Thermal dilepton rates and electrical conductivity of the QGP

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H.T.Ding et. al., Phys.Rev.D83 (2011) 034504, arXiv:1312.5609

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July 17, 2015

33rd International Symposium on Lattice Field Theory
Kobe International Conference Center, Kobe, Japan

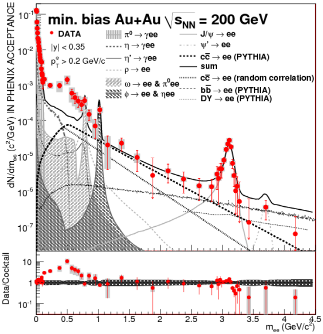
Outline

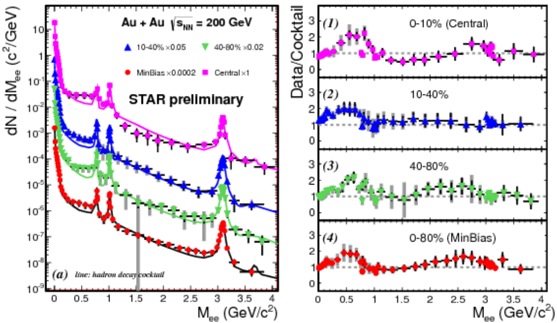
1. Introduction: spectral functions on the lattice
2. Continuum extrapolated Lattice data
3. Obtaining spectral functions for zero momenta
4. Comparing finite momentum correlators

Introduction

Experiments

- Dilepton rates in *pp* collisions well described by hadron cocktail model
- Enhancement in low energy region of *AuAu* collisions
  [PHENIX PRC81, 034911 (2010)]
  $\Rightarrow$ thermal modifications?





The spectral function (SPF)

QGP probes

- Dilepton and photon rates linked to the vector SPF:

$$\frac{\mathrm{d}W}{\mathrm{d}\omega\mathrm{d}^3p} \sim \frac{\alpha_{em}\rho_V(\omega,\vec{p},T)}{(\omega^2-\vec{p}^2)(e^{\omega/T}-1)}, \qquad \omega\frac{\mathrm{d}R_\gamma}{\mathrm{d}^3p} \sim \frac{\rho_V(\omega=|\vec{p}|,T)}{e^{\omega/T}-1}$$

- Goal is to compute vector channel SPF because
  1. Photons and dileptons are produced in the QGP
  2. Leave it almost undisturbed

  $\Rightarrow$ probes of all stages of a collision

Transport properties

- Here: electrical conductivity from spatial part: $\frac{\sigma}{T} = \frac{C_{em}}{6}\lim_{\omega\to 0}\frac{\rho_{ii}}{\omega}$

Lattice observables

Correlation function

- (Renormalized) local vector current $J_H = Z_V \bar{\psi}(x)\gamma_H \psi(x)$
- Euclidean correlator $G_H(\tau, \vec{x}) = \langle J_H(\tau, \vec{x}) J_H^\dagger(0, \vec{0}) \rangle$
- In momentum space $G_H(\tau, \vec{p}) = \sum_{\vec{x}} G_H(\tau, \vec{x}) e^{i\vec{p}\vec{x}}$

Polarizations

- Different polarizations of the vector SPF: sum over spatial ($H = ii$), temporal ($H = 00$) and full ($H = V$)
- Consider only correlators at zero momentum for now

Determining the SPF

Analytic continuation:

- Relation to SPF ρ_H:

$$G_H(\tau, \vec{p}) = \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \rho_H(\omega, \vec{p}, T) K(\omega, \tau, T)$$

with kernel $K(\omega, \tau, T) = \frac{\cosh(\omega(\tau - \frac{1}{2T}))}{\sinh(\frac{\omega}{2T})}$

- Inversion problematic: $\sim \mathcal{O}(10)$ data points $G(\tau)$, far finer resolution in ω required for $\rho(\omega)$
  $\Rightarrow$ ill posed problem

Two possible solutions

1. Bayesian methods
2. **Here:** use phenomenologically motivated ansatz and fit to correlator data

Choosing the Ansatz

Free case is known

- $\rho_{ii}(\omega, T) = 2\pi T^2 \omega \delta(\omega) + \frac{3}{2\pi}\omega^2 \tanh(\frac{\omega}{4T})$
- $\rho_{00}(\omega, T) = 2\pi T^2 \omega \delta(\omega)$
- $\rho_V(\omega, T) \equiv \rho_{ii}(\omega, t) - \rho_{00}(\omega, T) = \frac{3}{2\pi}\omega^2 \tanh(\frac{\omega}{4T})$

With interactions

- net quark number conservation: $\rho_{00} \rightarrow 2\pi \chi_q \omega \delta(\omega)$
- ρ_{ii} is modified: delta peak gets smeared out
  G.Aarts, J.M.Martinez Resco, JHEP 0204 (2002) 053
  J.Hong, D.Teaney, Phys.Rev.C82 (2010) 044908
  G.Moore, J.-M.Robert, arxiv:hep-ph/0607172
- Describe as Breit-Wigner peak + modified free part for large ω

⇒ Phenomenologically inspired ansatz

$$\rho_{ii}(\omega, T) = \chi_q c_{BW} \frac{\omega\Gamma}{\omega^2 + (\Gamma/2)^2} + \frac{3}{2\pi}(1+k)\omega^2 \tanh\left(\frac{\omega}{4T}\right)$$

with $k = \frac{\alpha_s}{\pi}$ at leading order

Lattice Data

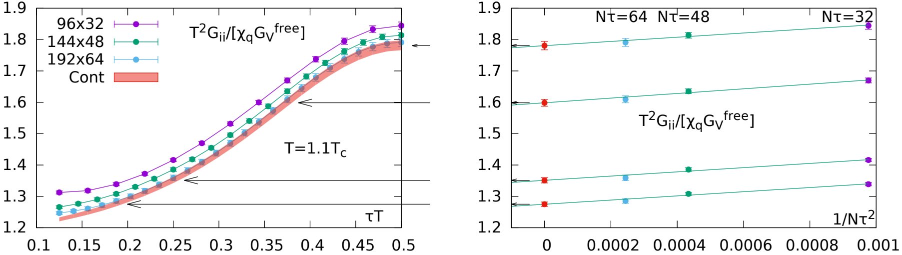
- Non-perturbatively improved Wilson-Clover Valence quark action
- No dynamical Sea Quarks
- Non-perturbative renormalization constants
- All quark masses $m_{\overline{MS}}(\mu = 2GeV) \sim \mathcal{O}(10MeV)$

$\mathbf{N_\tau}$	$\mathbf{N_\sigma}$	β	κ	**1/a[GeV]**	#
			$\mathbf{T = 1.1T_c}$		
32	96	7.192	0.13440	9.65	314
48	144	7.544	0.13383	13.21	358
64	192	7.793	0.13345	19.30	242
			$\mathbf{T = 1.2T_c}$		
28	96	7.192	0.13440	9.65	232
42	144	7.544	0.13383	13.21	417
56	192	7.793	0.13345	19.30	273
			$\mathbf{T = 1.45T_c}$		
24	128	7.192	0.13440	9.65	340
32	128	7.457	0.13390	12.86	255
48	128	7.793	0.13340	19.30	456

- Fixed aspect ratio $\frac{N_\sigma}{N_\tau} = 3$ and 3.43 for $T/T_c = 1.1$ and 1.2, resp.
- Finite volume effects under control

$\Rightarrow$ Continuum extrapolation of the vector correlator

Continuum extrapolation



- Use ratio $\frac{T^2}{\chi_q}\frac{G_{ii}(\tau T)}{G_V^{free,lat}(\tau T)}$ with $\chi_q = -G_{00}/T$
  $\Rightarrow$ Renormalization constant drops out
- Spline interpolate coarser lattices
  $\Rightarrow$ Continuum extrapolation for all N_τ points on the finest lattice
- Extrapolating for each τT in $1/N_\tau^2$

Fitting the SPF

Fit

- Ansatz $\rho_{ii}(\omega, T) = \chi_q c_{BW} \frac{\omega\Gamma}{\omega^2+(\Gamma/2)^2} + \frac{3}{2\pi}(1+k)\omega^2 \tanh\left(\frac{\omega}{4T}\right)$
- Electrical conductivity (Kubo): $\frac{\sigma}{T} = \frac{2}{3} C_{em}\chi_q \frac{c_{BW}}{\Gamma}$
- Fit ansatz to continuum extrapolated ratio $\frac{T^2}{\chi_q} \frac{G_{ii}(\tau T)}{G_V^{free}(\tau T)}$

Supply information

- From an individual analysis: second thermal moment of $G_{ii} \sim$ curvature at midpoint

  $\Rightarrow$ use as a constraint in the fit
- Correlated fit: continuum extrapolated Covariance matrix of the data (from bootstraping the extrapolation)

  $\longrightarrow \chi^2 \sim \sum_{jk}(G_j - f(t_j))C_{jk}^{-1}(G_k - f(t_k))$

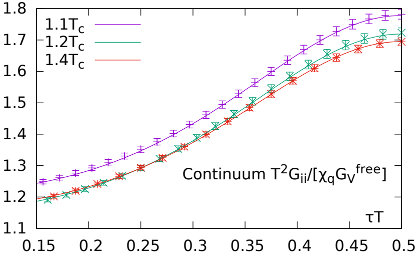
Results for the Breit-Wigner ansatz

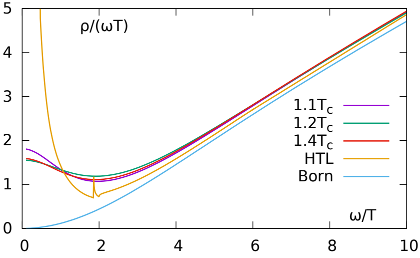
Left: Fit result

- G_{ii} almost degenerate for all T, difference in ratios due to χ_q
- Fit works well, $\chi^2/\text{dof} \sim 0.5 - 1.2$ for all T

Right: SPF result

- well developed transport peak
- HTL (LO thermal result, [E.Braaten, R.D.Pisarski, NP B337 (1990) 569]): small ω power law different

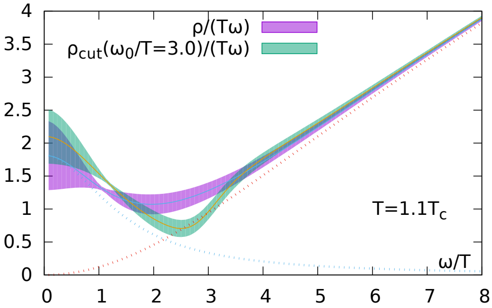




Systematics

Frequency cutoff

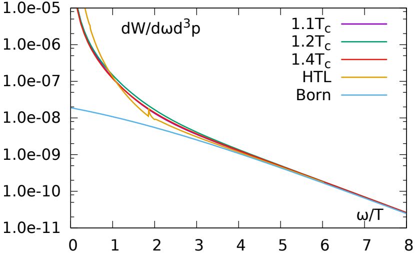
- Smeared step function $\Theta(\omega_0, \Delta_0) = \left(1 + e^{(\omega_0^2 - \omega^2)/(\omega \Delta_\omega)}\right)^{-1}$
- Change ansatz $\rho_{ii} \longrightarrow \rho_{BW} + (1 + k)\rho_V^{\text{free}} \Theta(\omega_0, \Delta_0)$
- Control influence of ρ_V on low ω region

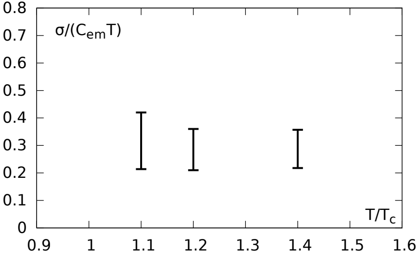


Left: SPF w/ and w/o cut

- Peak compensates cut off free part
- Peak height c_{BW} also increases
- take maximum increase of c_{BW}/Γ as systematic upper limit of conductivity σ

Electrical conductivity and Dileptonrate





Results

- Dileptonrate differs qualitatively from HTL for low and intermediate ω
- no visible temperature dependence

Similar studies: also Wilson fermions, but w/o continuum limit

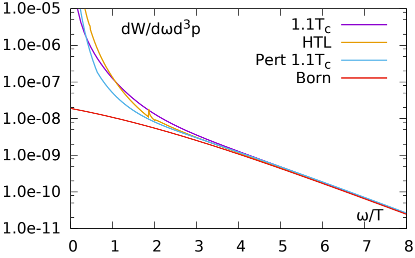
- A. Amato et al., Phys.Rev.Lett. 111 (2013) 17, 172001
  B. B. Brandt et al., JHEP 1303 (2013) 100
  G. Aarts et al., JHEP 1502 (2015) 186
- compares well within systematics

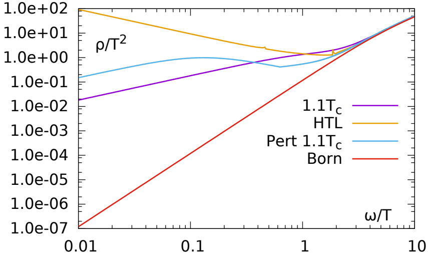
Comparison to perturbation theory

[J.Ghiglieri,O.Kaczmarek,M.Laine,FM, preliminary!]

Perturbation theory

- High order vacuum part [Burnier,Laine 2012]
- Thermal corrections at intermediate ([Altherr,Aurenche 1989]) and small ω ([Moore,Robert 2006], [Ghiglieri,Moore 2014]).
- Interpolate different perturbative regimes [Ghiglieri,Laine]
- Yields quite high intercept / conductivity





Correlators at finite momenta

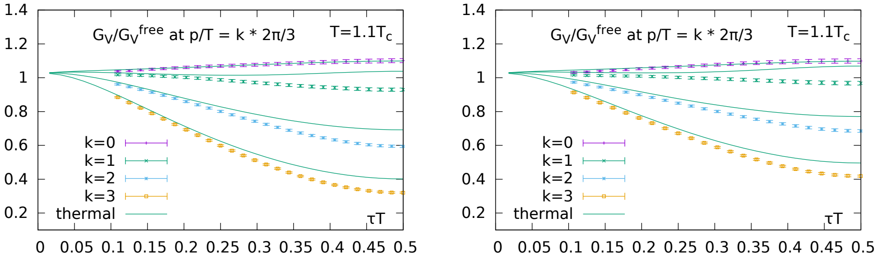
[J.Ghiglieri,O.Kaczmarek,M.Laine,FM, preliminary!]

Correlators at finite momentum

- Combined vector channel, normalized to free counterpart at zero momentum

Comparison to perturbation theory

- nonperturbative ratio vs. pert. thermal result [Ghiglieri,Laine,Moore et al.]
- agreement at small distances, more contribution at large distances
- Temperature dependence



Conclusion / Outlook

Conclusion

- Continuum extrapolation of correlator ratios at three temperatures above T_c
- No visible temperature dependence in the correlators at zero momentum
- Results for electrical conductivity and dilepton rate
- Comparison of perturbation theory with lattice for finite momenta

Outlook

- Use dynamical fermions
- Incorporate perturbative results in fit ansatz
- Finite momenta $\Rightarrow$ thermal photon rates from lattice correlators

Backup: Systematics II

Do we need a smeared out transport peak? $\longrightarrow$ mock testing our procedure

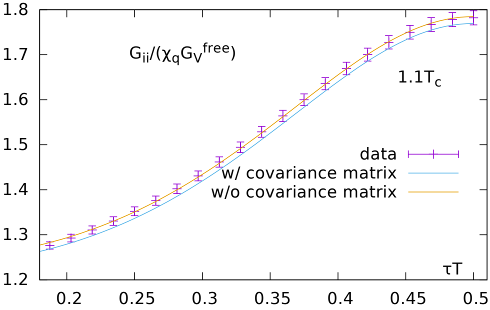
- set ansatz $\rho_{ii}(\omega, T) = A\chi_q \omega\delta(\omega) + (1+k)\rho_V^{\text{free}}(\omega, T)$: no width $\leftrightarrow$ infinite conductivity

  $\longrightarrow$ almost free SPF

  $\longrightarrow$ expect this ansatz to fail

- fails for correlated fit (but works for merely error weighted fit)

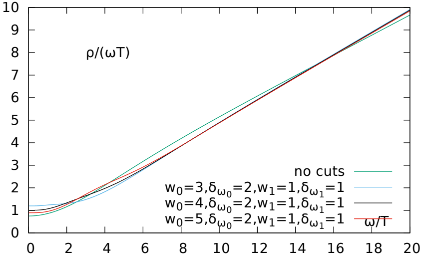
  $\Rightarrow$ need information of covariances to resolve width

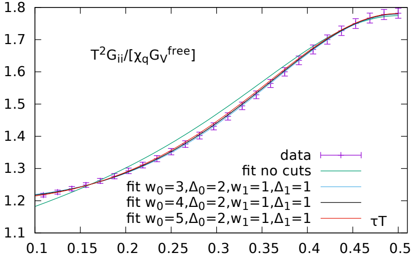


Backup: Systematics III

Strongly coupled case

- AdS/CFT predicts flat, 'featureless' low ω region
- (Roughly!) model this as
  $\rho_{ii}(\omega, T) = A\chi_q\omega(1 - \Theta(\omega_0, \Delta_0)) + (1+k)\rho_V^{\text{free}}(\omega, T)\Theta(\omega_1, \Delta_1)$
- cut with smoothed Θ functions
- compatible with our data for a range of cuts





Backup: Conclusion

Conclusion

- Fully correlated fit resolves resolves difference to free case
- Transport peak vs. 'featureless constant': cannot differentiate between two popular scenarios