Polyakov loop renormalization with gradient flow

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Introduction

- Polyakov loop UV divergent $\rightarrow$ needs renormalization.
- Use gradient flow as a direct method of renormalization.
- We use HotQCD HISQ $N_f = 2+1$ ensembles with $M_\pi = 160$ MeV on $24^3 \times 6$, $32^3 \times 8$, $40^3 \times 10$ and $48^3 \times 12$ lattices.
- Large temperature range from $T = 100$ up to $T = 1000$ MeV.
- Higher representations accessible with small errors.
- Can be used as check of large-N models/weak coupling.

Fundamental Polyakov loop

- **In the continuum:** Local Polyakov loop at a spatial point x defined as the trace of the path-ordered exponential

$$L(x) \;=\; \mathrm{Tr}\,\mathscr{P}\exp\left(i\int_0^{1/T} A_4(x,t)\,dt\right)\,,$$

with t the euclidean time.

- **On the lattice:**

$$L_3(x) = \mathrm{Tr}\prod_{t=1}^{N_t} U_4(x,t)\,,$$

with $U_4(x,t) \in SU(3)$.

Expectation value:

$$\langle|P_3|\rangle = \frac{1}{V}\left\langle\left|\sum_x L_3(x)\right|\right\rangle = Ce^{-F_3/T}\,.$$

Polyakov loop renormalization

- Multiplicative renormalization of the Polyakov loop:

$$P_N^{ren}(T) = (Z_N(T))^l P_N(T) ,$$

which depends on the coupling and the length of the contour l.

- On the lattice this is done as

$$P_N^{ren}(T) = e^{-c(a)N_t} P_N(T) ,$$

with some lattice spacing dependent constant c which has to be determined (e.g., use static potential at a certain distance O. Kaczmarek, et.al., Phys. Lett. B 543 (2002) 41, [arXiv:hep-lat/0207002]).

- For the free energy $F = -T \ln P_N$ this is a lattice spacing dependent shift.

Gradient flow

Gradient flow (or “Wilson flow” with Wilson gauge action):
M. Lüscher, JHEP 1008 (2010) 071, [arXiv:1006.4518 [hep-lat]]

$$\dot{V}_t(x,\mu) = -g_0^2(\partial_{x,\mu} S(V_t))\, V_t\ ,$$

with

$$V_t(x,\mu)|_{t=0} = U(x,\mu)\ .$$

$U(x,\mu)$ are our usual SU(3) link matrices and t is new index for the flow time.

Here we use Symanzik flow, i.e., Symanzik gauge action for $S(V_t)$. See also
Z. Fodor, et.al., (2014), arXiv:1406.0827 [hep-lat].

The flow equation is solved by a RK like scheme up to the desired value of t.

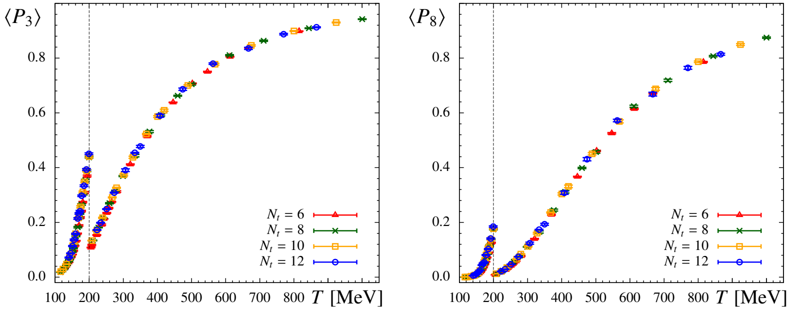
Renormalization procedure

- Evolution of the flow up to a certain value of the flow $f = \sqrt{8t}$ in physical units (fm) (t has dimension a^2).
- Choice corresponds to a certain renormalization scale.
- Different renormalization scales are related by a constant shift in the free energy (if cut-off effects under control).
- Casimir scaling: Different representations can be related to each other.

Higher representations

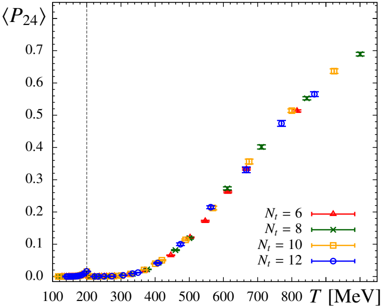
- From group theory one can derive the Polyakov loop in various representations. See, e.g., S. Gupta, et.al., Phys.Rev. D77 (2008) 034503, [arXiv:0711.2251 [hep-lat]].
- Relations make it easy to calculate P for arbitrary representations.
- We calculate it for sextet, adjoint, decuplet, ... up to 27-plet (in total 8 representations).
- The direct renormalization with gradient flow makes it easy to extract the free energy for higher representations at low T ($T \approx 120$ MeV).

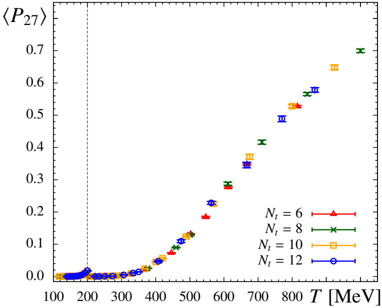
Renormalized Polyakov loop - 1



Note: We switch flow time at $T = 200$ MeV.
After continuum extrapolation: Match by constant shift of free energy.

Renormalized Polyakov loop - 2





Note: We switch flow time at $T = 200$ MeV.
After continuum extrapolation: Match by constant shift of free energy.

Casimir scaling - 1

- Casimir scaling:

$$(P_3)^{1/d_3} = (P_6)^{1/d_6} = \dots \,,$$

  with $d_N = C_2(N)/C_2(3)$, the ratio of the quadratic Casimirs of the fundamental and N-representation.

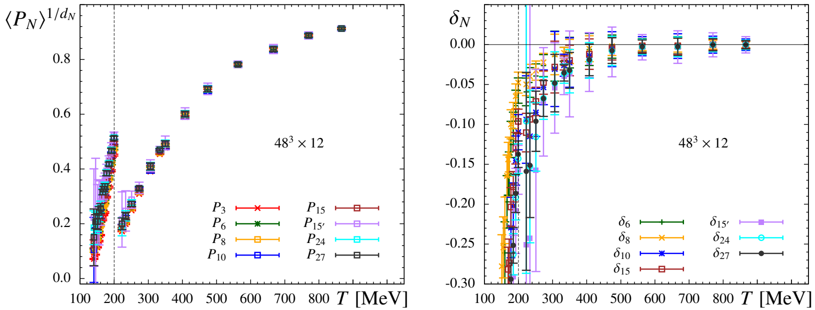
- This means: $(P_N)^{1/d_N}$ independent of the representation N.

- Shown to hold in perturbation theory at least up to $O(g^4)$
  Y. Schröder, Phys. Lett. B447 (1999) 321, [arxiv:hep-ph/9812205].

- Has been tested on the lattice, e.g.,
  S. Gupta, et.al., Phys.Rev. D77 (2008) 034503, [arXiv:0711.2251 [hep-lat]].

- Note: If Casimir scaling holds, we can relate the renormalization constants $c(a)$ from different representation with each other.

Casimir scaling - 2

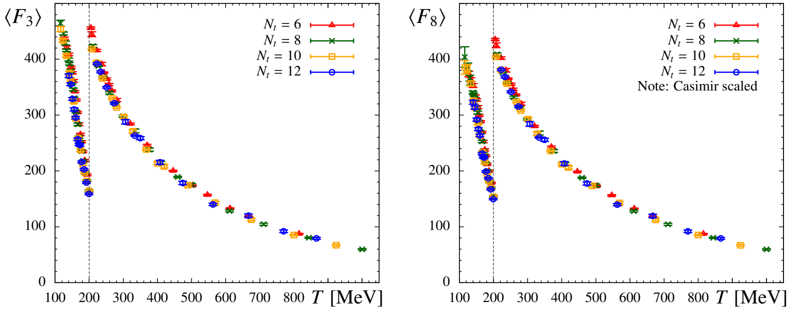


Measure for the Casimir scaling violation:

$$\delta_N = P_N^{1/d_N} / P_3 - 1$$

Note: Large error bars, but clear trend visible.

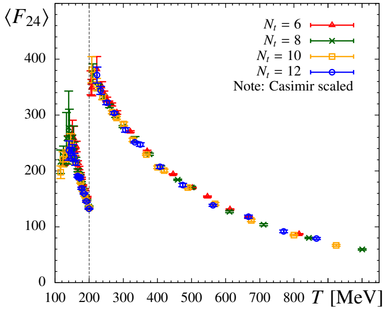
Free energy of a static quark - 1

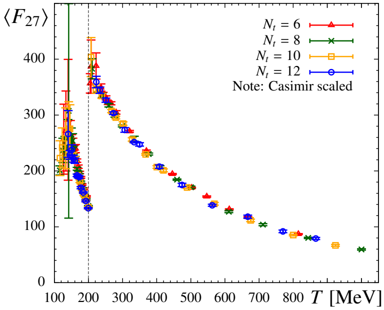


Recall the relation of the Polyakov loop to the “free energy”:

$$P_N = Ce^{-F_N/T}$$

Free energy of a static quark - 2





Recall the relation of the Polyakov loop to the "free energy":

$$P_N = Ce^{-F_N/T}$$

Conclusion:

- Direct way of renormalizing the Polyakov loop.
- Renormalization scale is set by the flow time t (in fm).
- Polyakov loop “easy target” for a test of that kind of renormalization.
- Comparison with usual renormalization procedures straightforward.

Outlook:

- Continuum extrapolation.
- Comparison with other renormalization procedures and model calculations.
- Look into other quantities:
  Gluon condensate, topological susceptibility, ...