The curvature of the crossover line in the (T, μ)-phase diagram of QCD

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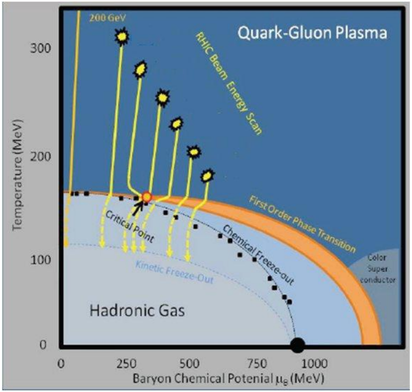
The (T, μ)-phase diagram of QCD



Aim: Determining T_c at finite μ_B. A parametrization which respects symmetry under charge conjugation at $\mu_B = 0$:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c(\mu_B)} \right)^2 + \mathcal{O}(\mu_B^4)$$

Determining κ on the lattice

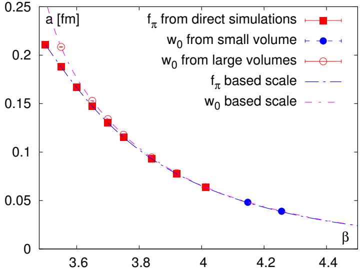
There are two main methods to determine the curvature κ from the lattice:

- taylor expansion form simulations at $\mu_B = 0$ ([1], [2])
- analytic continuation from simulation at imaginary μ_B ([3], [4])

The simulations are for $N_t = 10$ or coarser which might not be enough for a controlled continuum extrapolation. Also for a reliable error estimate an analysis of systematic influences is necessary.

[1] G. Endrödi et al (2011, arXiv:1102.1356), [2] O. Kaczmarek et al (2011, arXiv:1011.3130), [3] C. Bonati et al (2014, arXiv:1410.5758), [4] P. Cea (2014, arXiv:1403.0821)

Simulation details



- Action: tree-level Symanzik improved gauge action, with four times stout smeared staggered fermions
- 2+1+1 flavour, on LCP with pion and kaon mass
- Simulation at $\langle n_S \rangle = 0$ (as for heavy ion collisions, until now there are simulations with $\mu_s = 0$ or $\mu_S = 0$ where $\mu_S = \frac{1}{3}\mu_B - \mu_s$)
- Lattice sizes: $32^3 \times 8$, $40^3 \times 10$, $48^3 \times 12$ and $64^3 \times 16$
- $\frac{\mu_B}{T} = 0,\ 1.18i,\ 1.57i$ and $1.96i$
- Two methods of scale setting: f_π and w_0, $Lm_\pi > 4$

Tuning to $\langle n_S \rangle = 0$

Aim: For a given μ_B determine μ_S so that $\langle n_S \rangle = 0$. This means solving the differential equation

$$\langle n_S \rangle = 0 \Leftrightarrow \frac{\partial \log Z}{\partial \mu_S} = 0$$

Notation:

$$\chi_{udsc} = -\frac{1}{T^4} \frac{\partial^4}{\partial(\mu_u/T)\partial(\mu_d/T)\partial(\mu_s/T)\partial(\mu_c/T)} \frac{T}{V} \log Z$$

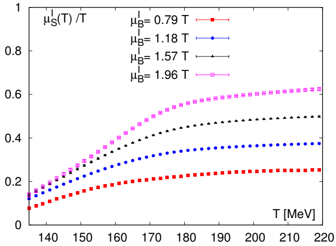
Assuming we know the value for $\mu_S(\mu_B)$ so that $\langle n_s \rangle = 0$ for $\mu_S(\mu_B^0)$ and $\mu_S(\mu_B^0 - \Delta\mu_B)$ with all the derivatives. Then (Runge-Kutta):

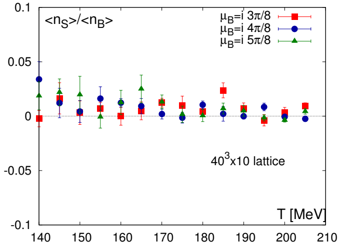
$$\mu_S(\mu_B^0 + \Delta\mu_B) = \mu_S(\mu_B^0 - \Delta\mu_B) + 2\Delta\mu_B \frac{\mathrm{d}\mu_S}{\mathrm{d}\mu_B}(\mu_B^0).$$

In the simulations with μ_B^0 and $\mu_B^0 - \Delta\mu_B$, μ_S might not precisely tuned. There we want to extrapolate to a better value. We assume that correct value of μ_S is $\tilde{\mu}_S = \mu'_S + \Delta\mu'_S$. Then:

$$\langle n_S \rangle = \frac{\partial \log Z}{\partial \tilde{\mu}_S} = \frac{\partial \log Z}{\partial \mu'_S} + \frac{\partial^2 \log Z}{\partial \mu'^2_S} \Delta\mu'_S = 0$$

Tuning to $\langle n_S \rangle = 0$





This yields

$$\Delta\mu'_S = -\frac{\chi_S}{\chi_{SS}}$$

Similar for the derivative we get:

$$\frac{\mathrm{d}\tilde{\mu}_S}{\mathrm{d}\mu_B} = -\frac{\tilde{\chi}_{SB}}{\tilde{Z}_{SS}}\Big|_{\langle n_S\rangle=0} = -\frac{\chi_{SB}}{\chi_{SS}} - \frac{\chi_{SSB}\chi_{SS} - \chi_{SSS}\chi_{SB}}{(\chi_{SS})^2}\Delta\mu'_S + \mathcal{O}(\Delta\mu'^2_S)$$

Observables

Chiral susceptibility:

$$\chi_{\bar{\psi}\psi} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial (m_q)^2}$$

$$\chi^r_{\bar{\psi}\psi} = \left(\chi_{\bar{\psi}\psi}(T,\beta) - \chi_{\bar{\psi}\psi}(0,\beta)\right) \frac{m_l^2}{m_\pi^4}$$

Chiral condensate:

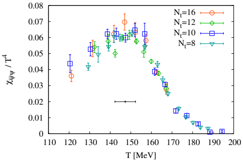
$$\langle\bar{\psi}\psi\rangle = \frac{T}{V} \frac{\partial \ln Z}{\partial m_q}$$

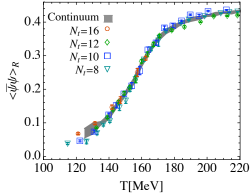
$$\langle\bar{\psi}\psi\rangle^r = -\left(\langle\bar{\psi}\psi\rangle(T,\beta) - \langle\bar{\psi}\psi\rangle(0,\beta)\right) \frac{m_l}{m_\pi^4}$$

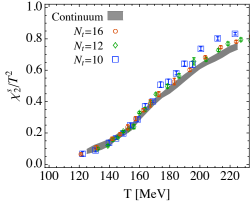
Strangeness susceptibility:

$$\chi_{SS} = \frac{T}{V} \frac{\partial^2 \ln Z}{\partial (\mu_S)^2}$$

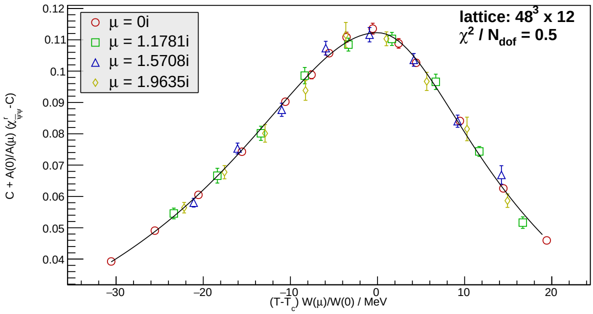
S. Borsányi et al (2010, arXiv:1005.3508)







$\chi_{\bar{\psi}\psi}$



Fit function:

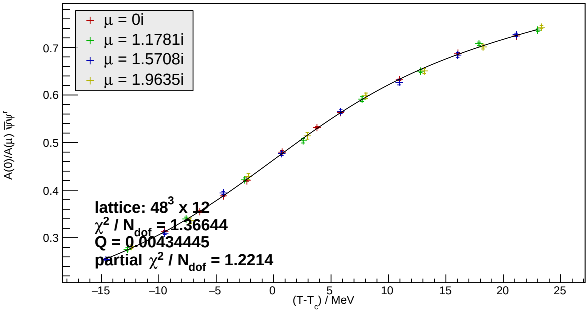
$$\chi^r_{\bar{\psi}\psi}(T) = \begin{cases} C + A^2(\mu)\left(1 + W^2(\mu)(T - T_c(\mu))^2\right)^{\alpha/2} & \text{for } T \leq T_c \\ C + A^2(\mu)\left(1 + b^2 W^2(\mu)(T - T_c(\mu))^2\right)^{\alpha/2} & \text{for } T > T_c \end{cases}$$

$$\left(\text{ or } \chi^r_{\bar{\psi}\psi}(T) = C + \frac{A(\mu)}{1 + W^2(\mu)(T - T_c(\mu))^2 + a_3 W^3(\mu)(T - T_c(\mu))^3}\right)$$

Zero temperature fit function:

$\chi_{\bar{\psi}\psi}(0,\beta) = \sum_{k=0}^{6} A_k \beta^k$ (or $\chi_{\bar{\psi}\psi}(0,\beta) = \sum_{k=-2}^{2} A_k \beta^k$)

$\bar{\psi}\psi$



Fit function:

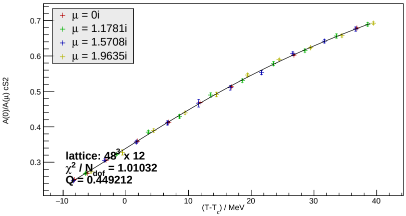
$$\langle\bar{\psi}\psi\rangle^r(\mu, T) = A(\mu)\,(1 + B \tanh\left[C\,(T - T_c(\mu))\right] + D\,(T - T_c(\mu)))$$

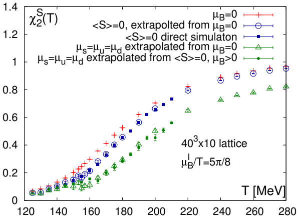
$$(\text{ or } \bar{\psi}\psi^r(\mu, T) = A(\mu)\,(1 + B \arctan\left[C\,(T - T_c(\mu))\right] + D\,(T - T_c(\mu))))$$

Zero temperature fit function:

$$\chi_{\bar{\psi}\psi}(0, \beta) = \sum_{k=0}^{K} A_k \beta^k \text{ with } K \in \{6, 7\}$$

χ_{SS}





Fit function:

$$\chi_{SS}(\mu, T) = A(\mu)\left(1 + B \tanh\left[C\left(T - T_c(\mu)\right)\right] + D\left(T - T_c(\mu)\right)\right)$$

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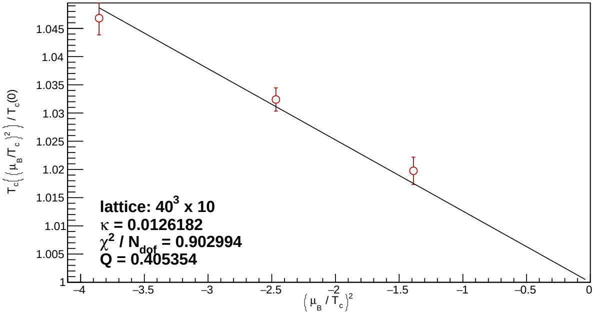
Curvature



Curvature function:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + \mathcal{O}(\mu_B^4)$$

For error analysis we also fit:

$$C_1(x) = 1 + ax + bx^2$$

$$C_2(x) = \frac{1 + ax}{1 + bx}$$

$$C_3(x) = \frac{1}{1 + ax + bx^2}$$

Continuum extrapolation

Continuum extrapolation:

$\kappa = \kappa^{\mathrm{c}} + A\left(\frac{1}{N_t}\right)^2$

Combined curvature fit and continuum extrapolation with:

$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \left(\kappa^{\mathrm{c}} + c_1\frac{1}{N_t^2}\right)\left(\frac{\mu_B}{T_c}\right)^2$

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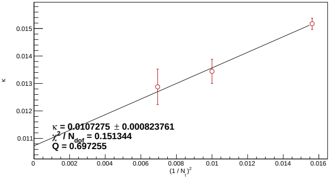
Extrap. with $Nt = 8,\ 10,\ 12$



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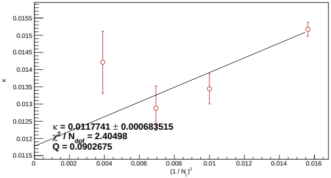
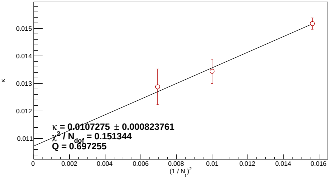
Extrap. with $Nt = 8,\ 10,\ 12,\ 16$



Extrap. with $Nt = 8,\ 10,\ 12$



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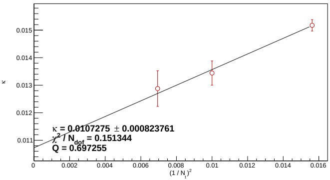
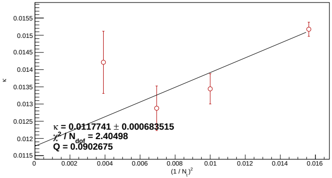
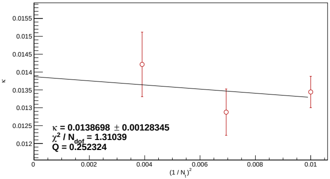


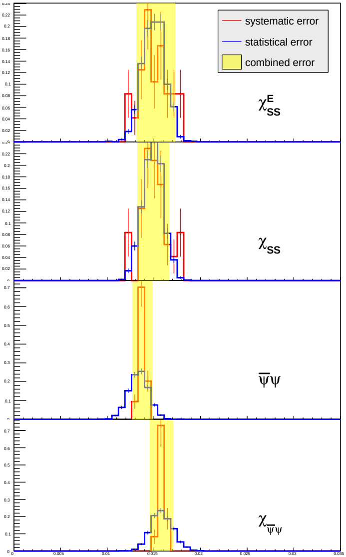
Extrap. with $Nt = 8,\ 10,\ 12,\ 16$



Extrap. with $Nt = 10,\ 12,\ 16$



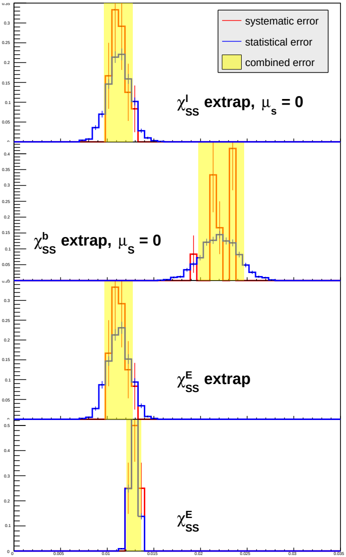
Comparison for different observables



χ^E_{SS}: $\langle n_S \rangle = 0$ and $0.5\langle B \rangle = \langle Q \rangle$

χ_{SS}: $\langle n_S \rangle = 0$ and $0.4\langle B \rangle = \langle Q \rangle$

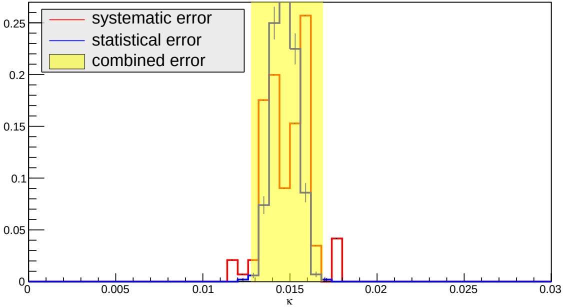
Comparison of Taylor expansion and analytic continuation



Comparison for results at $N_t = 10$, since here the precision is higher than in the continuum.

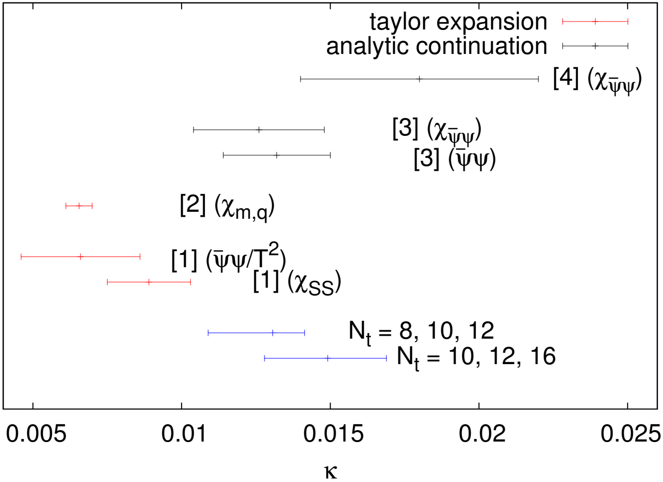
Result

Combined result from χ_{SS}, $\bar{\psi}\psi$ and $\chi_{\bar{\psi}\psi}$:



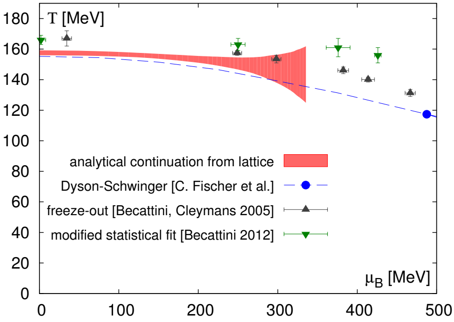
$$\kappa = 0.015 \pm 0.002^{+0.001}_{-0.001}$$

Comparison of results



T_c extrapolation

Determining $T_c(\mu_B)$ by solving the equation $\frac{T_c(\mu_B)}{T_c(0)} = C_i\left(-\frac{\mu_B^2}{T_c^2(\mu)}\right)$.



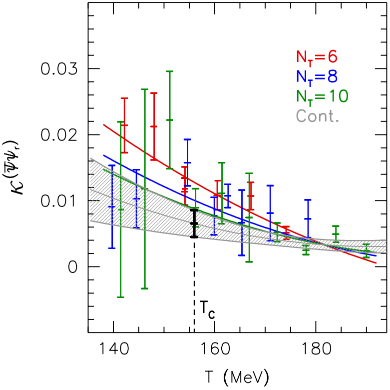
$$C_0(x) = 1 + ax$$

$$C_1(x) = 1 + ax + bx^2$$

$$C_2(x) = \frac{1 + ax}{1 + bx}$$

$$C_3(x) = \frac{1}{1 + ax + bx^2}$$

Analysis of systematic error sources



Sources for systematic error:

- There is a strong dependence on the crossover temperature
- Continuum extrapolation only from three relatively coarse lattices
- Systematics of fit functions

Analysis was done at $\mu_s = 0$ not $\langle n_S \rangle = 0$.