Neutral Kaon mixing beyond the Standard Model

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RBC-UKQCD Collaboration

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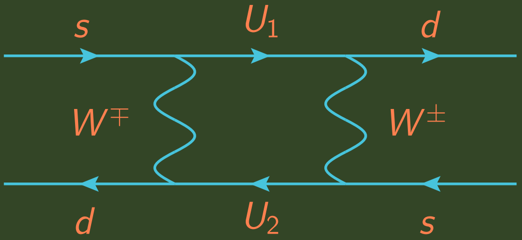
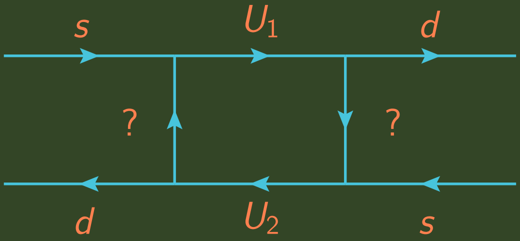
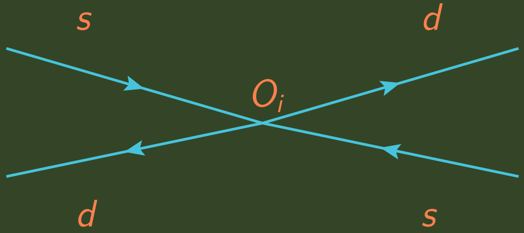


Standard model leading order contribution.



BSM picture is familiar.



OPE pinch is used to obtain low-energy hadronic matrix elements.
These are model-independent.

Operators

"SUSY" basis operators. [Gabbiani et. al. '96]

$$
\begin{aligned}
O_1 &= (\bar{s}_a\gamma_\mu(1-\gamma_5)d_a)(\bar{s}_b\gamma_\mu(1-\gamma_5)d_b),\\
O_2 &= (\bar{s}_a(1-\gamma_5)d_a)(\bar{s}_b(1-\gamma_5)d_b),\\
O_3 &= (\bar{s}_a(1-\gamma_5)d_b)(\bar{s}_b(1-\gamma_5)d_a),\\
O_4 &= (\bar{s}_a(1+\gamma_5)d_a)(\bar{s}_b(1+\gamma_5)d_b),\\
O_5 &= (\bar{s}_a(1+\gamma_5)d_b)(\bar{s}_b(1+\gamma_5)d_a).
\end{aligned}
$$

Interesting quantities

- Bag parameters:
$$B_K(\mu) = \frac{\langle \bar{K}^0 | O_1(\mu) | K^0 \rangle}{\frac{8}{3} m_K^2 f_K^2}, \quad B_i(\mu) = \frac{\langle \bar{K}^0 | O_i(\mu) | K^0 \rangle}{N_i f_K^2 m_K^2 \left(\frac{m_K}{(m_d(\mu)+m_s(\mu))} \right)^2}.$$
- Ratios: $R_i(\mu) = \left[\frac{f_K^2}{m_K^2} \right]_{\text{experiment}} \left[\frac{m_K^2}{f_K^2} \frac{\langle \bar{K}^0 | O_i(\mu) | K^0 \rangle}{\langle \bar{K}^0 | O_1(\mu) | K^0 \rangle} \right]_{\text{lattice}}.$
[Babic, Garron, Hoelbling, Howard, Lellouch, Rebbi '06].
- Golden combinations: $G_{23}(\mu) = \frac{3B_2(\mu)}{5B_2(\mu) - 2B_3(\mu)}$,
$G_{45}(\mu) = \frac{B_4(\mu)}{B_5(\mu)}, \quad G_{24}(\mu) = B_2(\mu)B_4(\mu), \quad G_{21} = \frac{B_2(\mu)}{B_K(\mu)}.$
[Becirevic and Villadoro '04. Bailey, Kim, Lee and Sharpe '12].

Previous recent results

Collaboration	B_K	B_2	B_3	B_4	B_5
RBC-UKQCD	0.53(2)	0.43(5)	0.75(9)	0.69(7)	0.47(6)
ETMC	0.51(2)	0.47(2)	0.78(4)	0.75(3)	0.60(3)
ETMC	0.51(2)	0.46(3)	0.79(5)	0.78(5)	0.49(4)
SWME	0.52(2)	0.53(2)	0.77(6)	0.98(6)	0.75(8)

Table : Previous collaboration results for the bag parameters in $\overline{\text{MS}}$ renormalised at 3 GeV, statistical and systematic errors have been added in quadrature.

RBC-UKQCD "Neutral kaon mixing beyond the standard model with $n_f = 2+1$ chiral fermions" : 1206.5737.
ETMC "Kaon Mixing Beyond the SM from N_f=2 tmQCD and model independent constraints from the UTA" : 1207.1287.
SWME "Calculation of BSM Kaon B-parameters using Staggered Quarks" : 1411.1501.
ETMC"$\Delta S = 2$ and $\Delta C = 2$ bag parameters in the SM and beyond from Nf=2+1+1 twisted-mass LQCD" : 1505.06639.

Where our differences lie

- Actions? DWF, Twisted mass, Staggered.
- Number of flavours? 2, 2+1, 2+1, 2+1+1.
- Discretisation? RBC-UKQCD had a single lattice spacing, ETMC and SWME had several.
- Renormalisation? ETMC and RBC-UKQCD used RI-MOM npr, SWME used one loop perturbation theory.

Previously ([PoS,Lattice2013]) presented RI-MOM results for second lattice spacing. Allowing investigation into discretisation effects.

Volume	a^{-1} [GeV]	am_{ud}^{sea} ($= am_{ud}^{\mathrm{val}}$)	m_π [MeV]
$24^3 \times 64 \times 16$	1.785(5)	0.005, 0.01, 0.02	340, 430, 560
	am_s^{sea}	am_s^{val}	am_s^{phys}
	0.04	0.04,0.035, 0.03	0.03224(18)
	a^{-1} [GeV]	am_{ud}^{sea} ($= am_{ud}^{\mathrm{val}}$)	m_π [MeV]
$32^3 \times 64 \times 16$	2.383(9)	0.004, 0.006, 0.008	300, 360, 410
	am_s^{sea}	am_s^{val}	am_s^{phys}
	0.03	0.03, 0.025	0.02477(18)

Table : Summary of our lattice ensembles. The heaviest mass of the coarse ensemble is not used in the chiral extrapolations. For the coarse lattice, we use $155, 152$ and 146 configurations for the $am = 0.005, 0.01$ and 0.02 ensembles respectively. For the fine lattice, we use $129, 186$ and 208 configurations for the $am = 0.004, 0.006$ and 0.008 ensembles respectively. a^{-1} from [Blum et. al. 1411.7017].

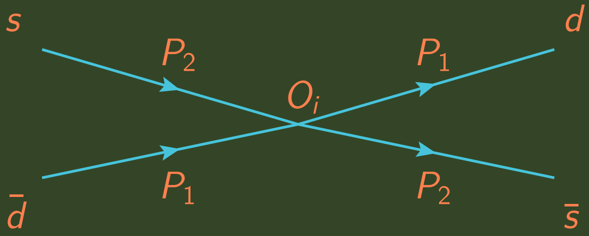
Renormalisation

Must renormalise our quantities, use intermediate lattice MOM-scheme (must fix the gauge),
[Martinelli, Pittori, Sachrajda, Testa, Vladikas '94]

$$O_i^{\overline{\mathrm{MS}}}(\mu) = C_{ij}^{\overline{\mathrm{MS}} \leftarrow \mathrm{MOM}}(\mu) \left(\lim_{a^2 \to 0} \frac{Z_{jk}^{\mathrm{MOM}}(\mu)}{Z_q^2} O_k(a) \right),$$

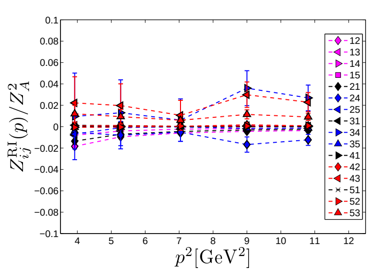
$$Z^{\mathrm{RI}}(\mu) P(\Lambda(p^2))|_{p^2=\mu^2} = \mathrm{tree}.$$

Non-exceptional (RI-SMOM) kinematics

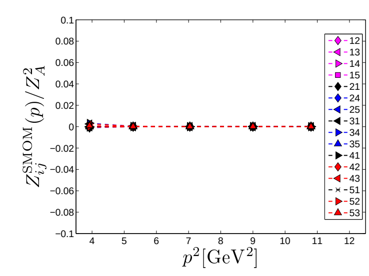


$$P_1 \neq P_2,\ P_1^2 = P_2^2 = (P_1 - P_2)^2 = \mu^2.$$

No zero momentum transfer between legs. Used for B_K in [RBC-UKQCD (Aoki et. al) '10, J.Frison (Wed, 14:40, 404)].



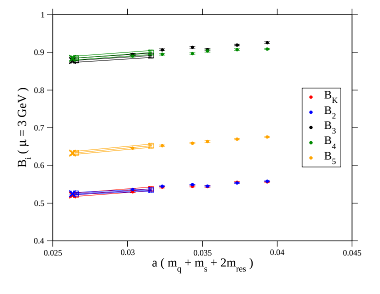
(a) RI-MOM



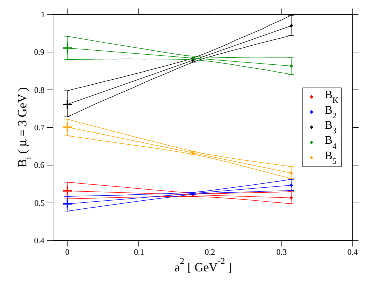
(b) SMOM

Figure : Chirally forbidden matrix elements in the Naive RI-MOM (left) and SMOM (right) in the chiral limit.

SMOM bag parameters



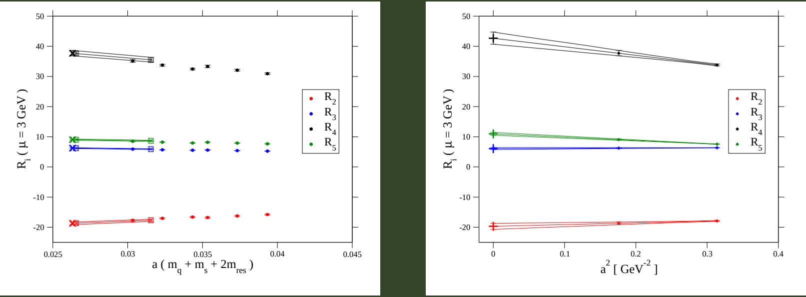
(a) Chiral limit, fine ensemble



(b) $a^2 \to 0$ extrapolation.

Figure : Chiral and $a^2 \to 0$ extrapolations of the bag parameters.

SMOM Ratios

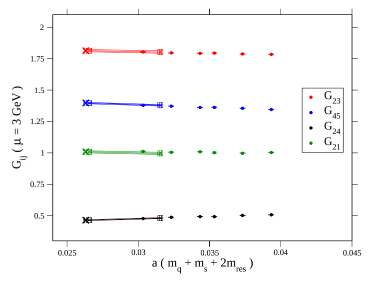


(a) Chiral limit, fine ensemble.

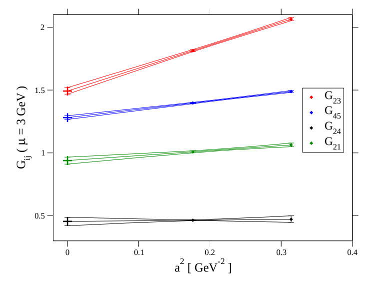
(b) $a^2 \to 0$ extrapolation.

Figure : Chiral and $a^2 \to 0$ extrapolations of the ratios.

SMOM Golden combinations



(a) Chiral limit, fine ensemble



(b) $a^2 \to 0$ extrapolation.

Figure : Chiral and $a^2 \to 0$ extrapolations of the Golden combinations.

Systematics

- Chiral, difference between linear and χ-pt.
- Discretisation, half difference between fine and $a^2 \to 0$ result.
- Perturbative, $\frac{1}{2}$ the 1-loop PT correction from tree level result.
- RI-MOM pole subtraction, large chirally forbidden matrix elements.

Results

Collaboration	B_K	B_2	B_3	B_4	B_5
RBC-UKQCD	0.53(2)	0.43(5)	0.75(9)	0.69(7)	0.47(6)
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ETMC	0.51(2)	0.46(3)	0.79(5)	0.78(5)	0.49(4)
RBC-UKQCD-RI	0.53(2)	0.44(2)	0.69(4)	0.76(6)	0.57(7)

Table : Our **preliminary** results for the bag parameters in the $\overline{\text{MS}}$ scheme, using the intermediate RI-MOM and SMOM schemes.

Results

Collaboration	B_K	B_2	B_3	B_4	B_5
SWME	0.52(2)	0.53(2)	0.77(6)	0.98(6)	0.75(8)
RBC-UKQCD-SMOM	0.53(2)	0.54(3)	0.79(7)	0.93(2)	0.68(5)

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Conclusion

Addressing conflict;
Renormalisation scheme's
Importance is shown