Polyakov line actions from SU(3) lattice gauge theory with dynamical fermions: first results via relative weights

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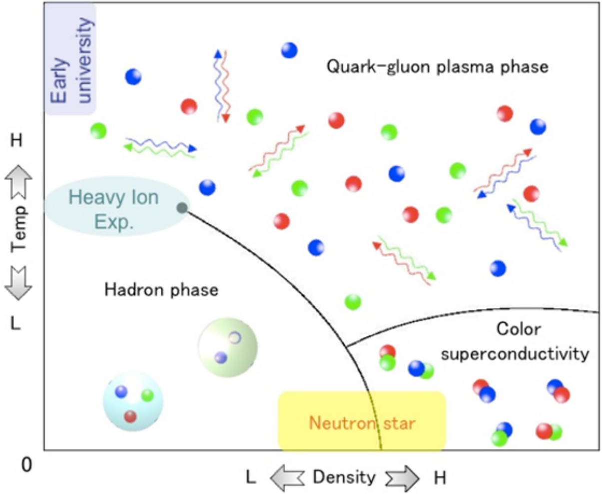
Agenda

Motivation

The Phase Diagram of QCD



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determinant is complex and satisfies

$$[\det\, M(\mu)]^* = \det\, M(-\mu^*)$$

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- finite chemical potential μ favors propagation of quarks

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- **Worldline formalism and strong coupling limit:**
  change order of integration, partial integration over loops and hopping parameter expansion

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- hard to compute $\exp[S_P(U_x)]$, use relative weights...

Relative Weights Method

- $S'_L \ldots$lattice action in temporal gauge with $U_0(\vec{x},0) = U'_x$, compute the ratio

$$\begin{aligned} e^{\Delta S_P} &= \frac{\exp[S_P(U'_x)]}{\exp[S_P(U''_x)]} = \frac{\int DU_k D\psi e^{S'_L}}{\int DU_k D\psi e^{S''_L}} \\ &= \frac{\int DU_k D\psi \exp[S'_L - S''_L] e^{S''_L}}{\int DU_k D\psi e^{S''_L}} \equiv \langle \exp[S'_L - S''_L] \rangle'' \end{aligned}$$

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$$U'_x = U_x(\lambda_0 + \Delta\lambda/2), U''_x = U_x(\lambda_0 - \Delta\lambda/2) \rightarrow (\frac{dS_P}{d\lambda})_{\lambda_0} = \frac{\Delta S}{\Delta\lambda}$$

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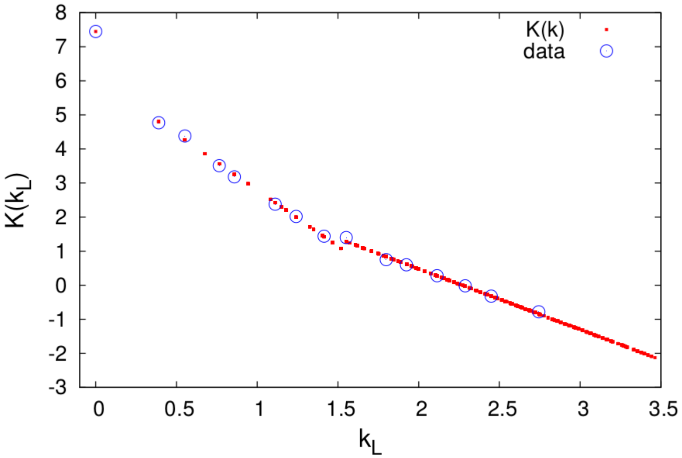
$$\frac{1}{L^3}\left(\frac{\partial S_P}{\partial a_k}\right)_{a_k=\alpha} = 2K(k)\alpha + \frac{p}{L^3}\sum_x (3he^{ikx} + 3h^2 e^{-ikx} + c.c.)$$

Preliminary Results



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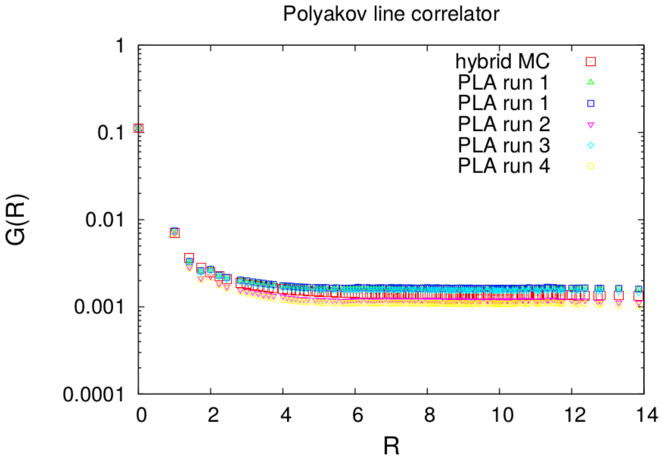

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- for details and comparison to complex Langevin see *Splittorff and Greensite (2012)*

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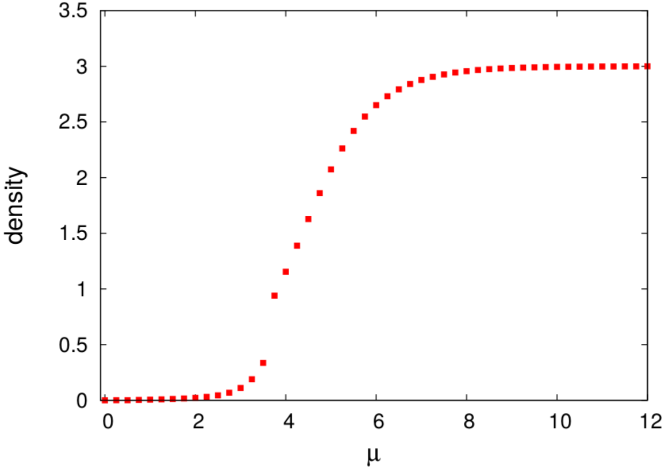

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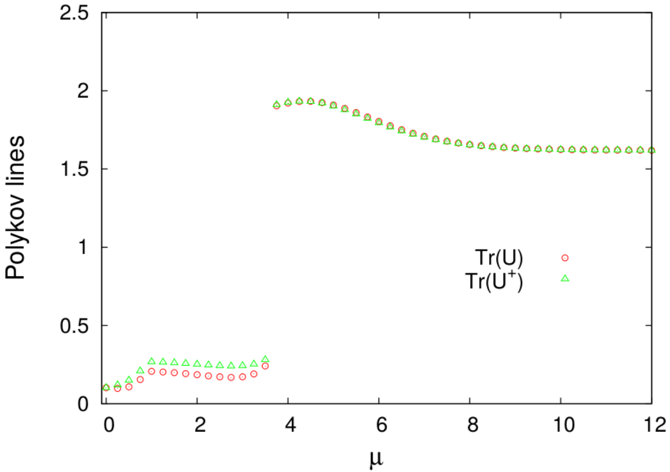

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- go on to smaller quark masses...

Questions?

Thank You &

Derar Altarawneh, Michael Engelhardt, Manfried Faber, Martin Gal, Jeff Greensite, Urs M. Heller, James Hettrick, Andrei Ivanov, Thomas Layer, Štefan Olejnik, Luis Oxman, Mario Pitschmann, Jesus Saenz, Thomas Schweigler, Wolfgang Söldner, David Vercauteren, Markus Wellenzohn

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