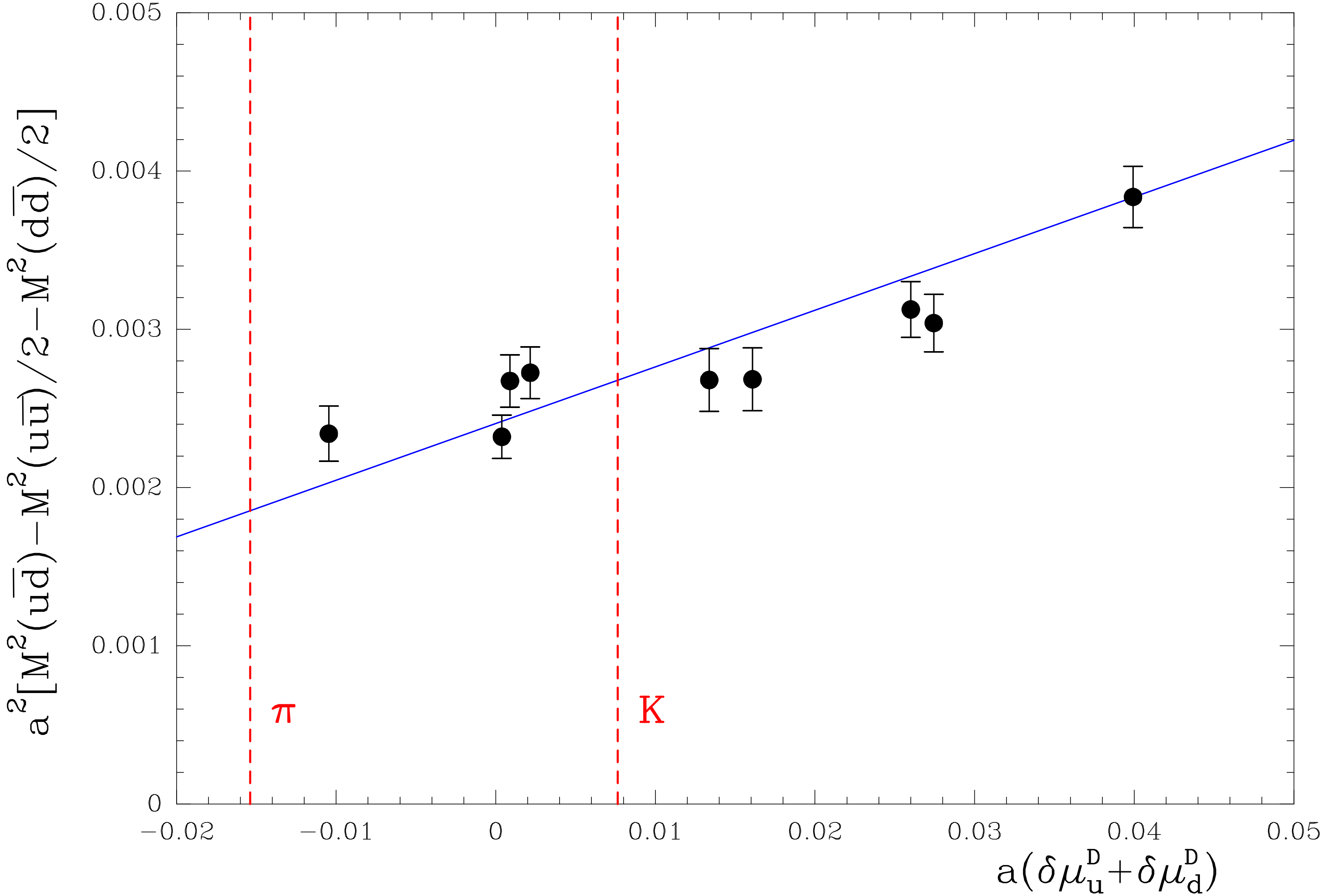
The QED contribution to the energy of the charged pseudoscalar mesons is constant

In our “Dashen scheme" the first statement is automatically true, the second needs testing.

# Dashen's Theorem(s)



# Dashen's Theorem(s)



# Results

Input:

$$M_{\pi^0} = 134.977 \text{ MeV}$$
$$M_{K^0} = 497.614 \text{ MeV}$$
$$M_{K^+} = 493.677 \text{ MeV}$$

Output:

$$M_{\pi^+} = 139.2(2) \text{ MeV}$$
$$M_{\pi^+} - M_{\pi^0} = 4.2(2) \text{ MeV}$$

(scheme independent)

# Results

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(scheme independent)
Real World:

$$\begin{aligned} M_{\pi^+} &= 139.570 \text{ MeV} \\ M_{\pi^+} - M_{\pi^0} &= 4.594 \text{ MeV} \end{aligned}$$

# $\epsilon$

Measure EM contributions in terms of $\pi^+$-$\pi^0$ splitting.

$$\Delta_\pi = M_{\pi^+}^2 - M_{\pi^0}^2$$

$$\begin{aligned} M_{\pi^0}^2(g^2, e^2) - M_{\pi^0}^2(g^2, 0) &= \epsilon_{\pi^0}\Delta_\pi \\ M_{K^0}^2(g^2, e^2) - M_{K^0}^2(g^2, 0) &= \epsilon_{K^0}\Delta_\pi \\ M_{\pi^+}^2(g^2, e^2) - M_{\pi^+}^2(g^2, 0) &= [1 + \epsilon_{\pi^0} - \epsilon_m]\Delta_\pi \\ M_{K^+}^2(g^2, e^2) - M_{K^+}^2(g^2, 0) &= [1 + \epsilon + \epsilon_{K^0} - \epsilon_m]\Delta_\pi \end{aligned}$$

$$[M_{K^+}^2 - M_{K^0}^2 - M_{\pi^+}^2 + M_{\pi^0}^2]_{QED} = \epsilon\Delta_\pi$$

FLAG review

Neglect $\epsilon_m$ (very small - disconnected diagrams)

# Scheme dependence of $\epsilon$

In QED plus QCD, each hadron will be surrounded by a photon cloud. As in pure QED, the total energy in the cloud will be UV divergent.

Crudely, we can think of two components of the cloud:

Short wave-length photons, with wave-lengths small compared with a hadron radius. These can be associated with particular quarks. If we look at the hadron with some finite resolution the photons with wavelengths shorter than this resolution are incorporated into the quark masses as self energies.

# Scheme dependence of $\epsilon$

Long wave-lengths photons, which can't be associated with particular quarks. these photons must be thought of as the photon cloud of the hadron as a whole, these are the photons that we include when we talk of the electromagnetic contribution to the hadron mass. We expect to see many more really long wave-length photons (large compared with the hadron radius) around a charged hadron than around a neutral hadron.

Clearly, in this picture, the value we get for the electromagnetic contribution to the hadron energy is going to depend on our resolution, i.e. on the scheme and scale that we use for renormalising QED.

# Scheme dependence of $\epsilon$

Use same $\overline{MS}$ quark masses for (QCD+QED) and for pure QCD (popular scheme).

What does it mean for masses in other schemes (eg Dashen)?

$$\begin{aligned} m_D(g^2, e^2) &= Z_m(g^2, e^2) m_{\overline{MS}} \\ m_D(g^2, 0) &= Z_m(g^2, 0) m_{\overline{MS}} \end{aligned}$$

$$m_D(g^2, 0) = \frac{Z_m(g^2, 0)}{Z_m(g^2, e^2)} m_D(g^2, e^2)$$

Use 1-loop estimate, plus generous error bars.
$m_D(g^2, e^2)$ stays as before (physical value), $m_D(g^2, 0)$ changes, so EM energy of a neutral meson no longer zero.

# Preliminary Results

$\overline{MS},\ \mu = 2$ GeV

$$\begin{aligned} \epsilon &= 0.48 \pm 0.05 \\ \epsilon_{\pi^0} &= 0.03 \pm 0.02 \\ \epsilon_{K^0} &= 0.2 \pm 0.1 \end{aligned}$$

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