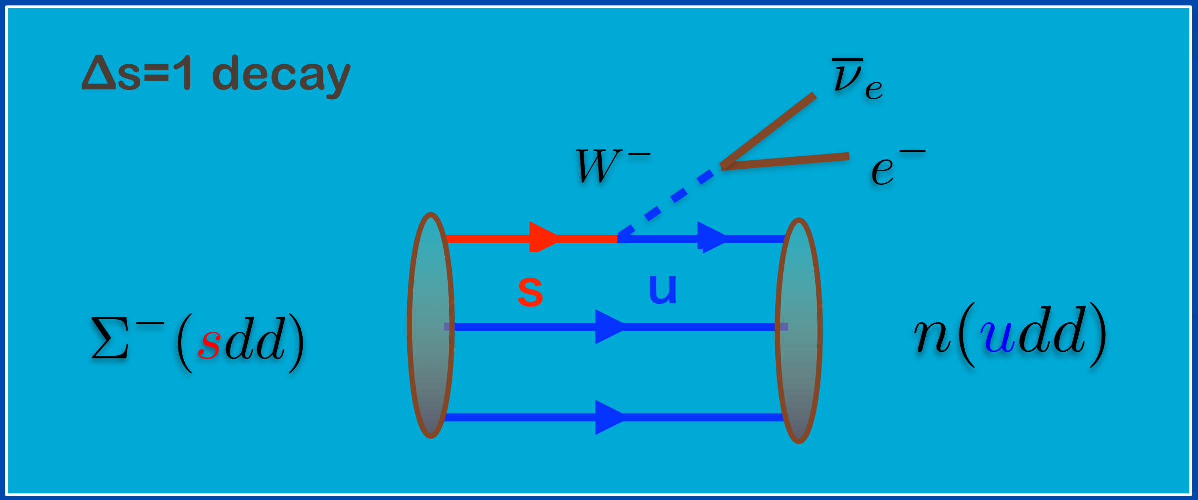
SU(3)-breaking effects and induced second-class form factors in hyperon beta decays from 2+1 flavor lattice QCD

Shoichi Sasaki (Tohoku Univ.)

Hyperon β-decay

Semi-leptonic decays of octet baryons (p, n, Λ, Σ, Ξ)

$$B_1 \rightarrow B_2 + l + \bar{\nu}_l$$



Weak transition process from s-quark to u-quark

Hyperon β-decay

Semi-leptonic decays of octet baryons (p, n, Λ, Σ, Ξ)

$$B_1 \to B_2 + l + \bar{\nu}_l$$

✓ Simple V-A structure (weak matrix element)

✓ Described by six form factors

$$\langle B_2|V_\alpha - A_\alpha|B_1\rangle = \bar{u}_{B_2}(p')[\gamma_\alpha f_1(q^2) + \sigma_{\alpha\beta}q_\beta \frac{f_2(q^2)}{M_{B_1}+M_{B_2}} + iq_\alpha \frac{f_3(q^2)}{M_{B_1}+M_{B_2}}$$

$$+\gamma_\alpha\gamma_5 g_1(q^2) + \sigma_{\alpha\beta}q_\beta\gamma_5 \frac{g_2(q^2)}{M_{B_1}+M_{B_2}} + iq_\alpha\gamma_5 \frac{g_3(q^2)}{M_{B_1}+M_{B_2}}]u_{B_1}(p)$$

$$q_\alpha = (p_{B_1} - p_{B_2})_\alpha = (p_l + p_\nu)_\alpha$$

Hyperon β-decay

Semi-leptonic decays of octet baryons (p, n, Λ, Σ, Ξ)

$$B_1 \to B_2 + l + \bar{\nu}_l$$

✓ Simple V-A structure (weak matrix element)

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✓ four independent channels (iso-spin limit : $m_u = m_d$)

$$\Lambda \to p \quad \Sigma \to n \quad \Xi \to \Lambda \quad \Xi \to \Sigma$$

Hyperon β-decay

Semi-leptonic decays of octet baryons (p, n, Λ, Σ, Ξ)

$$B_1 \to B_2 + l + \bar{\nu}_l$$

✓ Simple V-A structure (weak matrix element)

✓ Described by six form factors

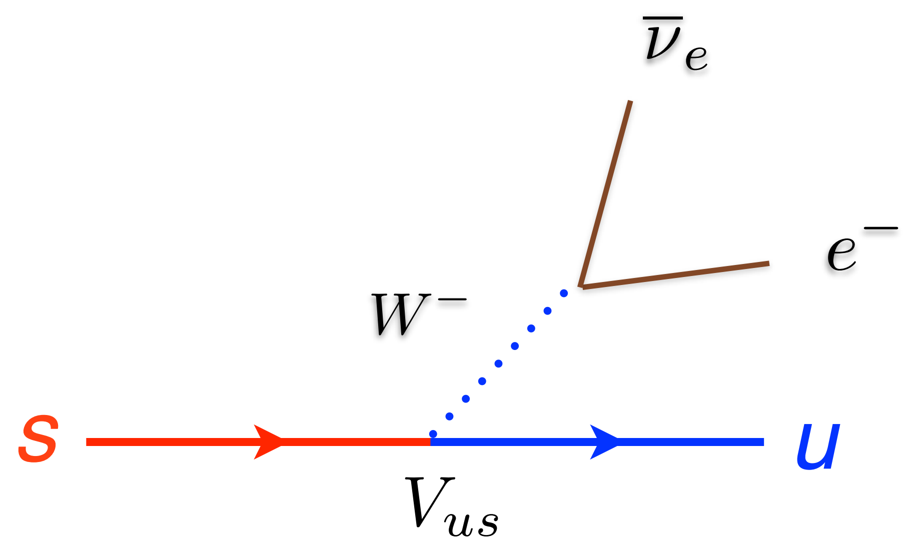
✓ four independent channels (iso-spin limit : $m_u = m_d$)

✳Unitarity of the CKM matrix ($|V_{us}|$)

✳Proton spin problem

CKM Unitarity

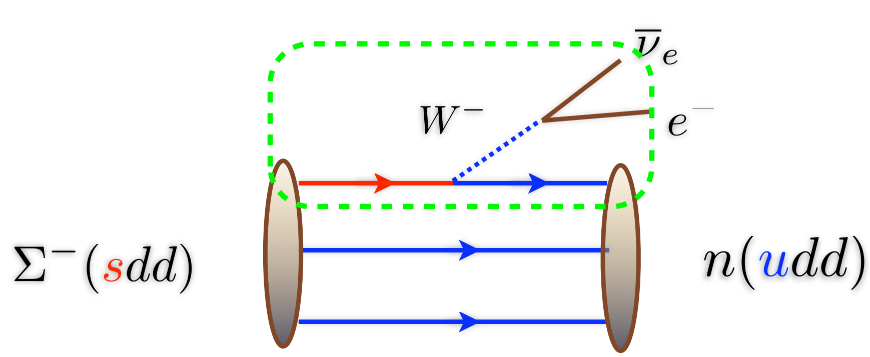
$s \longrightarrow u$ **transition**
(Weak process)



★ Hyperon beta decay provides a determination of $|V_{us}|$

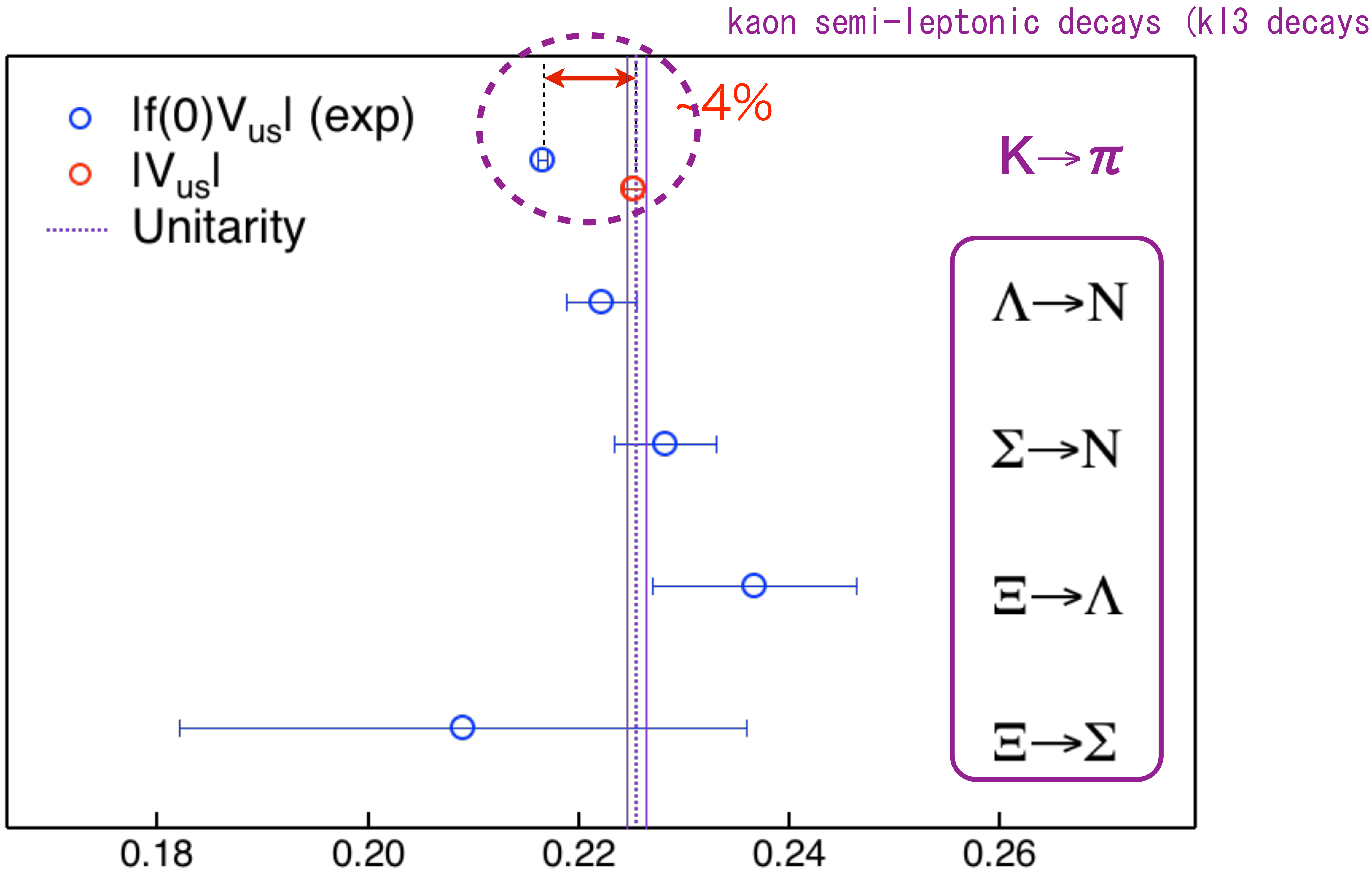
$$\text{Decay rate} \propto |V_{us}|^2 |f_1(0)|^2$$

Quantum correction by strong interaction



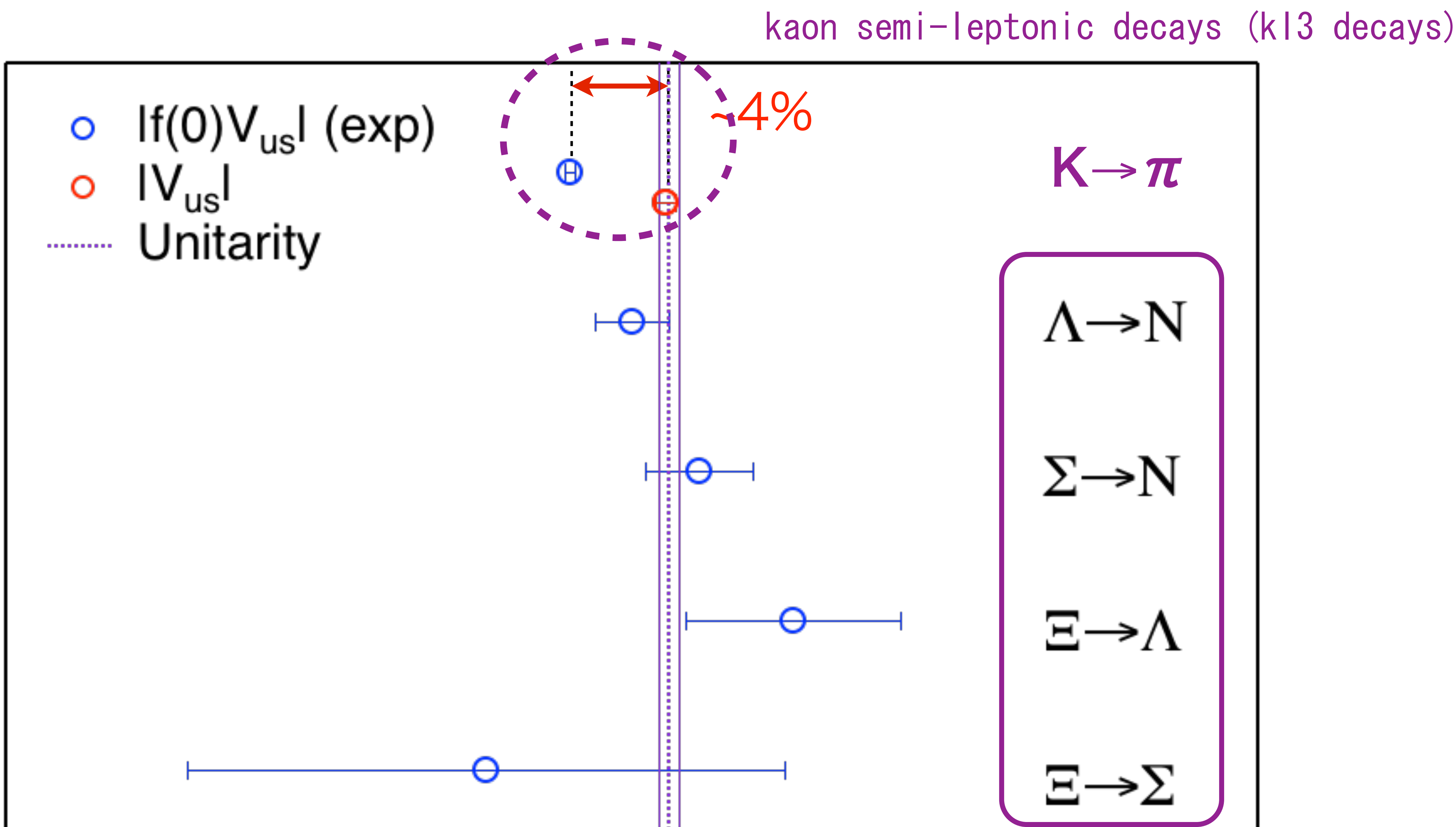
Recall:
In the exact flavor SU(3) limit, the weak vector coupling doesn't receive any quantum corrections even inside hadrons

SU(3) breaking corrections on V_{us}



SU(3) breaking effects are less known in hyperon decays

SU(3) breaking corrections on V_{us}



✳ Model independent evaluation of flavor SU(3)-breaking corrections is primarily required

✳ It can be achieved with high accuracy by lattice QCD

2+1 flavor DWF results

- 2+1 flavor RBC+UKQCD gauge configurations
  - **Domain wall fermions and Iwasaki gauge action**
  - **coarser lattice: L^3 x T x L_5 = 24^3 x 64 x 16 (β=2.13, 1/a~1.7 GeV)**
    - m_{ud}=0.005, 0.01, 0.02 (3 lightest u,d quark masses)
    - fixed strange quark masses at m_s=0.04
    - ✓ partly published in PRD86 (12) 114502

m_π [MeV]	# of meas.	src-sink sep.
330	240 x 4	12
420	120 x 4	12
570	80 x 4	12

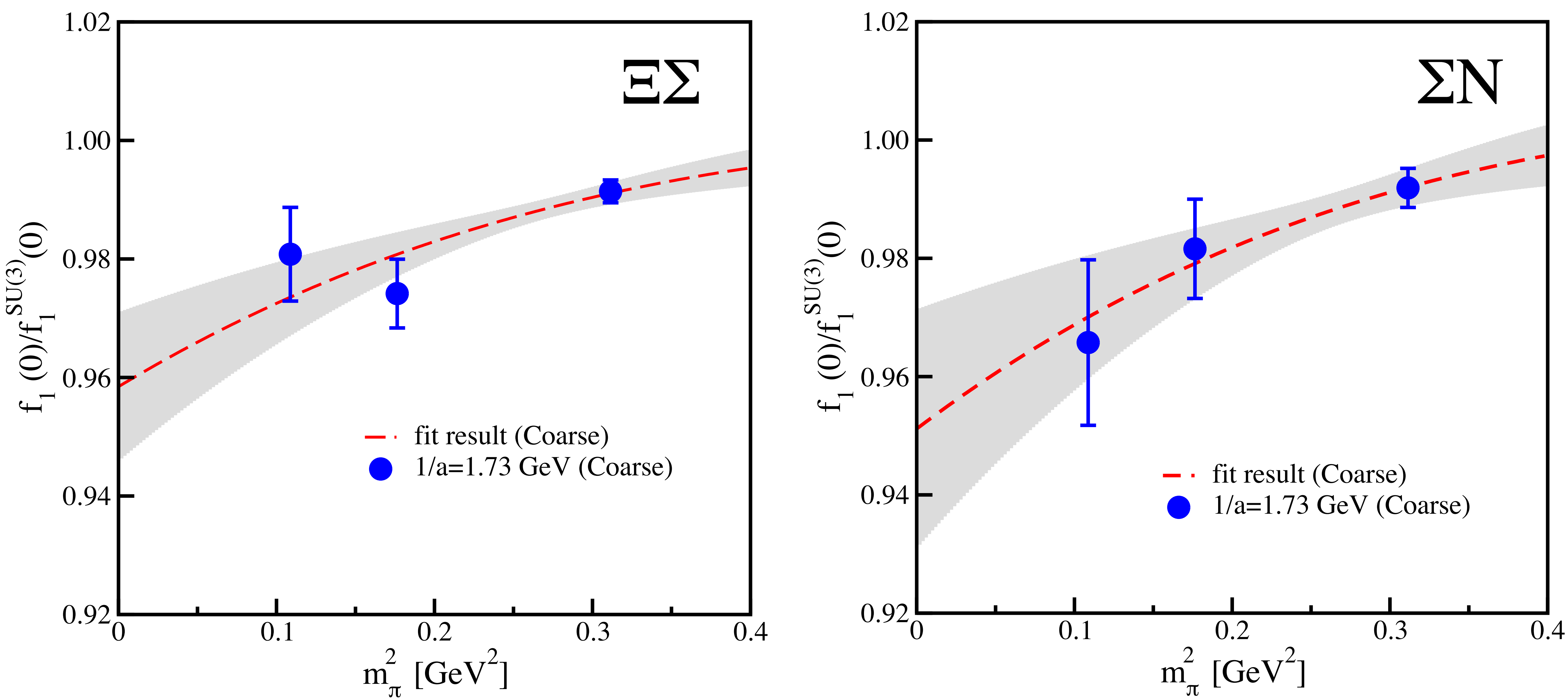
  - **finer lattice**: L^3 x T x L_5 = **32^3 x 64 x 16** (**β=2.25, 1/a~2.3 GeV**)
    - m_{ud}=0.004, 0.006, 0.008 (3 lightest u,d quark masses)
    - fixed strange quark masses at m_s=0.03
    - partly reported at Lattice 2013

m_π [MeV]	# of meas.	src-sink sep.
290	120 x 8	15
345	120 x 8	15
390	120 x 8	15

  - **$\Sigma \to N$ and $\Xi \to \Sigma$ decays**

Coarse lattice data (**published**)

SS, Phys. Rev. D86, (2012) 114502



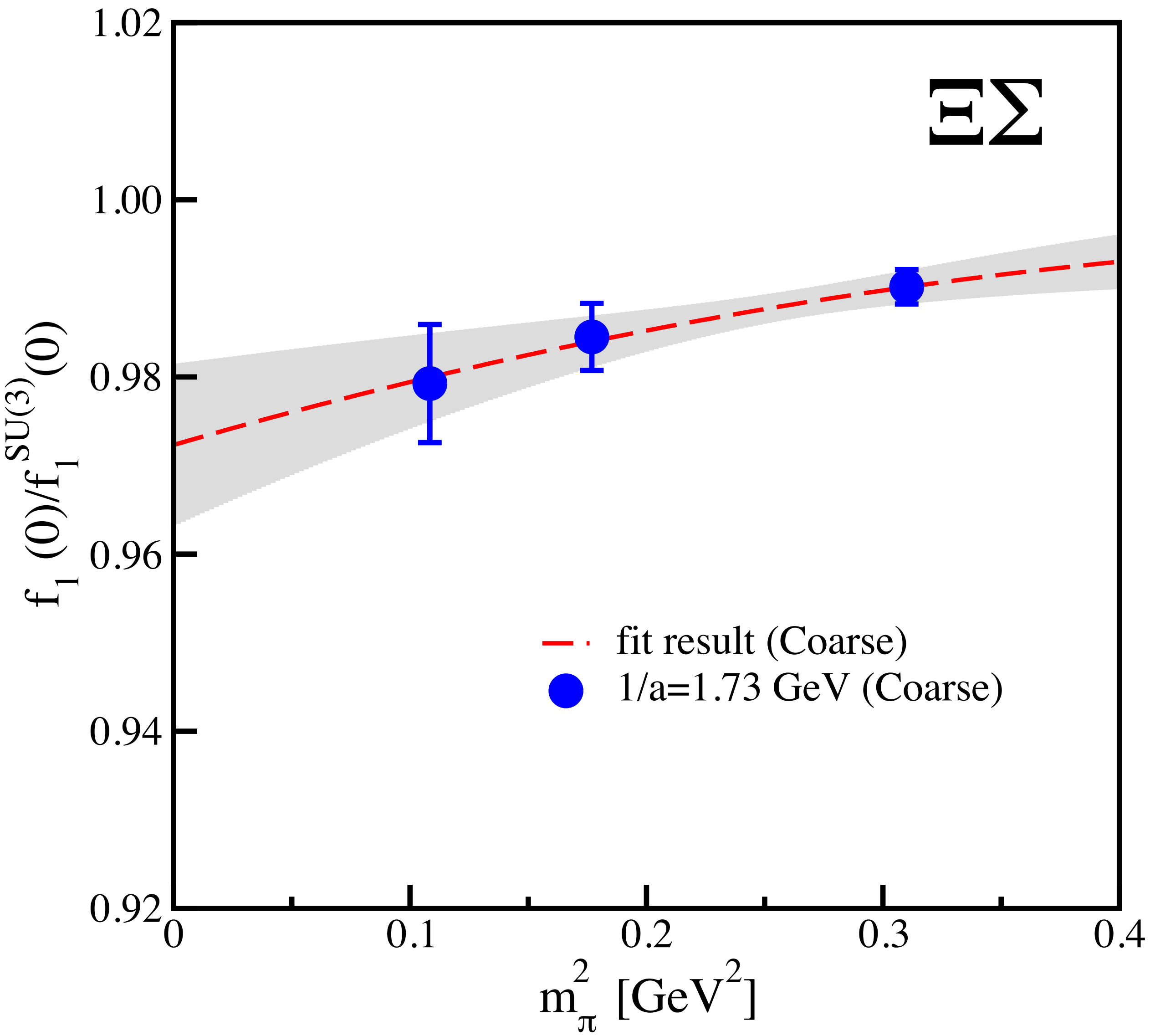
Fitting form:

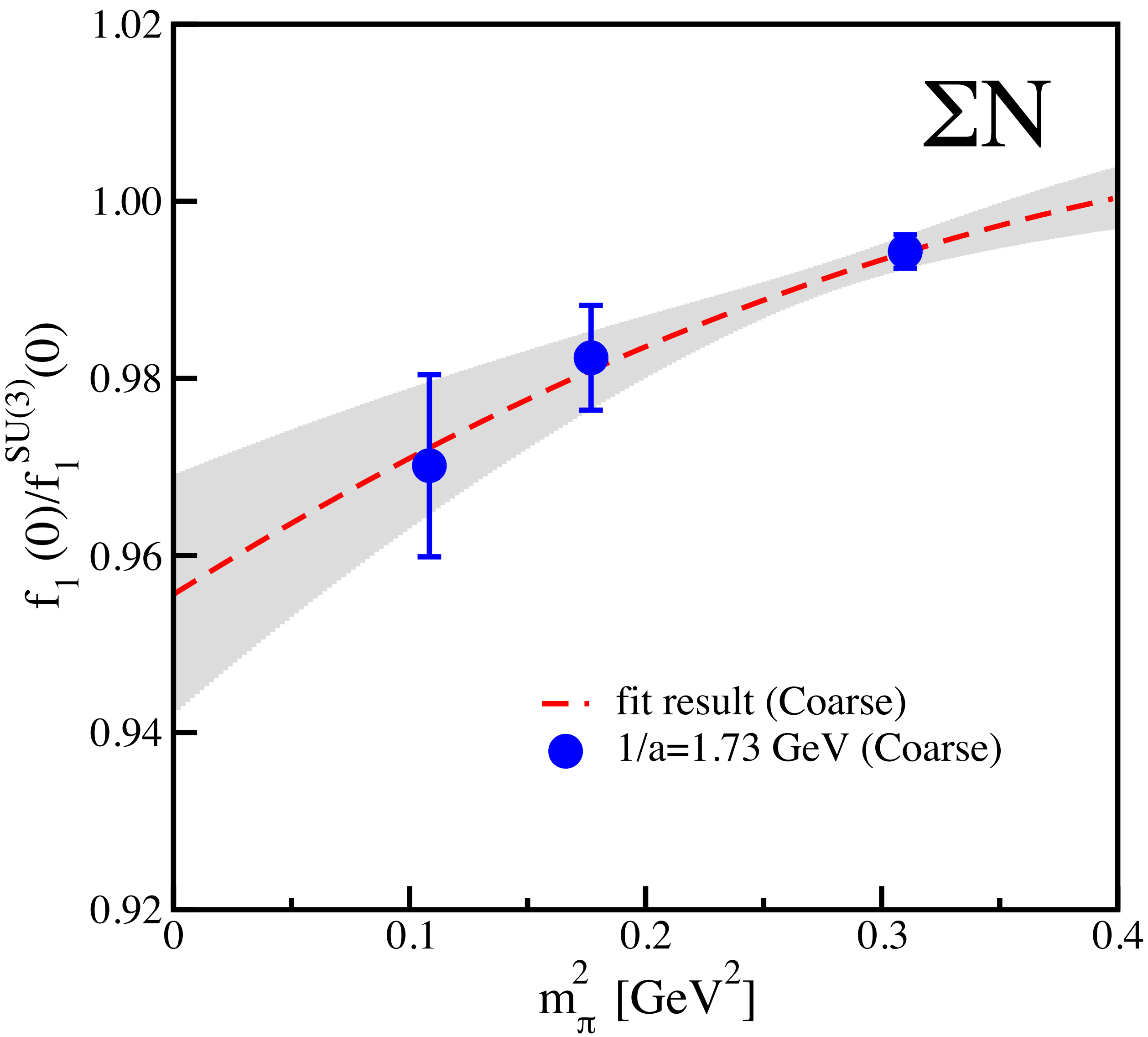
$$\tilde{f}_1(0) = C_0 + (C_1 + C_2 \cdot (M_K^2 + M_\pi^2)) \cdot (M_K^2 - M_\pi^2)^2$$

This functional form is motivated by the Ademollo-Gatto Theorem.

Coarse lattice data (updated)

increase # of src points from 2 to 4





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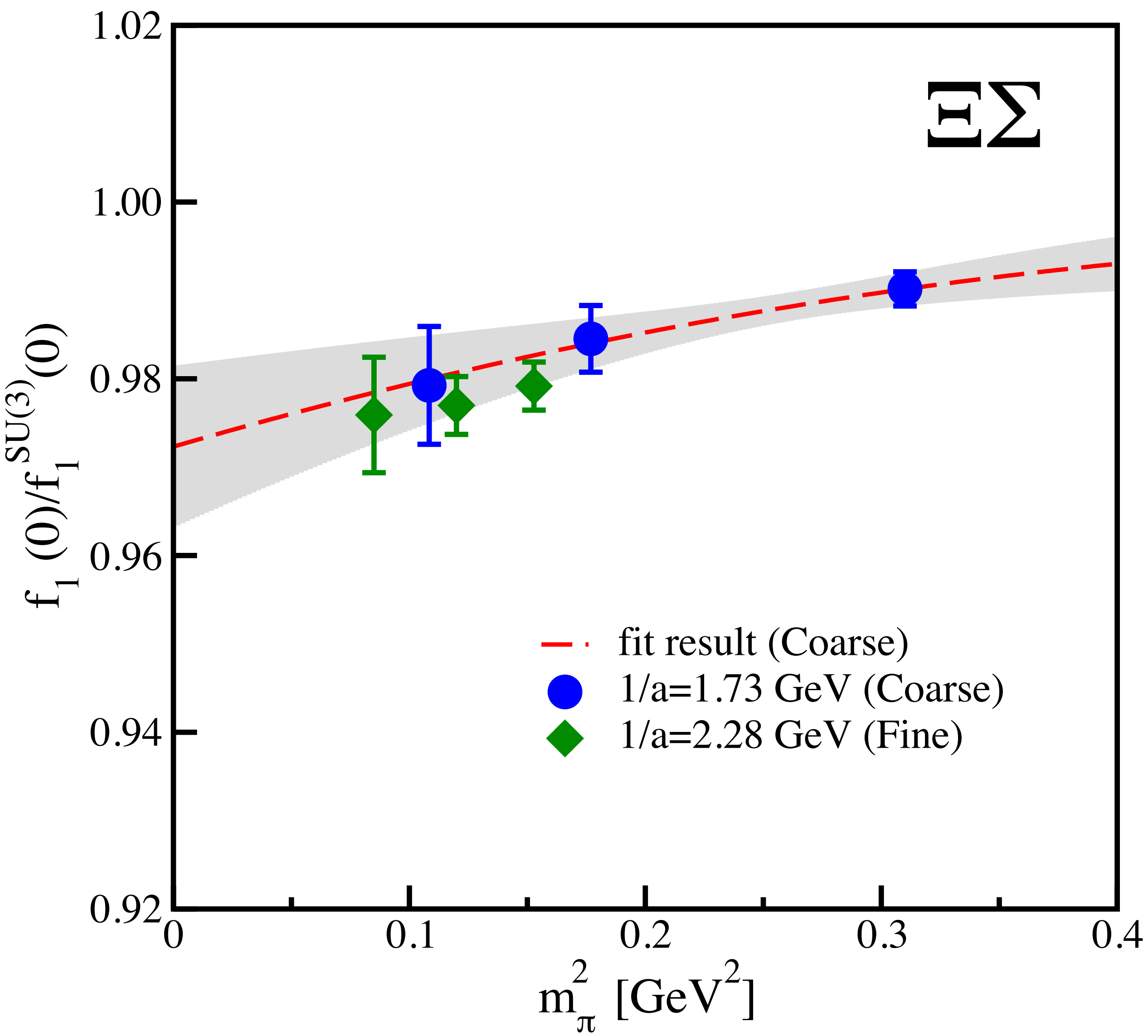
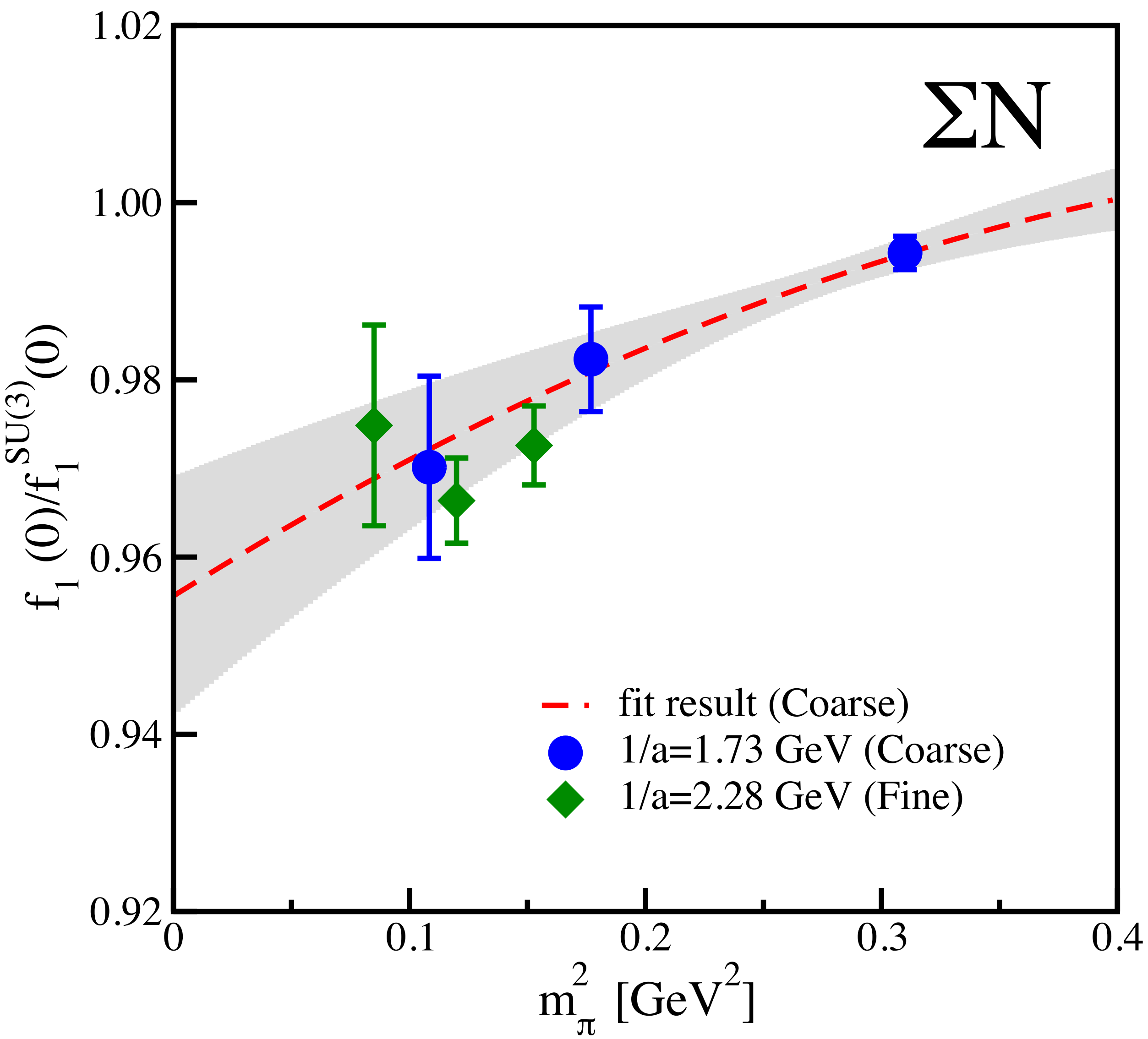
This functional form is motivated by the Ademollo-Gatto Theorem.

Comparison of $f_1(0)$ on coarse and fine lattices

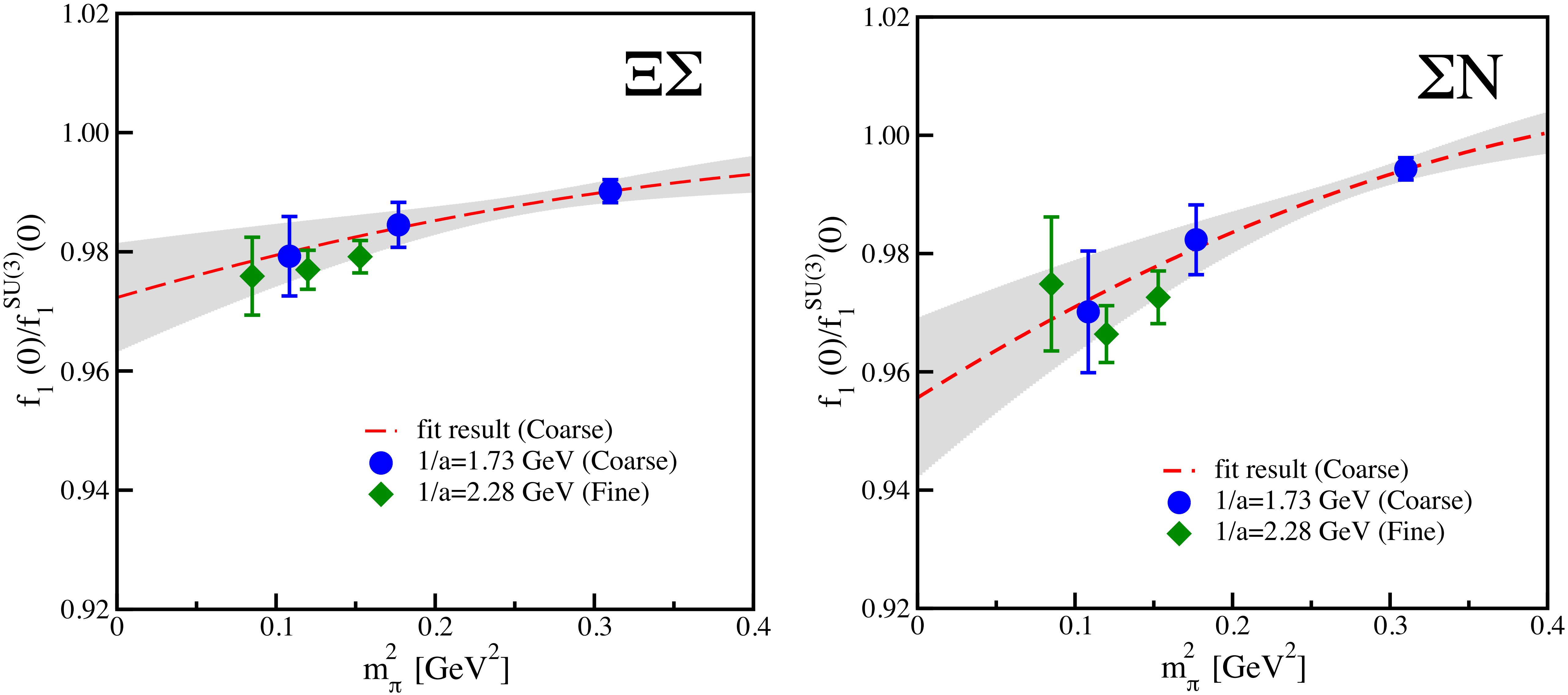
include fine lattice data





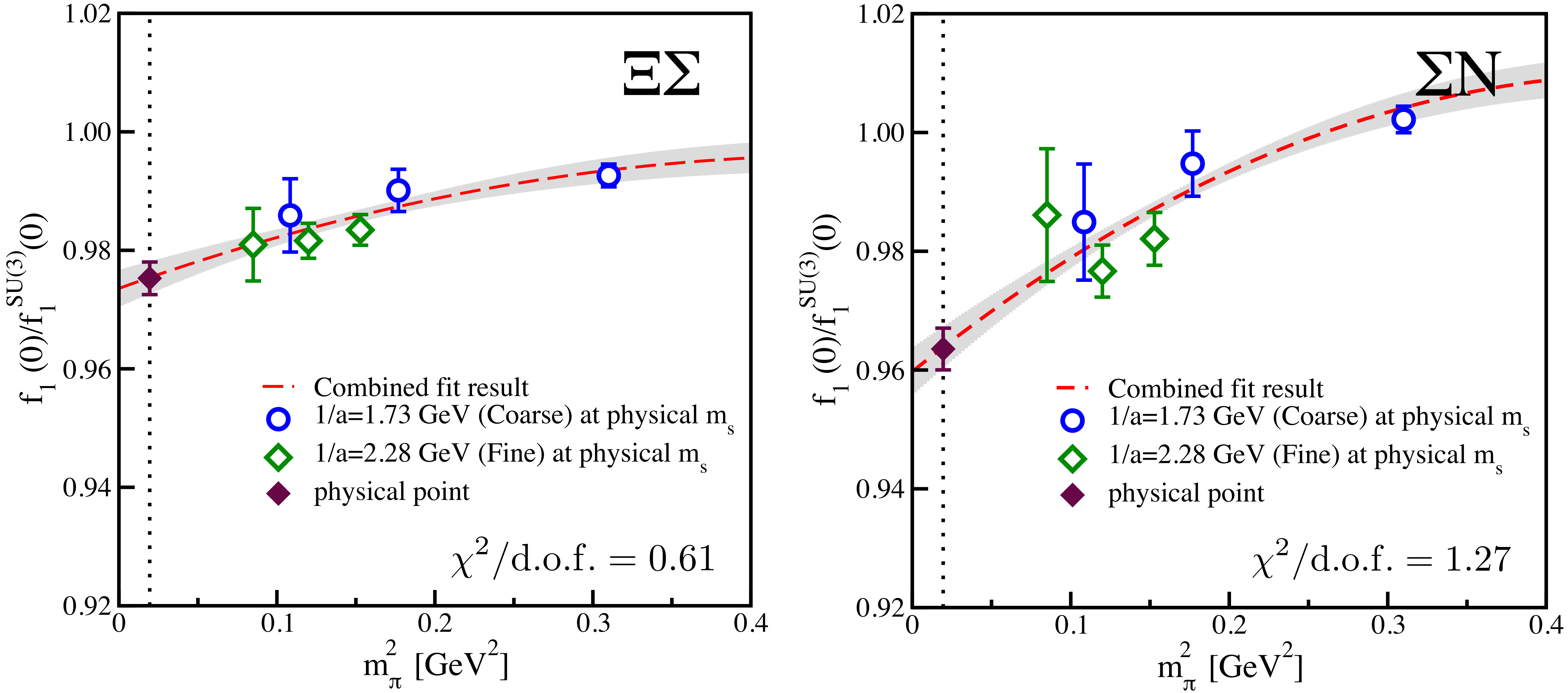
good scaling behavior (cutoff effect is small)

Comparison of $f_1(0)$ on coarse and fine lattices



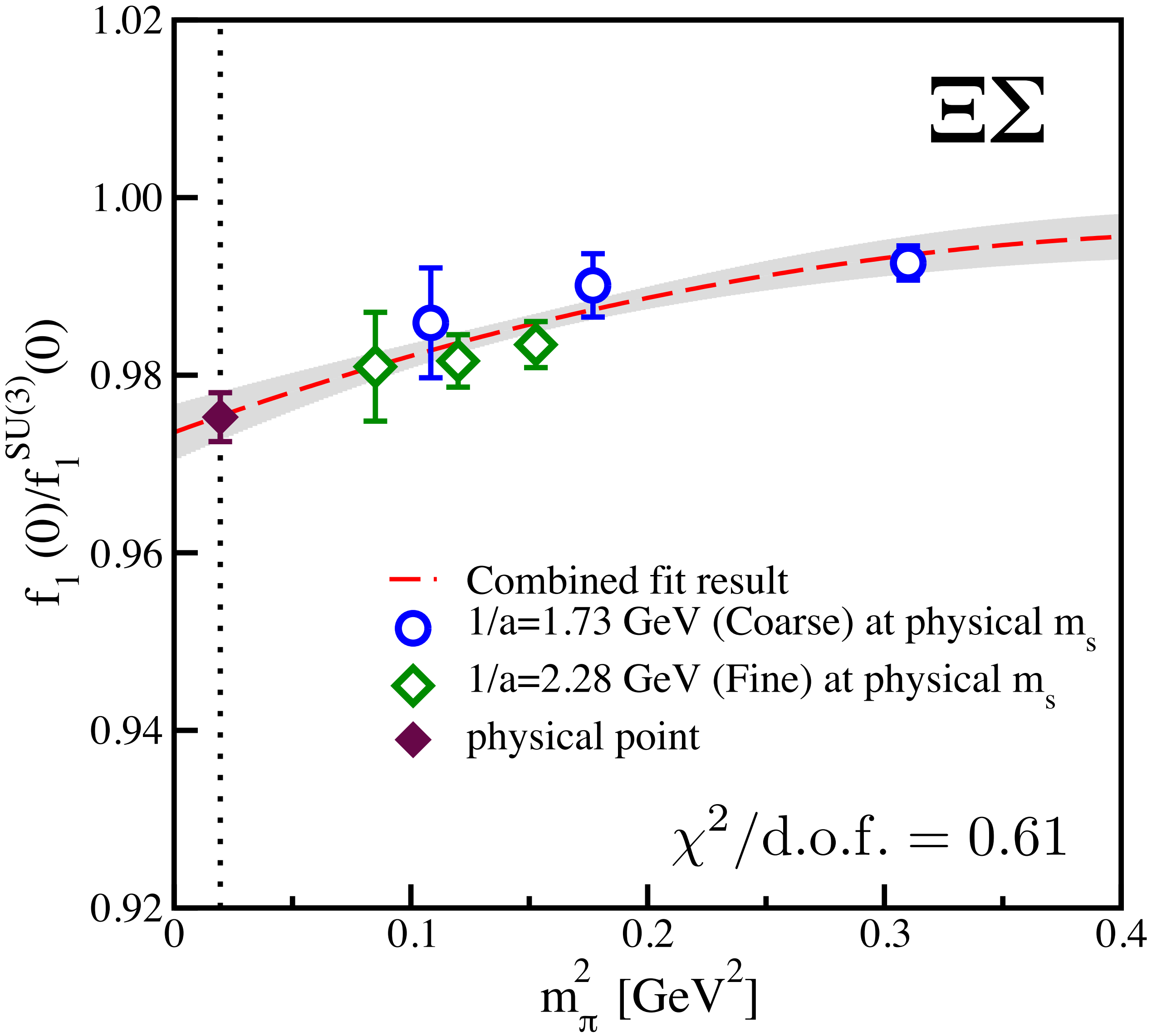
- needs a detailed study of **the strange quark mass dependence** since the simulated strange quark mass on both ensembles are not exactly at the physical point

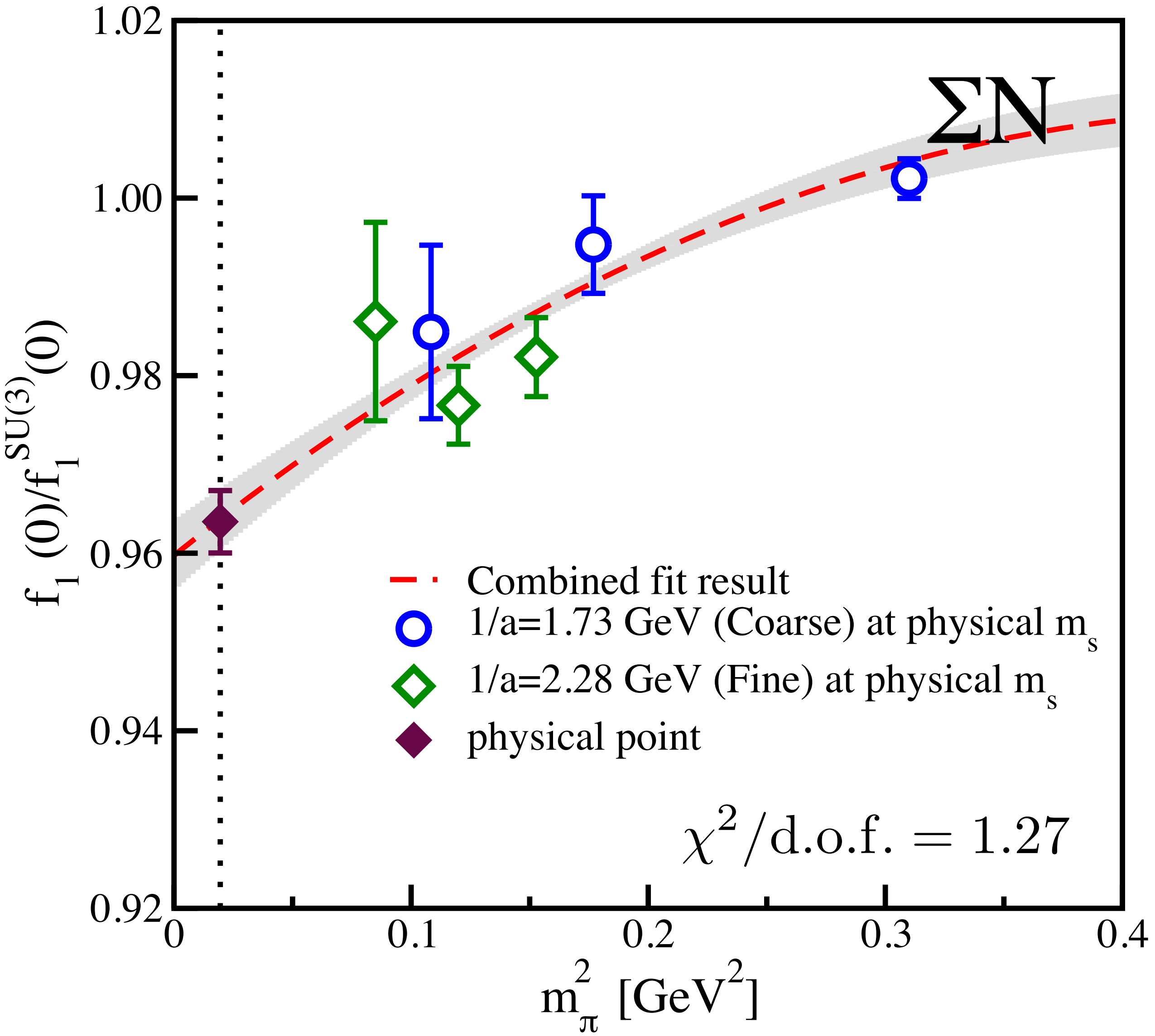
Simultaneous global fitting of both data sets



- Take into account the slight deviation of the strange mass from the physical one by using the leading order of ChPT form (**GMOR relation**) for the pion and kaon masses in **combined fits**.

Simultaneous global fitting of both data sets



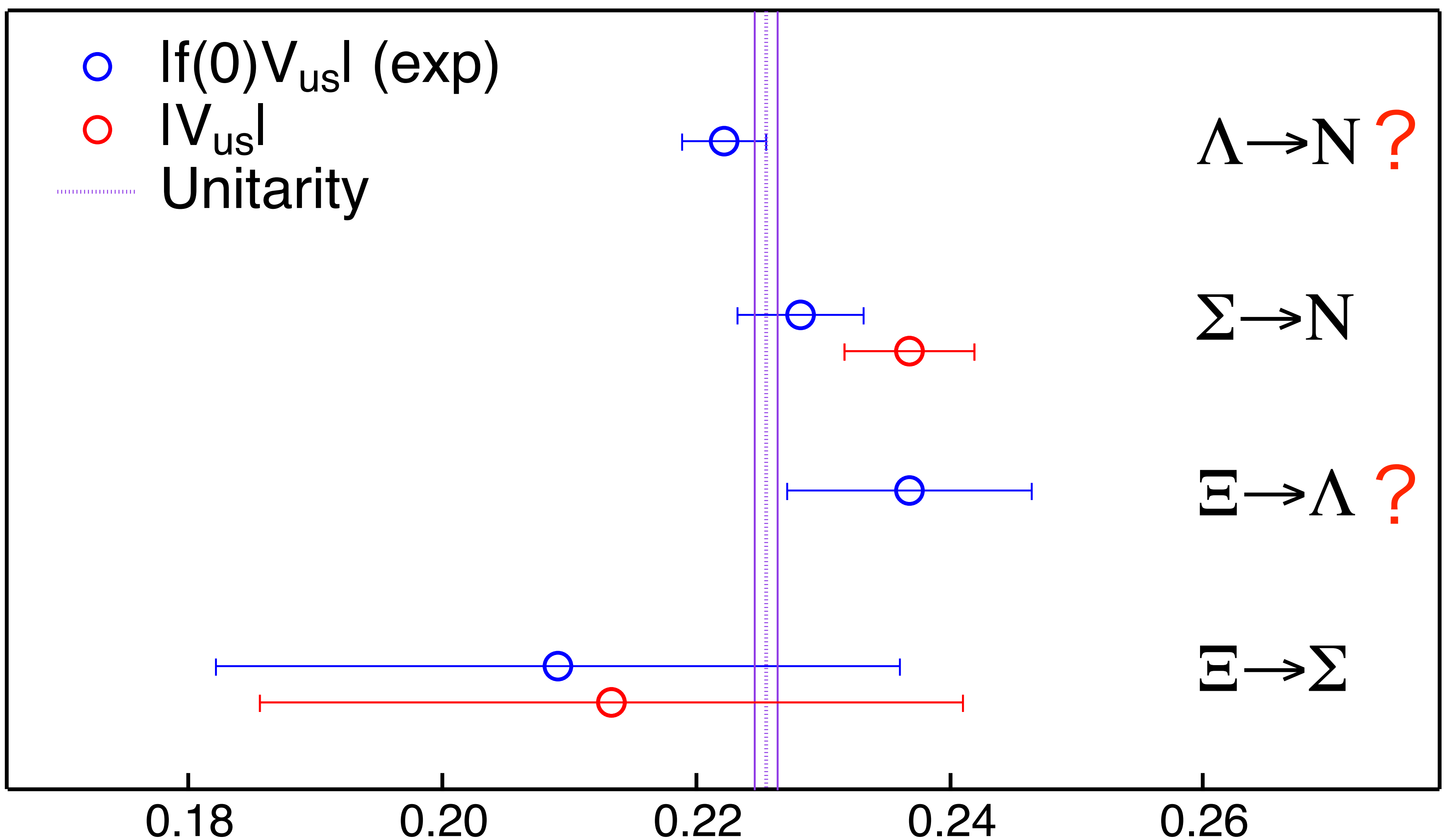


$$f_1^{\Xi\to\Sigma}(0) = +0.9753(28)_{\mathrm{stat.}}(2)_{q^2}(25)_{m_q}$$

$$f_1^{\Sigma\to N}(0) = -0.9635(35)_{\mathrm{stat.}}(38)_{q^2}(89)_{m_q}$$

less than 1% level accuracy

V_{us} is determined by combining the experimental values with **the lattice calculations of $f_1(0)$**



$$\Gamma \approx \frac{G_F^2}{60\pi^3}(M_{B_1}-M_{B_2})^5(1-3\delta)|V_{us}|^2|f_1(0)|^2\left[1+3\left|\frac{g_1(0)}{f_1(0)}\right|^2+\cdots\right]$$

$$\delta = \frac{M_{B_1}-M_{B_2}}{M_{B_1}+M_{B_2}} \sim 0.1-0.2$$

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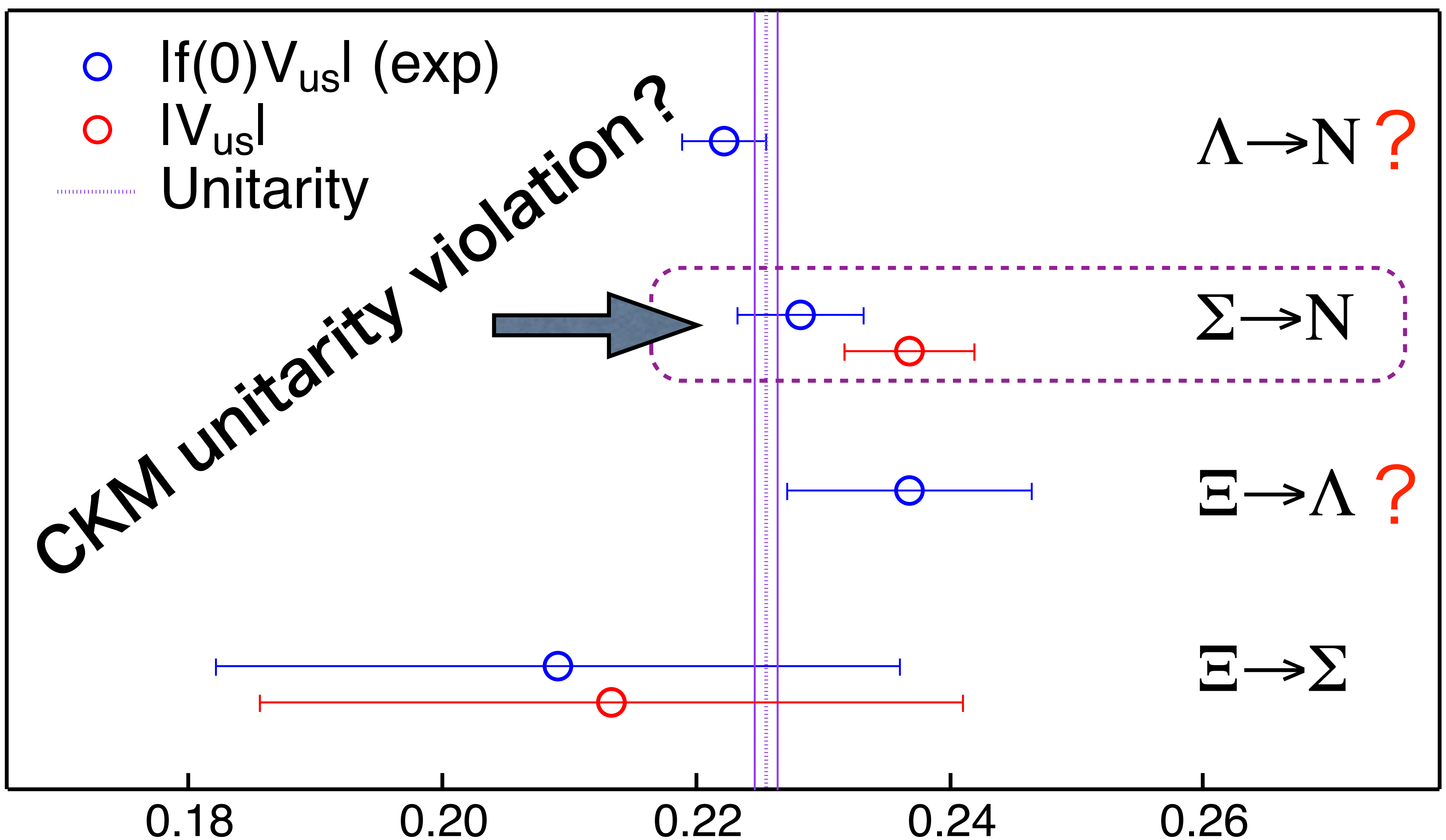

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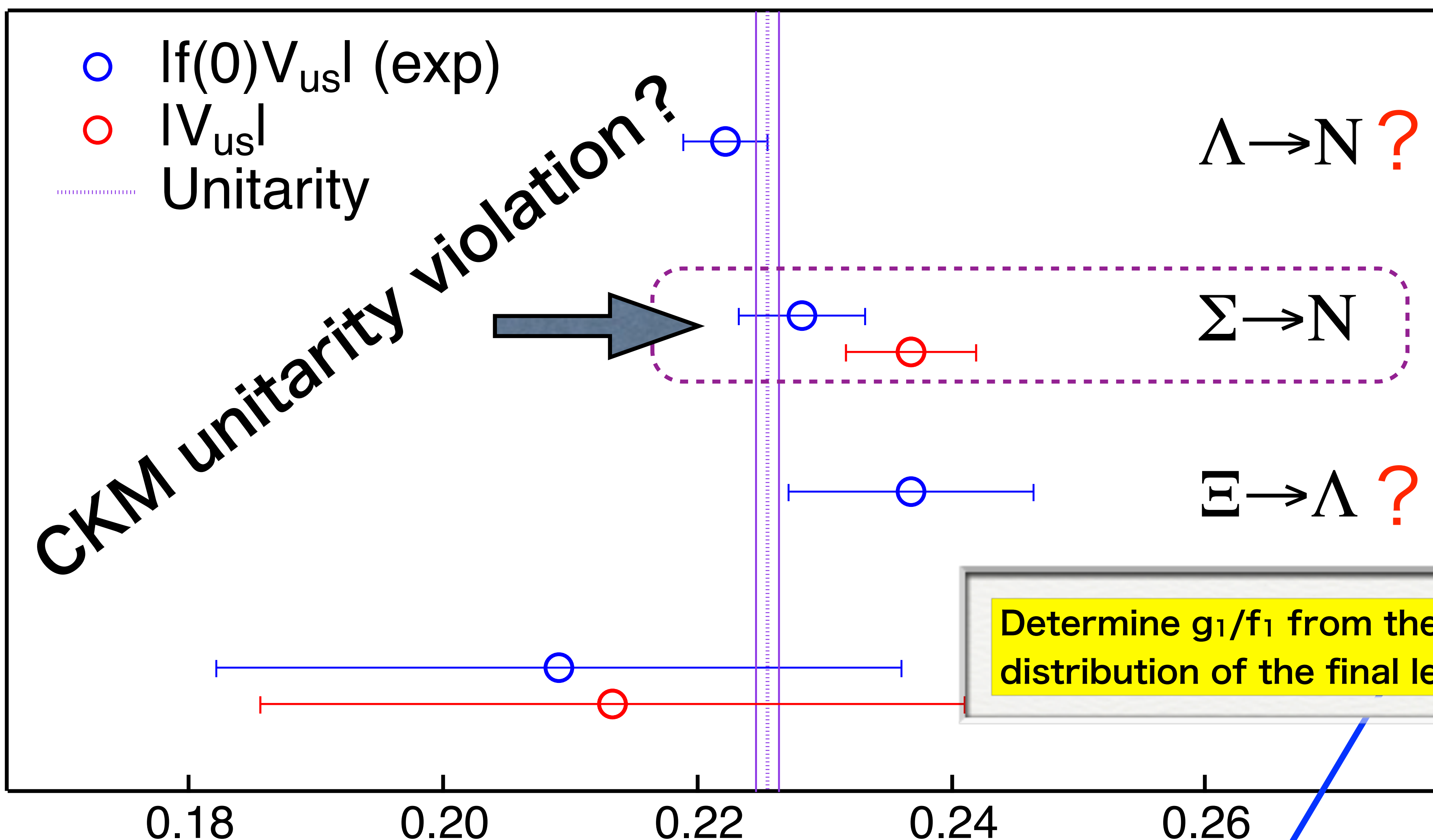

Determine g_1/f_1 from the angular distribution of the final lepton

$$\Gamma \approx \frac{G_F^2}{60\pi^3}(M_{B_1} - M_{B_2})^5(1-3\delta)|V_{us}|^2|f_1(0)|^2\left[1 + 3\left|\frac{g_1(0)}{f_1(0)}\right|^2 + \cdots\right]$$

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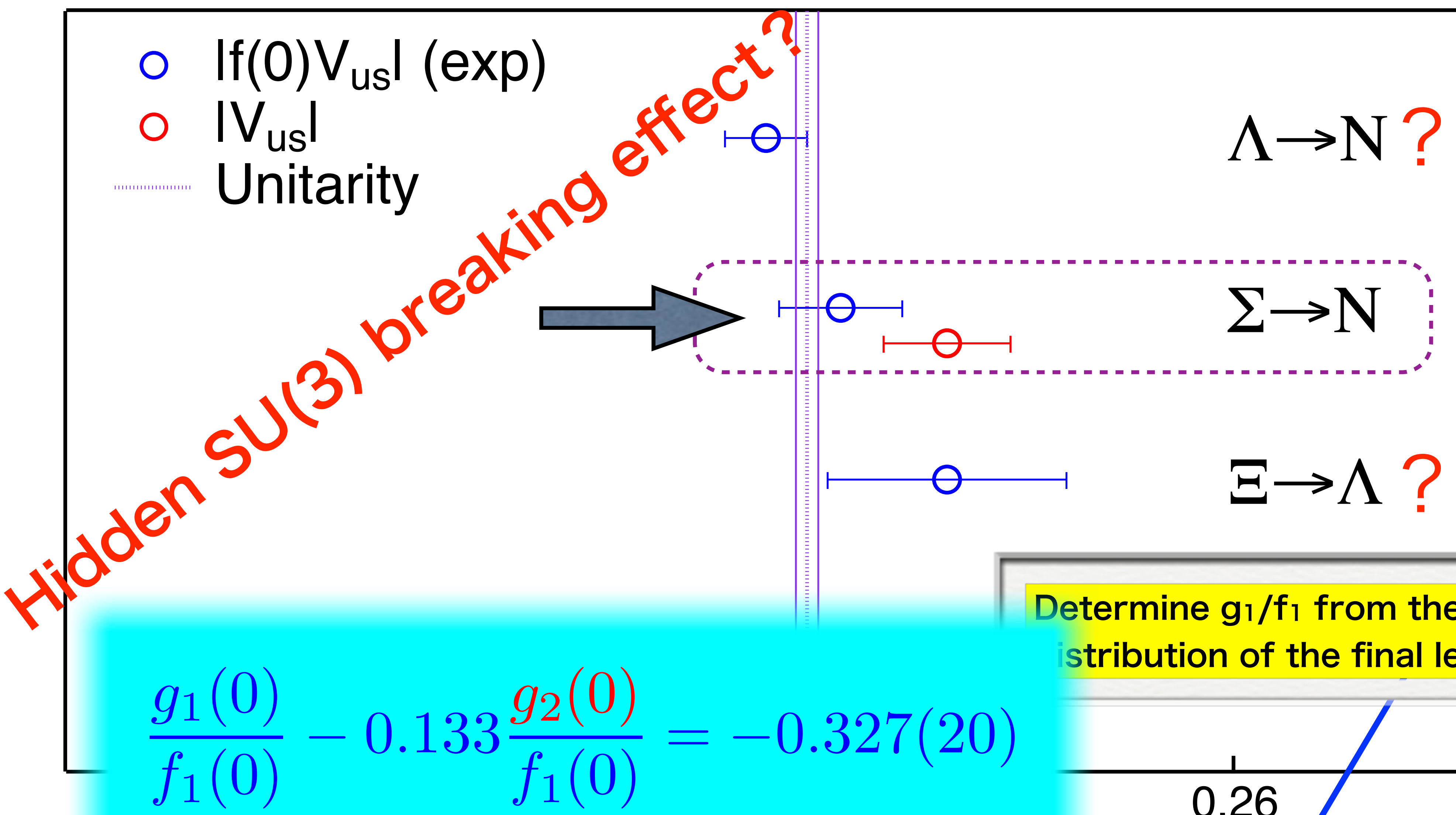

$$\frac{g_1(0)}{f_1(0)} - 0.133\frac{g_2(0)}{f_1(0)} = -0.327(20)$$

Determine g_1/f_1 from the angular distribution of the final lepton

$$\Gamma \approx \frac{G_F^2}{60\pi^3}(M_{B_1} - M_{B_2})^5(1-3\delta)|V_{us}|^2|f_1(0)|^2\left[1 + 3\left|\frac{g_1(0)}{f_1(0)}\right|^2 + \cdots\right]$$

$$\delta = \frac{M_{B_1} - M_{B_2}}{M_{B_1} + M_{B_2}} \sim 0.1 - 0.2$$

Exact SU(3) symmetry world

vector

$$\langle B_2|V_\alpha - A_\alpha|B_1\rangle = \bar{u}_{B_2}(p')[\gamma_\alpha f_1(q^2) + \sigma_{\alpha\beta}q_\beta \frac{f_2(q^2)}{M_{B_1}+M_{B_2}} + iq_\alpha \frac{f_3(q^2)}{M_{B_1}+M_{B_2}}$$

axial-vector

$$+\gamma_\alpha\gamma_5 g_1(q^2) + \sigma_{\alpha\beta}q_\beta\gamma_5 \frac{g_2(q^2)}{M_{B_1}+M_{B_2}} + iq_\alpha\gamma_5 \frac{g_3(q^2)}{M_{B_1}+M_{B_2}}]u_{B_1}(p)$$

- time reversal invariance requires all 6 form factor to be **real**
- transformation properties under the SU(3) analog of G-parity

First-class $Gf_{1,2}(q^2)G^{-1} = +f_{1,2}(q^2)$ $Gg_{1,3}(q^2)G^{-1} = -g_{1,3}(q^2)$

Second-class $Gf_3(q^2)G^{-1} = -f_3(q^2)$ $Gg_2(q^2)G^{-1} = +g_2(q^2)$

- SU(3) G-parity invariance requires

$$G = Ce^{-i\pi T_{2,5,7}}$$

$$f_3(q^2) = 0 \qquad g_2(q^2) = 0$$

e.g. neutron beta decay

induced scalar form factor — induced tensor form factor (weak electricity)

Induced 2nd-class form factors

vector

axial-vector

$$\langle B_2|V_\alpha - A_\alpha|B_1\rangle = \bar{u}_{B_2}(p')[\gamma_\alpha f_1(q^2) + \sigma_{\alpha\beta}q_\beta \frac{f_2(q^2)}{M_{B_1}+M_{B_2}} + iq_\alpha \frac{f_3(q^2)}{M_{B_1}+M_{B_2}}$$

$$+\gamma_\alpha\gamma_5 g_1(q^2) + \sigma_{\alpha\beta}q_\beta\gamma_5 \frac{g_2(q^2)}{M_{B_1}+M_{B_2}} + iq_\alpha\gamma_5 \frac{g_3(q^2)}{M_{B_1}+M_{B_2}}]u_{B_1}(p)$$

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Second-class	$Gf_3(q^2)G^{-1} = -f_3(q^2)$	$Gg_2(q^2)G^{-1} = +g_2(q^2)$

- Flavor SU(3) breaking induces

$$G = Ce^{-i\pi T_{2,5,7}}$$

$$f_3(q^2) \neq 0$$

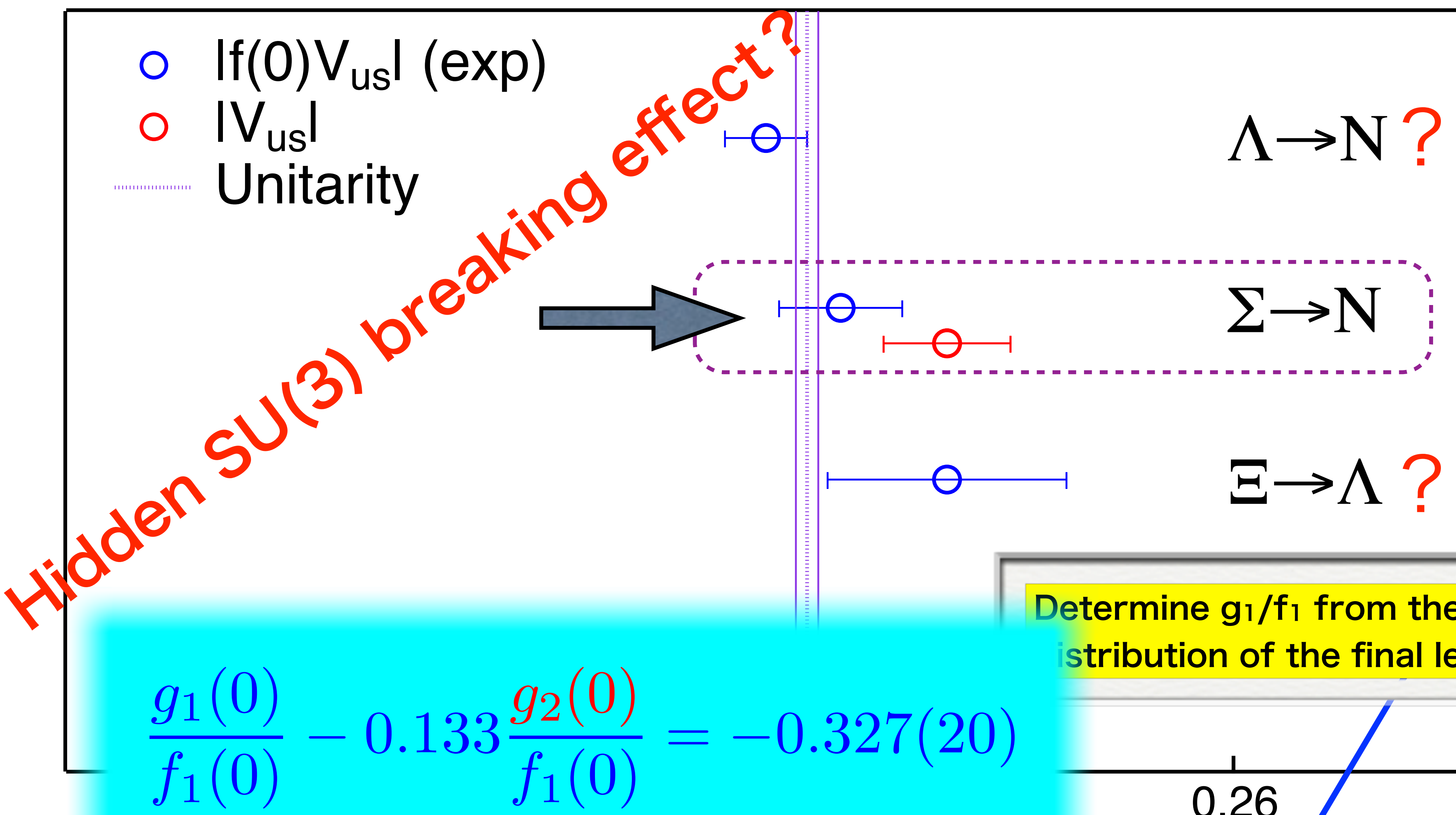
induced scalar form factor

$$g_2(q^2) \neq 0$$

induced tensor form factor (weak electricity)

e.g. neutron beta decay

V_{us} is determined by combining the experimental values with **the lattice calculations of $f_1(0)$**



Determine g_1/f_1 from the angular distribution of the final lepton

$$\frac{g_1(0)}{f_1(0)} - 0.133\frac{g_2(0)}{f_1(0)} = -0.327(20)$$

$$\Gamma \approx \frac{G_F^2}{60\pi^3}(M_{B_1}-M_{B_2})^5(1-3\delta)|V_{us}|^2|f_1(0)|^2\left[1+3\left|\frac{g_1(0)}{f_1(0)}\right|^2+\cdots\right]$$

$$\delta = \frac{M_{B_1}-M_{B_2}}{M_{B_1}+M_{B_2}} \sim 0.1-0.2$$

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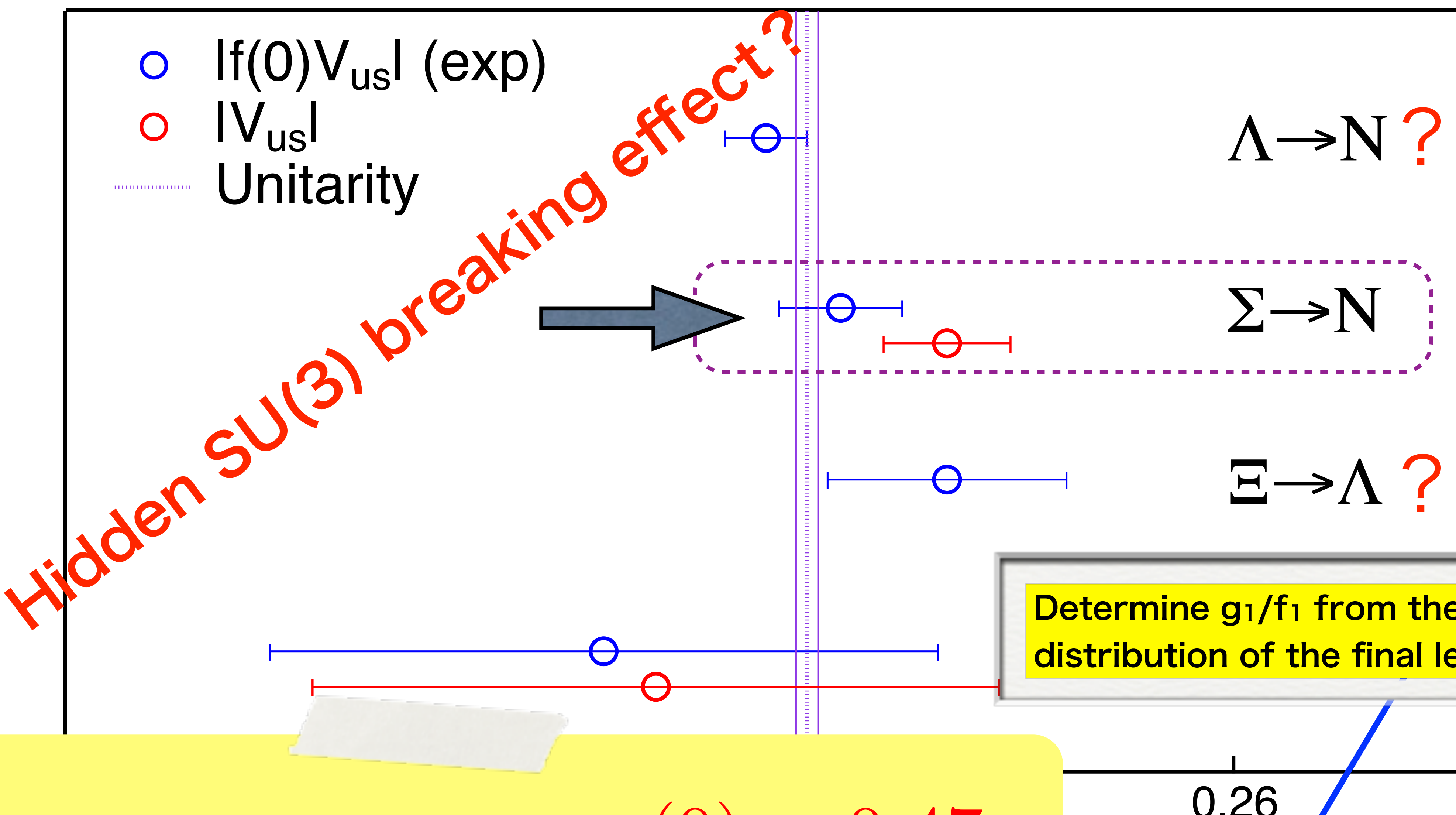


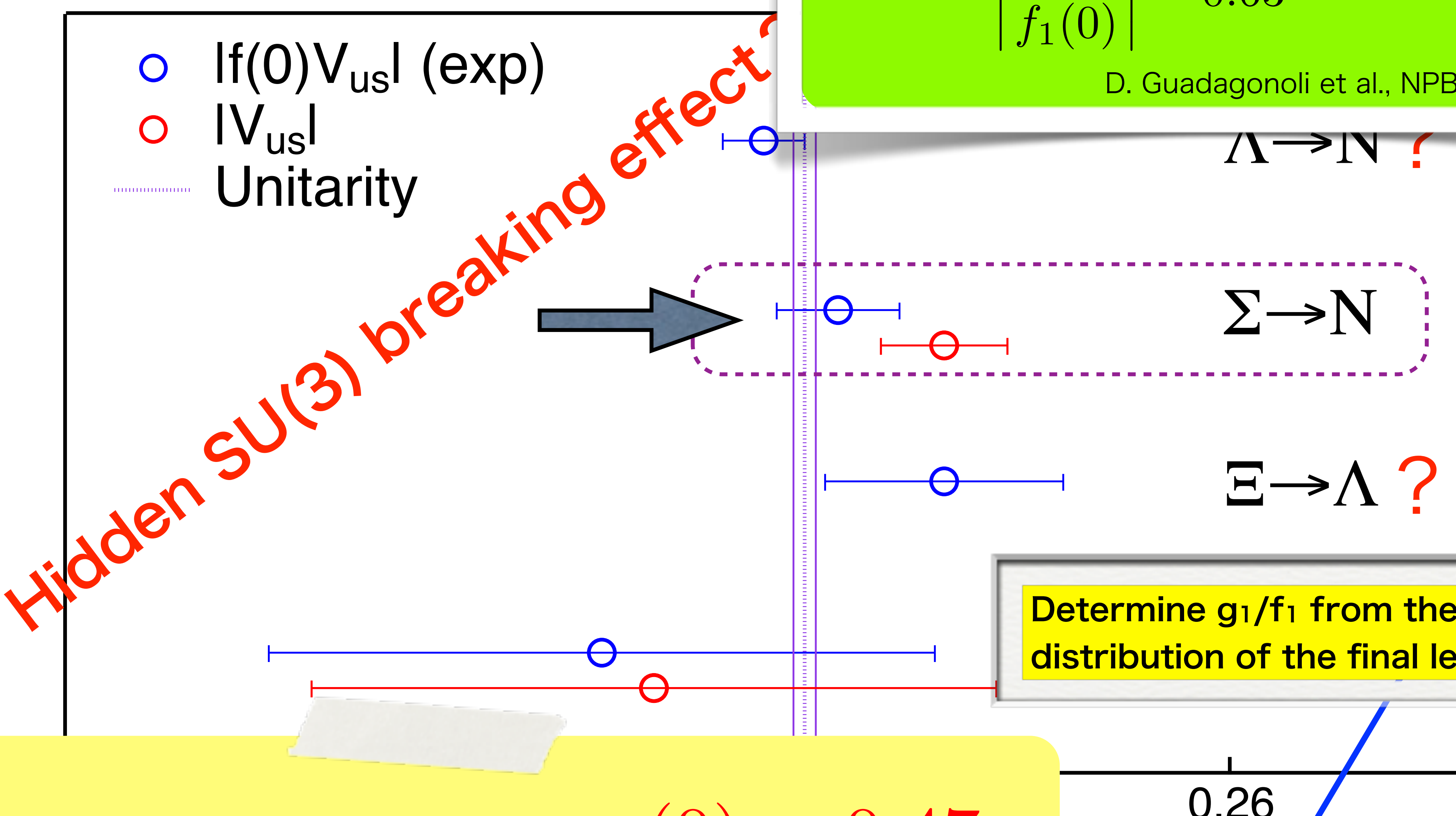
Determine g_1/f_1 from the angular distribution of the final lepton

CKM Unitarity ⇔ $g_2(0) \approx 0.47$

$$\Gamma \approx \frac{\cdots}{60\pi^3}(M_{B_1} - M_{B_2})^5(1-3\delta)|V_{us}|^2|f_1(0)|^2\left[1 + 3\left|\frac{g_1(0)}{f_1(0)}\right|^2 + \cdots\right]$$

$$\delta = \frac{M_{B_1} - M_{B_2}}{M_{B_1} + M_{B_2}} \sim 0.1 - 0.2$$

V_{us} is determined by combining the experimental values with **the lattice cal…**



Quenched simulations

$$\left|\frac{g_2(0)}{f_1(0)}\right| = 0.63$$

D. Guadagnoli et al., NPB761, 63 (07)

Determine g_1/f_1 from the angular distribution of the final lepton

CKM Unitarity ⇔ $g_2(0) \approx 0.47$

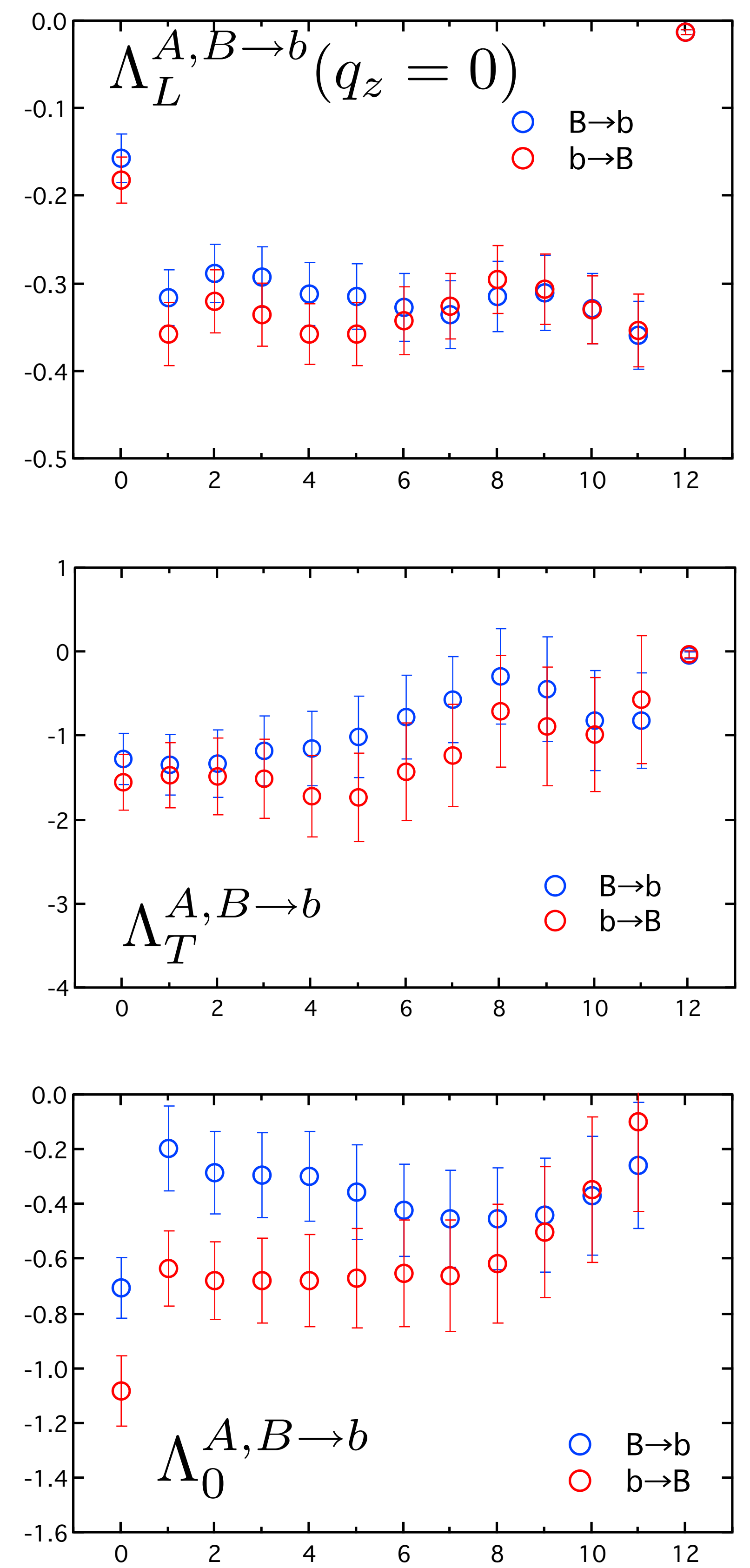
$$\Gamma \approx \frac{G_F^2}{60\pi^3}(M_{B_1} - M_{B_2})^5(1 - 3\delta)|V_{us}|^2|f_1(0)|^2\left[1 + 3\left|\frac{g_1(0)}{f_1(0)}\right|^2 + \cdots\right]$$

$$\delta = \frac{M_{B_1} - M_{B_2}}{M_{B_1} + M_{B_2}} \sim 0.1 - 0.2$$

Full QCD result of g_2

Three types of 3-pt functions

$$\mathbf{q} = (q_x, q_y, q_z) = \frac{2\pi}{L}(1,0,0)$$



$$\Lambda_L^{A,B\to b} \propto \mathrm{Tr}\left\{\mathcal{P}_z^5 \langle \mathcal{O}_b(t_{\mathrm{sink}}) A_z(t) \overline{\mathcal{O}}_B(t_{\mathrm{src}})\rangle\right\}$$

$$\mathcal{P}_z^5 = (1+\gamma_4)\gamma_5\gamma_z$$

$$\Lambda_T^{A,B\to b} \propto \mathrm{Tr}\left\{\mathcal{P}_z^5 \langle \mathcal{O}_b(t_{\mathrm{sink}}) A_{x,y}(t) \overline{\mathcal{O}}_B(t_{\mathrm{src}})\rangle\right\}$$

$$\Lambda_0^{A,B\to b} \propto \mathrm{Tr}\left\{\mathcal{P}_z^5 \langle \mathcal{O}_b(t_{\mathrm{sink}}) A_t(t) \overline{\mathcal{O}}_B(t_{\mathrm{src}})\rangle\right\}$$

SS, T.Yamazaki, PRD79 (09) 074508

Three types of 3-pt functions

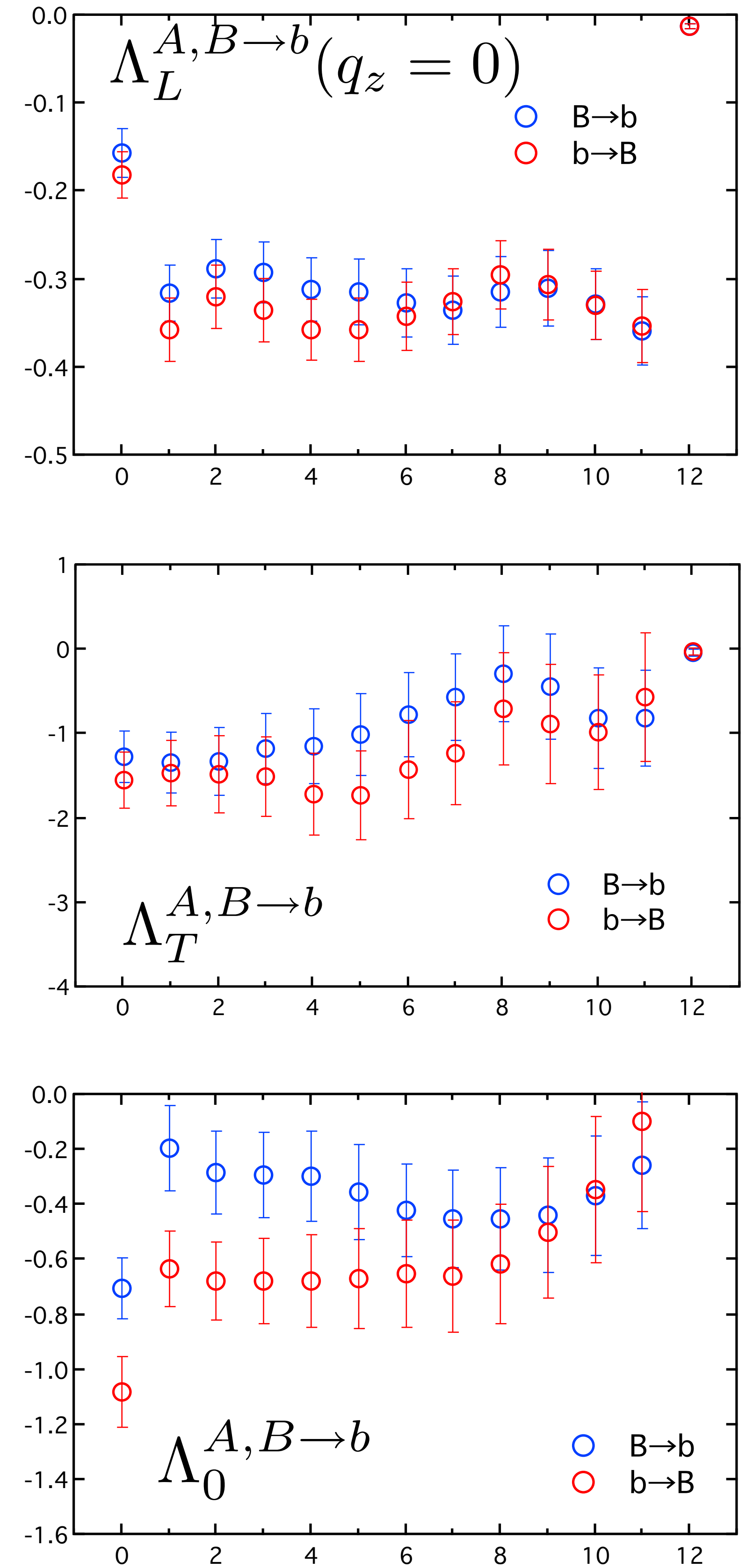
→Three types of form factors

$$\begin{pmatrix} 1 & -\frac{M_B - M_b}{M_B + M_b} & 0 \\ 0 & \frac{M_b}{M_B + M_b} & \frac{M_b}{M_B + M_b} \\ 1 & -\frac{E_B + M_B}{M_B + M_b} & -\frac{E_B - M_b}{M_B + M_b} \end{pmatrix} \begin{pmatrix} g_1^{B\to b}(q^2) \\ g_2^{B\to b}(q^2) \\ g_3^{B\to b}(q^2) \end{pmatrix} = \begin{pmatrix} \Lambda_L^{A,B\to b}(q_z = 0) \\ \Lambda_T^{A,B\to b} \\ \Lambda_0^{A,B\to b} \end{pmatrix}$$

$$g_2^{B\to b}(q^2) = \frac{M_B + M_b}{2M_b}\left[\Lambda_L^{A,B\to b}(q_z = 0) - \Lambda_0^{A,B\to b} - \frac{E_B - M_b}{M_b}\Lambda_T^{A,B\to b}\right]$$

SS, T.Yamazaki, PRD79 (09) 074508

$$\mathbf{q} = (q_x, q_y, q_z) = \frac{2\pi}{L}(1,0,0)$$



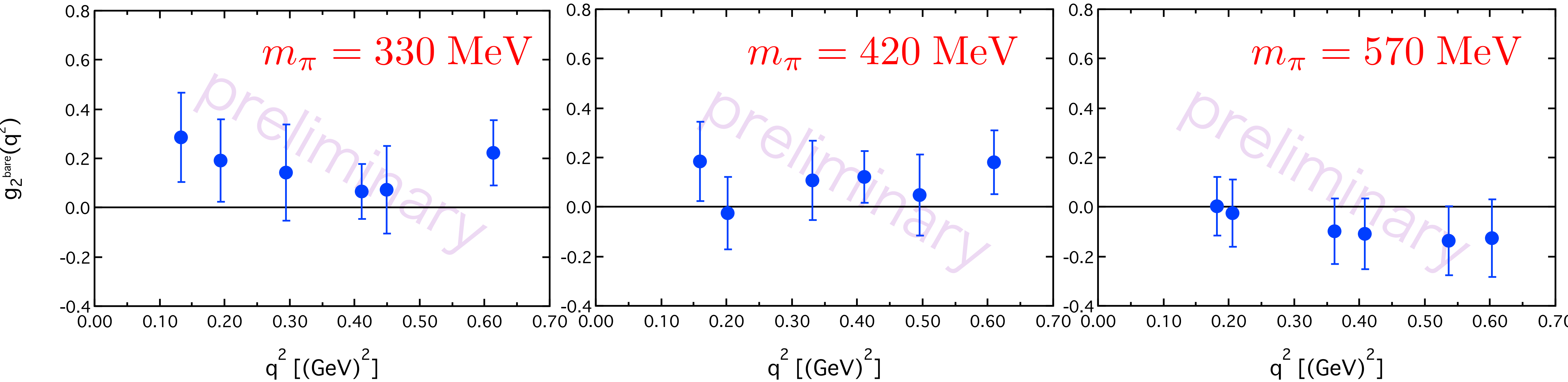
Direct measurement for g_2 form factor

2+1 flavor DWF simulations

coarser lattice: 24^3 x 64 x 16 (β=2.13, 1/a~1.7 GeV)

$$\delta = \frac{M_\Sigma - M_N}{M_\Sigma + M_N}$$

$\Sigma \rightarrow$N channel



δ=0.079(2) δ=0.054(1) δ=0.031(1)

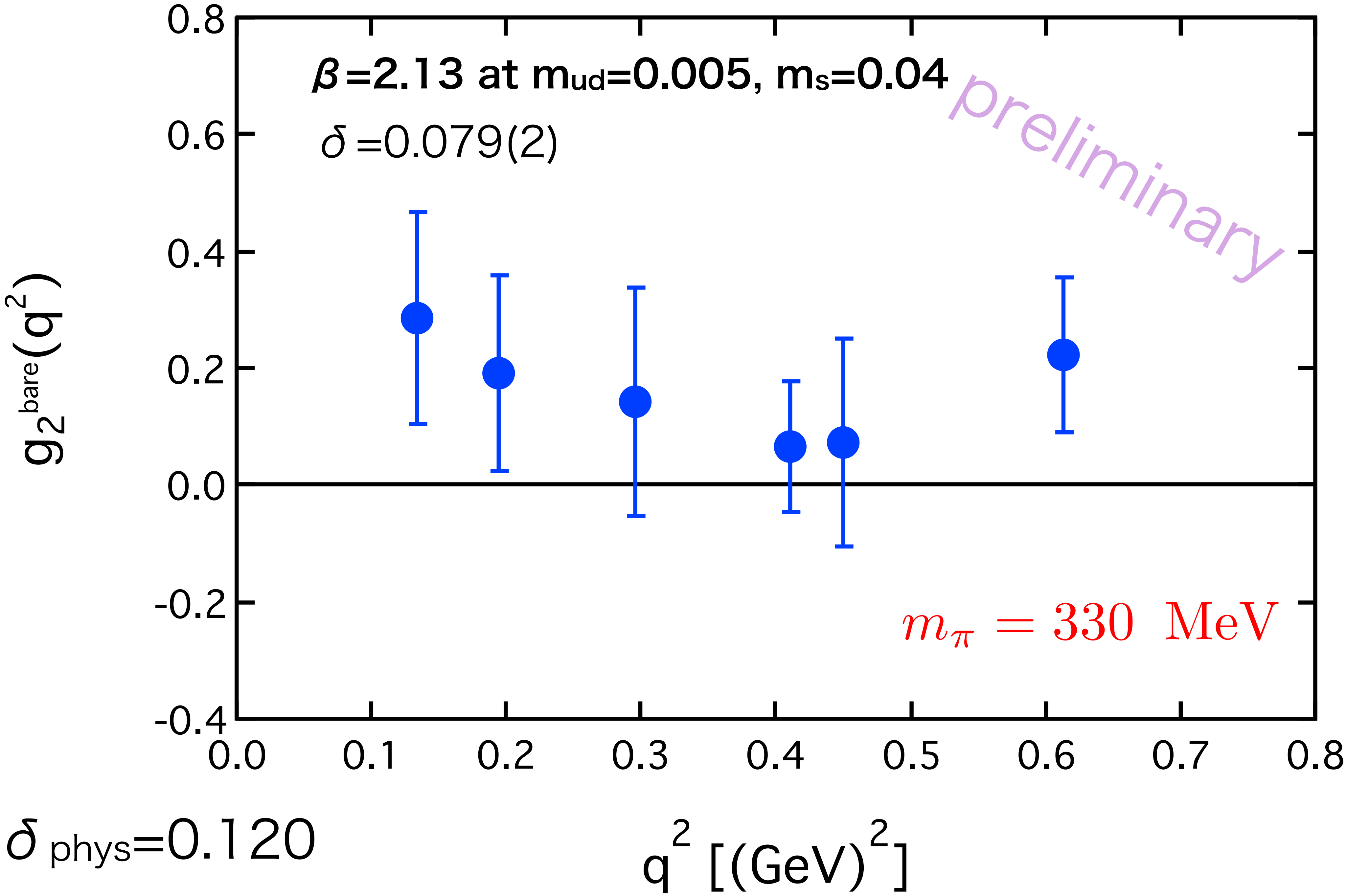
SU(3) breaking

cf. δ_{phys}=0.120

Direct measurement for g_2 form factor

CKM Unitarity $\Leftrightarrow$ $g_2(0) \approx 0.47$

$$Z_V = 0.7190(8) \approx Z_A$$



cf. δ_{phys}=0.120

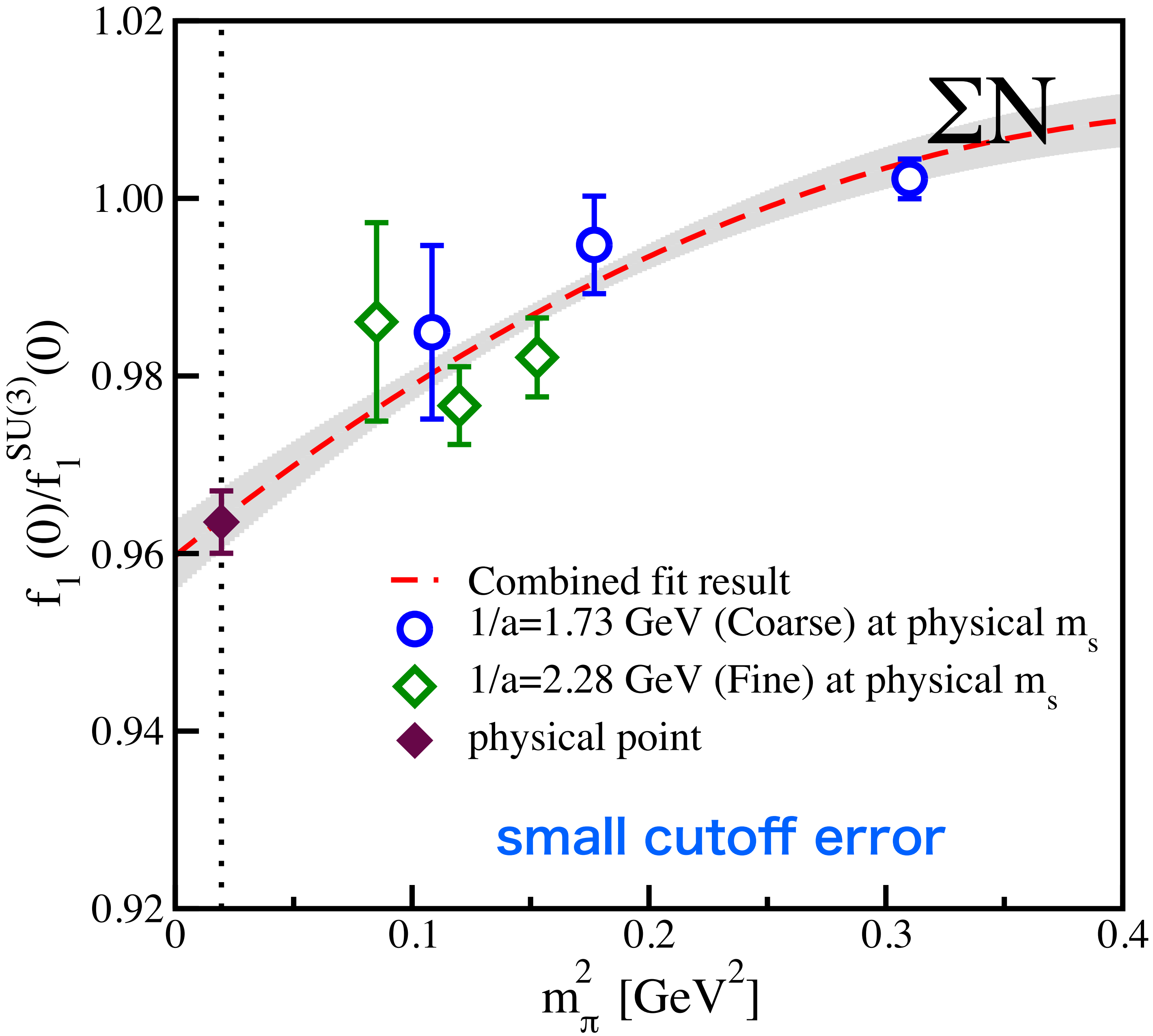
Summary

We have studied the SU(3) breaking effects on hyperon beta decays using 2+1 flavor dynamical lattice QCD.

- ✓ **Hyperon vector coupling $f_1(0)$** reaches a sub percent level accuracy.
  - The current **$\Sigma \rightarrow$N** data with lattice input of $f_1(0)$ moves slightly off the CKM unitarity condition.
- ✓ Conversely, **$f_1(0)$ + CKM unitarity** may expose a size of the induced 2nd-class form factor g_2, which was less-known and ignored in experiments.
  - **$g_2(0)$~0.4-0.5** ⇔ its size is consistent with the first-order SU(3) symmetry-breaking correction.
  - In lattice **direct** measurement, non-zero g_2 form factor is confirmed and its size is roughly consistent with the indirect estimation.

  ➡ The CKM unitarity could be satisfied in **$\Sigma \rightarrow$N decay.**

Update of $f_1(0)$ value



Only coarse lattice result

$$f_1(0) = -0.9698(106)$$

2+1 flavor DWF calculations
SS, PRD86, 114502 (12)

↓ statistics increases

$$f_1(0) = -0.9718(76)$$

Combined with both fine and coarse results

$$f_1(0) = -0.9635(35)$$

$$\tilde{f}_1(0) = C_0 + (C_1 + C_2 \cdot (M_K^2 + M_\pi^2)) \cdot (M_K^2 - M_\pi^2)^2$$

Comparison with ChPT results

✓ The tendency of the SU(3)-breaking correction **disagrees** with the latest **baryon ChPT** result.

SU(3) corrections [%]

	$\mathcal{O}(p^3)$ EOMS-CBChPT		$\mathcal{O}(p^4)$ EOMS-CBChPT		2+1f DWF-LQCD
	octet	+ decuplet	octet	+ decuplet	physical point
$\Lambda \to N$	-3.8	-3.1	$-3.6^{+1.2}_{-0.9}$	$+0.1^{+1.3}_{-1.0}$	—
$\Sigma \to N$	-0.8	-2.2	$+3.9^{+3.8}_{-2.8}$	$+8.7^{+4.2}_{-3.1}$	-3.7 ± 0.4
$\Xi \to \Lambda$	-2.9	-2.9	$-1.2^{+2.4}_{-1.8}$	$+4.0^{+2.8}_{-2.1}$	—
$\Xi \to \Sigma$	-3.7	-3.0	$-1.3^{+0.3}_{-0.2}$	$+1.7^{+2.2}_{-1.6}$	-2.5 ± 0.3

- suggests that baryon ChPT seems to have a serious convergence problem