Lattice calculation of the HVP contribution to the anomalous magnetic moment of muon

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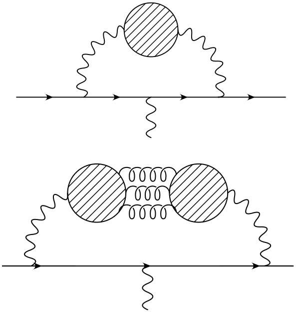
Current status of muon g-2

- BNL E821 0.54 ppm; planned 4-fold improvement in Fermilab E989.
- Overall theoretical uncertainty in a_μ: 0.42 ppm.

Contribution	Result (x 10^{-10})	Error
QED (leptons)	11658471.8	0.00 ppm
HVP(lo) [1]	692.3	0.36 ppm
HVP(ho)	-9.8	0.01 ppm
HLbL [2]	10.5	0.22 ppm
EW	15.4	0.02 ppm
Total SM	11659180.2	0.42 ppm

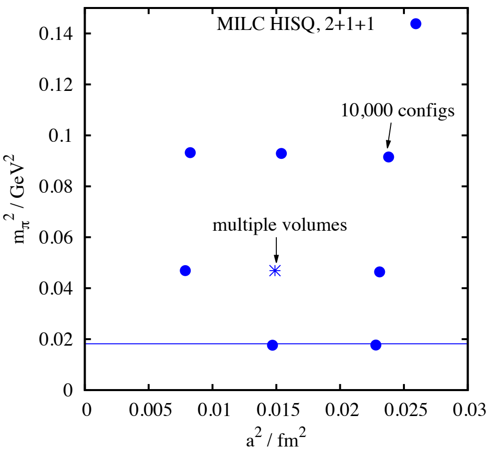
[1] Davier et. al.,Eur.Phys.J. C71 (2011) 1515 : dispersion relation + cross section for e^+e^- (and τ) $\rightarrow$hadrons with 0.7% uncertainty; [2] Prades, et. al., 0901.0306.

HVP (conn. + disc.)



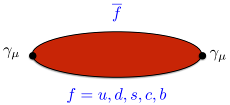
- g-2 discrepancy around 3σ between the SM and the expt.: $a_\mu^{exp} - a_\mu^{SM} = 25(9)x10^{-10}$.
- Calculate lowest order hadronic vacuum polarisation $a_\mu^{HVP,lo}$ from lattice QCD with $< 0.5\%$ uncertainty by 2017/18.

HPQCD analysis ingredients



- Light (up/down), strange, charm (2+1+1) quarks in sea: $m_l = m_s/5, m_s/10, m_s/27.5$ (physical).
- $a \approx 0.15$fm (very coarse), 0.12fm (coarse), 0.09fm (fine).
- Large box size: 5-6 fm on the finest lattices; finite volume effects tested.
- High statistics: random color wall sources, 1000 (or 10,000) configurations and 16 time sources.

HPQCD analysis method



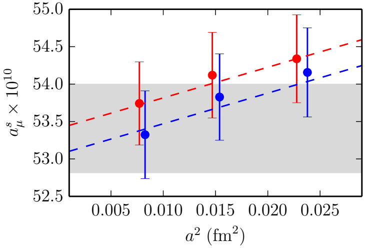
$$a_{\mu,\mathrm{HVP}}^{(\mathrm{f})} = \frac{\alpha}{\pi} \int dq^2 f(q^2)(4\pi\alpha Q_{\mathrm{f}}^2)\hat{\Pi}_{\mathrm{f}}(q^2).$$

(Lautrup,'72, T. Blum,'02)

- Reconstruct $\hat{\Pi}(q^2)$ from its derivatives at $q^2 = 0$ + Padé approximants [2,2] to calculate vacuum polarization at arbitrary Euclidean q^2: $\hat{\Pi}(q^2) = \sum_{j=1}^{\infty} q^{2j}\Pi_j$
- Time moments (n = 4,6,8,10) of (local-local) vector meson correlators give the derivatives:

$$G_{2n} \equiv a^4 \sum_t \sum_{\vec{x}} t^{2n} Z_V^2 \langle j^i(\vec{x}, t) j^i(0)\rangle$$
$$= (-1)^n \frac{\partial^{2n}}{\partial q^{2n}} q^2 \hat{\Pi}(q^2)\bigg|_{q^2=0}.$$

And $\Pi_j = (-1)^{j+1} \frac{G_{2j+2}}{(2j+2)!}$.



- 1% precision for strange (and charm) qurak case; vector correlators reproduce accurately physical properties of ϕ and J/ψ mesons [1208.2855, 1410.8466].

[Ref: Chakraborty et. al., PRD89(2014)114501, arXiv:1403.1778]

New issues for 1% uncertainty in u/d connected case: Handling ρ meson on lattice

- Correlators much noisier: Use data-fit hybrid correlator to control noise at large t:

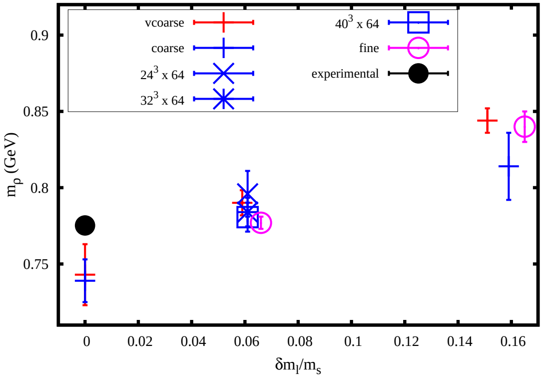
$$G(t) = \begin{cases} G_{data}(t) & \text{for } t <= t* \text{ from Monte Carlo,} \\ G_{fit}(t) & \text{for } t > t* \text{ from multi-exponential fit.} \end{cases}$$

  for t* = 1.5fm = $6/m_\rho$ (70% result from G_{data})
  (same results to within $\pm\sigma/4$ with 0.75fm).

- Gaussian smearings to obtain precise ρ ground state (25-30% gain in uncertainty in $a_\mu^{u/d}$).
- 80% of light quark vaccum polarisation contribution from ρ meson pole, need to understand ρ better.

New issues for 1% uncertainty in u/d connected case: Handling ρ meson on lattice

- Finite volume, finite lattice spacing errors: m_ρ and f_ρ too low; Significant impact on g-2.



- Rescale Π_j^{latt}: Reduces or eliminates errors from scale setting, Z factors, a^2, finite volume, m_l/m_s tuning (elaboration of ETMC idea in arXiv:1308.4327).

Rescaling to Taylor coefficients

- At large time we expect C_{ll} to behave as:

$$C_{ll,gs} = \frac{f_\rho^2 m_\rho}{2} e^{-m_\rho t}$$

- Π_j behaves as:

$$\Pi_{j,c} = (-1)^{j+1} \frac{f_\rho^2}{m_\rho^{2j+2}}$$

- Correction to the Taylor coefficients Π_j:

$$\Pi_j^{latt} \rightarrow \Pi_j^{latt} \left[\frac{m_\rho^{2+2j}}{f_\rho^2} \right]_{latt} \left[\frac{f_\rho^2}{m_\rho^{2+2j}} \right]_{expt}$$

- Experimental f_ρ is obtained from the decay width of $\rho \rightarrow e^+ e^-$ using:

$$\Gamma(\rho \rightarrow e^+ e^-) = \frac{4\pi}{3} \alpha_{QED}^2 \frac{f_\rho^2}{m_\rho} e_{u/d}^2$$

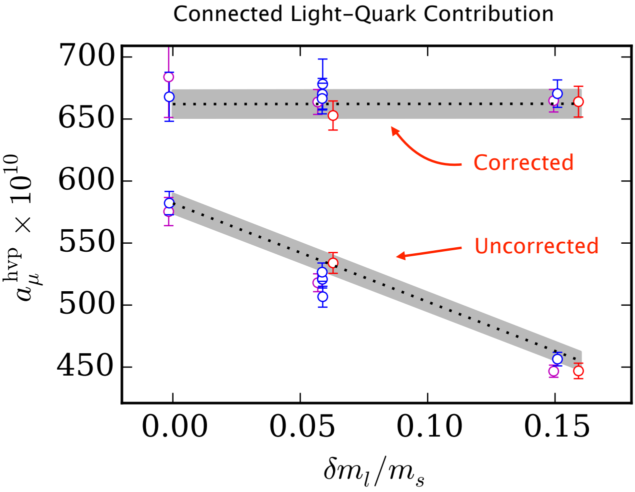
New issues for 1% uncertainty in u/d connected case: Handling $\pi\pi$ loop contribution on lattice

- $\pi\pi$ loop contribution $\sim$10% of the total light quark contribution.
- Highly sensitive to m_π (contribution roughly proportional to $1/m_\pi^2$) and finite volume.
- Staggered quark introduces extra discretisation artefacts from different taste π meson.
- Remove lattice $\pi\pi$ contribution using one-loop, staggered quark, finite-volume ChPT.
- Restore $\pi\pi$ contribution using one-loop continuum ChPT, with physical π mass.
- Scaling gives:

$$\Pi_j^{latt} \to (\Pi_j^{latt} - \Pi_j^{latt}(\pi\pi))\left[\frac{m_\rho^{2+2j}}{f_\rho^2}\right]_{latt}\left[\frac{f_\rho^2}{m_\rho^{2+2j}}\right]_{expt} + \Pi_j^{cont}(\pi\pi)$$

HPQCD preliminary Results: $a_\mu^{u/d}$ (Connected)

- Corrected results independent of m_l/m_s, a^2, finite volume.
- [2, 2] Padé approximants converged to better than 0.2%: negligible error.
- Total uncertainty is around 1.7%; statistics dominate.

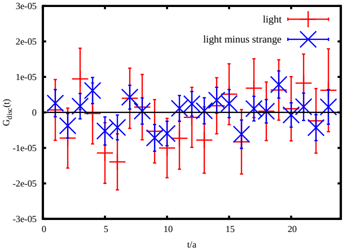


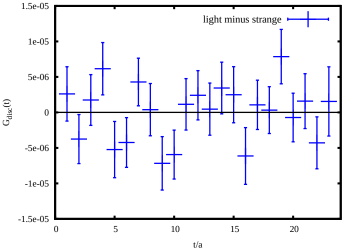
Disconnected correlators

- On lattice we calculate:

$$G_{disc}(x_0 - y_0) = -Z_V \langle (\sum_{\vec{x}} Tr[\gamma_k D^{-1}(x,x)])(\sum_{\vec{y}} Tr[\gamma_k D^{-1}(y,y)]) \rangle$$

- All-to-all propagator method with 50 stochastic noise vectors on each configuration, one-link spatial vector currents for (l-s) on both sides.





- correlators from $l - s$ vector currents at each end are much less noisy (consistent with Mainz group, arXiv:1411.7592); about 60% reduction in uncertainty compared to light quark case on $m_l/m_s = 0.2$ very coarse lattice.

HPQCD Estimation of Disconnected contribution to $a_\mu^{HVP,lo}$

- Disconnected u/d piece D_{ll} dominates (Hadspec collaboration (arXiv:1309.2608)).
- Assuming D_{ll} provide difference between ω and ρ, at large t,

$$2D_{ll,gs} = -\frac{f_\rho^2 m_\rho}{2} e^{-m_\rho t} + \frac{f_\omega^2 m_\omega}{2} e^{-m_\omega t}$$

- Ratio of the moments from D_{ll} to that from C_{ll}:

$$R_j = \frac{\Pi_{j,d}}{\Pi_{j,c}} = \frac{1}{2}\left[\frac{m_\rho^{2+2j} f_\omega^2}{m_\omega^{2+2j} f_\rho^2} - 1\right]$$

- From experiment, $m_\rho = 0.775$ GeV, $f_\rho = 0.217$, $m_\omega =$0.783 GeV and $f_\omega =$0.195 GeV:
  $R_1 =$-0.025; $R_2 =$-0.026; $R_3 =$-0.028 i.e. less than 3%.

$$a_{\mu,disc}/a_{\mu,conn} \approx -3\%.$$

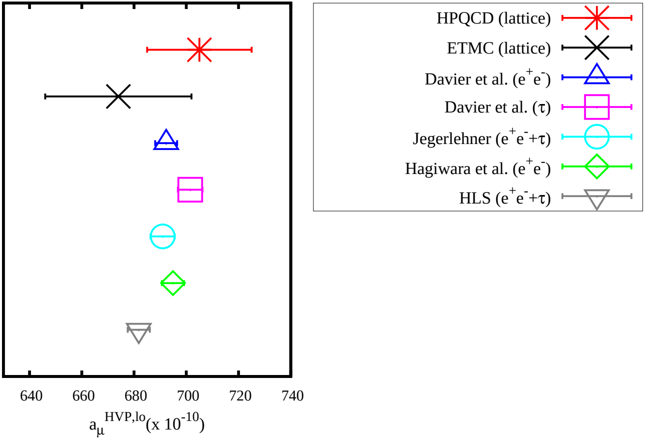
HPQCD Estimation for Total $a_\mu^{HVP,lo}$

Contribution	Result (x 10^{-10})	
light, conn	662(11)	(preliminary)
strange, conn	53.4 (6)	(1403.1778)
charm, conn	14.4(4)	(1403.1778, 1208.2855)
bottom, conn	0.27(4)	(1408.5768)
disconn. (estimate)	-25(15)	added -7 from $\pi\pi$ to simple estimate*
Total	705(20)	

[* $a_\mu^{(\pi\pi)} = 71x10^{-10}$ and we considered -10% of this following partially quenched ChPT.]

The uncertainty in the total $a_\mu^{HVP,lo}$ is dominated by our estimation of disconnected piece.

Comparison of total $a_\mu^{HVP,lo}$ with phenomenology



Conclusion

To summarize:

- HPQCD preliminary for total $a_\mu^{HVP,lo}$ using HISQ : 705(20) $\times 10^{-10}$($\sim$ 2.8% uncertainty)
- First principle result including 2+1+1 dynamical sea quarks, physical valence quarks and pion masses, takes finite volume effects into account.

Future:

- Collaborating with MILC for using large ensemble size (10,000 on $\sim$0.12fm lattices).
- Need to understand ρ meson better on lattice.
- Get a better precision from disconnected diagrams : collaborating with Hadron Spectrum collaboration.