Leading isospin breaking correction to the HVP

Marina Krstić Marinković

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Motivation I: computing IB correction to the HVP

- Discrepancy between $a_\mu^{exp} - a_\mu^{th,SM}$ mainly coming from hadronic contributions
- Once the aimed precision (<1%) for the connected HVP from the lattice is achieved (in the isosymmetric theory) —> the effects we neglected so far might become important:
  - disconnected contribution,
  - isospin breaking corrections,
  - charm in the sea, …
- In the phenomenological determination of a_μ^{had}, model calculation of [Jegerlehner,Szafron '11]
  - ➡ correctly applied IB correction reduced the discrepancy between e^+e^- and τ data
- Not clear how this translates to the Euclidean
- It would be good to have a model independent estimate of IB effects: lattice QCD+QED
- Note: systematic analysis based on the τ data may also benefit knowing how big/small this effect is

Motivation II: the method to compute IBE

- All necessary ingredients are, in principle, there
- R123 method [arXiv:1303.4896] for computing leading isospin breaking corrections (LIBE)
    - ➡ Expanding an observable (in the isospin broken theory) with respect to the isosymmetric QCD result
- For a start: applying it to the connected part of the HVP
- Main advantage w. respect to simulating QED+QCD:
    - ➡ Diagrams obtained individually (before multiplying with $O(\alpha_{em})$, $O(m_u - m_d)$ coeff.)
    - ➡ No extrapolation in α_{em}

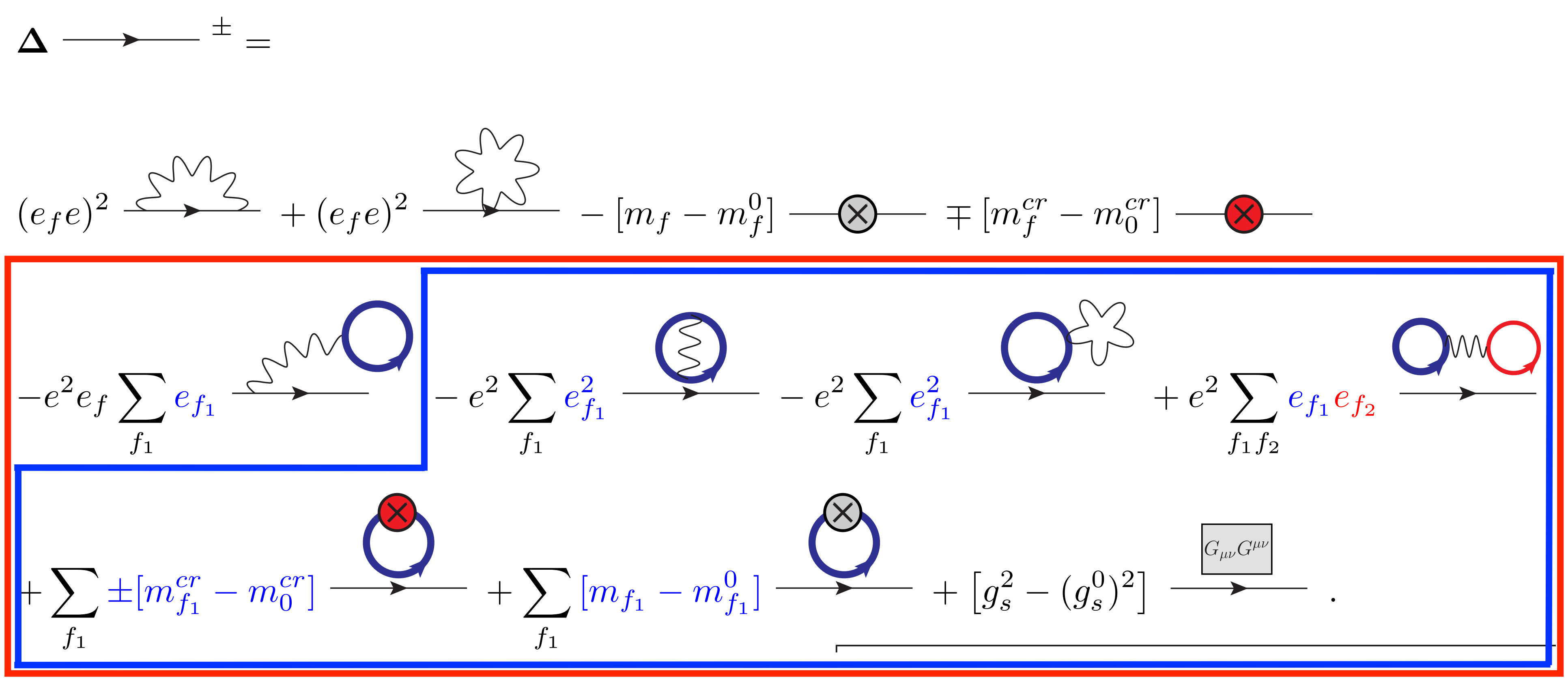
The method I: LIBE in practice (R123)

- Reusing the gauge configurations generated in the isosymmetric theory
- Reweighting:

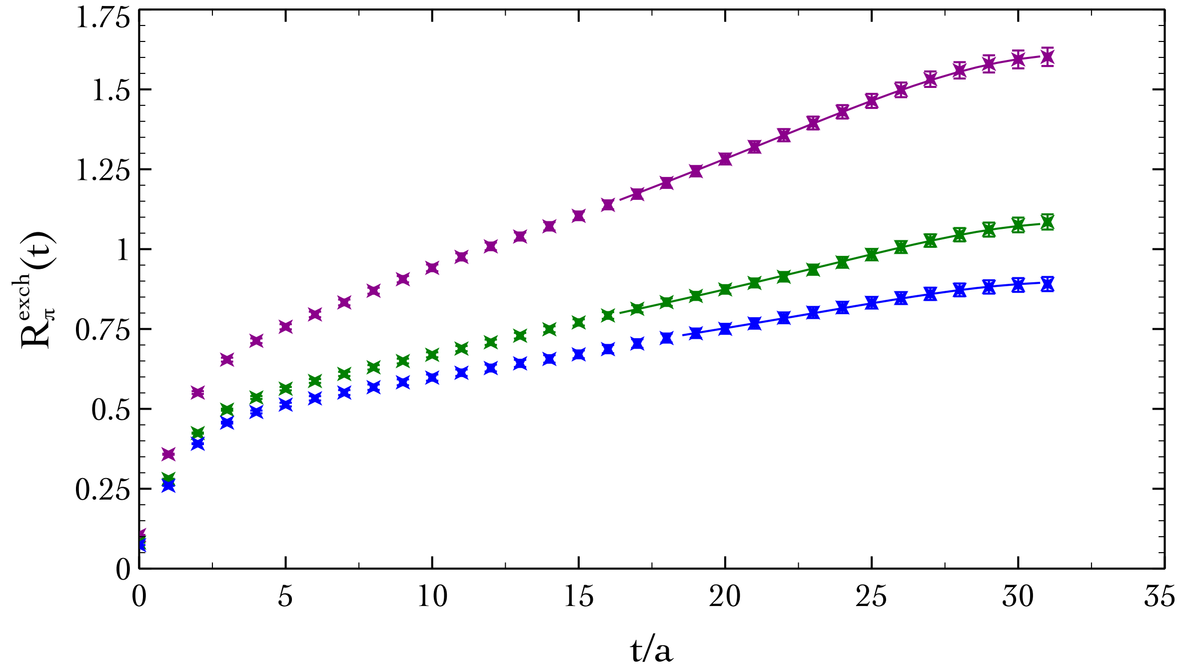
$$\langle O \rangle^{\vec{g}} = \frac{\left\langle R[U,A;\vec{g},\vec{g}^0]\; O[U,A;\vec{g}] \right\rangle^{A,\vec{g}^0}}{\left\langle R[U,A;\vec{g},\vec{g}^0] \right\rangle^{A,\vec{g}^0}}$$

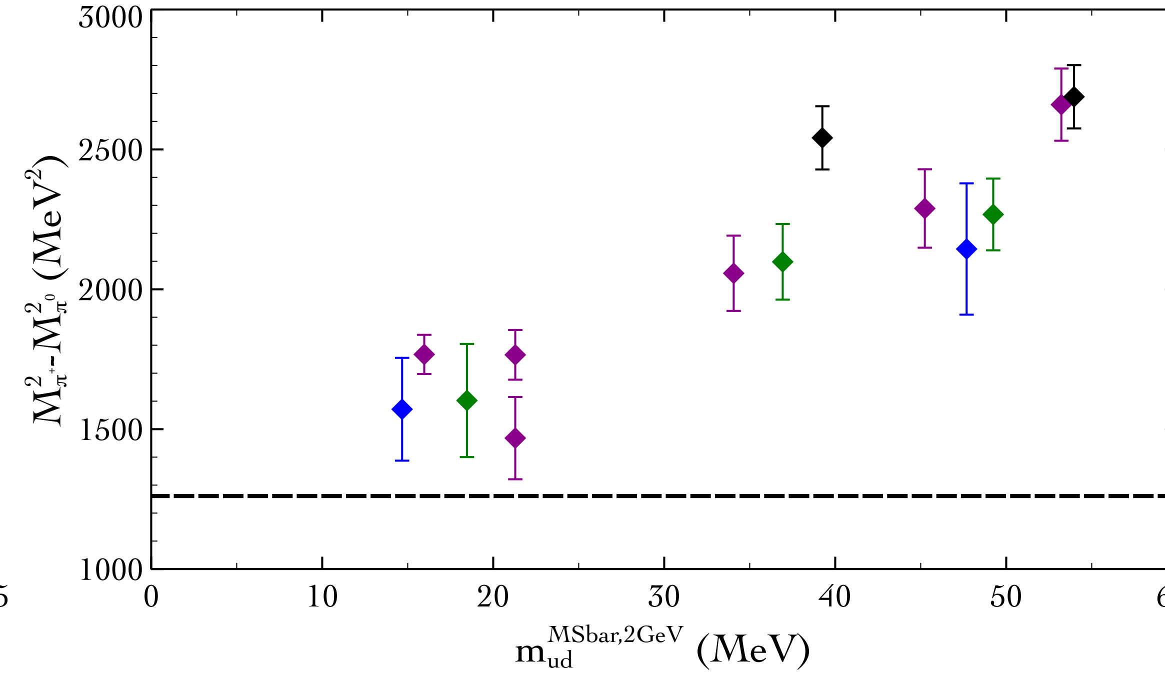
$\vec{g}^0$ - bare param. of isosymm. th
$\vec{g}$ - bare param. of the full th

- For simplicity, approximate sea quarks as electrically neutral: $R[U,A;\vec{g},\vec{g}^0] = 1$
- …once an appropriate renormalisation procedure is applied: $\Delta O = O(\vec{g}) - O(\vec{g}^0)$
- Example:



The method II: LIBE in practice (R123)





Review by N.Tantalo @Lat2013

- Previous results by Rome123 collaboration [arXiv:1303.4896, arXiv:1311.2797]
- Leading correction to different hadronic observables: pion/kaon mass splitting, Dashen theorem breaking parameter, u-d quark mass difference …
- Corrections function of the ratios of the correlators in the full and isosymmetric theory and give good numerical signal

Technicalities

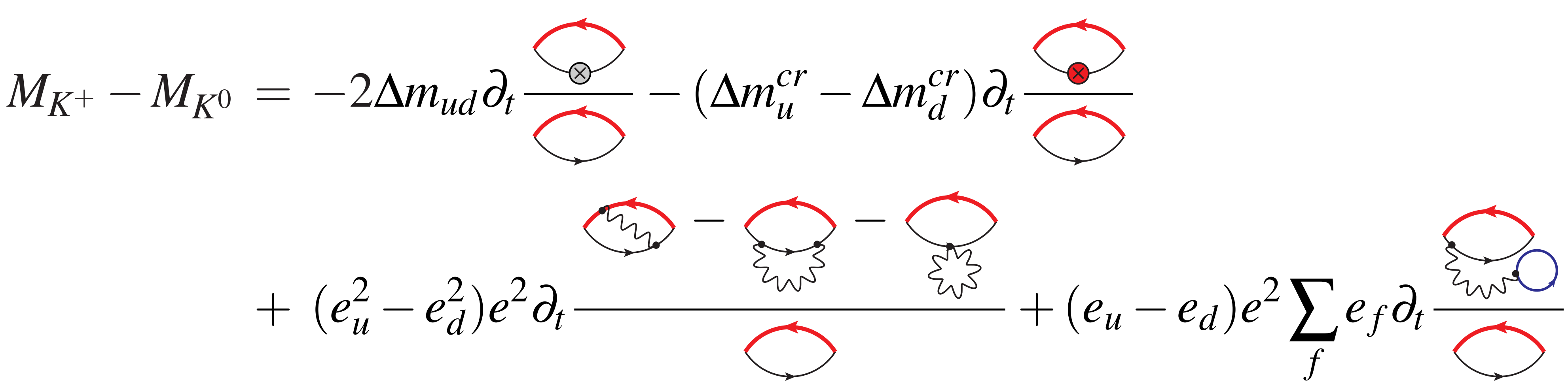
- Leading correction: expanding in powers of the difference between bare param. in full and isosymm. th:

$$\Delta O = \left\{ e^2 \frac{\partial}{\partial e^2} + \left[g_s^2 - (g_s^0)^2\right] \frac{\partial}{\partial g_s^2} + [m_f - m_f^0] \frac{\partial}{\partial m_f} + [m_f^{cr} - m_0^{cr}] \frac{\partial}{\partial m_f^{cr}} \right\} O$$

- Leading IB corrections are computed: also in QED+QCD simulations $O(\alpha(m_u - m_d))$ are neglected

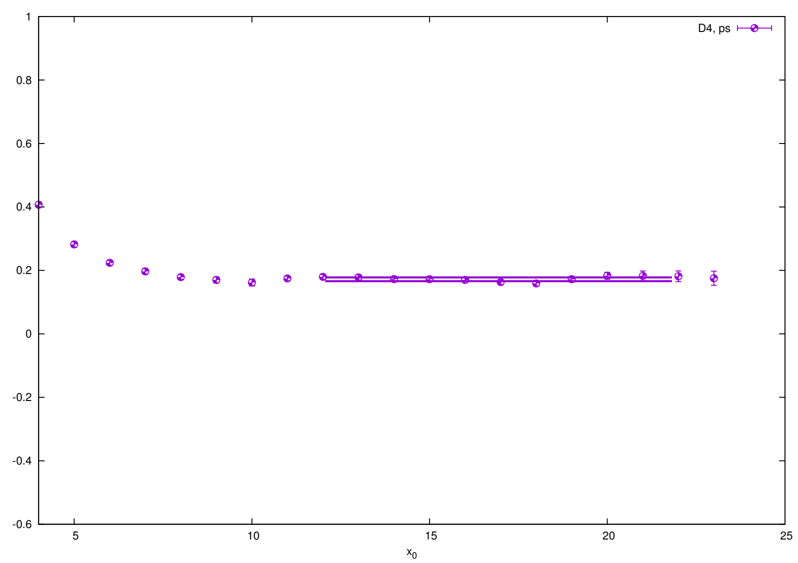
- Main general obstacle in implementing this method
  - ➡ many diagrams need to be computed
  - ➡ including the 3-pt, 4-pt functions and the disconnected ones (beyond el-quenched approximaton)

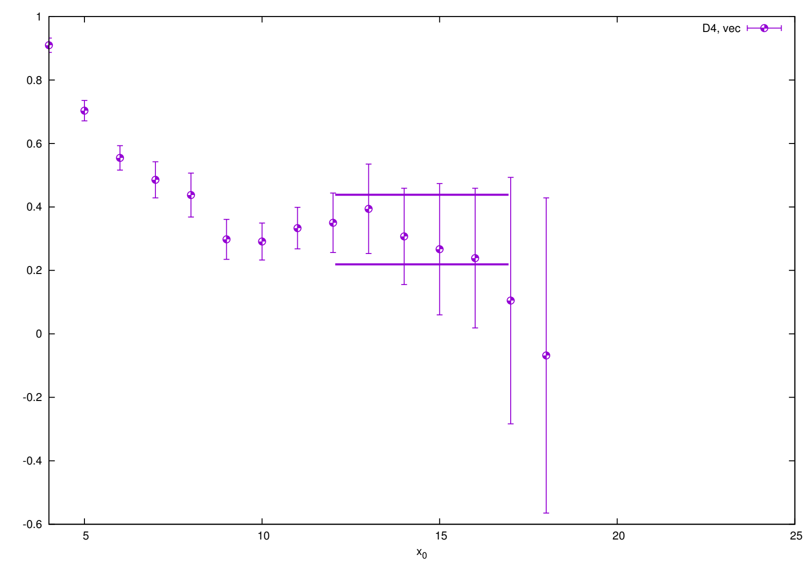
- Implementation: requires careful organisation of the computation of the diagrams:



Pseudo-scalar vs. vector

- We know that what works in pseudo-scalar channel
  - ➡ might not necessarily work that well in the vector one
- Example on two ensembles with Nf=2 O(a) improved Wilson fermions (CLS based configurations)

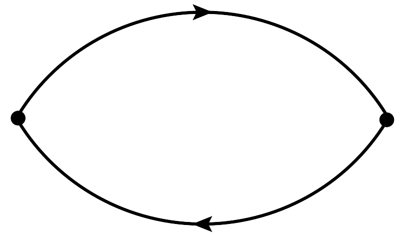




- Lattice spacing $a \approx 0.07 fm$, pion masses:
  - ➡ D4: 48×24^3, $m_\pi \approx 480\text{MeV}$
  - ➡ E5: 64×32^3, $m_\pi \approx 410\text{MeV}$
  - ➡ D5: 48×24^3, $m_\pi \approx 420\text{MeV}$

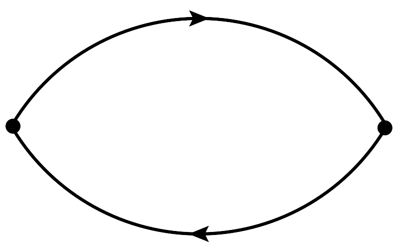
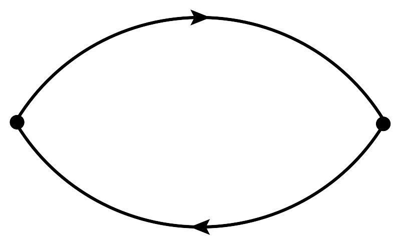
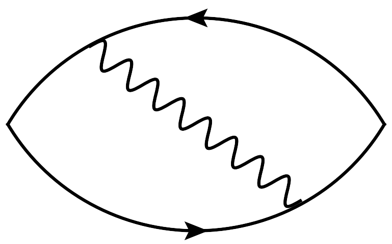
LIBE of the HVP in the electro-quenched approx.

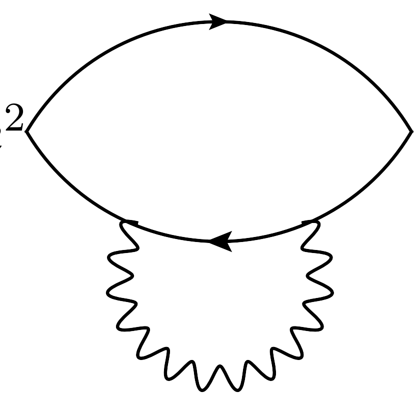
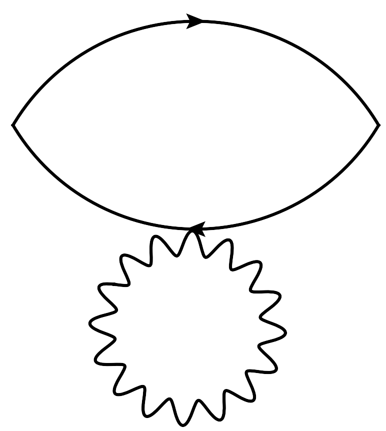
- Expanding the connected part of the HVP

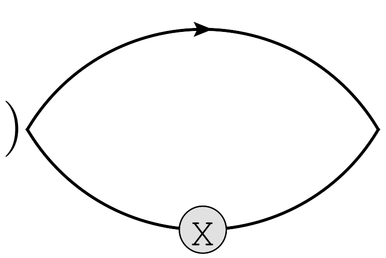


$$= \quad Tr\{\gamma_\mu S_f \gamma_\nu S_f\}$$

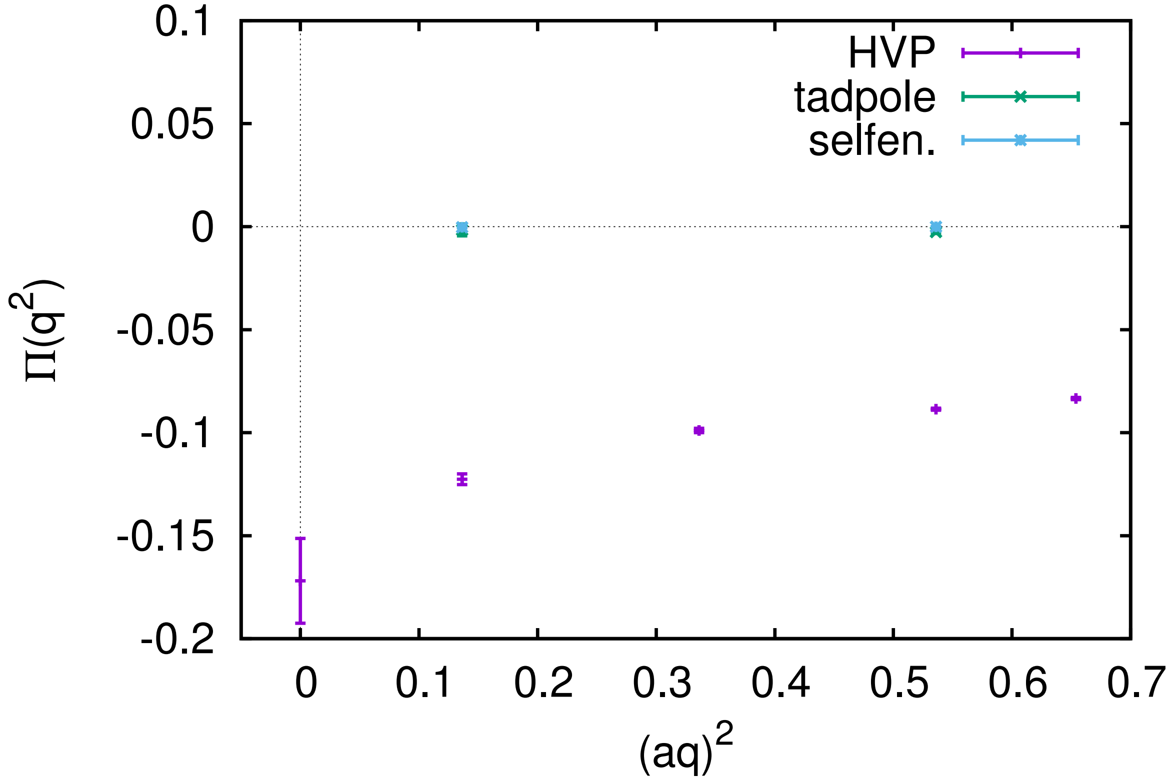
- Electro-quenched approximation:

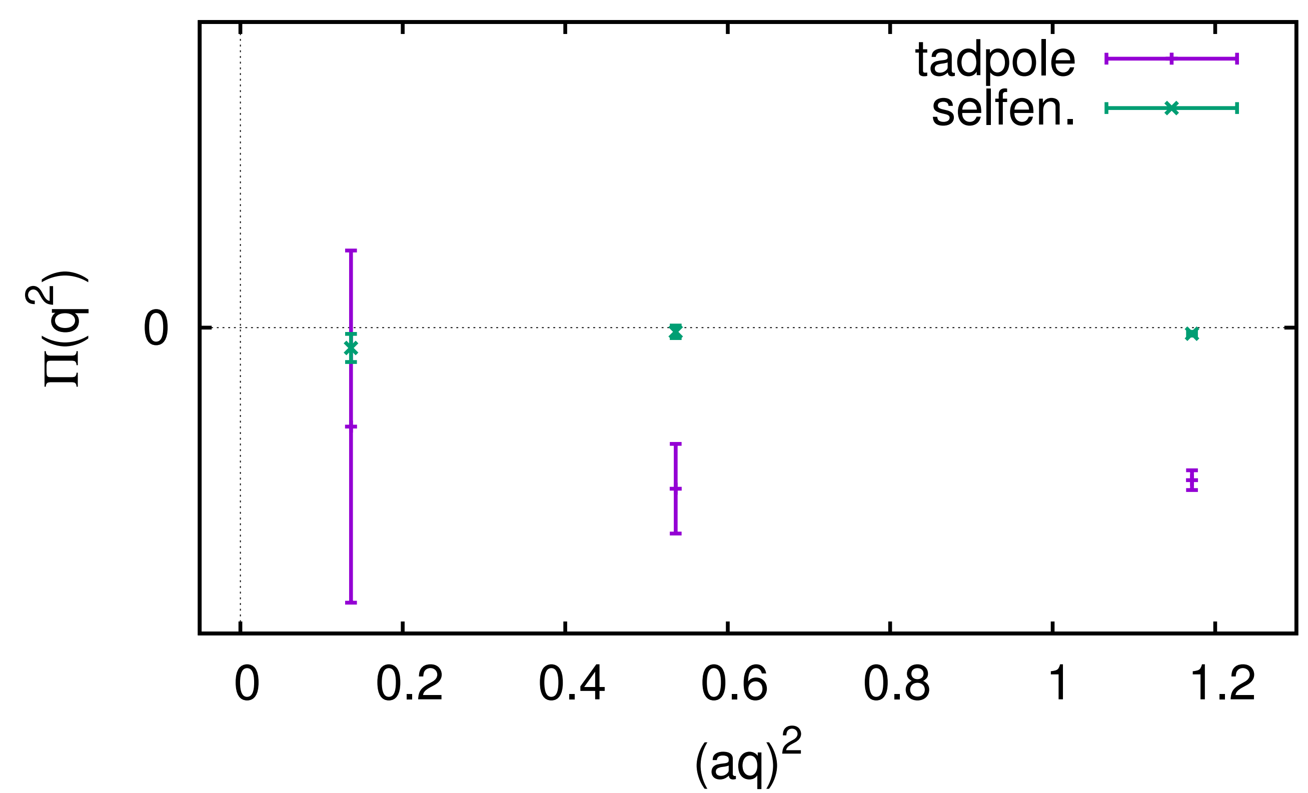
$(e_q^2$  $)$ $=$ $(e_q^2$  $)^0$ $- e_q^4 e^2$ 

$- e_q^4 e^2$  $- e_q^4 e^2$ 

$+ e_q^2 (m_f - m_f^0)$  $+ \quad \cdots$

A first look at the signal/noise

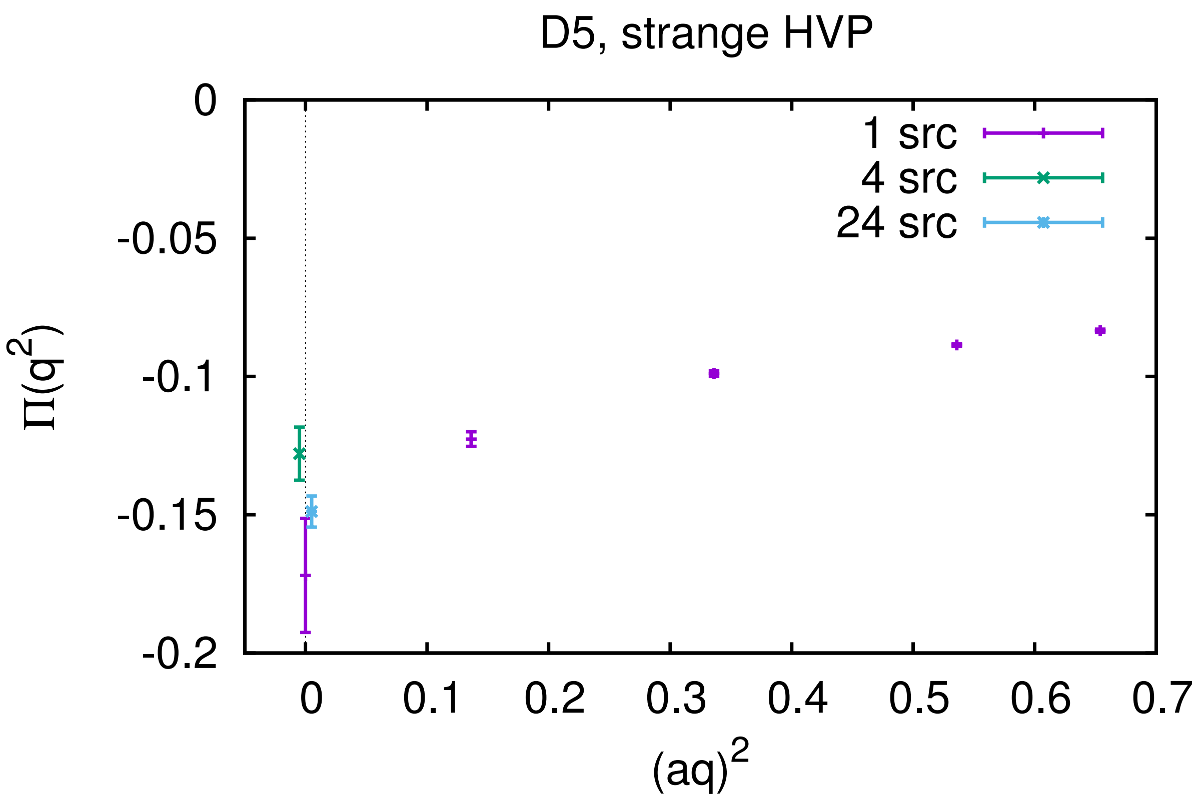




- Strange HVP and EM corrections (exploratory study, same bare parameters)
- Their sum makes sense only after the renormalisation:
  - ➡ intermediate renormalisation perscription and matching procedure
  - ➡ using experimental determinations of the charged mesons to fix quark masses and the lattice spacing in the isosymmetric theory

Outline I: technical improvements

- Performing this first test on a moderate size cluster was possible partially due to the deflated solver from DD-HMC code [M. Lüscher '07]
- Nevertheless, upgrading the code to use the new openQCD solver: speedup needed for accumulating the statistics for the light contribution
- This study is performed with point source
  - ➡ indications that replacing the point with some extended source might be advantageous



➡ 1src, 4src, 24src on 83, 64, 41 configurations, $\Pi(0)$ as prpopsed in [1208.5914]

Outline II: conceptual improvements

- Reducing finite volume effects - they are expected to be strong
- Currently: global zero mode subtracted: $A_\mu(k=0) \equiv 0$
  - ➡ Violates reflection positivity and does not have a well defined $T \to \infty$ limit [1406.4088]
- Removing the zero mode of the field on each time slice separately [Hayakawa, Uno '08]
  - ➡ this explicitly violates the hypercubic symmetry of the lattice -> no trace of the violation in the inf. vol limit [1406.4088]
- Charged particles in QED/QED+QCD with C* BCs —> FV effects even smaller
  - ➡ [talks by A. Patella and N. Tantalo: Thurs.,11.20, 11.40, s.402]
- This would be the way to go here as well, although the FV corrections for this quantity have not yet been explicitly computed in any of the above mentioned setups
- Getting the disconnected contributions (beyond el-quenched)

Conclusions

- Phenomenologically - IB plays an important role in the th.-exp. discrepancy
- Although the aimed precision of the HVP determinations from the lattice is not yet there to see this effect
  - ➡ useful to think ahead and work towards this estimate
  - ➡ all ingredients are there
- This is a first attempt to extract the IB correction to the HVP from first principles
- Difficult task, but R123 method should give better signal over simulating full theory (larger contributions are being computed)
  - ➡ QCD+QEDq certainly worth trying
- Even after (and if) getting the signal in the light sector —> many things to understand before a definite answer is known
  - ➡ 1. Finite volume effects
- Stay tuned …

Thanks!

- Nazario for the (great!) code basis, discussions and motivation to look at the IB effects in this way
- lattice@CERN for managing and providing the computer cluster, CLS based for the gauge configurations
- Fred Jegerlehner and RBC-UKQCD colleagues (esp. HVP working group) for various discussions on HVP and isospin breaking related issues

The RBC & UKQCD collaborations

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Amarjit Soni
Sergey Syritsyn

CERN

Marina Marinkovic

Columbia University

Ziyuan Bai
Norman Christ
Xu Feng
Luchang Jin
Bob Mawhinney
Greg McGlynn
David Murphy
Daiqian Zhang

University of Connecticut

Tom Blum

Edinburgh University

Peter Boyle
Luigi Del Debbio
Julien Frison
Richard Kenway
Ava Khamseh
Brian Pendleton
Oliver Witzel
Azusa Yamaguchi

Plymouth University

Nicolas Garron

University of Southampton

Jonathan Flynn
Tadeusz Janowski
Andreas Juettner
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Chris Sachrajda
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Matthew Spraggs
Tobias Tsang

York University (Toronto)

Renwick Hudspith

BACKUP I: Different strategy than Pi(0)

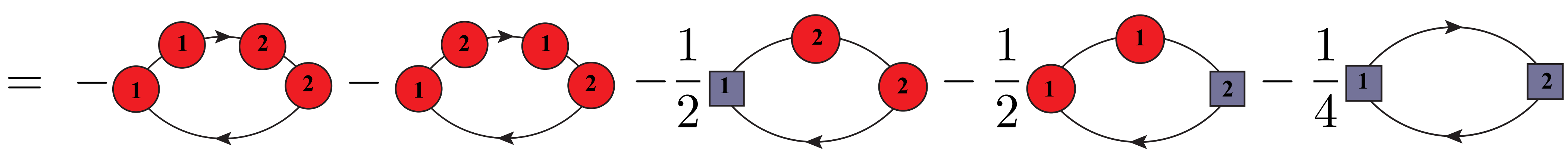
- But same machinery needs to be implemented [de Divitis, Petronzio, Tantalo1208.5914]:

- $\Pi_{12}(Q) = \sum_x \langle Tr\{S[y,x;U]\Gamma_V^1(x,\vec{q})S[x,y;U,\lambda^p]\Gamma_V^2(y,\vec{0})\}\rangle.$

- $\Pi(0) = -\frac{\partial \Pi_{12}(Q)}{\partial Q_1 \partial Q_2}\Big|_{Qs=0}$

$$= -\frac{1}{(TL^3)^2}\sum_{x,y}\langle Tr\left[S\Gamma_V^1 \frac{\partial^2 S}{\partial Q_1 \partial Q_2}\Gamma_V^2\right] - \frac{i}{2}\, Tr\left[S\Gamma_T^1 \frac{\partial S}{\partial Q_2}\Gamma_V^2\right]$$

$$-\frac{i}{2}\, Tr\left[S\Gamma_V^1 \frac{\partial S}{\partial Q_1}\Gamma_T^2\right] - \frac{1}{4}\, Tr\left[S\Gamma_T^1 S\Gamma_T^2\right]\rangle$$

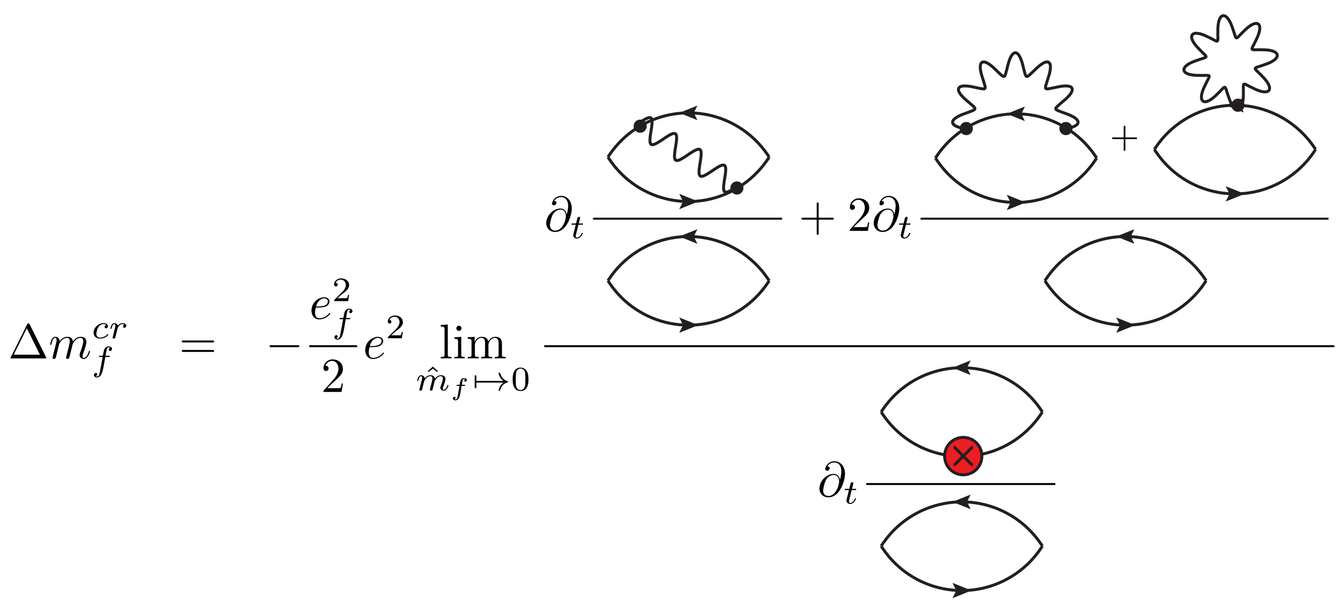


- Consequently: tricks learned in one computation can be used in another and vice versa

BACKUP II: Tuning the critical mass

- Using WTI :
  - ➡ Dashen theorem: $\hat{m_f}={\hat{m_u},\hat{m_d},\hat{m_s}}=0$
  - ➡ Also with EM, in the massless theory:
    - $M_{\Pi^0}=M_{K^0}=0$
    - $M_{\Pi^+}=M_{K^+}=0$
    - $\kappa^{crit}_s = \kappa^{crit}_d$

- Need to be done with high accuracy, in order to cancel linear ultraviolet divergencies



BACKUP III: Subtracting the zero mode

- Illustration of the reduced finite-volume effects, once zero mode of the photon field subtracted time-slice by time-slice (L) instead of the global zero mode subtraction (TL)
- Note that translation invariance is violated here, but it is recovered in the continuum limit [BMW,1406.4088]