Higher order net baryon number cumulants in the strong coupling lattice QCD

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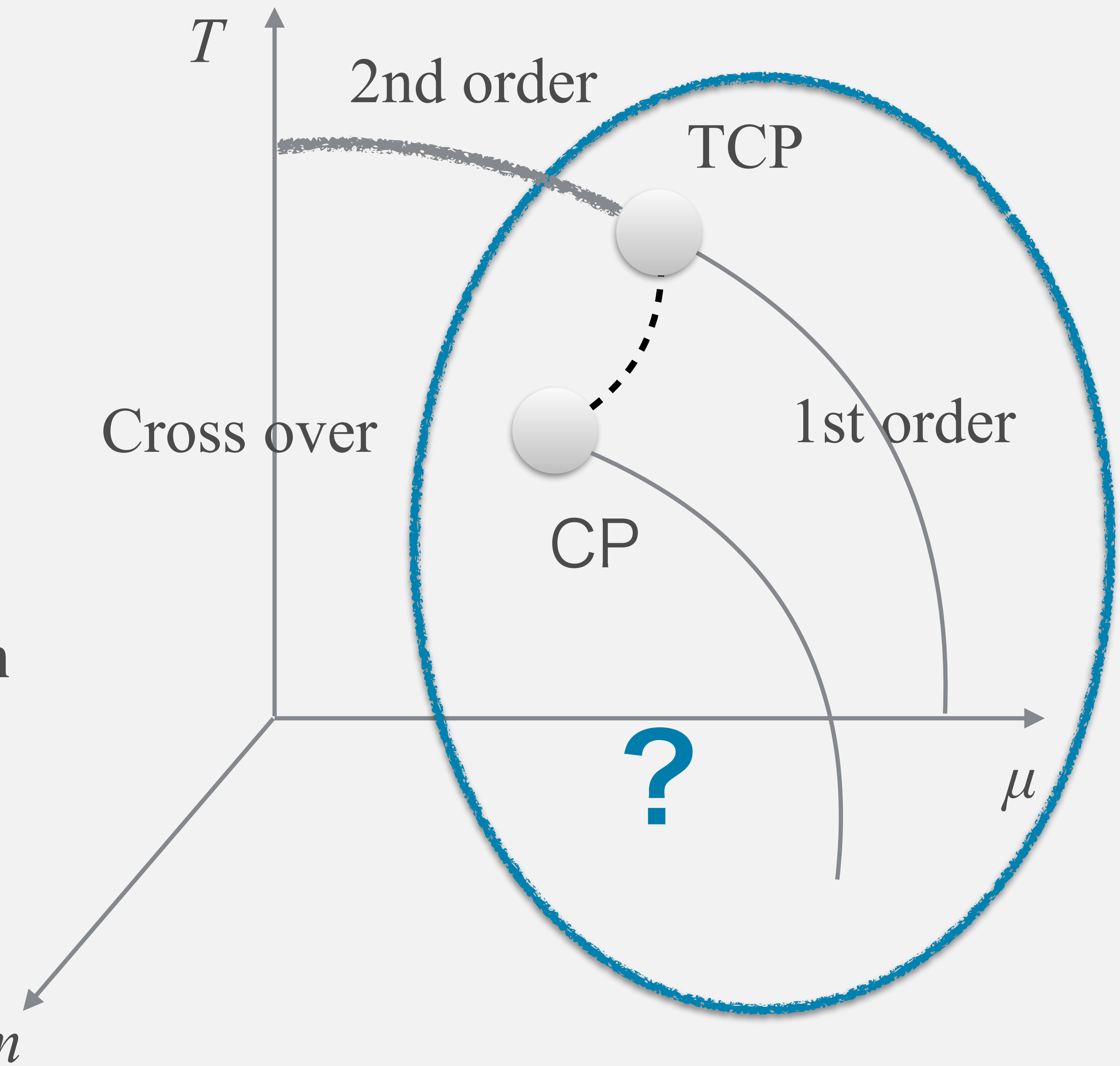
News !
We got an arXiv number yesterday. arXiv:1507.04527

QCD phase diagram

- Theory
  - Model analyses
    : Model dependent
  - **Lattice QCD**
    : **Sign problem** at finite μ
    not easy to study

- **Experiment**
  - **Beam energy scan program**
    Examining finite mu region
    by heavy ion collision



Higher order cumulant ratios

- **Search for CP or phase transition**

  higher-order cumulant ratios of net baryon number e.g. Sσ, κσ^2, ….

$$S\sigma = \chi_\mu^{(3)}/\chi_\mu^{(2)} \quad \kappa\sigma^2 = \chi_\mu^{(4)}/\chi_\mu^{(2)}$$

$$\chi_\mu^{(n)} = \frac{1}{VT^3}\frac{\partial^n \log Z}{\partial(N_c\mu/T)^n}$$

1. Experimental data: net proton

   Non-monotoic behavior

   (Skellam distribution :

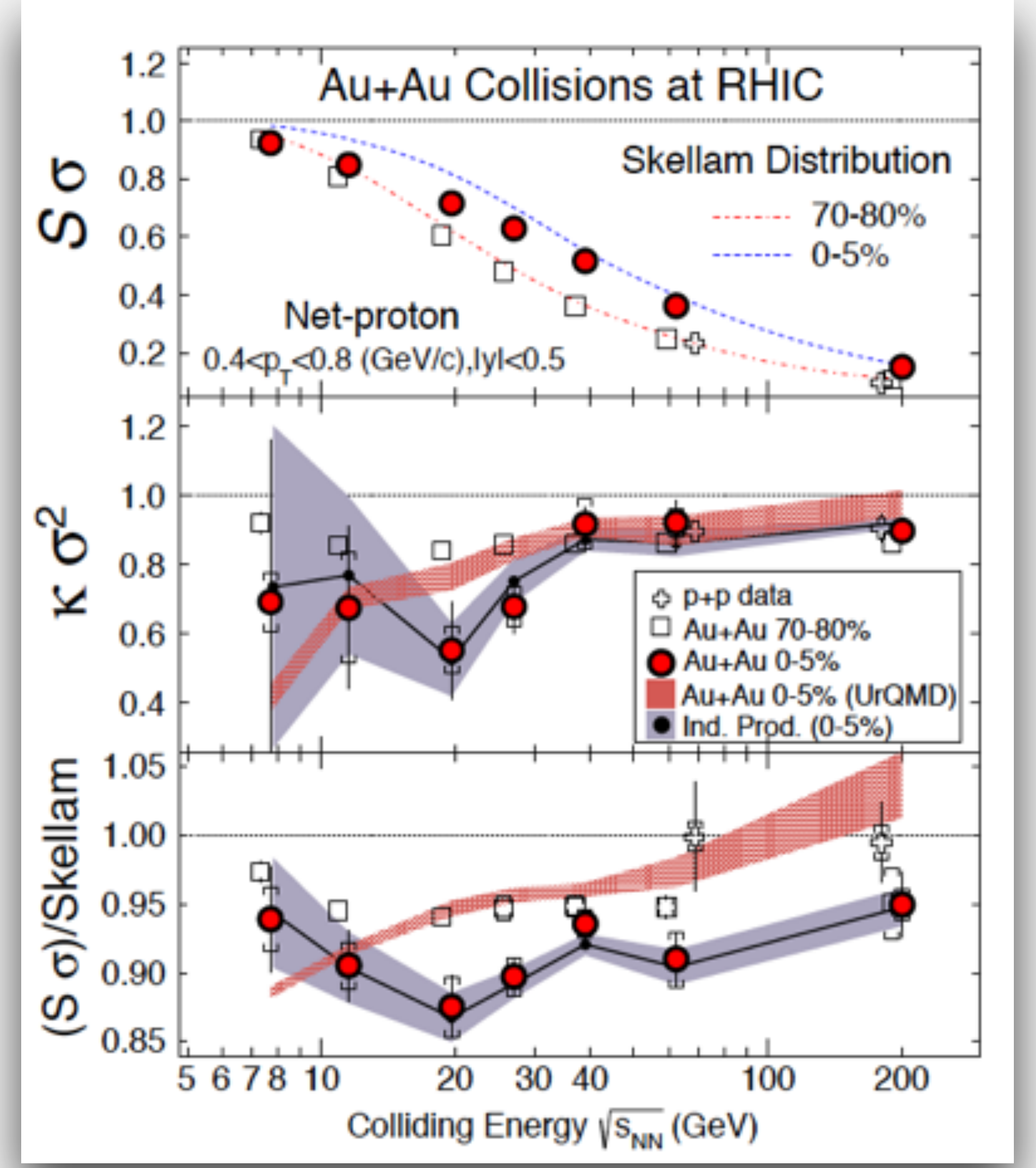
   Poisson - Poisson)

   may be lots of mechanism

2. Theory

   We can study higher order cumulants by using chiral models or lattice QCD

The net proton number



L. Adamczyk, et al. (STAR Collaboration), Phys. Rev. Lett. 112 032302 (2014)

Theory : Toward physical results

- **Important ingredients for high-order cumulants**

1. **Fluctuation effects beyond mean field (MF) analysis**

   Functional renormalization group

   B. Friman, et. al., Eur. Phys. J. C 71, 1694 (2011); V. Skokov, et. al., Phys. Rev. C 83, 054904 (2011), etc.

2. **Based on Lattice QCD (LQCD) at large μ**

   - finite μ : R.V. Gavai, S. Gupta. Phys. Lett. B696, 459 (2011); A. Bazavov, et al. (BNL-Bielefeld), Phys. Rev. Lett. 109, 192302 (2012); X. Y. Jin, Y. Kuramashi, et. al, Phys. Rev. D 88, 094508 (2013), etc.

3. **Light mass or chiral limit**

   Heavy mass: X. Y. Jin, Y. Kuramashi, et. al., Phys. Rev. D 88, 094508 (2013), A. Suzuki's talk the day before yesterday, etc.

   Light mass : R.V. Gavai, S. Gupta. Phys. Lett. B696, 459 (2011)

4. **Continuum limit & finite size scaling**

Is it possible to satisfy above ingredients?

- **Partly YES. except for continuum limit**

  → **Strong coupling lattice QCD**

  1. is an approximation method of LQCD action

  2. could study even in the chiral limit beyond MF analysis.

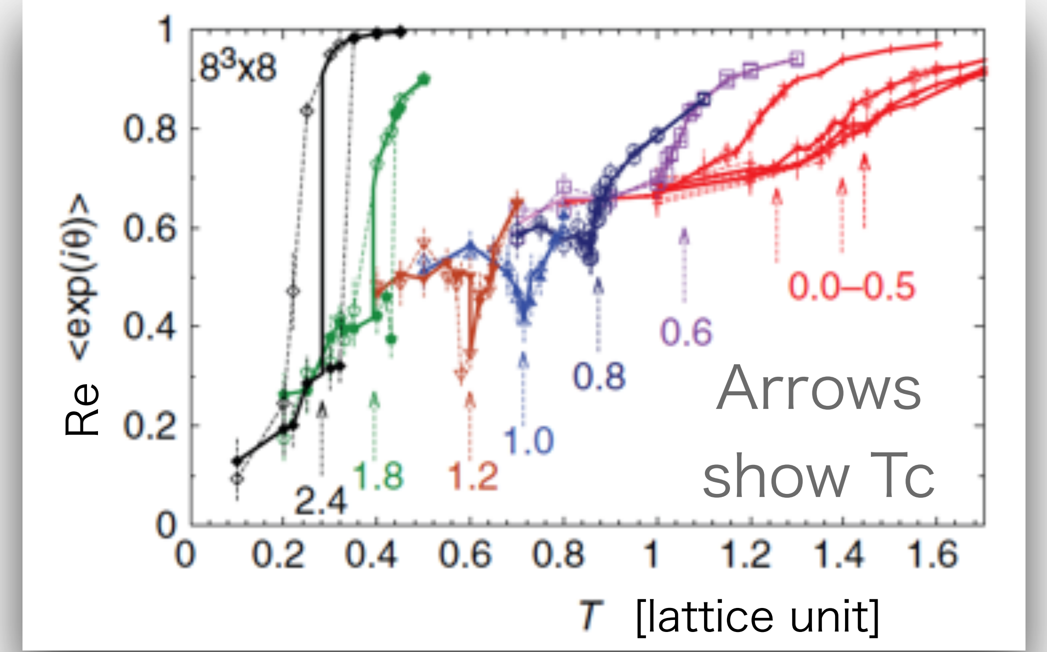
  Demerits : large coupling constant, coarse lattice, confined phase

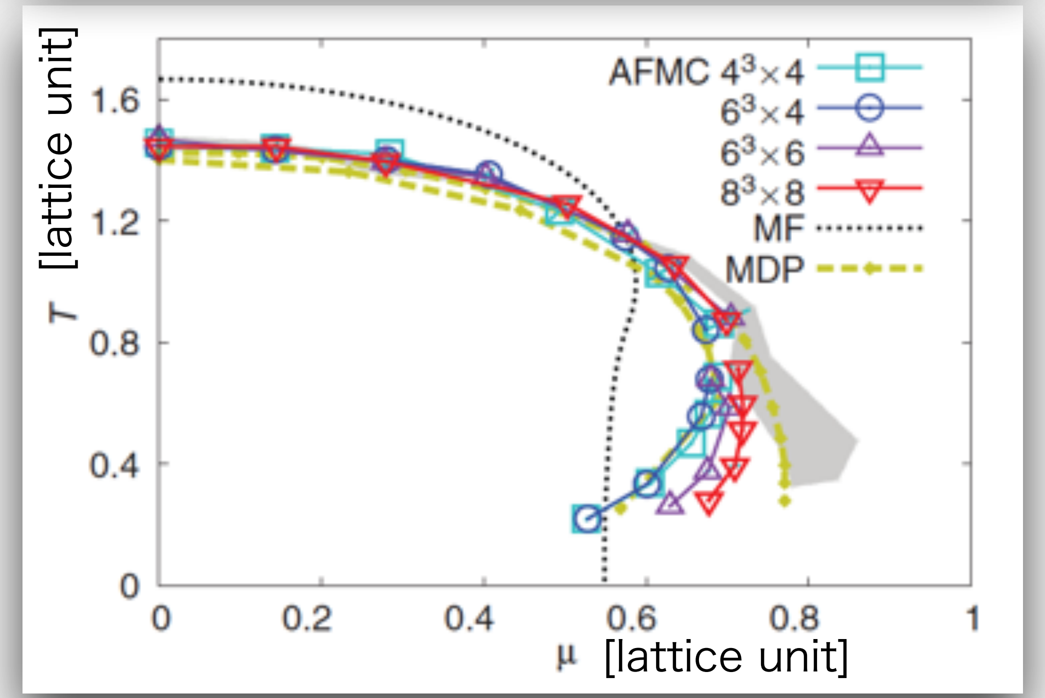
Strong coupling lattice QCD

- Lattice QCD action with sufficiently large coupling
  Wilson ('74), Creutz ('80), Munster ('81), Kawamoto, Smit ('81), Ichinose ('84), Karsch, Mutter ('89) , Bilic, Karsch, Redlich ('92), Faldt, Petersson ('86), Damggar, Fukushima ('03), Nishida ('03), de Forcrand, Fromm ('10), de Forcrand, Unger ('11), Nakano, Miura, Ohnishi ('10,'11), Tomboulis ('13,'14),…
  Approximation in lattice QCD
- Recent developments
  - **Fluctuations beyond MF**
    Monomer-Dimer-Polymer (MDP) simulation
    W. Unger and P. de Forcrand, J. Phys. G 38, 124190 (2011), P. de Forcrand and M. Fromm, Phys. Rev. Lett. 104, 112005 (2010)
    Auxiliary field Monte-Carlo (AFMC) method
    T. I. , A. Ohnishi, and T. Z. Nakano, PTEP 2014 12, 123D02 (2014).
  - Less severe weight cancellation than lattice QCD
    → QCD phase diagram

Strong coupling and chiral limit

Average phase factor 8^4 lattice : AFMC





T. I. , A. Ohnishi, and T. Z. Nakano, PTEP 2014 12, 123D02 (2014).

Strong coupling lattice QCD

- Lattice QCD action with sufficiently large coupling
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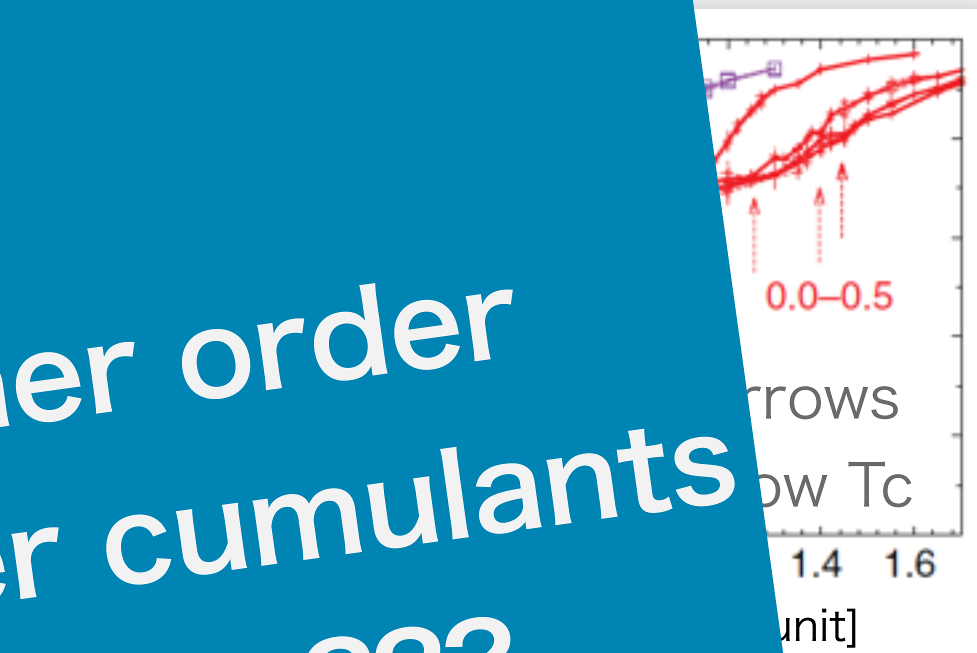
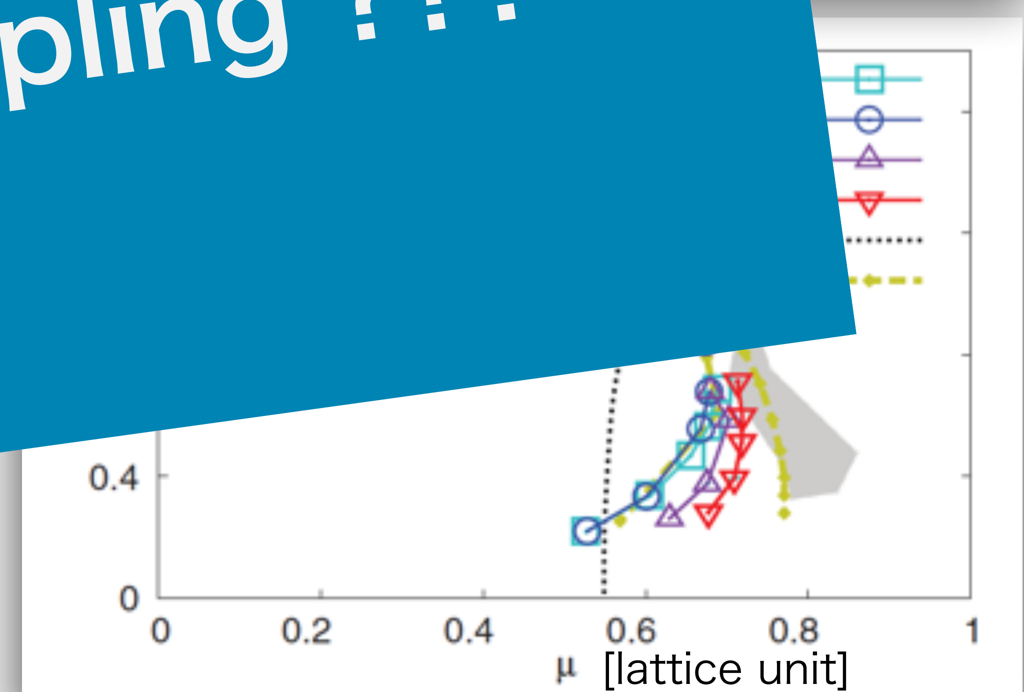
    (2014)
  - Less ... weight cancellation than lattice QCD
    → QCD phase diagram

Strong coupling and chiral limit

Aver... ...ice : AFMC





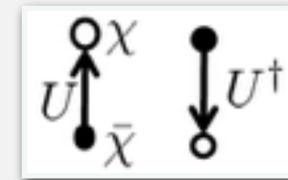
How about higher order net baryon number cumulants at strong coupling ???

T. I. , A. Ohnishi, and T. Z. Nakano, PTEP 2014 12, 123D02 (2014).

Formalism

- Lattice QCD action in **strong coupling limit : no plaquette term**
  - unrooted staggered fermion with d(=3)+1 dimension for Nc=3
  - anisotropic lattice assumption : $a_\tau = a_s/\gamma^2$ due to quantum correction. Tc(μ=0) does not depend on γ in MF. N. Bilic, F. Karsch, and K. Redlich, Phys. Rev. D 45, 3228 (1992); N. Bilic and J. Cleymans, Phys. Lett. B 355, 266 (1995)

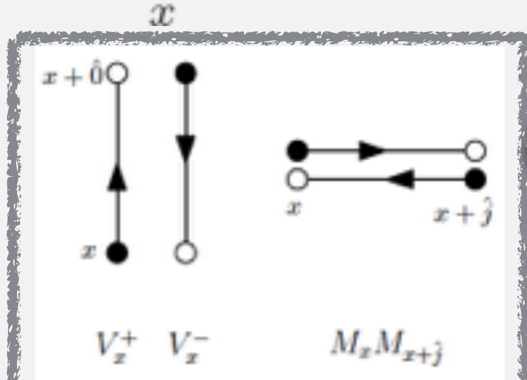
$$S_{\rm SCL} = \frac{1}{2}\sum_{x,\nu=0}^{d}\left[\eta^{+}_{\nu,x}\bar{\chi}_x U_{\nu,x}\chi_{x+\hat{\nu}} - \eta^{-}_{\nu,x}(\mathrm{H.C.})\right]$$



$$+m_0\sum_x \bar{\chi}_x\chi_x \qquad \eta^{\pm}_{\nu,x} = (\gamma e^{\pm\mu a_\tau}, (-1)^{x_1+\cdots+x_{\nu-1}})$$

- Effective action after integrating out spatial link variables and1/d expansion H. Kluberg-Stern, A. Morel, B. Petersson, (1983), etc.

$$S_{\rm eff} = \frac{1}{2}\sum_x[V^{+}_x - V^{-}_x] - \frac{1}{4N_c}\sum_{x,j} M_x M_{x+\hat{j}} + m_0\sum_x M_x \qquad V^{\pm}_x = \gamma e^{\pm\mu a/\gamma^2}\bar{\chi}_x U_{0,x}\chi_{x+\hat{0}}$$



K. Miura, T. Z. Nakano, A. Ohnishi and N. Kawamoto, Phys. Rev. D 80, 074034 (2009).

4-fermi interaction

Bosonization

Auxiliary fields are integrated by Monte-Carlo technique (AFMC method)

Results

- Set up
  unrooted staggered fermion : O(2) symmetry
  chiral and strong coupling limit
  Auxiliary field Monte-Carlo method

- Higher-order cumulant ratios
  - Normalized **skewness**

    $$S\sigma = \chi_\mu^{(3)} / \chi_\mu^{(2)}$$

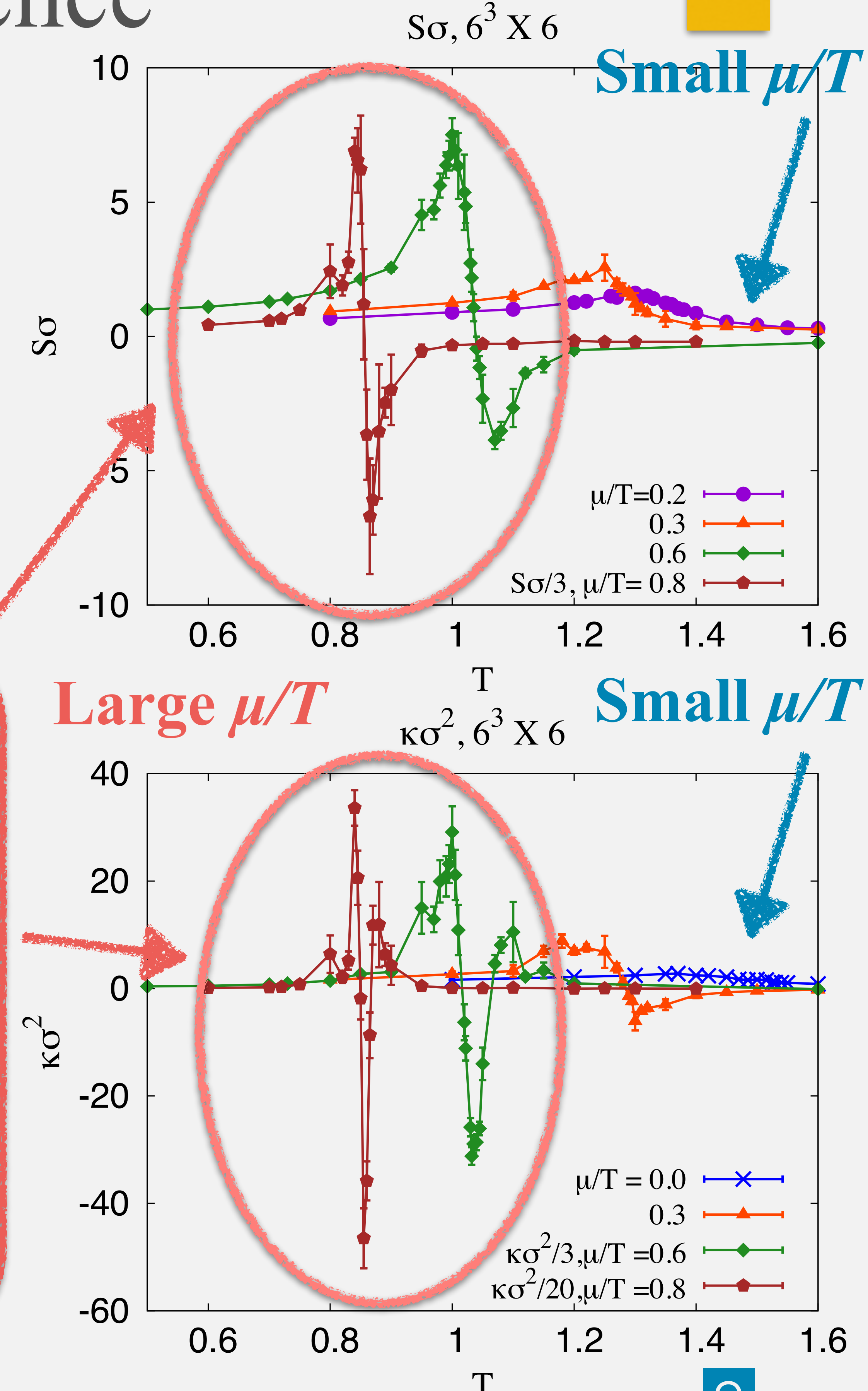
  - Normalized **kurtosis**

    $$\kappa\sigma^2 = \chi_\mu^{(4)} / \chi_\mu^{(2)}$$

$$\chi_\mu^{(n)} = \frac{1}{VT^3} \frac{\partial^n \log Z}{\partial (N_c \mu / T)^n}$$

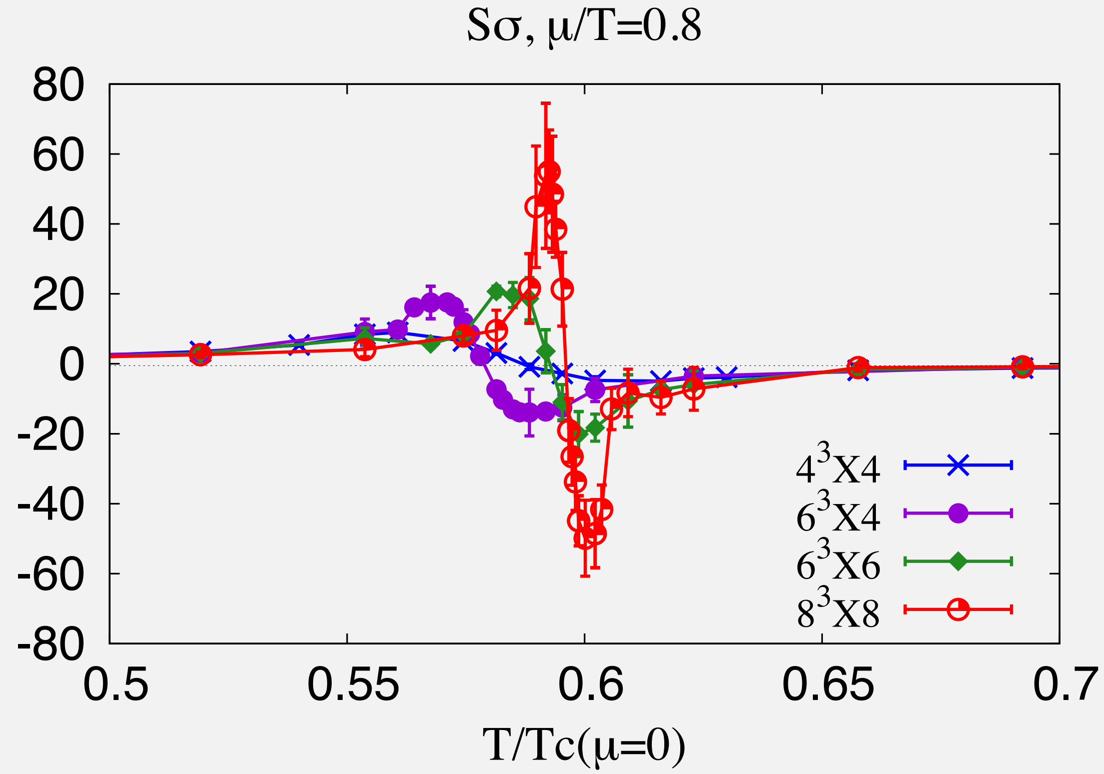
μ, T dependence

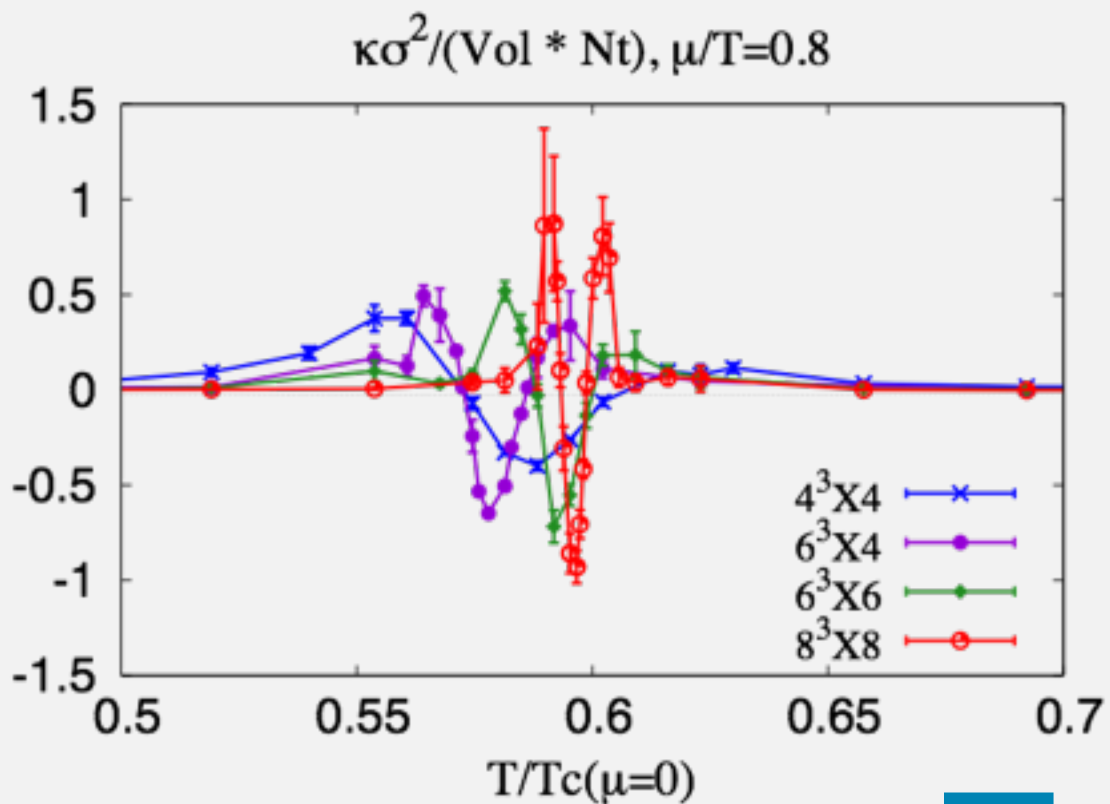
- Small μ/T
  Both cumulant ratios
  - have small positive peak around phase boundary.
  - stay positive,
- Large μ/T
  Kurtosis (Skewness)
  - shows **oscillatory behavior**.
  - has two (one) positive peaks and one negative valley.
  - has large amplitude approaching to TCP.



Size dependence

- **Amplitude**
  - becomes larger with increasing lattice size.
    - divergent behavior
- **Positive peaks and negative valley**
  - **shrink on larger lattices**, which is consistent with O(4) scaling function analysis.
    Friman, Karsch, Redlich, Skokov, Eur. Phys. J. C71, 1694 (2011).
  - Positive divergent behavior in the chiral limit.
    - Qualitative behavior is not different between O(4) and O(2)



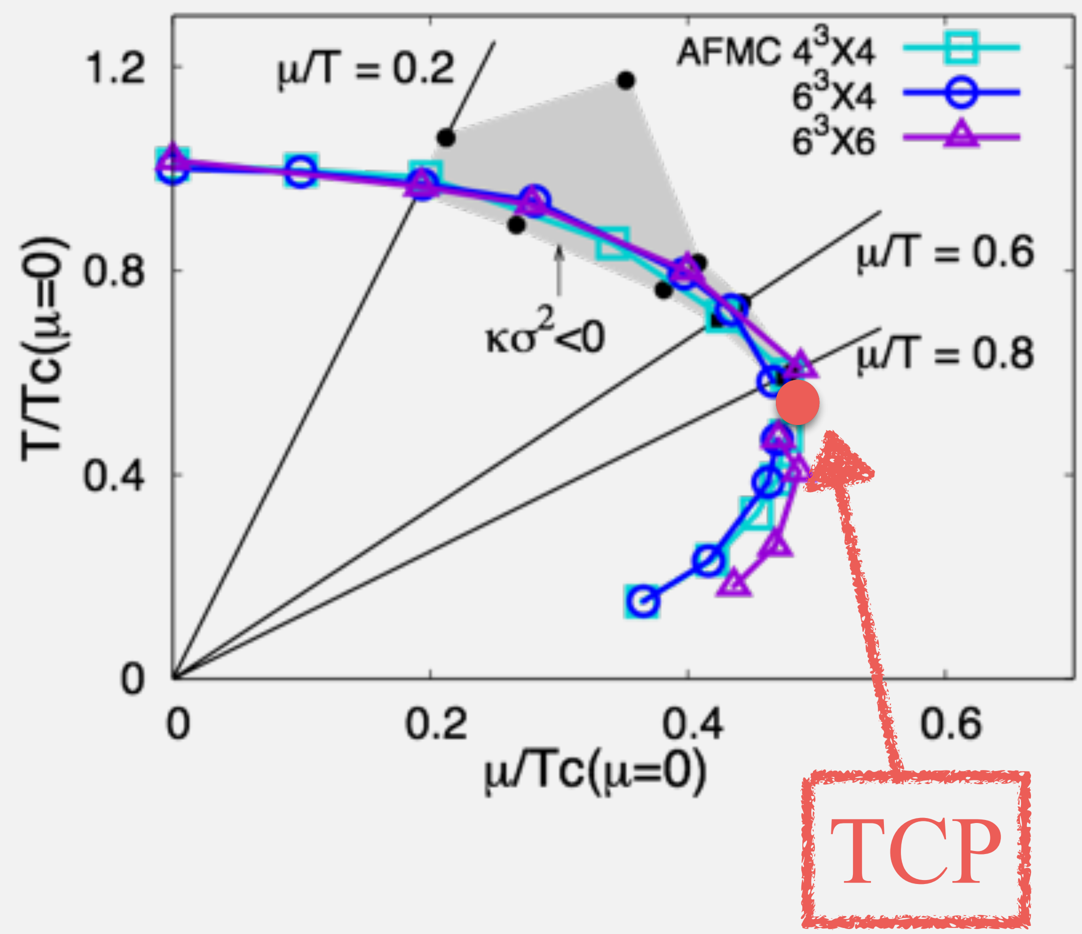


Negative normalized kurtosis region in the QCD phase diagram

- Set up
  Chiral and strong coupling limit
  Lattice size : 6^4
  Subtracting artifact at high T
  - no spatial baryonic hopping

- Phase boundary
  Determined by chiral susceptibility peak at $\mu/T < 0.8$ (would be 2nd order)

- Negative kurtosis valley - shaded area
  - **Consistent with phase boundary**
  - Expected to shrink in the thermodynamic limit



- Important next steps
  - **Finite mass simulation**
  - Negative region may survive in the thermodynamic limit

Summary

- We investigate normalized kurtosis and skewness in the chiral and strong coupling limit.
  - We find
    - **oscillatory behavior** at high μ/T and **negative kurtosis valley due to the finite size effect**.
    - increasing peak height of skewness and kurtosis and shrinking negative valley of kurtosis with larger lattice.
    - consistency with O(4) scaling analysis.
- Important steps is that we investigate cumulant ratios
  - on larger lattice.
  - with finite mass.
    $\rightarrow$ Negative region may remain in the thermodynamic limit.

Effective action

- **Bosonization** - extended Hubbard-Stratonovich transformation
  K. Miura, T. Z. Nakano, A. Ohnishi and N. Kawamoto, Phys. Rev. D 80, 074034 (2009).
  - bosonization when (A≠B)
  - an imaginary number → sign problem

$$e^{\alpha AB} = \int d\varphi d\phi e^{-\alpha\{[\varphi-(A+B)/2]^2+[\phi-i(A-B)/2]^2\}+\alpha AB}$$

- **Auxiliary field Monte-Carlo method after Grasmann and U_0 integral**
  - integrating over auxiliary fields(σ,π) by MC
  - **Mesonic fluctuation effects taken into account**

$$S_{\rm eff}^{\rm AF} = \sum_{\mathbf{k},\tau,f(\mathbf{k})>0} \frac{L^3 f(\mathbf{k})}{4N_c}\left[|\sigma_{\mathbf{k},\tau}|^2 + |\pi_{\mathbf{k},\tau}|^2\right] - \sum_{\mathbf{x}} \log R(\mathbf{x})$$

$$R(\mathbf{x}) = X_{N_\tau}(\mathbf{x})^3 - 2X_{N_\tau}(\mathbf{x}) + 2\cosh(N_c\mu/T)$$

X_{N_τ} : known function G. Faldt and B. Petersson, Nucl. Phys. B 265, 197 (1986)

$X_{N_\tau} = 2\cosh(N_\tau\ \mathrm{arcsinh}\ (m_x/\gamma))$: Mean Field case