Thermal modifications of mesons and restoration of broken symmetries from spatial correlation functions in HISQ

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in collaboration with

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Introduction

Thermal fluctuation in QCD

Modifications of hadrons
- sequential melting pattern of quarkonium and open-flavor mesons
  e.g. $J/\psi$ suppression
  Matsui and Satz (1986)

Restorations of broken symmetries
- restored pattern of chiral and $U_A(1)$ symmetries
  Pisarski and Wilczek (1984)

Theoretical understanding in lattice QCD simulations from spatial correlation functions

Previous: strange-charm PRD91 (2015) 5, 054503
This work: including up/down at widely $T$ range
Hadronic excitation on Lattice

Temporal correlation function:

\[ G^T(\tau, T) = \int d^3x \langle \hat{J}_H^+(0, 0) J_H(\tau, x) \rangle \xrightarrow{\tau \to \infty} Ae^{-m_0 \tau} \]

...difficult due to the limitation \( \tau < 1/T \)

Spatial correlation function:

\[ G^S(z, T) = \int_{0}^{1/T} d\tau \int dx dy \langle \hat{J}_H^+(0, 0) J_H(\tau, x) \rangle \xrightarrow{z \to \infty} Ae^{-M(T)z} \]

No limitation to spatial direction: more sensitive to in-medium modification

\( M(T) \): screening mass
Hadronic excitation on Lattice

**Temporal correlation function:**

\[ G^T(\tau, T) = \int d^3x \langle J^\dagger_H(0, 0) J_H(\tau, x) \rangle \xrightarrow{\tau \to \infty} A e^{-m_0 \tau} \]

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**Spectral function**

\[ G^T(\tau, T) = \int_0^\infty d\omega \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)} \sigma(\omega, T) \]  

\[ G^S(z, T) = \int_0^\infty \frac{2d\omega}{\omega} \int_0^\infty dp_z e^{ip_z z} \sigma(\omega, p_z, T) \]

No \( T \) dependence in Kernel: direct probe of thermal modification of \( \sigma \)

\[ G^S(z, T)/G^S(z, T = 0) \]
Hadronic excitation on Lattice

Parity partner of meson states

**Vector** (vector and axial-vector)

\[ \bar{\psi} \gamma_i \psi \]
\[ \bar{\psi} \gamma_4 \psi \]

\[ 1^+ \]
\[ 1^- \]

\[ M_V(T) \]

Chiral

**Scalar** (pseudo-scalar and scalar)

\[ \bar{\psi} \gamma_5 \psi \]
\[ \bar{\psi} \psi \]

\[ 0^+ \]
\[ 0^- \]

\[ M_S(T) \]

\[ U_A(1) \]

Degeneracy of parity partners: indicator of symmetry restorations
**Hadronic excitation on Lattice**

### Parity partner of meson states

**Vector** (vector and axial-vector)

\[
\begin{align*}
\bar{\psi} \gamma_i \psi & \quad 1^+ \\
\bar{\psi} \gamma_4 \psi & \quad 1^- \\
\end{align*}
\]

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\end{align*}
\]

*Chiral* \[\Rightarrow\] Degeneracy of parity partners: indicator of symmetry restorations

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### Behavior in limiting cases:

**At low** \(T\), **bound state**: \(M(T) \sim m_0\) pole mass at \(T=0\)

\[
\sigma(\omega, 0, 0, p_z, T) \sim \delta(\omega^2 - p_z^2 - m_0^2)
\]

**At** \(T \sim T_c\), **in-medium modification and/or dissolution**

degeneracy of parity partner states

**At** \(T \to \infty\), **free quark-antiquark pair**: \(M \to 2\sqrt{m_q^2 + (\pi T)^2}\)

with the lowest Matsubara frequency
Lattice simulations

- Setup in HISQ
- Modifications of Mesons
- Restorations of broken symmetries
Highly Improved Staggered Quark

Reduction of taste violation
Control of cutoff effects

Bazavov et al. '11, Hot-QCD '11, '14

Lattice parameters

- 2+1 flavor QCD
  (charm quenched)
- $m_s$: physical, $m_I/m_s = 1/20$
  ($m_{\pi} \sim 160$ MeV, $m_K \sim 504$ MeV)
- $N_\tau = 8$ ($T = 110—207$ MeV)
  10 ($T = 139—166$ MeV)
  12 ($T = 149—400$ MeV)
  keeping $N_s/N_\tau = 4$
- $32^4--48^3\times64$ at $T = 0$
- scale: $f_k$ input
- calculating quark-line connected part of meson correlators

<table>
<thead>
<tr>
<th>$\Gamma$</th>
<th>$J^P$</th>
<th>$u\bar{d}$</th>
<th>$u\bar{s}$</th>
<th>$u\bar{c}$</th>
<th>$s\bar{s}$</th>
<th>$s\bar{c}$</th>
<th>$c\bar{c}$</th>
</tr>
</thead>
<tbody>
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<td>$\gamma_5$</td>
<td>0$^-$</td>
<td>$\pi$</td>
<td>$K$</td>
<td>$D$</td>
<td>$(\eta_{s\bar{s}})$</td>
<td>$D_s$</td>
<td>$\eta_c$</td>
</tr>
<tr>
<td>1</td>
<td>0$^+$</td>
<td>$K^*_0$</td>
<td>$D_0^*$</td>
<td>$-$</td>
<td>$D_{s0}^*$</td>
<td>$\chi_{c0}$</td>
<td></td>
</tr>
<tr>
<td>$\gamma_i$</td>
<td>1$^-$</td>
<td>$\rho$</td>
<td>$K^*$</td>
<td>$D^*$</td>
<td>$\phi$</td>
<td>$D_s^*$</td>
<td>$J/\psi$</td>
</tr>
<tr>
<td>$\gamma_i\gamma_5$</td>
<td>1$^+$</td>
<td>$K_1$</td>
<td>$D_1$</td>
<td>$f_{1}(1420)$</td>
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Reduction of taste violation
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### Mesons contents

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### Meson spectra at \(T = 0\) (input: ★)

- \([\text{MeV}]\) ud
- \([\text{MeV}]\) u\(\bar{s}\)
- \([\text{MeV}]\) u\(\bar{c}\)
- \([\text{MeV}]\) s\(\bar{s}\)
- \([\text{MeV}]\) s\(\bar{c}\)
- \([\text{MeV}]\) c\(\bar{c}\)
Ratio of spatial correlation functions

Probe of thermal modifications of spectral function

\[ \frac{G^S(z, T)}{G^S(z, T = 0)} \simeq 1 \text{ the same } \sigma \text{ at } T = 0, \text{ or } \neq 1 \text{ modified} \]

Pseudo-scalar $J^P = 0^-$

- $G^S(z, T)/G^S(z, 0) \simeq 1$ at short distance \( T_c = (154 \pm 9) \text{ MeV} \)
- $G^S(z, T)/G^S(z, 0) \neq 1$ at large distance physics: not sensitive to $T$
- $G^S(z, T)/G^S(z, 0) \neq 1$ at large distance thermal modification of $\sigma$
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Ratio of spatial correlation functions

Probe of thermal modifications of spectral function

\[ G^S(z, T)/G^S(z, T = 0) \approx 1 \] the same \( \sigma \) at \( T = 0 \), or \( \neq 1 \) modified

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modification at \( T < T_c \), explicit flavor dependence at \( T > T_c \)
probe of thermal modifications of spectral function

\[ \frac{G^S(z, T)}{G^S(z, T = 0)} \simeq 1 \quad \text{the same } \sigma \text{ at } T = 0, \text{ or } \neq 1 \text{ modified} \]

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- modification at \( T < T_c \), explicit flavor dependence at \( T > T_c \)
Mass difference

$\Delta M(T) = M(T) - m_0 \sim \text{change of “binding energy”}$

Pseudo-scalar $J^P = 0^-$

$\bullet$ $u\bar{d}$, $u\bar{s}$, $u\bar{c}$: explicit thermal modification below $T_c$, similar modification pattern at $T < T_c$, explicit flavor dependence at $T > T_c$
Mass difference

\[ \Delta M(T) = M(T) - m_0 \sim \text{change of “binding energy”} \]

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- $u\bar{d}$, $u\bar{s}$, $u\bar{c}$: explicit thermal modification below $T_c$,
  similar modification pattern at $T < T_c$,
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- $s\bar{s}$, $s\bar{c}$: slight modification below $T_c$
- $c\bar{c}$: stable beyond $T_c$

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Restoration of broken symmetries

Degeneracy of vector partners $\rightarrow$ restoration of chiral symmetry
Degeneracy of scalar partners $\rightarrow$ (effective) restoration of $U_A(1)$ symmetry

$G^S(z, T)$

$T = 139$ MeV = 0.90 $T_c$
Restoration of broken symmetries

Degeneracy of vector partners $\Rightarrow$ restoration of chiral symmetry
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$G^S(z, T)$

$T = 149 \text{ MeV} = 0.97 \ T_c$
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$G^S(z, T)$

Vector partner degenerates at $T \sim 1.0T_c - 1.1T_c$
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\[ G^S(z, T) \]

\[ T = 171 \text{ MeV} = 1.11 T_c \]

- Vector partner degenerates at \( T \approx 1.0 T_c \rightarrow 1.1 T_c \)
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\[ G^S (z, T) \]

- Vector partner degenerates at $T \sim 1.0T_c -- 1.1T_c$
- Scalar partner degenerates at $T \sim 1.4T_c -- 1.6T_c$

T = 220 MeV = 1.43 $T_c$
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- Vector partner degenerates at $T \sim 1.0 T_c \sim 1.1 T_c$
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Restoration of broken symmetries

Large distance behavior of spatial correlator $G^S(z, T) \xrightarrow{z \to \infty} A e^{-M(T)z}$

Light-unflavored $u \bar{d}$

- Vector partner degenerates at $T \sim 1.0T_c$--$1.1T_c$
- Scalar partner degenerates at $T \sim 1.4T_c$--$1.6T_c$

chiral: restored, $U_A(1)$: broken at $T_c$, no dependence on lattice spacing
Restoration of broken symmetries

Large distance behavior of spatial correlators

\[ \langle S(z, T) \rangle \sim z \rightarrow \infty, A e^{-M(T)z} \]

Light-unflavored \( u \bar{u} \) Vector partner degenerates at \( T \sim 1.0 \sim 1.1 T_c \)

Scalar partner degenerates at \( T \sim 1.4 \sim 1.6 T_c \)

Chiral: restored, \( U_{A}(1) \): broken at \( T_c \), no dependence on lattice spacing.
Summary

In-medium mesons from spatial correlation function

- Sensitive to thermal effect at finite $T$ on lattice
  - Direct probe of modification of meson spectral function
  - Indicator of restorations of broken symmetries

$(2+1)$-flavor QCD lattice simulations with HISQ of

\[ \frac{G^S(z, T)}{G^S(z, T = 0)} \text{, screening mass: } G^S(z, T) \xrightarrow{z \to \infty} A e^{-M(T)z} \]

- $u\bar{d}$, $u\bar{s}$, $u\bar{c}$: explicit thermal modification below $T_c$
  - similar modification pattern below $T_c$
  - explicit flavor dependence above $T_c$

- $s\bar{s}$, $s\bar{c}$: slight modification below $T_c$

- $c\bar{c}$: stable beyond $T_c$

Degeneracies of chiral partners

- chiral: restored, $U_A(1)$: broken at $T_c$
  - in continuum and physical quark mass (preliminary)

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Backup slides
Including negative (non-oscillating) and positive (oscillating) parity states

\[ G(z) = A_{NO}^2 e^{-M_-z} - (-1)^z A_O^2 e^{-M_+z} \]

Parameters: obtained by four successive data

\[ g_i \equiv G(z + i) \text{ with } i = 0, 1, 2, 3 \]

Effective masses \( x_\pm \equiv e^{-M_\pm} \)

in quadratic equations:

\[ Ax^2 + Bx + C = 0 \]

\[ x_\pm = \pm \frac{B}{2A} + \frac{\sqrt{B^2 - 4AC}}{2|A|} \]

\[ A = g_1^2 - g_2g_0, \quad B = g_3g_0 - g_2g_1, \quad C = g_2^2 - g_3g_1 \]

Then the effective correlators reconstructed:

\[ G_{NO}(z) \equiv A_{NO}^2 \bar{g}(z) e^{-M_-z} = \frac{g_1 + g_0x_+}{x_- + x_+} \]

\[ G_O(z) \equiv A_O^2 \bar{g}(z) e^{-M_+z} = (-1)^z \frac{g_1 - g_0x_-}{x_- + x_+} \]
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Degeneracy of vector partners $\rightarrow$ restoration of chiral symmetry
Degeneracy of scalar partners $\rightarrow$ (effective) restoration of $U_A(1)$ symmetry

$G^S(z, T)$

Vector partner degenerates at $T \sim 1.0 T_c - 1.1 T_c$
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$G^S(z, T)$

- Vector partner degenerates at $T \sim 1.0T_c--1.1T_c$
- Scalar partner degenerates at $T \sim 1.4T_c--1.6T_c$
- Spin dependence explicit at $T \sim 2.6T_c$

cf.) Thermal perturbation: no channel dependence  

Laine et al. 2004