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# Curvature of the QCD chiral peusdocritical line from analytic continuation

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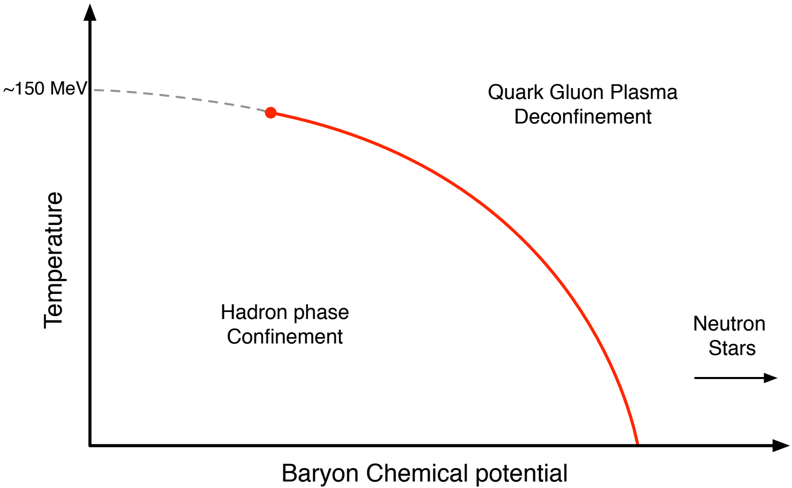
# Outline

- The critical line of QCD
- The method of analytic continuation
- Renormalized observables
  - Chiral condensate, renormalization I
  - Chiral condensate, renormalization II
  - Chiral susceptibility
- Numerical setup
- Numerical results
  - Finite size effects
  - Effects of $\mu_s \neq 0$
  - Continuum limit extrapolations.
- Conclusions

# QCD at nonzero $\mu_B$



# The pesudocritical line and analytic continuation

At lowest order in $\mu$, the pseudocritical line can be parametrized as:

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa \left( \frac{\mu_B}{T_c(\mu_B)} \right)^2 + O(\mu^4)$$

**The sign problem and analytic continuation**

For purely imaginary $\mu$, the fermion determinant is real positive, and the sign problem is non existent.

With the transformation $\mu \rightarrow i\mu$, the pseudocritical line parametrization is modified as:

$$\frac{T_c(\mu_B)}{T_c} = 1 + \kappa \left( \frac{\mu_B}{T_c(\mu_B)} \right)^2 + O(\mu^4)$$

**Renormalization of the chiral condensate**

$$\langle\bar\psi\psi\rangle_u d = \frac{T}{V}\frac{\partial \log Z}{\partial m_u d} = 2\frac{T}{V}\langle \mathrm{Tr} M_l^{-1}\rangle = \langle\bar u u\rangle + \langle\bar d d\rangle$$

We have considered two renormalizations:

1. As in [Cheng *et al.*, 08]:

$$\langle\bar\psi\psi\rangle^r_{(1)} \equiv \frac{\langle\bar\psi\psi\rangle_{ud}(T) - \frac{2m_{ud}}{m_s}\langle\bar s s\rangle(T)}{\langle\bar\psi\psi\rangle_{ud}(0) - \frac{2m_{ud}}{m_s}\langle\bar s s\rangle(0)}$$

2. Alternatively [Endrodi *et al.*, 11 ]:

$$\langle\bar\psi\psi\rangle^r_{(2)} \equiv \frac{m_{ud}}{m_\pi^4}\left(\langle\bar\psi\psi\rangle_{ud} - \langle\bar\psi\psi\rangle_{ud}(T=0)\right)$$

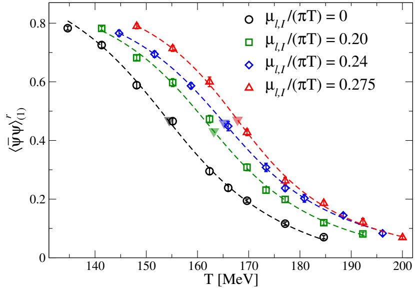
**Renormalized chiral susceptibility**

$$\chi_{\bar{\psi}\psi} \equiv \frac{\partial \langle \bar{\psi}\psi \rangle_{ud}}{\partial m_l}$$

We have chosen this renormalization[ Y.Aoki *et al.*, 06 ]:

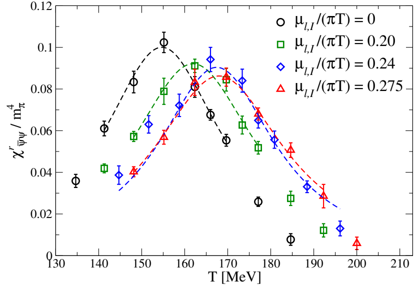
$$\chi^r_{\bar{\psi}\psi}(T) \equiv m_{ud}^2 \left[ \chi_{\bar{\psi}\psi}(T) - \chi_{\bar{\psi}\psi}(0) \right]$$

We use the dimensionless quantity $\chi^r_{\bar{\psi}\psi}(T)/m_\pi^4$.




Fit for the **chiral condensates (I) and (II)**:

$$\langle\bar{\psi}\psi\rangle^r(T) = A_1 + B_1 \arctan\left[C_1\left(T - T_c\right)\right]$$




Fit at the peak for the **renormalized chiral susceptibility**:

$$\chi^r_{\bar{\psi}\psi}(T) = \frac{A_2}{(T - T_c)^2 + B_2^2}$$

# Numerical setup

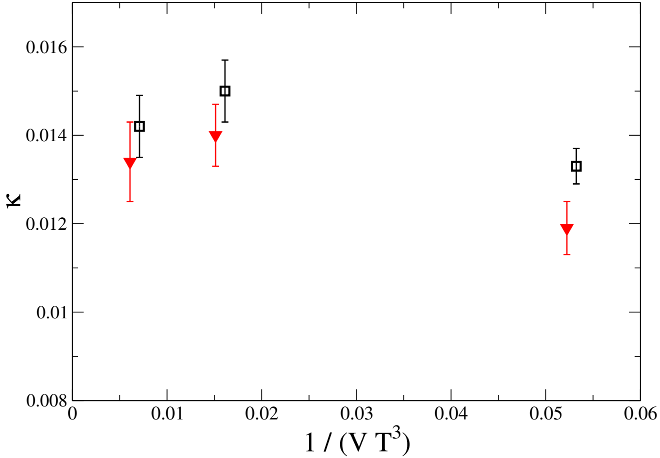
- Study of the $\mu_s = \mu_l \neq 0$ ($32^3$x8 only) and $\mu_s = 0$ cases.
- Tree level Symanzik improved gauge action with $N_f = 2 + 1$ flavours of 2-stouted staggered fermions.
- At the physical point (line of constant physics, parameters taken from [Aoki *et al.*, 09] ) $N_t = 6, 8, 10, 12$ lattices.
- Also performed simulations at zero temperature for subtractions ($32^4$,$48^3$x96).
- Observables evaluated with noisy estimators, with 8 random vectors per quark.

Simulations run on IBM BG-Q at CINECA (Bologna, Italy).

| Lattice | $16^3 \times 6$ | $24^3 \times 6$ | $32^3 \times 6$ |
|---|---|---|---|
| $i\mu/(\pi T)$ | 0.00 0.20 0.24 0.275 | 0.00 0.24 0.275 | 0.00 0.24 0.275 |
| Lattice | $32^3 \times 8$ | $40^3 \times 10$ | $48^3 \times 12$ |
| $i\mu/(\pi T)$ | 0.00 0.10 0.15 0.20 0.24 0.275 0.30 | 0.00 0.20 0.24 0.275 | 0.00 0.20 0.24 0.275 |

# Finite size effects
## On $N_t = 6$ lattices

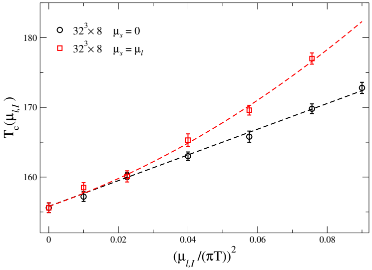
From [Bonati *et al.*, 14] ($16^3$x6, $24^3$x6 and $32^3$x6 lattices)




Black : Renormalized Chiral Condensate (1), Red : Renormalized Chiral Susceptibility

$\Rightarrow$ Aspect ratio = 4 is enough.

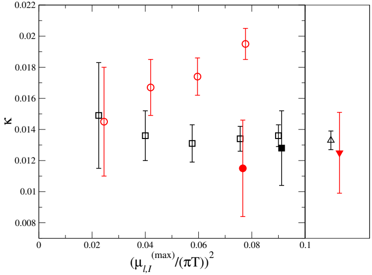
# Effects of $\mu_s$
## $32^3 \times 8$ Lattice




(Data also from [Bonati *et al.*, 14])

(Renormalized chiral susceptibility)

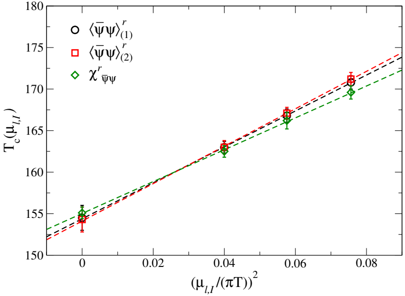
Results for $\kappa$ varying the $\mu$ fit range:



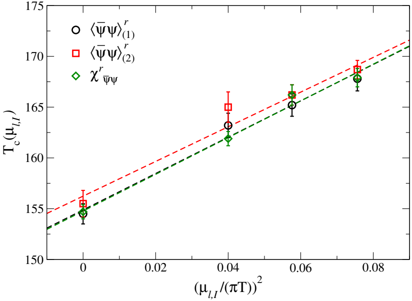

Empty Red: $\kappa$, linear fit ($\mu_s = \mu_l$ data)
Full Red: $\kappa$, lin+quad fit ($\mu_s = \mu_l$)
Empty Black: $\kappa$, linear fit ($\mu_s = 0$)
Empty Black: $\kappa$, lin+quad fit ($\mu_s = 0$)
Right: $\kappa$ from combined (lin+quad) fit

# Critical line
## Finer Lattices

$40^3 \times 10$




$48^3 \times 12$

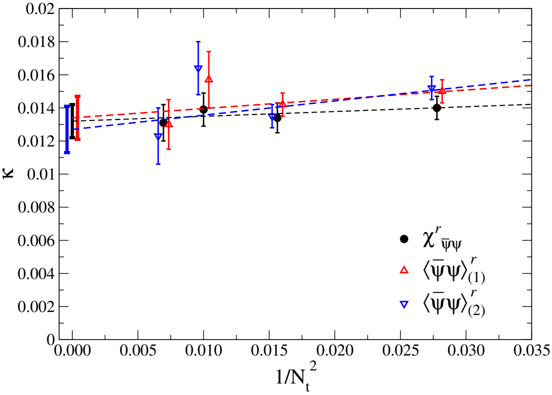



$$\kappa_{\bar\psi\psi,1} = 0.0157(17)$$
$$\kappa_{\bar\psi\psi,2} = 0.0164(16)$$
$$\kappa_{\chi} = 0.0139(10)$$

$$\kappa_{\bar\psi\psi,1} = 0.0130(15)$$
$$\kappa_{\bar\psi\psi,2} = 0.0123(17)$$
$$\kappa_{\chi} = 0.0131(11)$$

# Continuum Limit of $\kappa$

We evaluated the curvature $\kappa$ for each lattice spacing and then performed the continuum limit extrapolation on $\kappa$ itself.




$$\kappa_{\bar\psi\psi,1} = 0.0134(13)$$
$$\kappa_{\bar\psi\psi,2} = 0.0127(14)$$
$$\kappa_{\chi} = 0.0132(10)$$

# Continuum limit of Observables

- For the **renormalized chiral condensates**, we used the formula

$$\langle\bar{\psi}\psi\rangle^r(T) = A_1 + B_1 \arctan\left[C_1\left(T - T_c\right)\right]$$

to fit the data from all values of $N_t$ simultaneously. We added a $N_t$ dependency to $T_c$ ($T_c(N_t) = T_c(N_t = \infty) + const./N_t^2$) and a similar one to $C_1$.

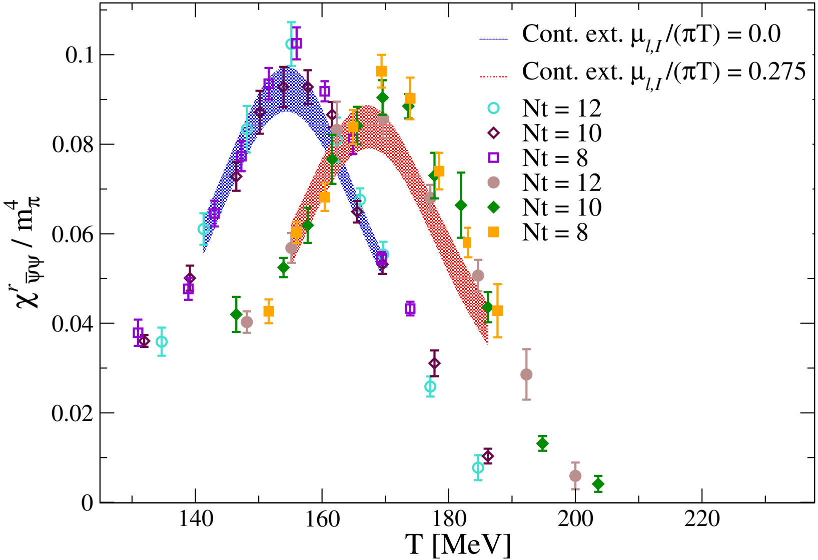
- For the **renormalized chiral susceptibility**, we used the formula

$$\chi^r_{\bar{\psi}\psi}(T) = \frac{A_2}{(T - T_c)^2 + B_2^2}$$

where we added a dependency on $N_t$ similar to $T_c(N_t) = T_c(N_t = \infty) + const./N_t^2$ for all parameters.

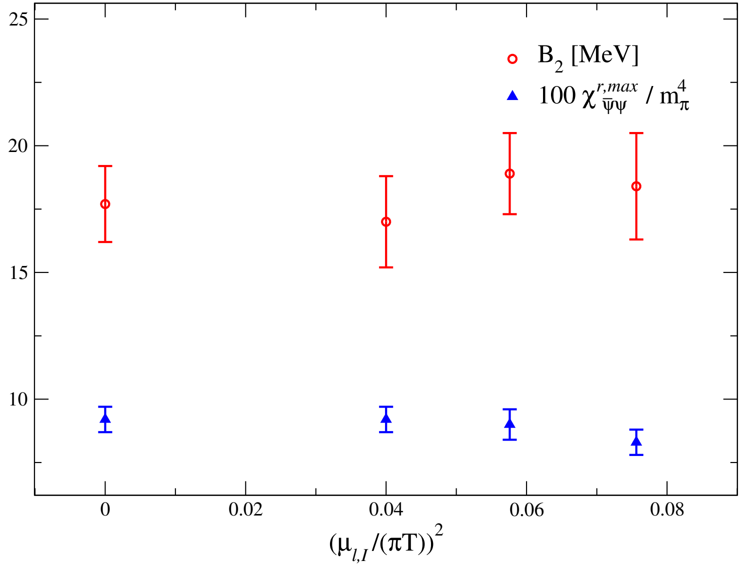
# Continuum limit of Observables

## Renormalized chiral susceptibility

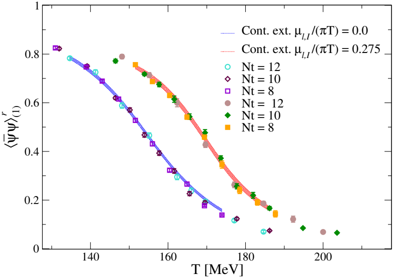


# Continuum limit of Observables

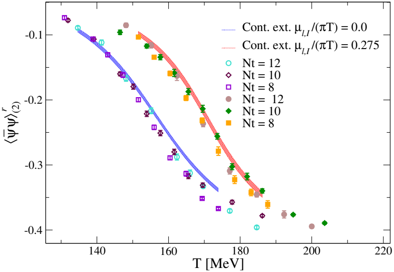
## Renormalized chiral susceptibility - Width and height of the peak ($B_2$ and $\chi^{r,max}_{\bar{\psi}\psi}$)



## Chiral condensate 1




## Chiral condensate 2

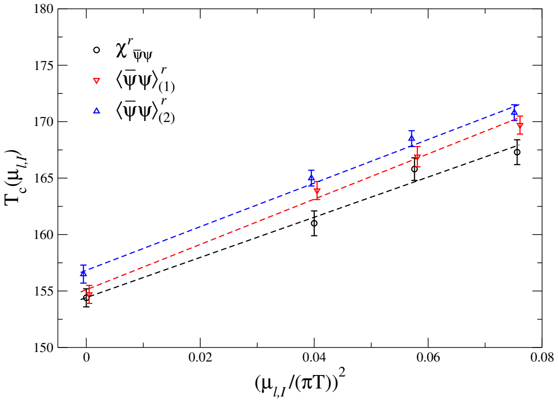



$$\langle\bar{\psi}\psi\rangle^r_{(1)} \equiv \frac{\langle\bar{\psi}\psi\rangle_{ud}(T) - \frac{2m_{ud}}{m_s}\langle\bar{s}s\rangle(T)}{\langle\bar{\psi}\psi\rangle_{ud}(0) - \frac{2m_{ud}}{m_s}\langle\bar{s}s\rangle(0)}$$

$$\langle\bar{\psi}\psi\rangle^r_{(2)} \equiv \frac{m_{ud}}{m_\pi^4}\left(\langle\bar{\psi}\psi\rangle_{ud} - \langle\bar{\psi}\psi\rangle_{ud}(T=0)\right)$$

# Continuum limit of Observables

## Critical line from continuum extrapolated $T_c$s




2nd method:

$$\kappa_{\bar{\psi}\psi,1} = 0.0145(11)$$
$$\kappa_{\bar{\psi}\psi,2} = 0.0138(10)$$
$$\kappa_{\chi} = 0.0131(12)$$

1st method:

$$\kappa_{\bar{\psi}\psi,1} = 0.0134(13)$$
$$\kappa_{\bar{\psi}\psi,2} = 0.0127(14)$$
$$\kappa_{\chi} = 0.0132(10)$$

# Conclusions

- We investigated the effects of including a nonzero strange quark potential ($\mu_s = \mu_l = \mu$). We have confirmed the presence of a quartic contribution. Considering such contribution, the curvature of the critical line for $\mu_s = \mu_l$ or $\mu_s = 0$ is compatible within errors.
- We performed a continuum scaling analysis in two ways, directly on $\kappa$ and on the observables. The resulting estimates of $\kappa$ are in agreement. Our prudential estimate is $\kappa = 0.0135(20)$.