Mass and Axial current renormalization in the Schroedinger functional scheme for the RG-improved gauge and the stout smeared O(a)-improved Wilson quark actions

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For PACS collaboration

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1. Schroedinger Functional

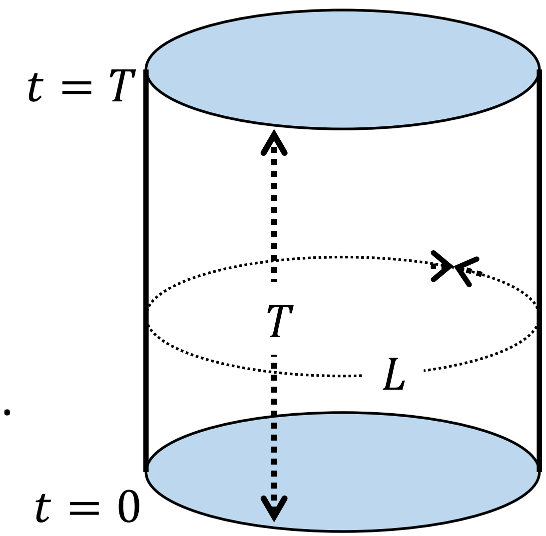
- The Schroedinger functional scheme is one of the renormalization scheme for the QFT.
- For Lattice QCD, the running coupling constant and the various renormalization constants have been computed for various gauge+fermion actions.
- The PACS collaboration have generated the configurations at the physical ud and strange quark masses on 96^4 lattices with $a^{-1} \sim 2.3\ \mathrm{GeV}$ using K computer since 2012 in the Strategic Field Program 5.
- [The hadron spectroscopy presented by Naoya Ukita on 14 (Tue) @Room 402]
- The RG-Improved Iwasaki gauge and the Stout smeared NPT-O(a)-improved Wilson quark actions with Nf=2+1 are used.

1. Schroedinger Functional

- We determine the quark masses and the pion and Kon decay constants as the basic observables of the configurations.
- Since the bare quark masses and the decay constants need the renormalization, we employ the Schroedinger Functional (SF) scheme to renormalize them for the action combination:

The RG-Improved Iwasaki gauge and the Stout smeared NPT-O(a)-improved Wilson quark actions.

1. Schroedinger Functional



- 4D finite space-time box
- Dirichlet boundary condition in Temporal direction.
- Periodic in spatial directions.
- Dirichlet boundary = Initial and Final Wave functional.
- Transition amplitude (Continuum)

$$Z[C', C] = \langle C' | e^{-HT} | C \rangle = \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\overline{\psi} \Psi_f[C', \vec{A}(t = T)]^* \Psi_i[C, \vec{A}(t = 0)] e^{-S[A,\overline{\psi},\psi]+G.F.}$$

- Transition amplitude (Lattice)

$$C_k = \frac{1}{L}\begin{pmatrix} \phi_1 & & \\ & \phi_2 & \\ & & \phi_3 \end{pmatrix}, \; C'_k = \frac{1}{L}\begin{pmatrix} \phi'_1 & & \\ & \phi'_2 & \\ & & \phi'_3 \end{pmatrix}$$

$$Z[C', C] = \langle C' | e^{-HT} | C \rangle = \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\overline{\psi} \Psi_f[C', \vec{U}(t = T)]^* \Psi_i[C, \vec{U}(t = 0)] e^{-S[U,\overline{\psi},\psi]}.$$

- They can be used to define the renormalization constants combined with other techniques.
- The Z_A and Z_P for the axial current and the pseudo scalar operators can be determined combined with the PCAC relation/W-T. identities.

2. Z_A and Z_P

- The axial current and the pseudo scalar operator renormalization constants are determined via the PCAC relation/W-T identities to hold them in the SF setup.
- Z_A and Z_P are required to renormalize the quark masses m_q.
- For the renormalized quark masses with the MS-bar scheme $m_q^{\overline{MS}}(\mu = 2\mathrm{GeV})$, we need
  - The physical box size L, the renormalization scale for m_q (Z_P) in the SF.
  - The box size L must be related to the low energy observables in QCD. ($ML = (aM)(L/a)$ at a bare coupling g_0 , large scale simulation)
  - The box size L is implicitly determined via the SF coupling.
  - Running and matching between the SF and the MS-bar scheme via the RGI form.

2. Z_A and Z_P

- Z_A : Axial current (massless)
  - From the Axial W-T identity :

$$if^{abc}\int d\vec{y}\langle P^a(\vec{u},T)V_4^b(y)P^c(\vec{v},0)\rangle = -\frac{2}{N_f}f^{abc}f^{cde}\int\int d\vec{y}d\vec{x}\langle P^d(\vec{u},T)A_4^a(x)A_4^b(y)P^e(\vec{v},0)\rangle$$

  - We define Z_A to hold

$$Z_A^2[f_{AA}^I(s+t,s)] = f_1$$

$$f_{AA}^I(t,s) = f_{AA}(t,s) + (ac_A)[\tilde{\partial}_t f_{PA}(t,s) + \tilde{\partial}_s f_{AP}(t,s)] + (ac_A)^2\tilde{\partial}_t\tilde{\partial}_s f_{PP}(t,s)$$

$$f_{XY}(t,s) = -\frac{2}{N_f^2(N_f^2-1)N_V^2}\sum_{\vec{x},\vec{y},\vec{u'},\vec{v'},\vec{u},\vec{v}} f^{abc}f^{cde}\langle O'^d(\vec{u'},\vec{v'})X^a(\vec{x},t)Y^b(\vec{y},s)O^e(\vec{u},\vec{v})\rangle$$

$$f_1 = -\frac{1}{(N_f^2-1)N_V^2}\sum_{\vec{u'},\vec{v'},\vec{u},\vec{v}}\langle O'^a(\vec{u'},\vec{v'})O^a(\vec{u},\vec{v})\rangle$$

$$A_\mu^a(x) = \bar{\psi}(x)\gamma_\mu\gamma_5T^a\psi(x), \qquad P^a(x) = \bar{\psi}(x)\gamma_5T^a\psi(x),$$

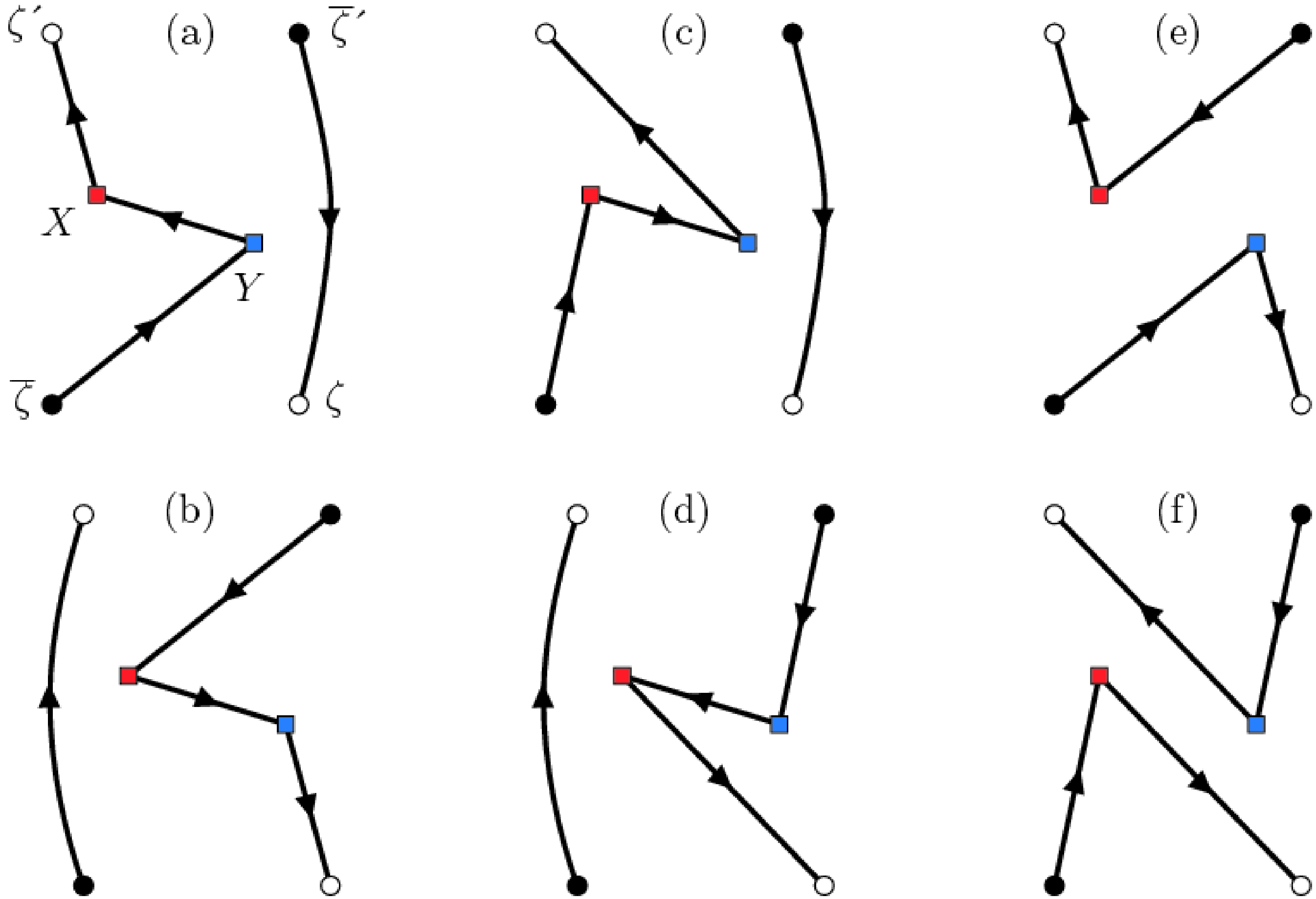
$$O^a(\vec{y},\vec{z}) = \bar{\zeta}(\vec{y})\gamma_5T^a\zeta(\vec{z}), \qquad O'^a(\vec{y},\vec{z}) = \bar{\zeta}'(\vec{y})\gamma_5T^a\zeta'(\vec{z})$$

$$Z_A = \sqrt{\frac{f_1}{n_A}}\left[f_{AA}^I\left(\frac{2T}{3},\frac{T}{3}\right)\right]^{-1/2}$$

  - Where n_A is a constant to normalize $Z_A = 1$ at the tree level.

2. Z_A and Z_P

- Wick contraction for $f_{XY}(s,t)$



2. Z_A and Z_P

- Z_P : Pseudo scalar (massless)
  - From the two point function

$$f_P(t) = -\frac{1}{(N_f^2-1)N_V}\sum_{\vec{x},\vec{y},\vec{z}}\langle P^a(\vec{x},t)O^a(\vec{y},\vec{z})\rangle$$

  - Z_P is defined by

$$Z_P = \frac{\sqrt{N_c f_1}}{n_P f_P(T/2)}$$

  - Where n_P is a constant to normalize $Z_P = 1$ at the tree-level.
- The scale dependence via the finite box scaling (step scaling) has been determined for Nf=3 case. [Quenched and Nf=2: Alpha collaboration]

- Z_V : Vector current renormalization is also measured.

$$Z_V = \frac{f_1}{n_V f_V(T/2)} \qquad f_V(t) = \frac{1}{N_f(N_f^2-1)N_V^2}\sum_{\vec{u}',\vec{v}',\vec{x},\vec{u},\vec{v}} if^{abc}\langle O^a(\vec{u}',\vec{v}')V_4^b(\vec{x},t)O^c(\vec{u},\vec{v})\rangle$$

3. Scale setting via the SF coupling

- The bare physical quark masses and lattice cutoff a are determined in the large scale simulation at $\beta = 1.82$ on 96^4 lattices.
- At the same beta, we have to determine L_{max} from which the running evolution to high energy starts.
- L_{max} is implicitly determined by g_{SF} the SF scheme coupling.
- The running of the SF coupling via the finite box scaling has been determined for $N_f = 3$ case.
- We need the SF scheme coupling with the RG-improved Iwasaki gauge and the Stout smeared O(a)-improved Wilson quark action to determine L_{max}.

3. Scale setting via the SF coupling

- The SF scheme coupling $$\frac{1}{g_{SF}^2} = -\frac{1}{k}\frac{\partial}{\partial\eta}\log Z[C',C]\Big|_{\eta=0}$$
- The Stout smeared O(a)-improved Wilson fermion
  - O(a)-improved Wilson + All Link variables are replaced with the stout smeared one.

[*cf.* Clover term unsmeared, Stout or SLiNC action: SQCDSF+UKQCD, N.Cundy et al, PRD 79(2009) 094507]

  - Stout smearing : [Morningstar and Peardon, PRD 69 (2004) 054501]

$$U_\mu^{(k+1)}(n) = \exp\left[i\alpha Q_\mu^{(k)}(n)\right] U_\mu^{(k)}(n)$$
$$Q_\mu^{(k)}(n) = \frac{i}{2}\left[\left(\Omega_\mu^{(k)}(n) - \Omega_\mu^{(k)}(n)^\dagger\right) - \frac{1}{N_c} Tr\left(\Omega_\mu^{(k)}(n) - \Omega_\mu^{(k)}(n)^\dagger\right)\right]$$
$$\Omega_\mu^{(k)}(n) = U_\mu^{(k)}(n)\sum_{\nu\neq\mu}\left[U_\nu^{(k)}(n+\hat\mu)U_\mu^{(k)}(n+\hat\nu)^\dagger U_\nu^{(k)}(n)^\dagger + U_\nu^{(k)}(n-\hat\nu+\hat\mu)^\dagger U_\mu^{(k)}(n-\hat\nu)^\dagger U_\nu^{(k)}(n-\hat\nu)\right]$$

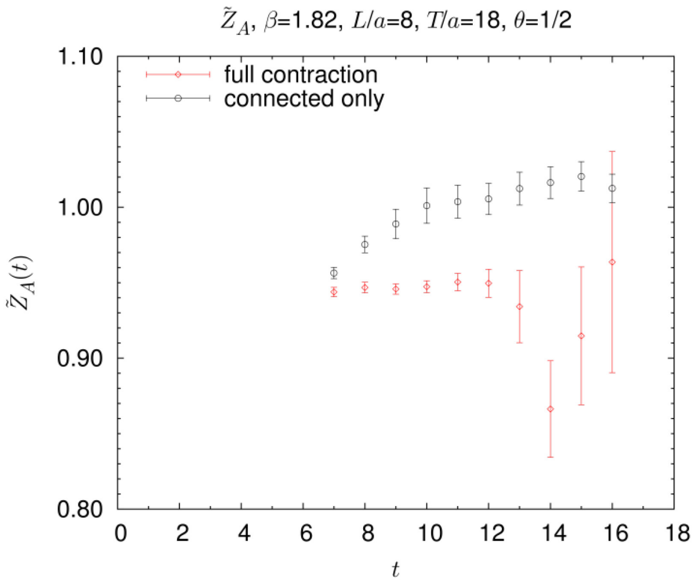
    - The boundary link variables are kept fixed during the smearing steps.
    - $\alpha = 0.1, n_{step} = 6$ is used.

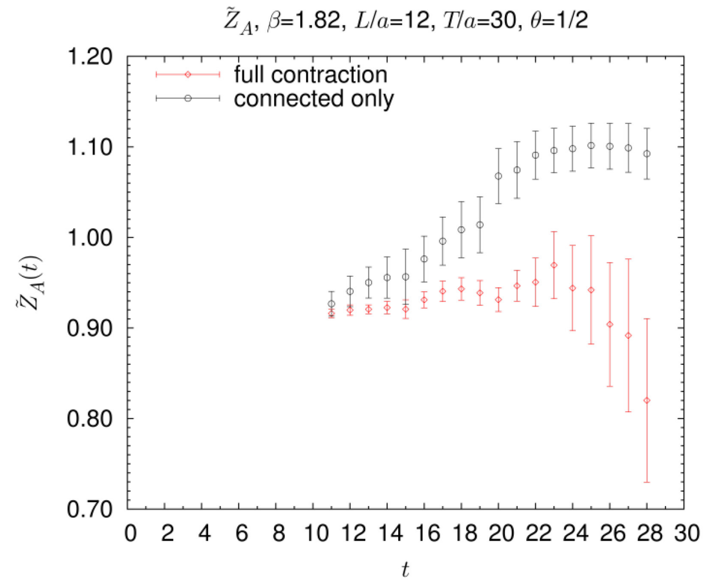
- There is the quark contribution to the SF coupling in the bulk region through the boundary derivative $\partial/\partial\eta$ on the smeared link variables. The boundary parameter dependence spreads out in the bulk region.
- The chain rule on $\partial/\partial\eta$ is required. This is almost identical to the force computation for the Stout smeared action in the HMC algorithm.

4. Results

- Z_A:
  - $\beta = 1.82$, $c_{SW} = 1.110$, $c_A = 0$, $\tilde{c}_t = 1$, $\theta = 1/2$ (Zero boundary field) (HMC(Nf=2)+RHMC(Nf=1))
  - $c_{SW} = 1.110$ has been non-perturbatively determined. [Y. Taniguchi, PoS Lattice 2012, 236]
  - $\tilde{Z}_A(t) \equiv \frac{f_1}{n_A}\left[f_{AA}\left(t, \frac{T}{3}\right)\right]$ is computed on 8^3x18 and 12^3x30 lattices.
  - The PCAC mass is monitored for vanishing masses.

$$am_{PCAC} \equiv \frac{1}{3}\sum_{t=\frac{T}{2}-a}^{\frac{T}{2}+a} \frac{a\tilde{\partial}_t f_A(t)}{2f_P(t)}$$





$$Z_A = \sqrt{\tilde{Z}_A(2T/3)}$$

- A plateau is observed around $t = 2T/3$.
- The discrepancy is used to estimate the systematic errors (O(a/L)).

LxT	Statistics(#traj)	am_{PCAC}	Z_V	Z_A
8x18	20000	0.00041(61)	0.9664(20)	0.9745(48)
12x30	34700	-0.00080(33)	0.9650(68)	0.9650(68)

4. Results

- Z_P and g^2_{SF} for Z_m:
  - The non-perturbative step scaling function for Nf=3 QCD has been determined in [S. Aoki et al. [PACS-CS], JHEP0910(2009)053, JHEP1008(2010)101].
  - g^2_{SF} :
    - The largest coupling $g^2_{SF}(L_{\mathrm{max}})$ is about 5.2.
    - At $\beta = 1.82$, $c_{SW} = 1.110$, $c_A = 0$, $\tilde{c}_t = 1$, $\theta = \pi/5$ (standard boundary field), We compute $g^2_{SF}(L)$ on L/a=4 and 6 lattices.

LxT	κ	Statistics(#traj)	am_{PCAC}	g^2_{SF}
4x4	0.126110	70000	-0.04550(26)	3.662(17)
4x4	0.125120	80000	0.00042(25)	3.776(16)
6x6	0.126110	40000	-0.01009(27)	6.022(117)

    - The scale L = 4a can be used as the start point of the evolution (L_max).
  - Z_P:
    - At the same box size (L/a=4) and $\beta = 1.82$, $c_{SW} = 1.110$, $c_A = 0$, $\tilde{c}_t = 1$, $\theta = 1/2$ (Zero boundary field), we compute Z_P.

LxT	κ	Statistics(#traj)	am_{PCAC}	Z_P
4x4	0.126110	100000	-0.021859(94)	1.01317(43)
4x4	0.125120	80000	0.013241(99)	1.00670(45)

    - The discrepancy between two kappa's on 4x4 lattices is used to estimate the systematic errors.

4. Results

- Z_m:

Values at $\beta = 1.82$

$$Z_m^{\overline{MS}}(g_0,\mu) = \left(\frac{\overline{m}_{\overline{MS}}(\mu)}{M_{RGI}}\right)\left(\frac{M_{RGI}}{\overline{m}_{SF}(1/L_{\max})}\right)\left(\frac{Z_A(g_0,a/L)}{Z_P(g_0,a/L_{\max})}\right)$$

$$\left(\frac{M_{RGI}}{\overline{m}_{\overline{MS}}(\mu)}\right) = \left[(2b_0\bar{g}^2(\mu))^{-\frac{d_0}{2b_0}}\exp\left(-\int_0^{\bar{g}(\mu)} dg\left(\frac{\tau(g)}{\beta(g)}-\frac{d_0}{b_0 g}\right)\right)\right]_{\overline{MS}}$$

Evolution by PT MS-bar Nf=3 4-loop

$$\left(\frac{M_{RGI}}{\overline{m}_{SF}(1/L_{\max})}\right) = \left(\prod_{j=1}^{n}\sigma_P(u_j)\right)\left[(2b_0\bar{g}^2(1/L_n))^{-\frac{d_0}{2b_0}}\exp\left(-\int_0^{\bar{g}(1/L_n)} dg\left(\frac{\tau(g)}{\beta(g)}-\frac{d_0}{b_0 g}\right)\right)\right]_{SF}$$

Evolution by NPT SF-SSF Nf=3 $\sigma_P(u)$, $\sigma(u)$

Evolution by PT SF Nf=3 $\tau(g)$:two-lop, $\beta(g)$: three-loop

[S. Aoki et al. [PACS-CS], JHEP0910(2009)053, JHEP1008(2010)101].

[ALPHA collaboration: A. Bode, P.Weisz, U.Wollf, NPB 576(2000)517; B 600(2001)453;B 608(2001)481]

4. Results

Preliminary Results

- Axial current normalization for the RG-improved Iwasaki gauge and three-flavor of the Stout smeared O(a)-improved Wilson quark actions.

$$Z_A = 0.9650(68)(95) \quad \text{at } \beta = 1.82$$

- From the Hadron spectroscopy at $\beta = 1.82$, the lattice cutoff is determined as $a^{-1} = 2.332(18)$ GeV [π,K,Ω] at the physical point. Using

$$Z_A = 0.9650(68), Z_P = 1.01317(43), g_{SF}^2 = 3.662(17),$$

$$[Z_A = 0.9650(68), Z_P = 1.00670(45), g_{SF}^2 = 3.776\ (16)],$$

- We obtain

$$\Lambda_{\overline{MS}}^{(3)} = 284.6(17.0)[\mathrm{MeV}], \qquad Z_m^{\overline{MS}}(g_0, \mu = 2\mathrm{GeV}) = 0.9950(111).$$

$$[\Lambda_{\overline{MS}}^{(3)} = 302.4(18.1)[\mathrm{MeV}],\ Z_m^{\overline{MS}}(g_0, \mu = 2\mathrm{GeV}) = 1.0039(117)].$$

- Taking the discrepancy as the systematic error, we obtain

$$\Lambda_{\overline{MS}}^{(3)} = 284.6(17.0)(17.8)[\mathrm{MeV}],\ \ Z_m^{\overline{MS}}(g_0, \mu = 2\mathrm{GeV}) = 0.9932(111)(89).$$

All Preliminary

5. Summary

- The Mass and Axial current renormalizations for the RG-improved Iwasaki gauge and three-flavor of the Stout smeared NPT-O(a)-improved Wilson quark actions are determined using the Schroedinger functional method.

- With the Stout smearing, the renormalization constants, Z_A and Z_m, become closer to unity.

References

SF scheme

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[2] M. Luscher, R. Sommer, P. Weisz, U. Wolff, NPB 413(1994)481.

[3] S. Sint, NPB 421 (1994) 135.

[4] ALPHA collaboration: A. Bode, P.Weisz, U.Wolff, NPB 576(2000)517; B 600(2001)453;B 608(2001)481.

SF running coupling

[5] ALPHA collaboration, M. Della Morte et al., NPB 713(2005) 378.

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O(a)improvements

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[15] SQCDSF-UKQCD collaboration: N.Cundy et al, PRD 79(2009) 094507; PoS(LAT2008)132.

[16] Y. Taniguchi, PoS LATTICE2012 (2012) 236.

[17] ALPHA collaboration: J. Bulava et al., NPB896(2015)555.

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Stout smearing

[23] C. Morningstar and M Peardon, PRD69(2004)054501.

In this conference

Strategic Field Program 5

- Naoya Ukita , 14 (Tue) @Room 402.
- Takeshi YAMAZAKI, 15(Wed) @ Room 402.
- Takumi DOI, 15(Wed) @ Room 402.
- Noriyoshi ISHII, 15 (Wed) @ Room 402.
- Hidekatsu NEMURA, 17 (Fri) @ Room 405.
- Kenji SASAKI, 15(Wed) @ Room 402.

Renormalization

- Stefan SINT, 14 (Tue) @ Room 404.
- Patrick FRITZSCH, 14 (Tue) @ Room 404.
- Andrew LYTLE, 14 (Tue) @ Room 404
- David PRETI , 14 (Tue) @ Room 404
- Arwed SCHILLER, 18 (Sat) @ Room 403.
- Jangho KIM, Poster #15
- *and many presentations……*