NLO and NNLO Low Energy Constants for SU(3) Chiral Perturbation Theory

Lattice 2015
Kobe, Japan
July 16, 2015

Robert Mawhinney
Columbia University

This work done in conjuction with the RBC and UKQCD Collaborations

Current Members of the RBC and UKQCD Collaborations

BNL and RBRC
Tomomi Ishikawa
Taku Izubuchi
Chulwoo Jung
Christoph Lehner
Meifeng Lin
Shigemi Ohta (KEK)
Taichi Kawanai
Christopher Kelly
Amarjit Soni
Sergey Syritsyn

CERN
Marina Marinkovic

University of Connecticut
Tom Blum

Columbia University
Ziyuan Bai
Norman Christ
Xu Feng
Luchang Jin
Bob Mawhinney
Greg McGlynn
David Murphy
Daiqian Zhang

Edinburgh University
Peter Boyle
Luigi Del Debbio
Julien Frison
Richard Kenway
Ava Khamseh
Brian Pendleton
Oliver Witzel
Azusa Yamaguchi

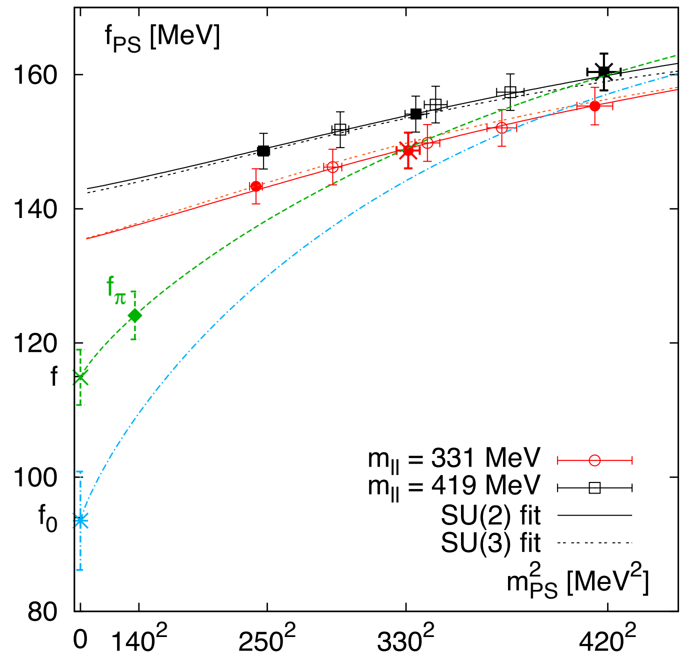
Plymouth University
Nicolas Garron

University of Southampton
Jonathan Flynn
Tadeusz Janowski
Andreas Juettner
Andrew Lawson
Edwin Lizarazo
Antonin Portelli
Chris Sachrajda
Francesco Sanfilippo
Matthew Spraggs
Tobias Tsang

York University (Toronto)
Renwick Hudspith

Some ChPT Observations

- The RBC and UKQCD Collaborations have been using NLO SU(2) ChPT to extrapolate to physical quark masses, since observing (PRD 78 (2008) 114509) that SU(3) NLO fits gave small f_0 (and hence large NLO corrections) when fit to m_{PS} in 250-400 MeV range.



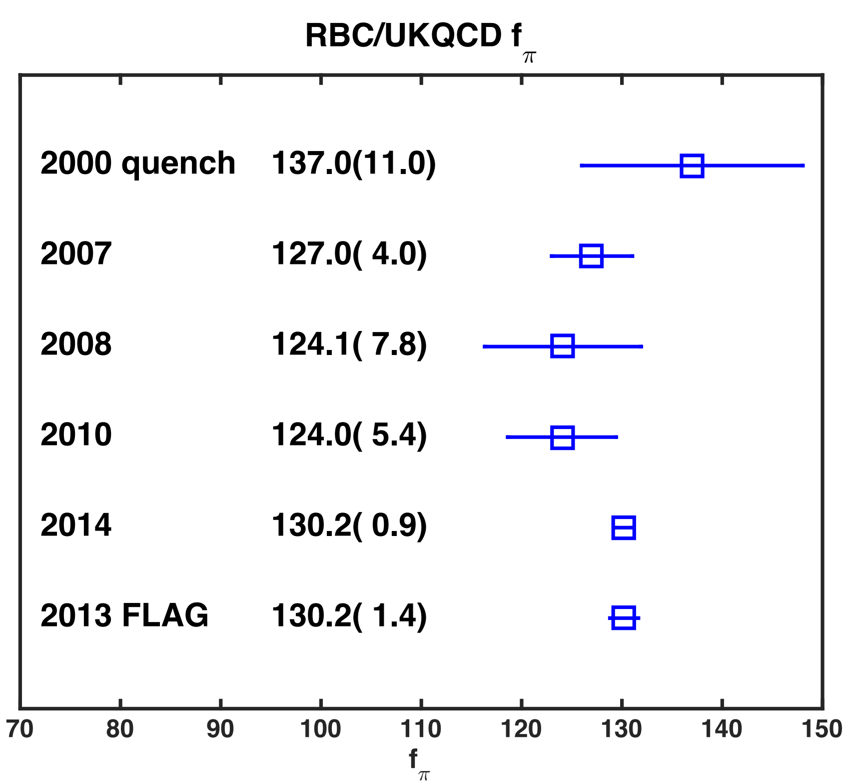
V

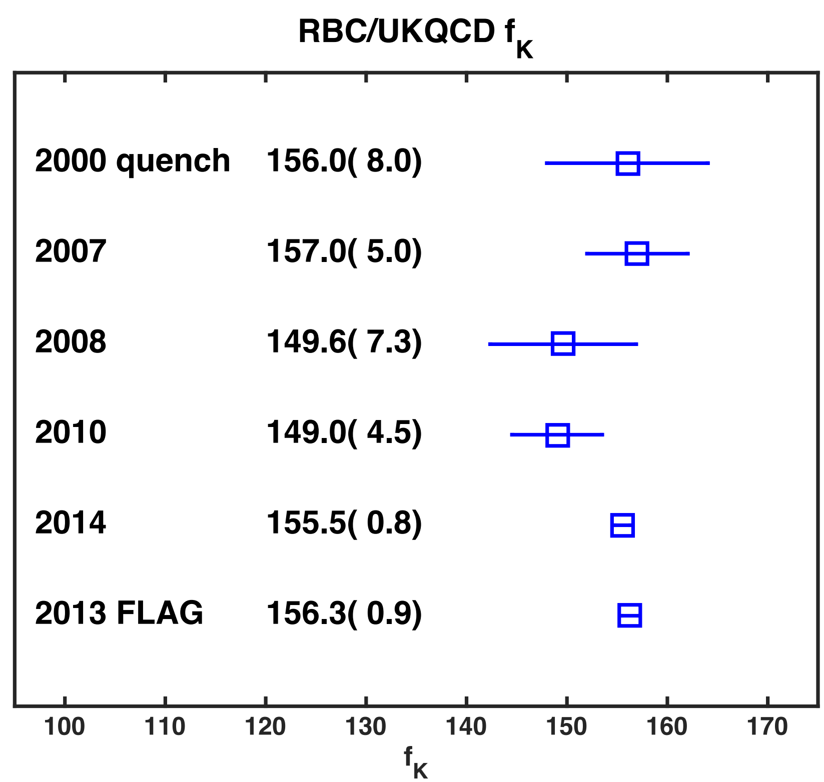
- David's talk showed that we can fit our current, much larger data set to either NLO SU(2) with $m_\pi \leq 350$ MeV or NNLO with $m_\pi \leq 450$ MeV. The NNLO expansion is quite robust and a straightforward, least squares minimization without any priors or constraints is numerically stable and reproduces the lattice data at the 1% level

Essentially Physical Quark Mass Ensembles

- Use SU(2) ChPT to make small extrapolation (arXiv:1411.7017).
- Inputs: m_π, m_K and m_Ω. Outputs: f_π and f_K.

Quantity	Physical Value	Ens. 10 Value	Deviation	Ens. 11 Value	Deviation
m_π/m_K	0.2723	0.2790	2.4%	0.2742	0.7%
m_π/m_Ω	0.0807	0.0830	2.8%	0.0822	1.9%
m_K/m_Ω	0.2964	0.2974	0.3%	0.2998	1.2%





ChPT Fits to m_{PS} and f_{PS}

- Now focus on ChPT fits for their own sake, not just to make small corrections.
- We can simultaneously fit lattice data for different lattice spacings, actions and volumes using expansions of the form (SU(2) NLO example):

$$(m_{ll}^{\mathbf{e}})^2 = \chi_l^{\mathbf{e}} + \chi_l^{\mathbf{e}} \cdot \left\{ \frac{16}{f^2}\Big((2L_8^{(2)} - L_5^{(2)}) + 2(2L_6^{(2)} - L_4^{(2)})\Big)\chi_l^{\mathbf{e}} + \frac{1}{16\pi^2 f^2}\chi_l^{\mathbf{e}} \log\frac{\chi_l^{\mathbf{e}}}{\Lambda_\chi^2} \right\}$$

$$f_{ll}^{\mathbf{e}} = f\left[1 + c_f (a^{\mathbf{e}})^2\right] + f \cdot \left\{ \frac{8}{f^2}(2L_4^{(2)} + L_5^{(2)})\chi_l^{\mathbf{e}} - \frac{\chi_l^{\mathbf{e}}}{8\pi^2 f^2}\log\frac{\chi_l^{\mathbf{e}}}{\Lambda_\chi^2} \right\}$$

with

$$\chi_l^{\mathbf{e}} = \frac{Z_l^{\mathbf{e}}}{R_a^{\mathbf{e}}}\frac{B^{\mathbf{1}}\tilde{m}_l^{\mathbf{e}}}{(a^{\mathbf{e}})^2}$$

- At NNLO order, using codes from Bijnens and collaborators, we fit to

$$X(\tilde{m}_q, L, a^2) \simeq X_0 \left(1 + \underbrace{X^{\mathrm{NLO}}(\tilde{m}_q) + X^{\mathrm{NNLO}}(\tilde{m}_q)}_{\text{NNLO Continuum PQChPT}} + \underbrace{\Delta_X^{\mathrm{NLO}}(\tilde{m}_q, L)}_{\text{NLO FV corrections}} + \underbrace{c_X a^2}_{\text{Lattice spacing}} \right)$$

- For SU(2), we use m_π, m_K and m_Ω to set the scale. There are a^2 corrections to the decay constant, whose coefficient depends on whether the Iwasaki or Iwasaki+DSDR gauge action is used

Revisiting SU(3) Fits

- We now have (M)DWF data with m_π (unitary) in the range 116 to 432 MeV. After correcting for the residual mass, we can fit to the complete continuum ChPT formula to NNLO SU(3), without any approximation in the theoretical expansion.

- For SU(3), we don't know how reliable the expansion is at the scale of m_K, so we use m_π, f_π and m_Ω set the scale. m_K and f_K are then outputs. Caveat: all of our simulations are with 2+1 flavors and have a heavy sea quark within 20% of m_s.

- We use two types of uncorrelated fits (too much data for a reliable correlation matrix): 1) all data is uncorrelated and 2) each partially quenched data point (N_{data} total points) from a given ensemble is given weight $1/N_{data}$ to keeps highly correlated data from dominating the fit. Use the difference to estimate fit systematics.

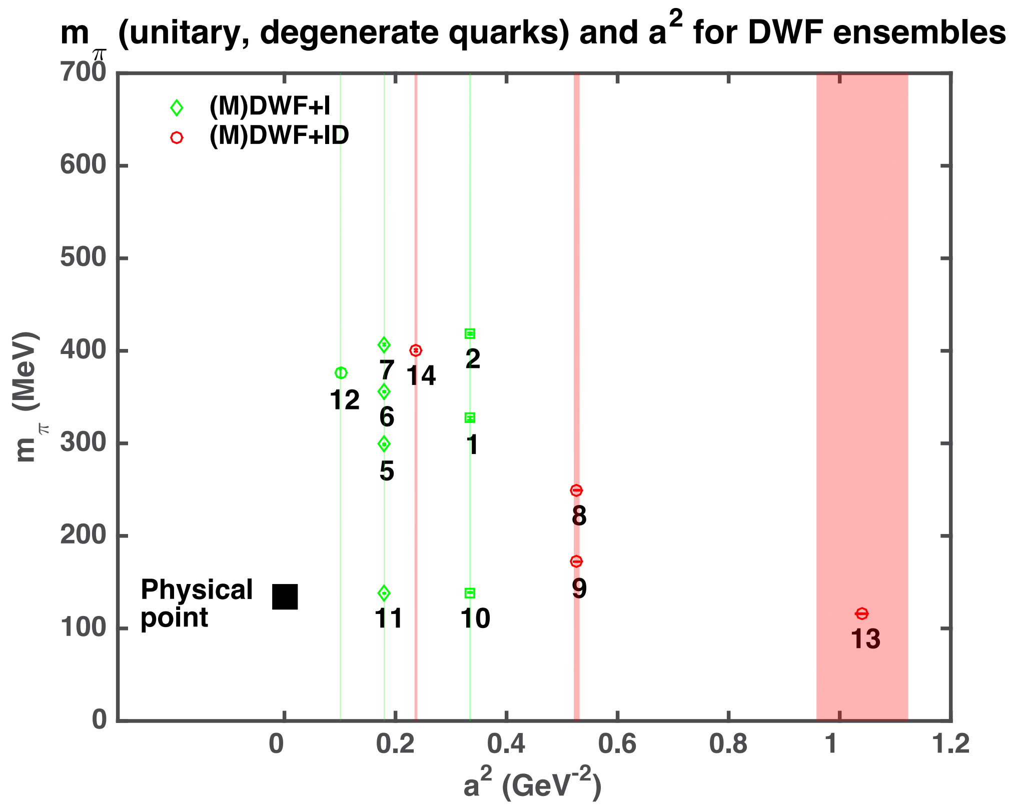
- Since ChPT at NLO or NNLO is an approximation to the exact values, χ^2/dof is not the best measure of the quality of the fit, since this will become arbitrarily large as the accuracy of the data improves.

- A better measure is a histogram of the percent deviation between the fit and the data, for all the data points. This immediately shows whether the fits are reproducing the data at the 1%, 5% or 10% level, for example.

- Will show both χ^2/dof and percent deviation histograms.

Ensembles Used

- 2+1 flavor, (M)DWF ensembles of the RBC and UKQCD collaborations used in these fits.
- Volumes: $(2.0\ \mathrm{fm})^3$ to $(5.5\ \mathrm{fm})^3$
- $3.8 \leq m_\pi L \leq 5.8$.
- ~100 quark mass combinations for $m_{PS} \leq 510$ MeV
- m_{PS} and f_{PS} have statistical errors in the 0.1-0.4% range.
- Standard least-squares fitting
- Superjacknife for errors
- w_0 and t_0 not included in fit
- m_π cuts of 370 and 510 MeV used for both NLO and NNLO



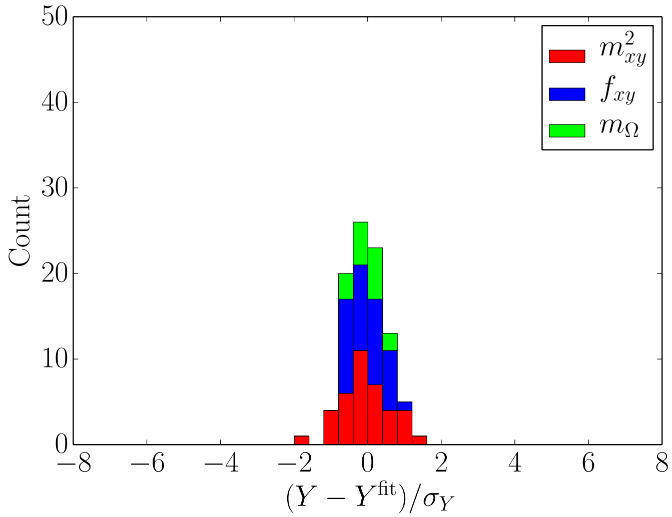
For NLO fits, $f_0 \sim 113$ MeV

For NNLO fits, $f_0 \sim 128$ MeV

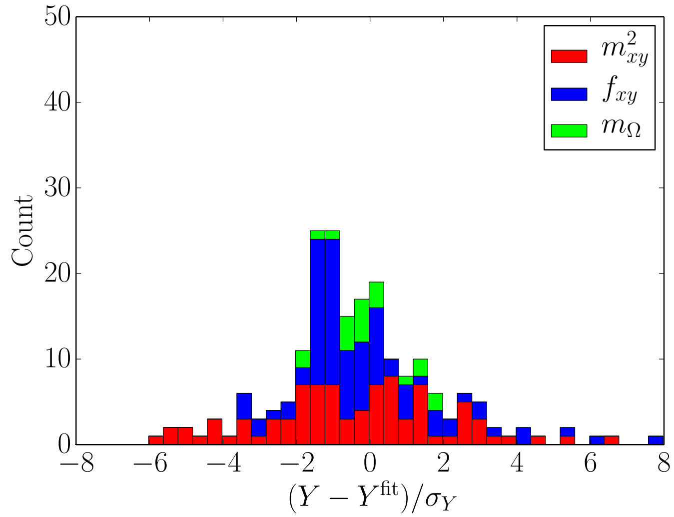
Indicates not enough data or not very convergent or both.

Our choice: freeze NNLO values for f_0 and B_0 to NLO result for each jacknife block

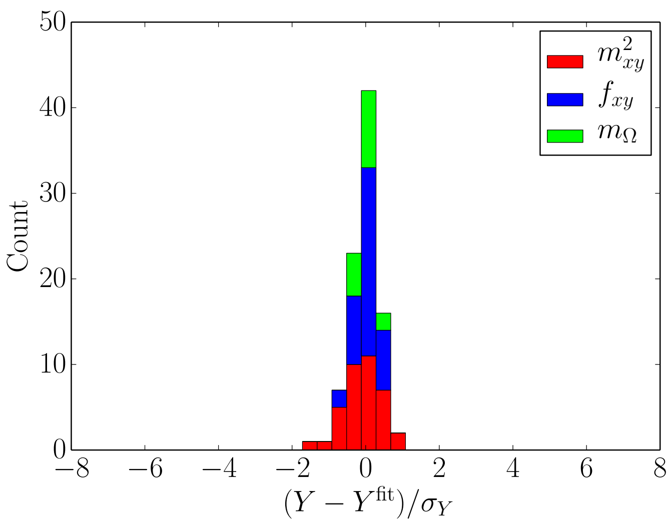
Histograms of Deviations in Units of σ



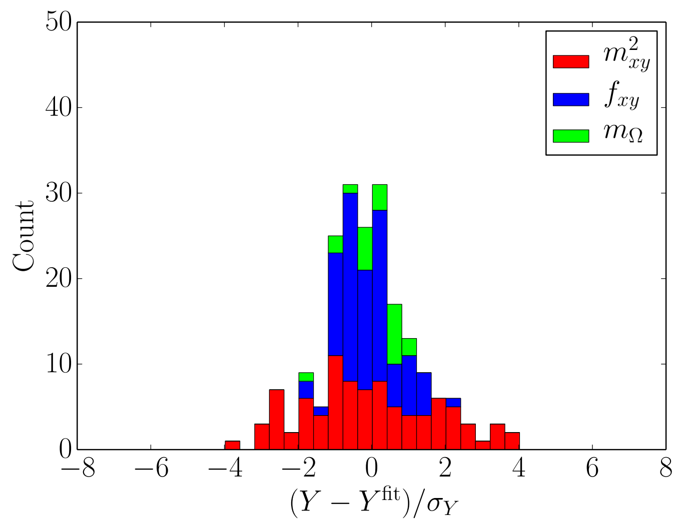
(a) NLO, $m_{xy}^{\text{cut}} = 370\,\text{MeV}$



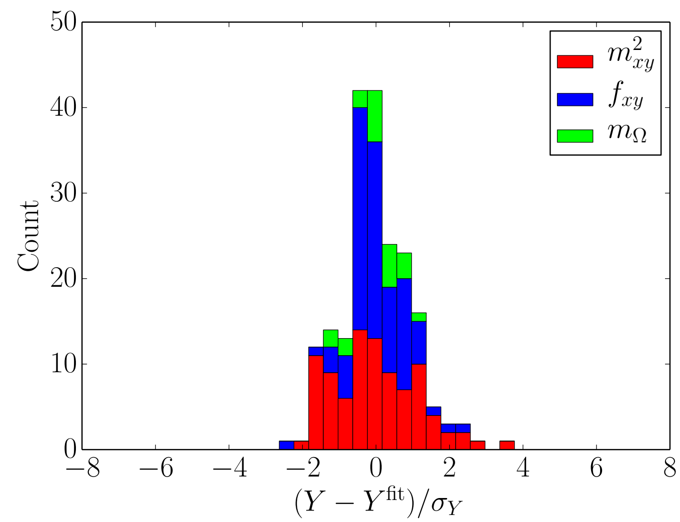
(b) NLO, $m_{xy}^{\text{cut}} = 510\,\text{MeV}$



(c) NNLO, $m_{xy}^{\text{cut}} = 370\,\text{MeV}$

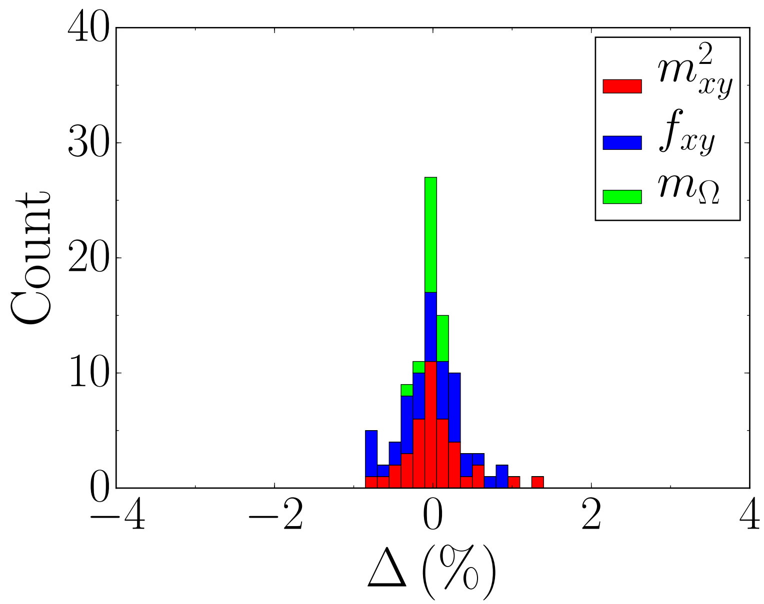


(d) NNLO with frozen LO LECs, $m_{xy}^{\text{cut}} = 510\,\text{MeV}$

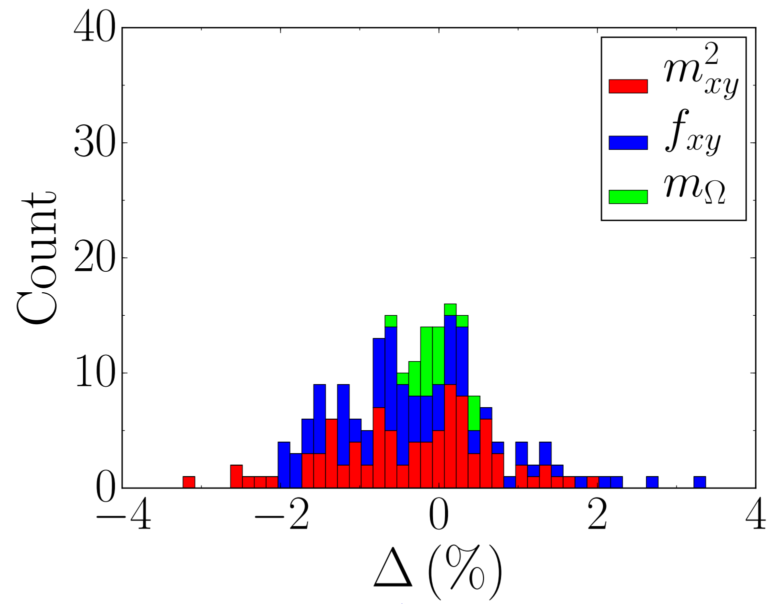


(e) NNLO, $m_{xy}^{\text{cut}} = 510\,\text{MeV}$

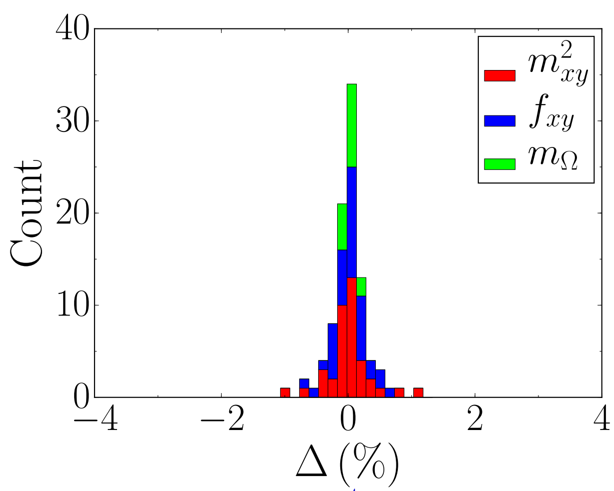
Histograms of Deviations in %



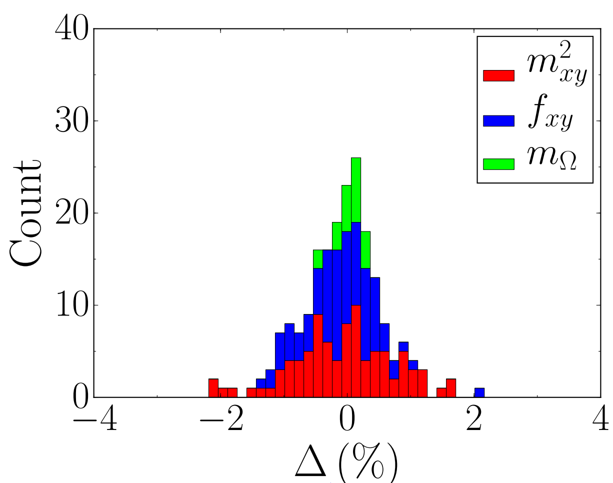
NLO, m^{cut} = 370 MeV



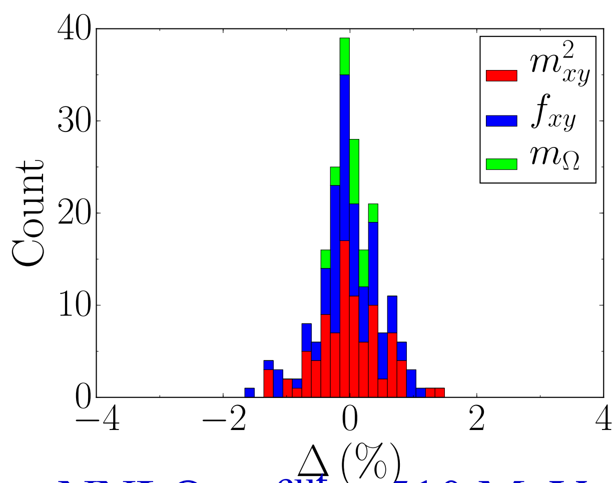
NLO, m^{cut} = 510 MeV



NNLO, m^{cut} = 370 MeV

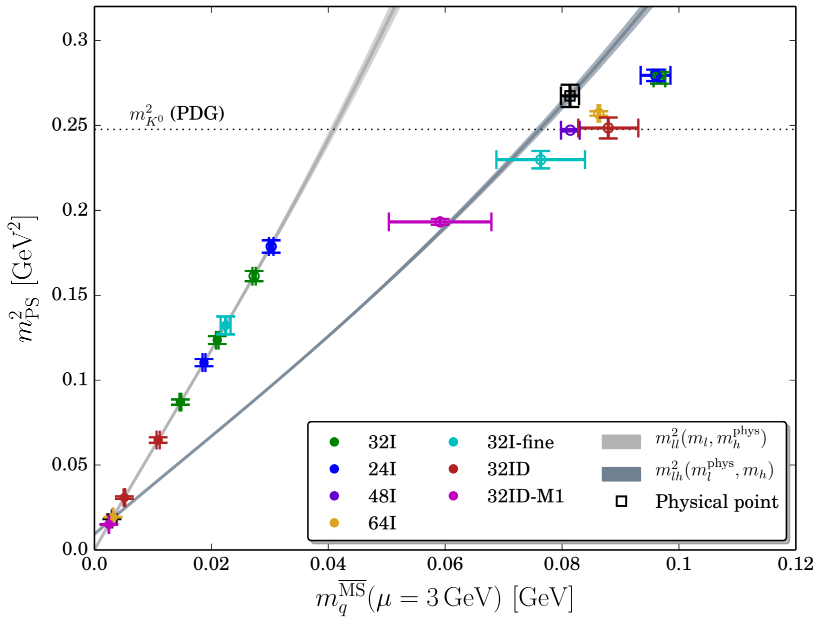


NNLO, m^{cut} = 510 MeV, Frozen LO LECs

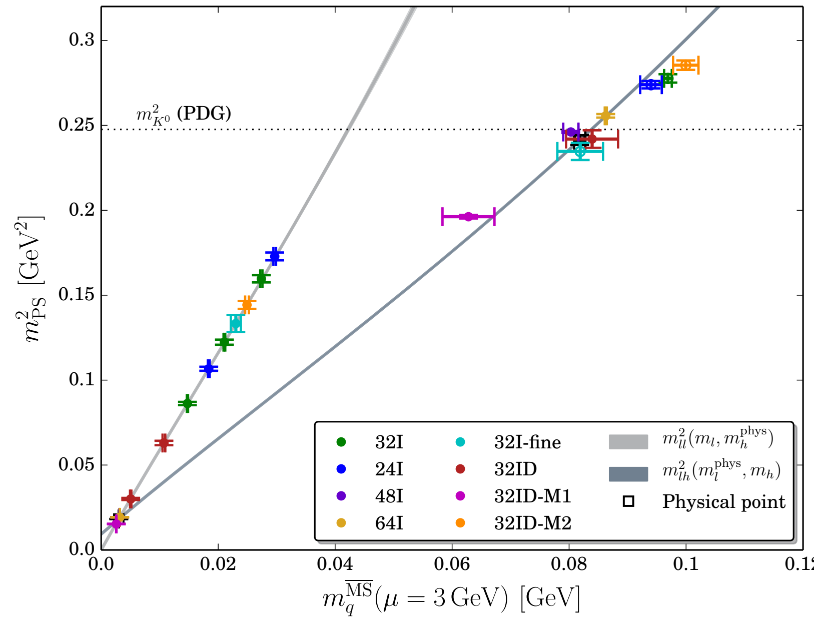


NNLO, m^{cut} = 510 MeV

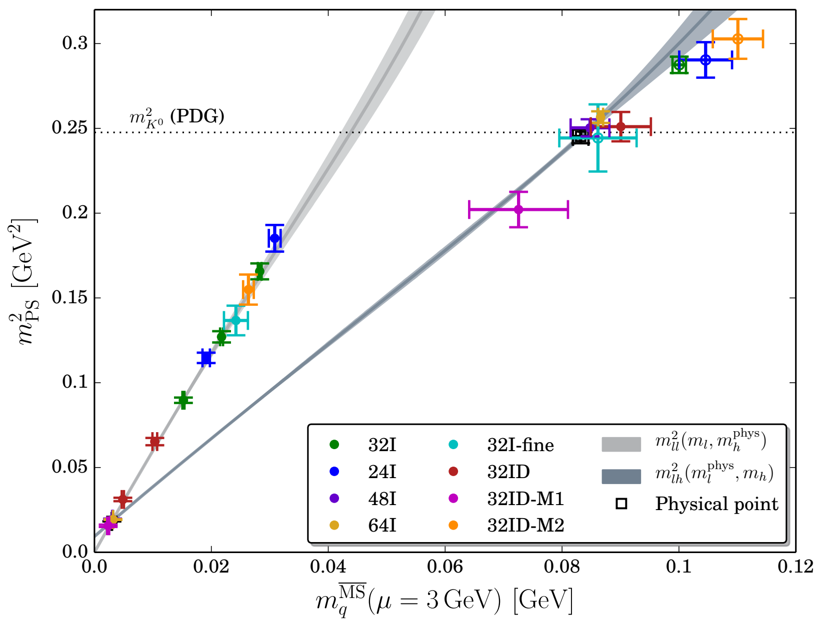
Plots of Pseudoscalar Masses



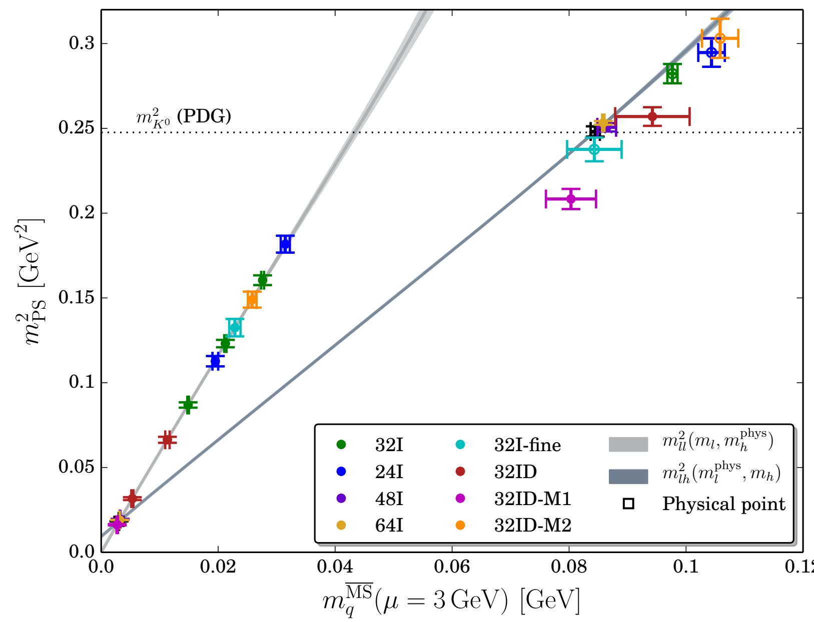
(a) NLO, $m_{xy}^{\mathrm{cut}} = 370\,\mathrm{MeV}$



(b) NLO, $m_{xy}^{\mathrm{cut}} = 510\,\mathrm{MeV}$

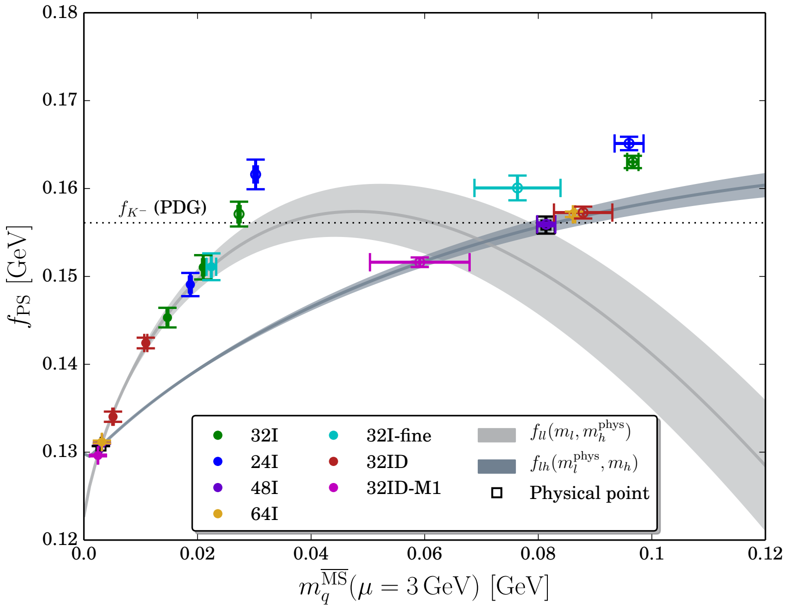


(c) NNLO with frozen LO LECs, $m_{xy}^{\mathrm{cut}} = 510\,\mathrm{MeV}$

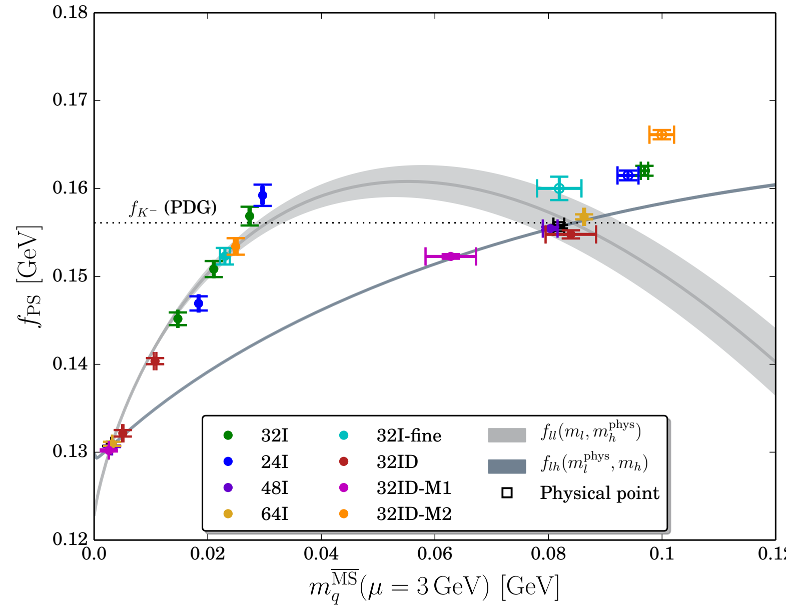


(d) NNLO, $m_{xy}^{\mathrm{cut}} = 510\,\mathrm{MeV}$

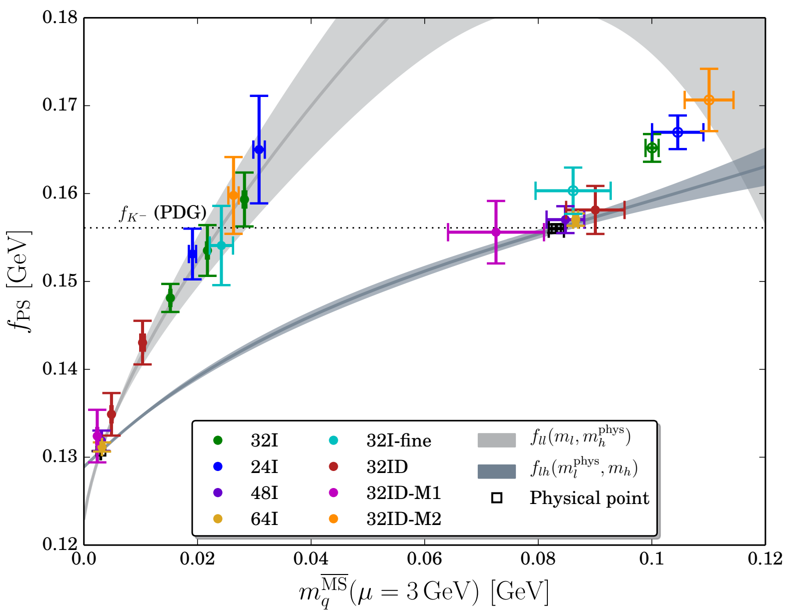
Plots of Pseudoscalar Decay Constants



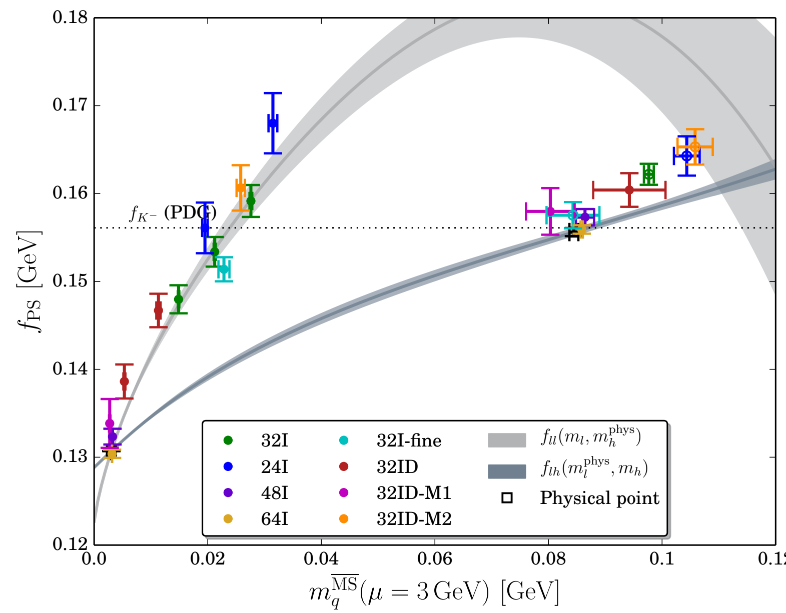
(a) NLO, $m_{xy}^{\rm cut} = 370\,{\rm MeV}$



(b) NLO, $m_{xy}^{\rm cut} = 510\,{\rm MeV}$



(c) NNLO with Frozen LO LECs, $m_{xy}^{\rm cut} = 510\,{\rm MeV}$



(d) NNLO, $m_{xy}^{\rm cut} = 510\,{\rm MeV}$

Assessing Reliability of Expansion

The data shows $m_\pi^2 \sim m_f$ to be very accurate over a large range of masses

Therefore NLO corrections to m_π^2 must be small

For NNLO fits must have either or both:

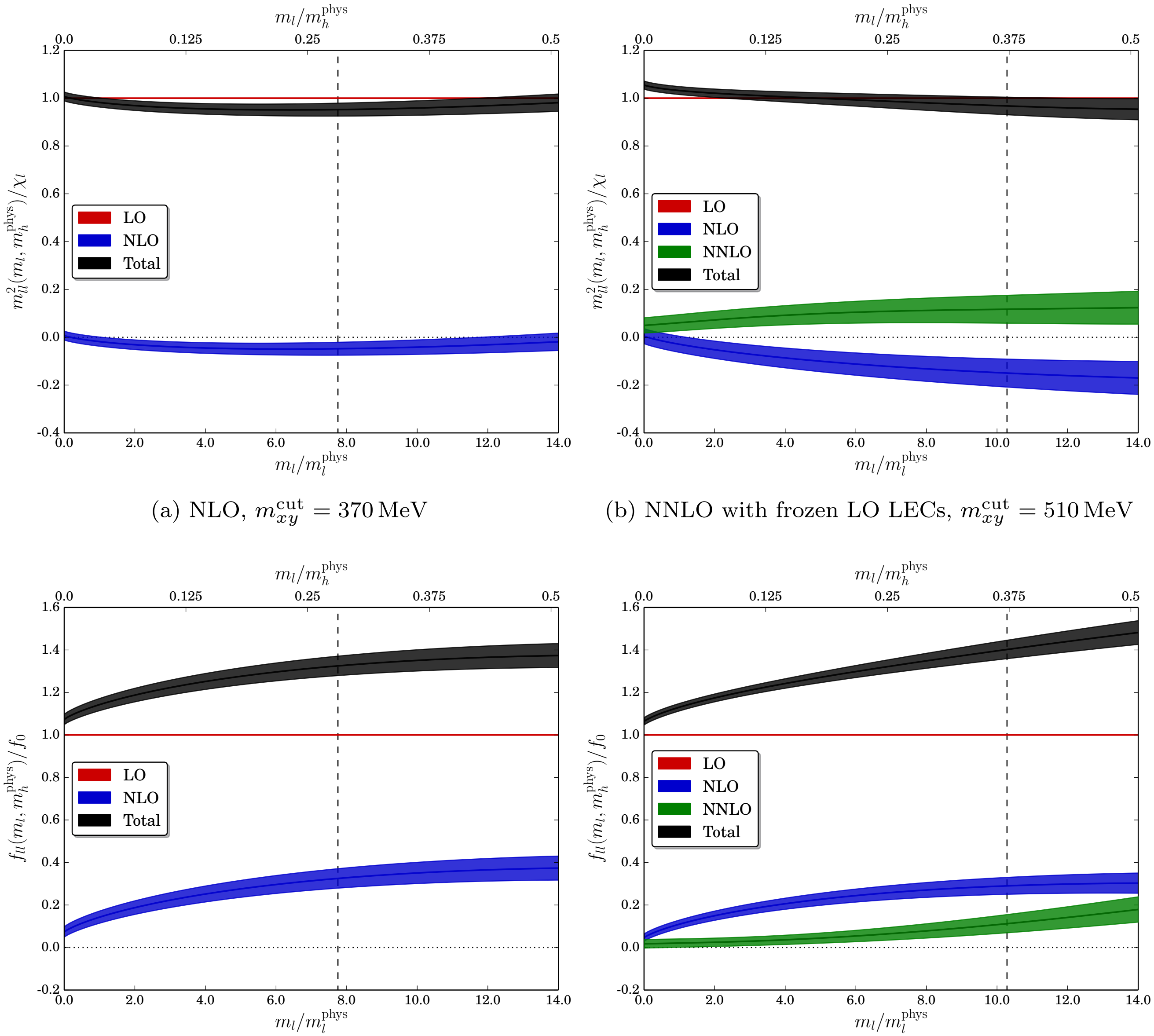
* NLO terms ~ -(NNLO terms)
* Both NLO and NNLO terms small

This means the series for m_π^2 will not have $|\mathrm{LO}| > |\mathrm{NLO}| > |\mathrm{NNLO}|$

To be reasonably reliable, it should have $|\mathrm{LO}| > |\mathrm{NLO} + \mathrm{NNLO}|$

f_π can then be used to judge reliability of expansion. If reliable, then should find $|\mathrm{LO}| > |\mathrm{NLO}| > |\mathrm{NNLO}|$ and $|\mathrm{LO}| > |\mathrm{NLO} + \mathrm{NNLO}|$

Decomposition of ChPT Expansion: $m_h = m_s$



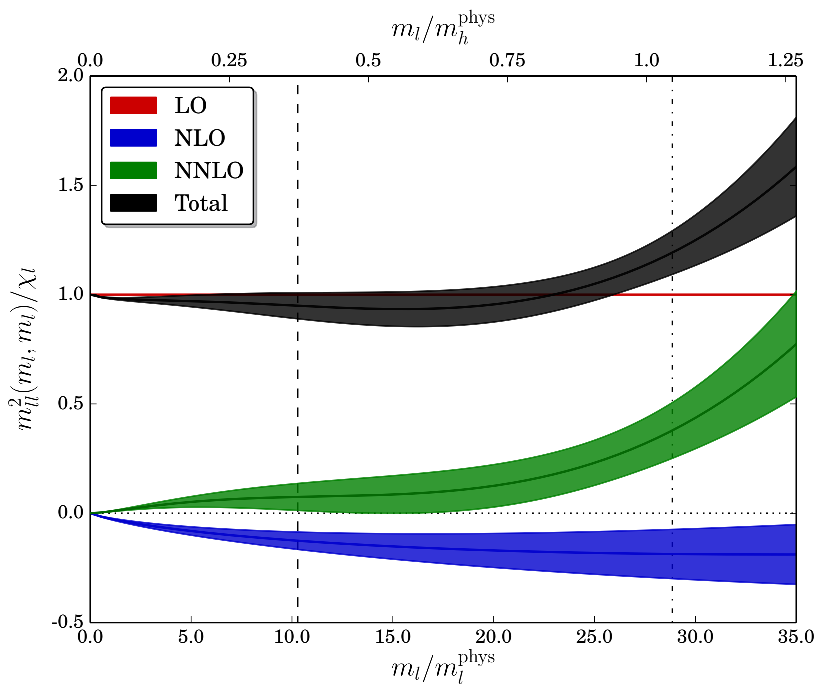
(a) NLO, $m_{xy}^{\rm cut} = 370\,\text{MeV}$

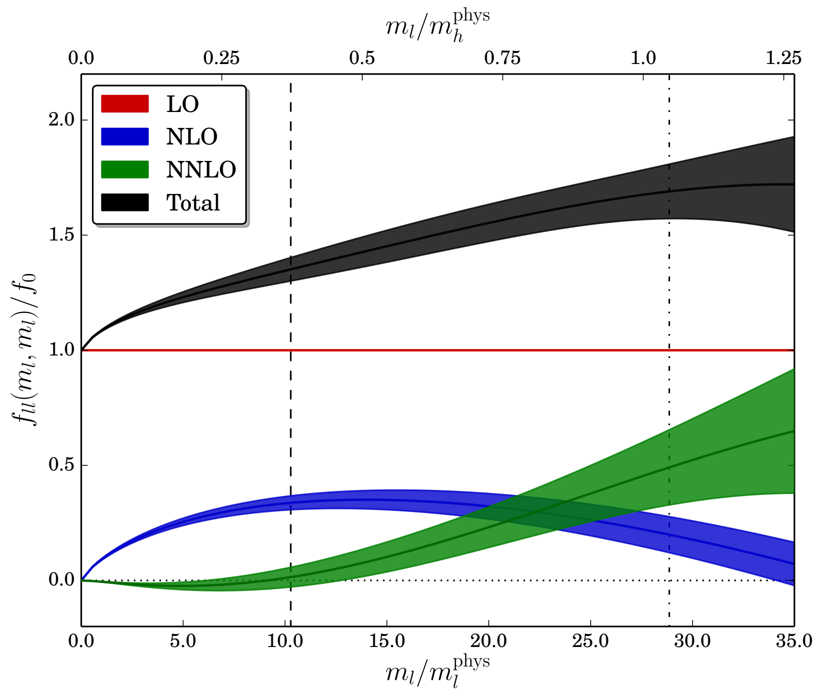
(b) NNLO with frozen LO LECs, $m_{xy}^{\rm cut} = 510\,\text{MeV}$

(c) NLO, $m_{xy}^{\rm cut} = 370\,\text{MeV}$

(d) NNLO with frozen LO LECs, $m_{xy}^{\rm cut} = 510\,\text{MeV}$

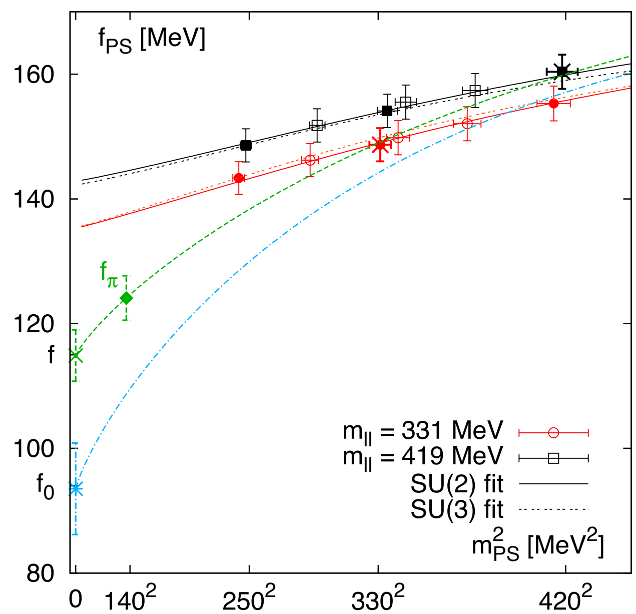
Decomposition of ChPT Expansion: SU(3) symmetric



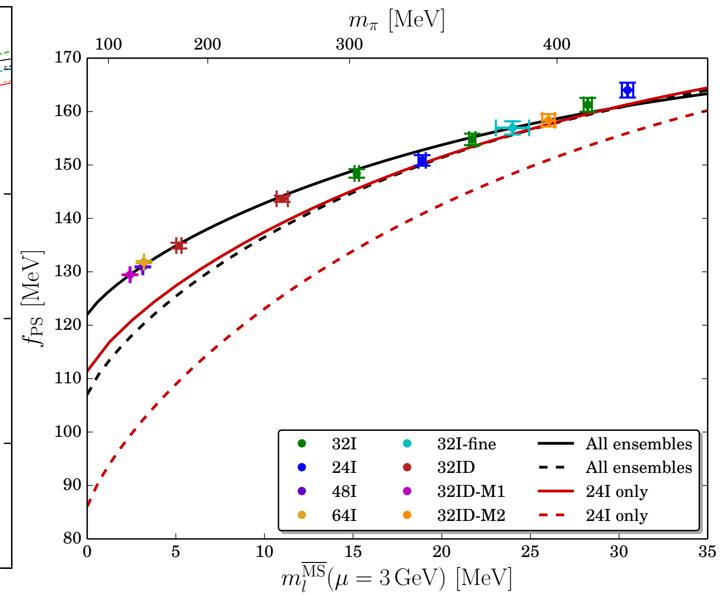


- For f_π see NLO = NNLO at $m_l/m_l^{\mathrm{phys}} = 20$.
- Series appears well behaved until $m_l/m_l^{\mathrm{phys}} = 15$, which is slightly above m_K.

Comparison with 2008 RBC/UKQCD Fits



(a) Reprinted from [5]



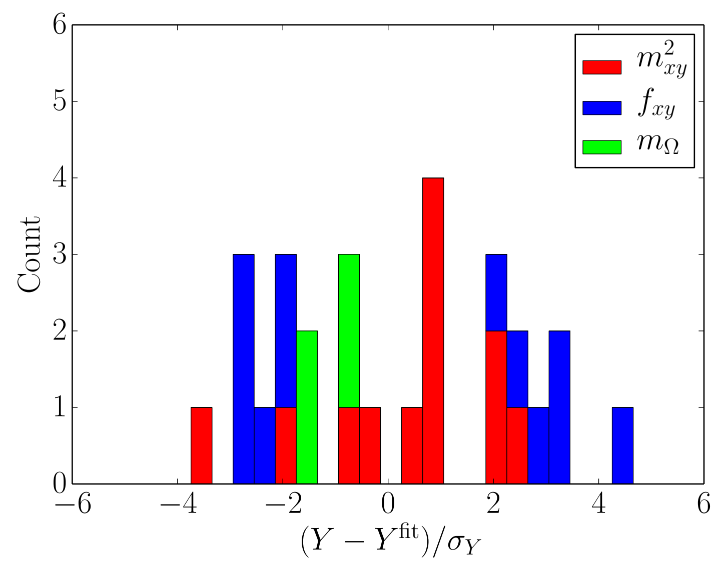
(b) This work

Fit all data to NLO, up to 420 MeV

Extrapolate to SU(3) chiral limit (black dashed line in upper right)

Remove light masses from fit then extrapolate to SU(3) chiral limit (red dashed line in upper right)

Histogram shows that heavy data wants to steepen slope.



(c) 24I data from the "all ensembles" fit

Results for LO LECs

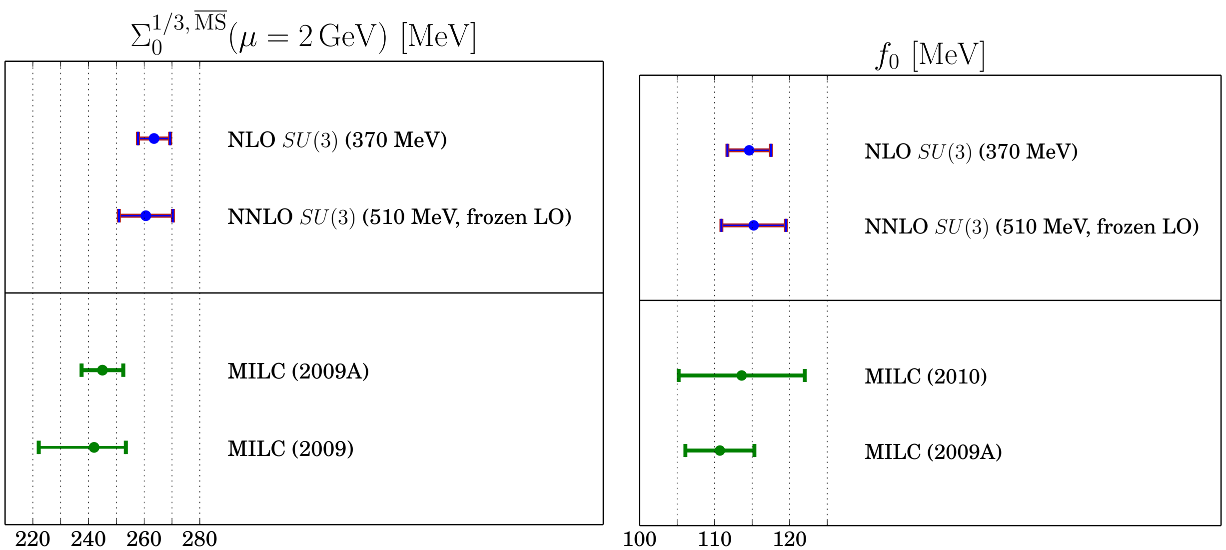


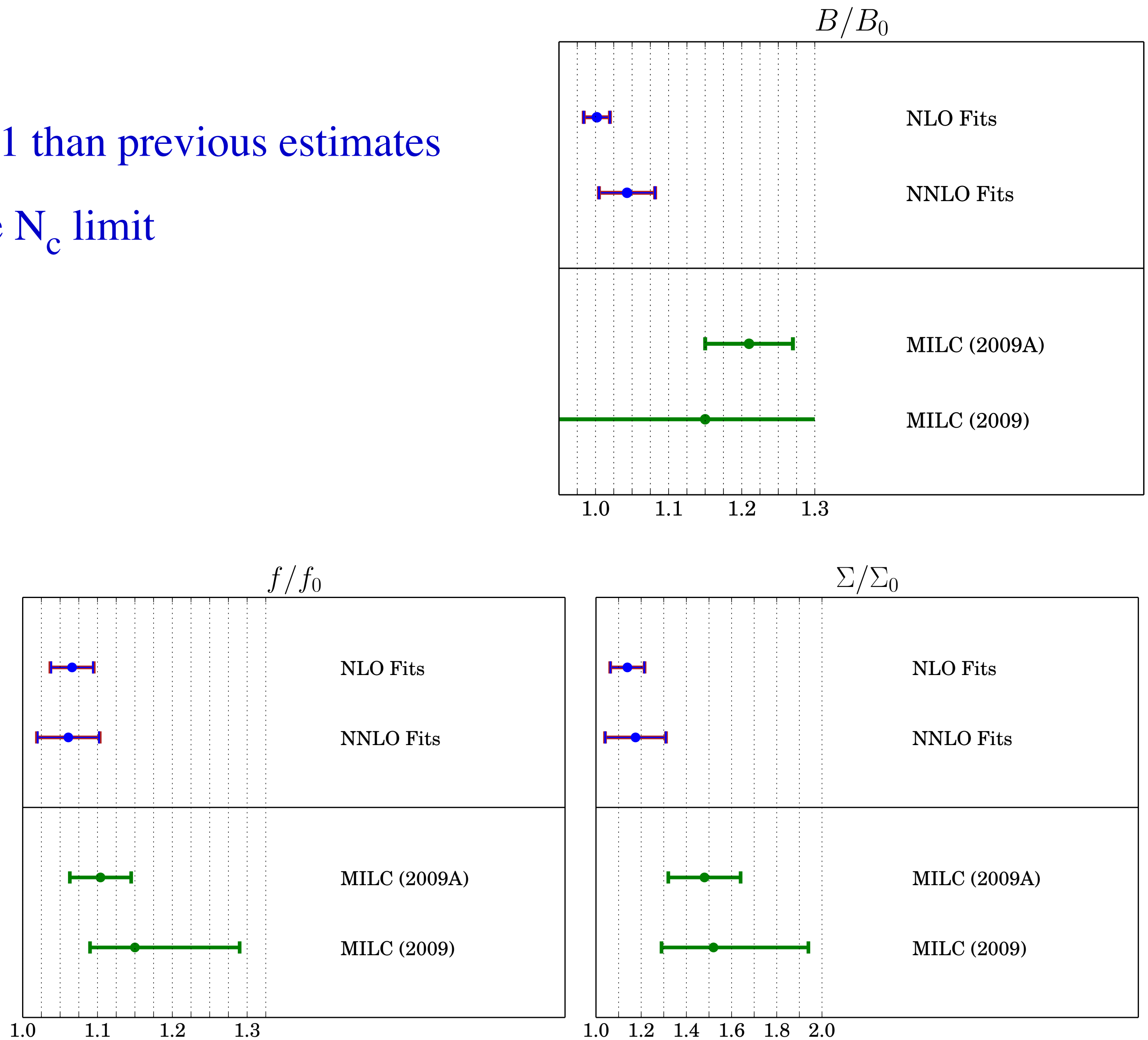
Figure 18: Leading order $SU(3)$ ChPT LECs from this work compared to other lattice results [12, 13, 51]. The blue error bands are statistical, while the red error bands also include a systematic error associated with our use of uncorrelated fits.

Results for NLO LECs

Ratios of SU(2) LO LECs to SU(3)

Ratios are closer to 1 than previous estimates

Should be 1 in large N_c limit



Predictions from SU(3) ChPT Fits

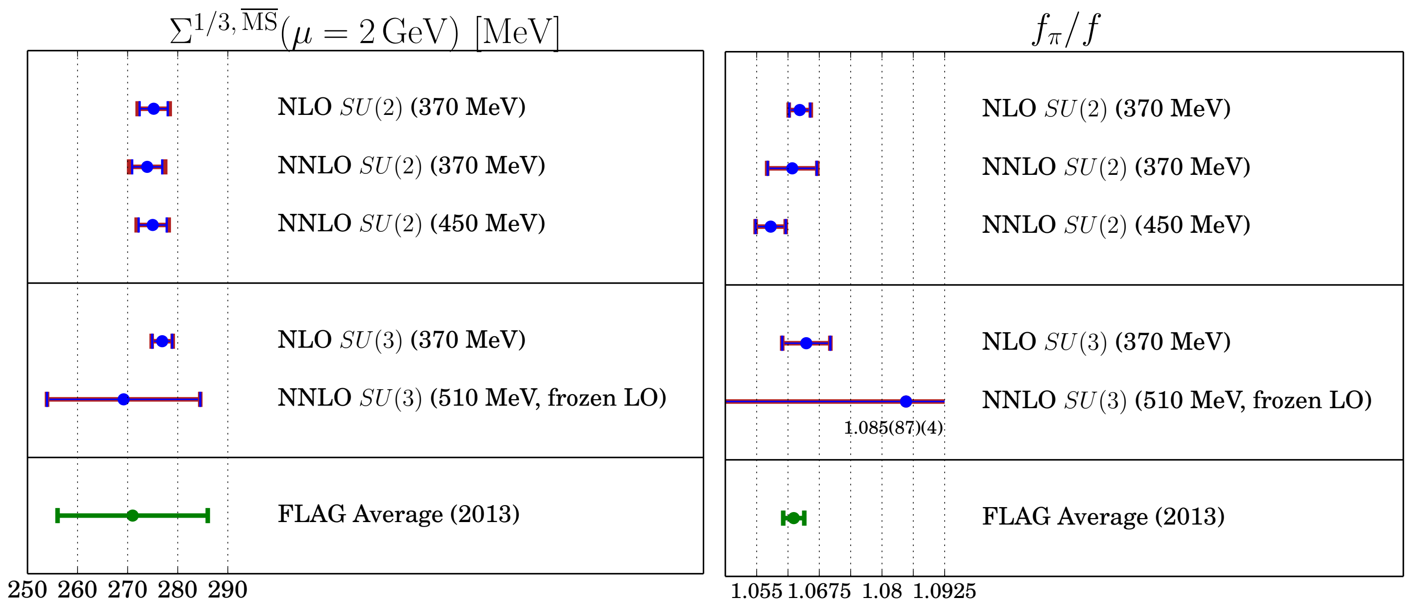
			Free		Frozen LO LECs
	NLO+FV (370 MeV)	NLO+FV (510 MeV)	NNLO+FV (370 MeV)	NNLO+FV (510 MeV)	NNLO+FV (510 MeV)
m_K	0.5171(64) GeV	0.4913(29) GeV	0.479(70) GeV	0.4982(30) GeV	0.4952(41)
f_K	0.15584(97) GeV	0.15566(20) GeV	0.160(42) GeV	0.15562(47) GeV	0.15601(49) GeV
f_K/f_0	1.363(36)	1.390(20)	1.25(39)	1.221(22)	1.349(22)
$[m^2_{K^0} - m^2_{K^\pm}]_{\mathrm{QCD}}/\Delta m_{du}$	5.44(24) GeV	3.658(62) GeV	1.75(93) GeV	3.46(28) GeV	2.74(39) GeV
$[\frac{f_{K^0}}{f_{K^\pm}} - 1]_{\mathrm{QCD}}/\Delta m_{du}$	3.01(13) GeV^{-1}	3.068(32) GeV^{-1}	1.9(1.9) GeV^{-1}	2.48(19) GeV^{-1}	2.72(27) GeV^{-1}
$[m^2_{\pi^\pm} - m^2_{\pi^0}]_{\mathrm{QCD}}/\Delta m^2_{du}$	—	—	45(45)	18(14)	11(16)
$m_\pi a^{I=0}_{\pi\pi}$	—	—	0.153(21)	0.1610(86)	0.1991(65)
$m_\pi a^{I=2}_{\pi\pi}$	—	—	-0.0376(58)	-0.0402(17)	-0.0449(18)
$m_\pi a^{I=1/2}_{\pi K}$	—	—	0.124(18)	0.1435(56)	0.1376(92)
$m_\pi a^{I=3/2}_{\pi K}$	—	—	-0.067(14)	-0.0781(47)	-0.0671(84)

Table 18: Predictions from NLO and NNLO fits and $SU(3)$ ChPT. $\Delta m_{du} \equiv m_d - m_u$.

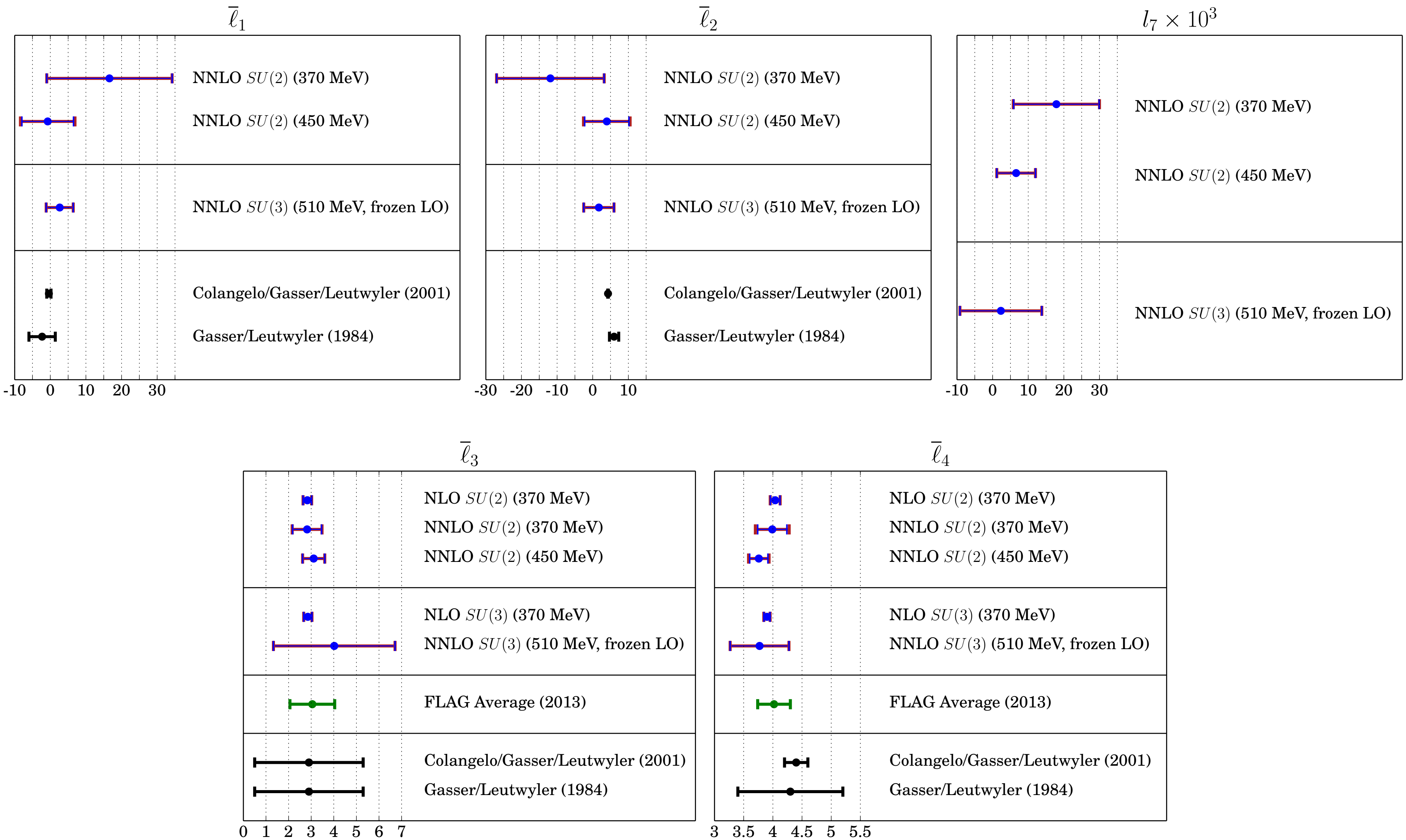
Since m_π, f_π and m_Ω are used to set the scale, m_K and f_K are predictions

π-K scattering lengths are also being calculated directly by RBC/UKQCD - see talk by T. Janowski. The preliminary results are $m_\pi a_{\pi K}^{I=1/2} = 0.16(2)$ and $m_\pi a_{\pi K}^{I=3/2} = -0.06(1)$. Good agreement between these methods.

Predictions for LO SU(2) LECs from SU(3) ChPT Fits



Predictions for NLO SU(2) LECs from SU(3) ChPT Fits



Conclusions

SU(2) NLO (350 MeV cut) and NNLO (450 MeV cut) fits are quite robust and accurate at the 1% level

SU(3) NLO (370 MeV cut) and NNLO (510 MeV) cut well represent data and appear reasonably reliable.

SU(3) NLO represents data at 1% level to 370 MeV.

SU(3) NNLO represents data at 2% level to 510 MeV. Needed to freeze LO LECs for NNLO fit - an indication of needing more data and/or lack of reliability of the expansion.

Good agreement between SU(3) NLO LECs and SU(2)

Find ratio of LO SU(2)/SU(3) LECs close to 1

$SU(2)$	$\boldsymbol{B^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV})}$	**4.229(35)(11) GeV**
	$\boldsymbol{f}$	**122.2(1.5)(0.9) MeV**
	$\bar{\ell}_1$	$-0.7(7.2)(2.5)$
	$\bar{\ell}_2$	$4.0(6.2)(2.1)$
	$\boldsymbol{\bar{\ell}_3}$	**2.97(19)(14)**
	$\boldsymbol{\bar{\ell}_4}$	**3.90(8)(14)**
	$10^3 l_7$	$6.6(5.4)(0.1)$
$SU(3)$	$\boldsymbol{B_0^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV})}$	**4.138(93)(93) GeV**
	$\boldsymbol{f_0}$	**114.9(2.9)(1.9) MeV**
	$10^3 L_1$	$-0.4(1.7)(0.0)$
	$10^3 L_2$	$-0.9(2.2)(0.2)$
	$10^3 L_3$	$0.4(5.3)(0.1)$
	$\boldsymbol{10^3 L_4}$	**−0.149(62)(42)**
	$\boldsymbol{10^3 L_5}$	**0.909(87)(20)**
	$\boldsymbol{10^3 L_6}$	**−0.094(40)(21)**
	$10^3 L_7$	$-0.13(25)(1)$
	$\boldsymbol{10^3 L_8}$	**0.51(4)(12)**

Find self-consistently reliable results.

Will results presist as even lighter mass data becomes available?