Non-perturbative renormalization of the static quark theory in a large volume

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Introduction

Flavor physics

Heavy quark physics are believed to hold the keys to BSM physics. Many studies try to provide results for the heavy mesons' properties. The simulations involving heavy quarks are however difficult to perform straigthforwardly.

HQET: an effective field theory for QCD

HQET provides an effective description of QCD processes with initial and final states containing a single heavy quark.

- high momentum components of the massive quark field are integrated out; their contribution is summarized in the HQET parameters,
- low momentum components are present as a new two-component effective field ψ_h,
- out of which the explicit dependence on the quark mass is removed: all the masses computed within HQET must be shifted by m_{bare} ,

Introduction

HQET: an effective field theory for QCD

- the leading contribution of HQET is described by the static quark theory, which assumes that the heavy quark is infinitely heavy

$$\mathscr{L}_{\rm stat} = \bar{\psi}_h D_0 \psi_h$$

- $\bar{\psi}_h D_0 \psi_h$ has the same quantum numbers as the mass operator $\delta_m \bar{\psi}_h \psi_h$, dimensional analysis shows that $\delta_m \sim 1/a$,
- renormalization must be done non-perturbatively otherwise uncancelled divergent terms can combine with lattice artefacts giving finite, non-vanishing contributions
- available solution: renormalize HQET and match it to QCD in a small volume and evolve the parameters non-perturbatively (step scaling) to the large volume using SF boundary conditions,
- possible alternative: this talk.

Method

The primary object of interest is the correlator of two heavy-light currents:

$$C_\Gamma(t) = \langle \bar{\psi}_h(t)\Gamma\psi(t)\bar{\psi}(0)\Gamma\psi_h(0)\rangle$$

Static Lagrangian

We use the large distance behavior of $C_\Gamma(t)$ to subtract the linear divergence non-perturbatively by fitting and subtracting the slope:

$$C_\Gamma(t) \to e^{m_{\mathrm{bare}}\, t} C_\Gamma(t), \qquad m_{\mathrm{bare}} \sim \frac{1}{a} + \text{finite part}$$

Heavy-light currents

We use the short distance bahavior of $C_\Gamma(t)$ to define renormalization constants of heavy-light currents:

$$\left(Z_\Gamma^{\mathrm{X}}(t_0)\right)^2 C_\Gamma(t_0) = C_\Gamma^{\text{lattice tree-level}}(t_0)$$

Feasibility study

Ensemble

RBC's $16^3 \times 32$ lattice ensemble with $m_\pi \approx 420$ MeV, Iwasaki gauge action and domain wall light fermions, $a = 0.11\mathrm{fm} = 1.73\mathrm{GeV}^{-1}$

Details

- the test study was done using 20 configurations separated by 200 MDU
- stochastic wall source, one per configuration $\rightarrow$ we observed improvement of precision of the correlator at small distances
- two additional masses were measured $\rightarrow$ $am = 0.005, 0.01, 0.02$, naive chiral extrapolation for the renormalization constants

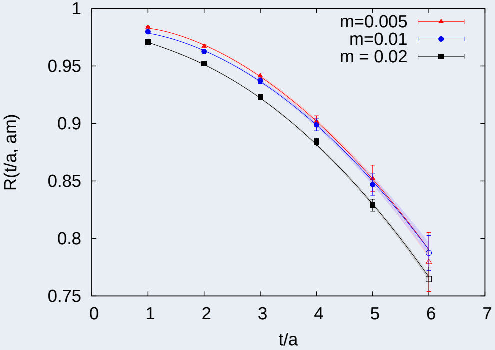
Preliminary results

$Z_A^{\mathrm{stat}} / Z_V^{\mathrm{stat}}$ as a check of precision

In the ratio

$$R(t_0/a, am) = Z_A^{\mathrm{stat}}(t_0/a, am)/Z_V^{\mathrm{stat}}(t_0/a, am)$$

the factor with m_{bare} cancels and the ratio can be evaluated without additional renormalization conditions.



Preliminary results

$Z_A^{\rm stat} / Z_V^{\rm stat}$ as a check of precision

Combined, correlated fit was performed using the ansatz

$$R(t_0/a, am) = R + \alpha/(t_0/a)^2 + \beta(t_0/a)^2 + \gamma am$$

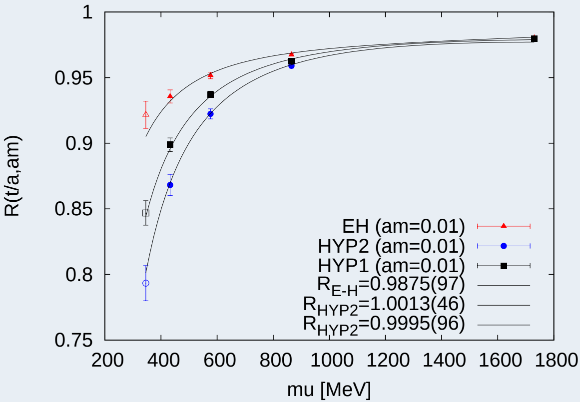
→ the discretization errors accounted in the fit ansatz turned out to be very small.

	R	α	β	γ
E-H	0.9875(9)(97)	0.0041(7)(422)	-0.0030(2)(5)	-0.79(4)(40)
HYP1	0.9995(14)(96)	-0.0033(12)(50)	-0.0057(2)(3)	-1.16(9)(37)
HYP2	1.0013(27)(38)	-0.0075(32)(40)	-0.0076(3)(2)	-0.92(12)(10)

→ first error is statistical, second systematic

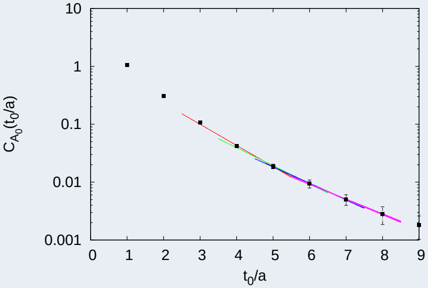
Preliminary results

$Z_A^{\rm stat} / Z_V^{\rm stat}$ as a check of precision



Preliminary results

Mass renormalization

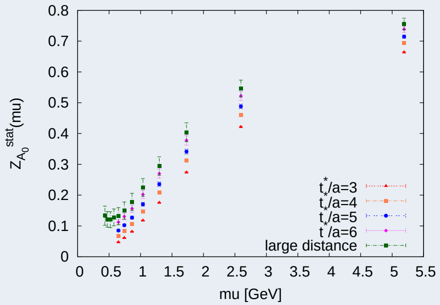


We renormalize the heavy mass by setting the local slope to 0:

$$\delta m(t^*) \sim -\log\left(\frac{C_\Gamma(t^*+1)}{C_\Gamma(t^*)}\right)$$

Preliminary results

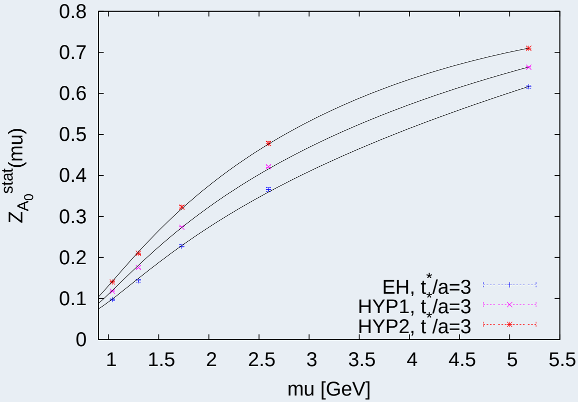
$Z_A^{\rm stat}$ and $Z_V^{\rm stat}$ in position space scheme



- HYP1,
- in the chiral limit,
- mass renormalization condition at m=0.005,
- different colors correspond to different t^*.

Preliminary results

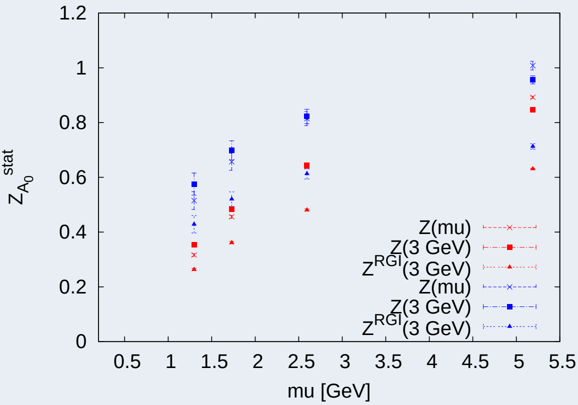
Z_A^{stat} and Z_V^{stat} in position space scheme



→ estimated size of discretization errors ~ 0.005

Z_A^{stat} and Z_V^{stat} in $\overline{MS}$ scheme at 3 Gev

We use a one-loop convertion factor between continuum HQET position space scheme and continuum HQET $\overline{MS}$ scheme and two-loop running.



Preliminary results

Z_A^{stat} and Z_V^{stat} in $\overline{MS}$ scheme at 3 Gev

We use a one-loop convertion factor between continuum HQET position space scheme and continuum HQET $\overline{MS}$ scheme.

scale [GeV]	α_s	$Z_{A_0}^{\mathrm{stat},\overline{MS}}(3GeV)$	$Z_{A_0}^{\mathrm{stat},\overline{MS},\mathrm{RGI}}$
5.44	0.1967	0.956(15)	0.712(10)
2.72	0.2563	0.822(26)	0.613(19)
1.81	0.3166	0.698(35)	0.520(27)
1.36	0.3864	0.575(40)	0.428(32)

Table: Numerical values of renormalization constants in $\overline{MS}$.

Discussion and conclusions

The main ingredients of our proposal are

- position space renormalization condition
- reduction of cut-off effects through a tree-level improvement
- stochastic wall source

It's main advantages are

- gauge invariance
- on-shell
- no need to compute field renormalization constants

Possible issues

- usual window problem $\rightarrow$ finer lattices, step scaling

Next steps

- full RI-MOM study
- four-fermion operators