Computing the long-distance contributions to ε_K

Lattice 2015 – Kobe

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RBC and UKQCD Collaborations

Overview

- ε_K : Indirect CP violation in $K^0 \rightarrow \pi\pi$ decay.
  - Experiment: 2.228(11) x 10^{-3}
  - Theory: 2.13(23) x 10^{-3} [SWME arXiv:1503.06613]
- Theory error dominated by $(V_{cb})^4$ factor
- ε_K decomposition:
  - 2/3 top quark loop
  - 1/3 top and charm

  } short distance
  - 2 - 4% long distance contribution

RBC Collaboration

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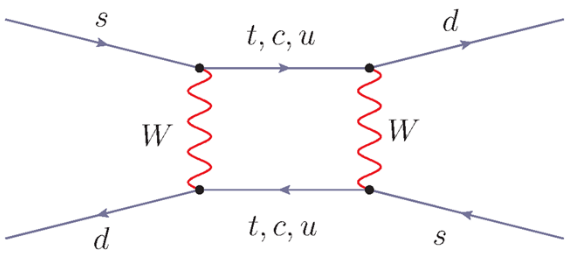
  - Marina Marinkovic

$K^0 - \overline{K^0}$ mixing

- ΔS=1 weak interactions allow $\overline{K^0}$ and K^0 to mix.



- CP violation leads to decay eigenstates K_L and K_S which are not CP eigenstates:

$$K_S = \frac{K_+ + \overline{\epsilon} K_-}{\sqrt{1 + |\overline{\epsilon}|^2}}$$

$$K_L = \frac{K_- + \overline{\epsilon} K_+}{\sqrt{1 + |\overline{\epsilon}|^2}}$$

- Here ε is closely related to:

$$\epsilon_K = \overline{\epsilon} + i \frac{\mathrm{Im} A_0}{\mathrm{Re} A_0}$$

Conventional calculation of ε_K

- Introduce $\Delta S = 2$ effective Hamiltonian:

ΔM_K **top** **top-charm**

$$\mathcal{H}_{\text{eff}}^{\Delta S=2} = \frac{G_F^2}{16\pi^2}\Big[\lambda_c^2\eta_{cc}S_0(x_c) + \lambda_t^2\eta_{tt}S_0(x_t) + 2\lambda_c\lambda_t\eta_{ct}S_0(x_c,x_t)\Big]b(\mu)Q_{LL}$$

$$\lambda_i = V_{i,d}^*V_{i,s}$$

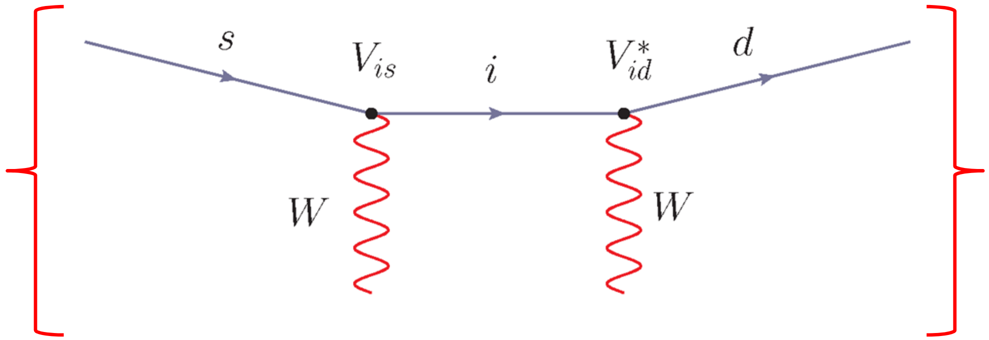
- S_0 is the 1-loop Inami-Lim function and $x_q = \left(\frac{m_q}{M_W}\right)^2$
- Lattice QCD gives the K^0-$\overline{K}^0$ matrix element of Q_{LL}:

$$B_K = \frac{3\langle\overline{K}^0|Q_{LL}|K^0\rangle}{8f_K^2M_K^2} \quad \text{where} \quad Q_{LL} = \overline{s}(1-\gamma^5)d\,\overline{s}(1-\gamma^5)d$$

Standard Model Review

- Three up-type propagators:

$$\sum_{i=u,c,t}$$



- GIM subtraction:

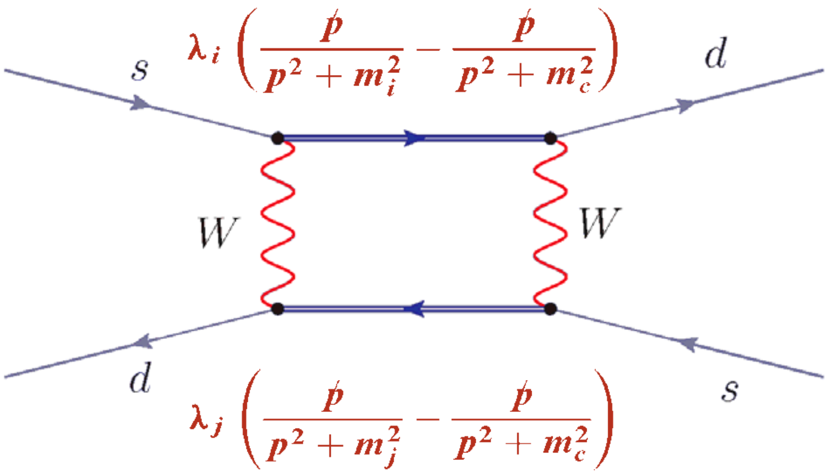
$$\sum_{i=u,c,t}\left\{V^*_{i,d}\frac{\not p}{p^2+m_i^2}V_{i,s}-V^*_{i,d}\frac{\not p}{p^2+m_c^2}V_{i,s}\right\}$$

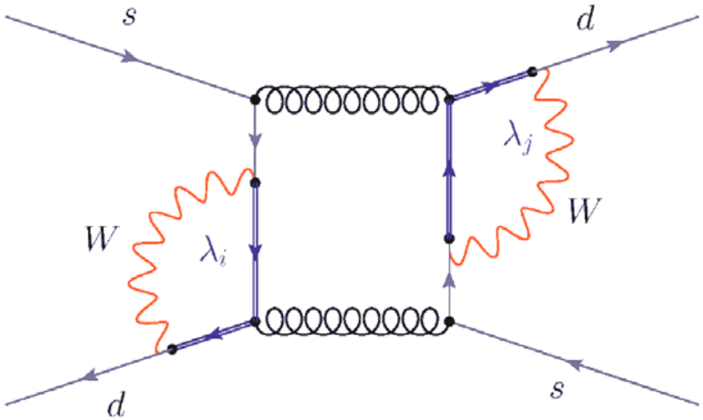
$$=\lambda_t\left\{\frac{\not p}{p^2+m_t^2}-\frac{\not p}{p^2+m_c^2}\right\}+\lambda_u\left\{\frac{\not p}{p^2+m_u^2}-\frac{\not p}{p^2+m_c^2}\right\}$$

$$\lambda_i = V^*_{i,d}\,V_{i,s}$$

Six contributions to ΔM_K and ε_K

- Six types of diagram:

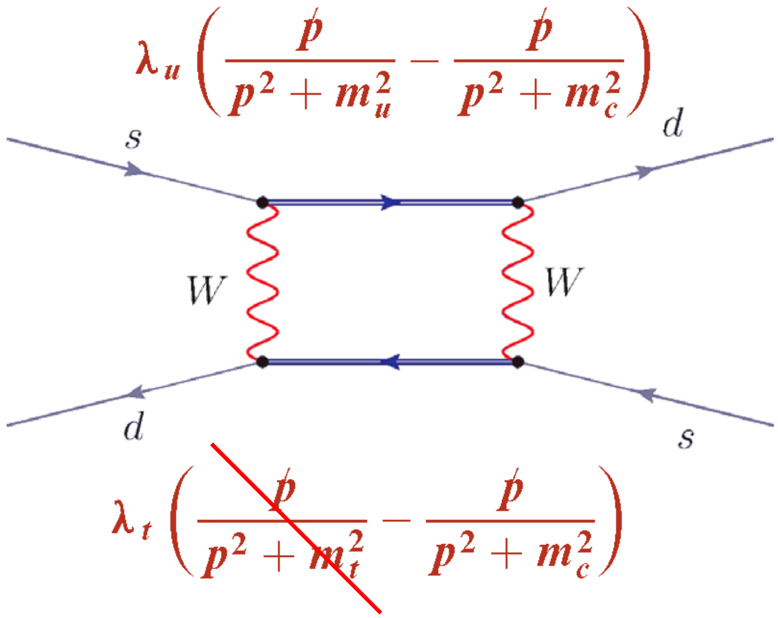


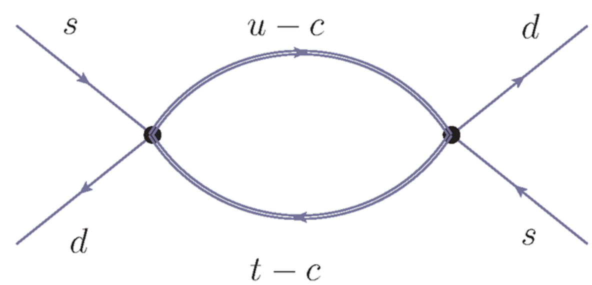


$\lambda_i\lambda_j = \lambda_t\lambda_t$, $\lambda_u\lambda_u$ and $\lambda_t\lambda_u$

- ΔM_K: $\lambda_u \lambda_u$ **term**
- ε_K: $\lambda_t \lambda_t$ **and** $\lambda_u \lambda_t$ **term**

$\lambda_u\ \lambda_t$ Box diagram





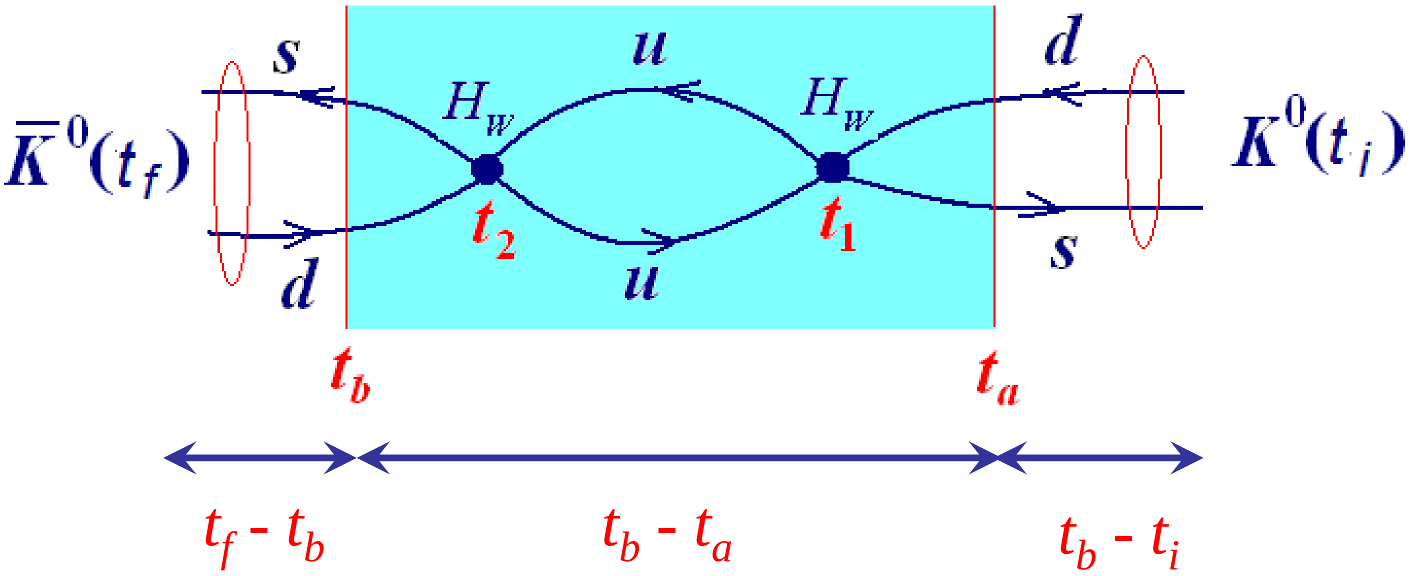
$\lambda_u\ \lambda_t$ ~0.22x(3.2 –*i* 1.34)x10^{-4}

- Enhanced by $\lambda_u / \lambda_t \sim 10^3$
- Suppressed by $(m_c/m_t)^2 \sim 4.5 \times 10^{-5}$
- Long distance contributes ~ 4% to ε_K

Lattice Version

- Evaluate standard, Euclidean, 2nd order $K^0 - \overline{K^0}$ amplitude:

$$\mathcal{A} = \langle 0|T\left(K^0(t_f)\frac{1}{2}\int_{t_a}^{t_b} dt_2 \int_{t_a}^{t_b} dt_1 H_W(t_2) H_W(t_1) K^{0'}(t_i)\right)|0\rangle$$



Interpret Lattice Result

$$\mathcal{A} = N_K^2 e^{-M_K(t_f-t_i)} \sum_n \frac{\langle \overline{K}^0 | H_W | n\rangle\langle n | H_W | K^0\rangle}{M_K - E_n}\left(\underbrace{-(t_b - t_a)}_{1.} \underbrace{- \frac{1}{M_K - E_n}}_{2.} \underbrace{+ \frac{e^{(M_K-E_n)(t_b-t_a)}}{M_K - E_n}}_{3.}\right)$$

1. Δm_K^{FV}
2. Uninteresting constant
3. Growing or decreasing exponential: states with $E_n < m_K$ must be removed!

- Finite volume correction:

$$M_{K_L} - M_{K_S} = 2\sum_n \frac{\langle \overline{K}^0 | H_W | n\rangle\langle n | H_W | K^0\rangle}{M_K - E_n} - 2\frac{d(\phi + \delta_0)}{dk}\Big|_{m_K} |\langle n_0 | H_W | K^0\rangle|^2 \cot(\phi + \delta_0)\Big|_{M_K}$$

(N.H. Christ, X. Feng, G. Martinelli, C.T. Sachrajda, (Phys.Rev. D91 (2015) 11, 114510 [arXiv:1504.01170])

ΔS = 1, four-flavor operators

- Choose appropriate N_f=4 effective Hamiltonian:

$$H_W^{\Delta S=1;\Delta C=\pm 1,0} = \frac{G_F}{\sqrt{2}} \left\{ \sum_{q,q'=u,c} V^*_{q's} V_{qd} \sum_{i=1}^{2} C_i Q_i^{q'q} + V^*_{ts} V_{td} \sum_{i=3}^{6} C_i Q_i \right\}$$

$$\begin{aligned} Q_1^{q'q} &= (\bar{s}_i q'_j)_{V-A} (\bar{q}_j d_i)_{V-A} \\ Q_2^{q'q} &= (\bar{s}_i q'_i)_{V-A} (\bar{q}_j d_j)_{V-A} \end{aligned}$$

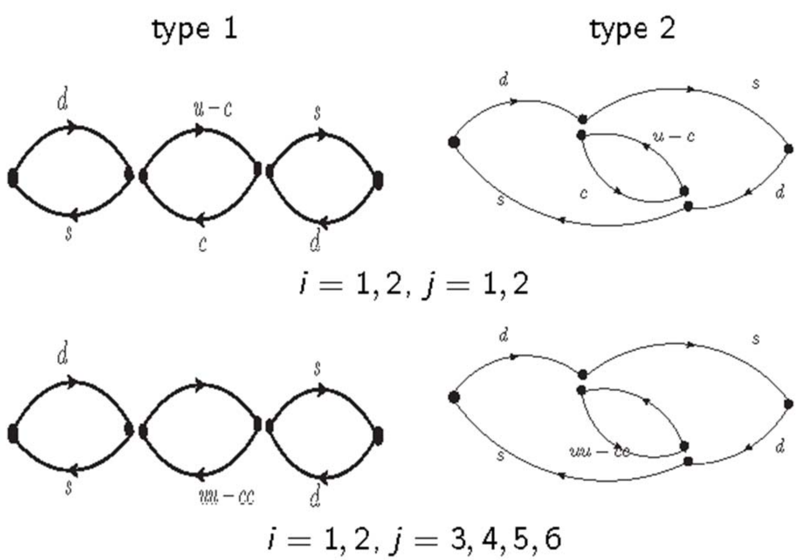
current x current

$$\begin{aligned} Q_3 &= (\bar{s}_i d_i)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_j)_{V-A} \\ Q_4 &= (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_i)_{V-A} \\ Q_5 &= (\bar{s}_i d_i)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_j)_{V+A} \\ Q_6 &= (\bar{s}_i d_j)_{V-A} \sum_{q=u,d,s,c} (\bar{q}_j q_i)_{V+A} \end{aligned}$$

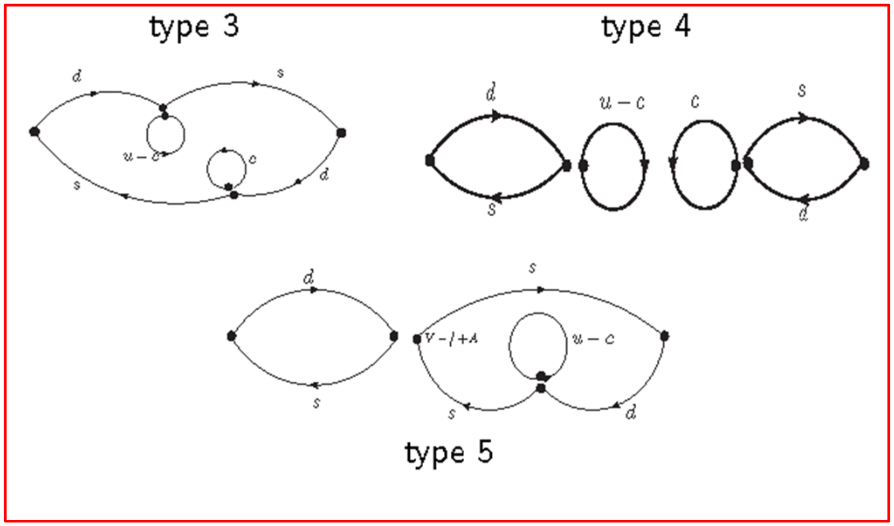
QCD penguin

Diagrams for $\lambda_t \lambda_u$ contribution to ε_K

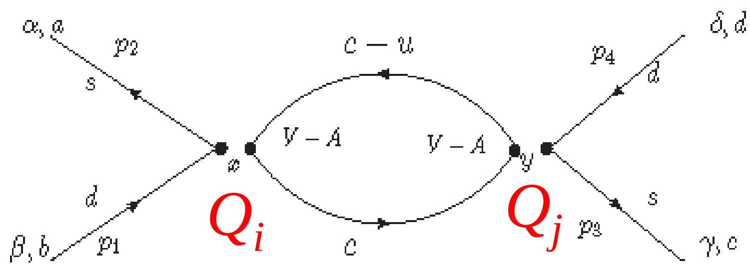
- Identify five types of diagrams



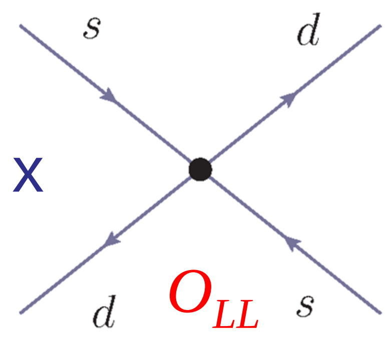
Omit from 1st study



Correcting short distance part



$+ \quad E_{ij}(\mu)\ \times$



$- E_{ij}^{\text{lat}}(\mu) + E_{ij}^{\text{pert}}(\mu)$

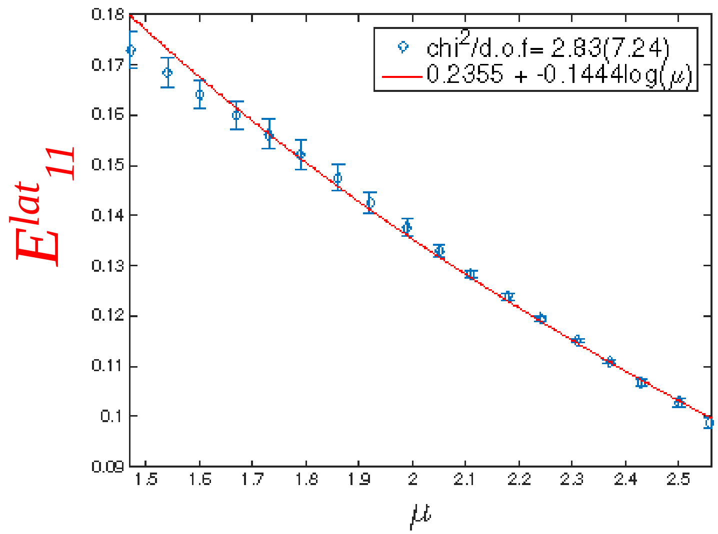
- Add $E_{ij}(\mu)\ \big(\bar{s}\gamma^{\nu}(1\text{-}\gamma^5)d\big)\ \big(\bar{s}\gamma^{\nu}(1\text{-}\gamma^5)d\big)$
- Evaluate off-shell Green's function at $\boldsymbol{p_i^2} = \mu^2$
- Forces internal momentum also to the scale μ or greater
- μ must obey:
  - $\alpha_S(\mu^2) << 1$ so perturbation theory is accurate.
  - $a\,\mu << 1$ to avoid lattice artifacts.

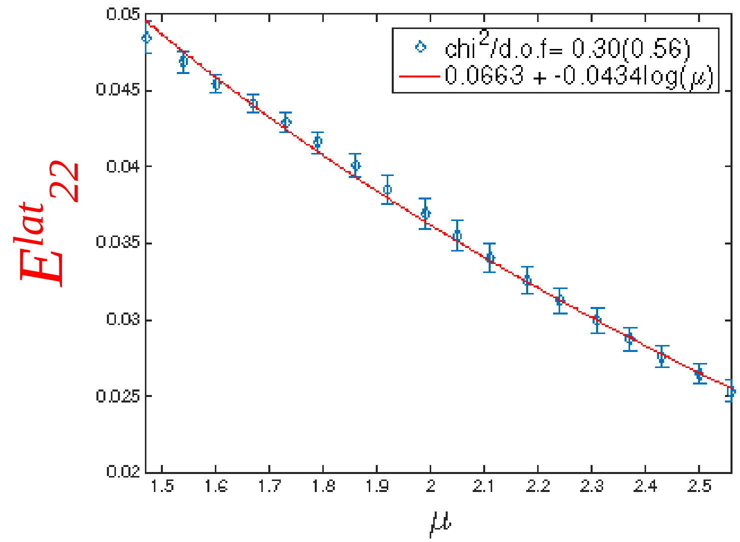
Simulation details

- Use 24^3 x 64, $1/a$ = 1.73 GeV ensemble
- m_π = 329 MeV, m_K = 575 MeV, m_c = 941 MeV (0.363/a)
- Average over 64 separate, time-translated measurements on 200 configurations.
- Use low-mode deflation with 300 Lanczos eigenvectors
- Study 1.4 GeV $\leq \mu \leq$ 2.6 GeV

Short-distance lattice correction

- Results for short-distance coefficient E_{11} and E_{22} of O_{LL} for the products Q_1Q_1 and Q_2Q_2:





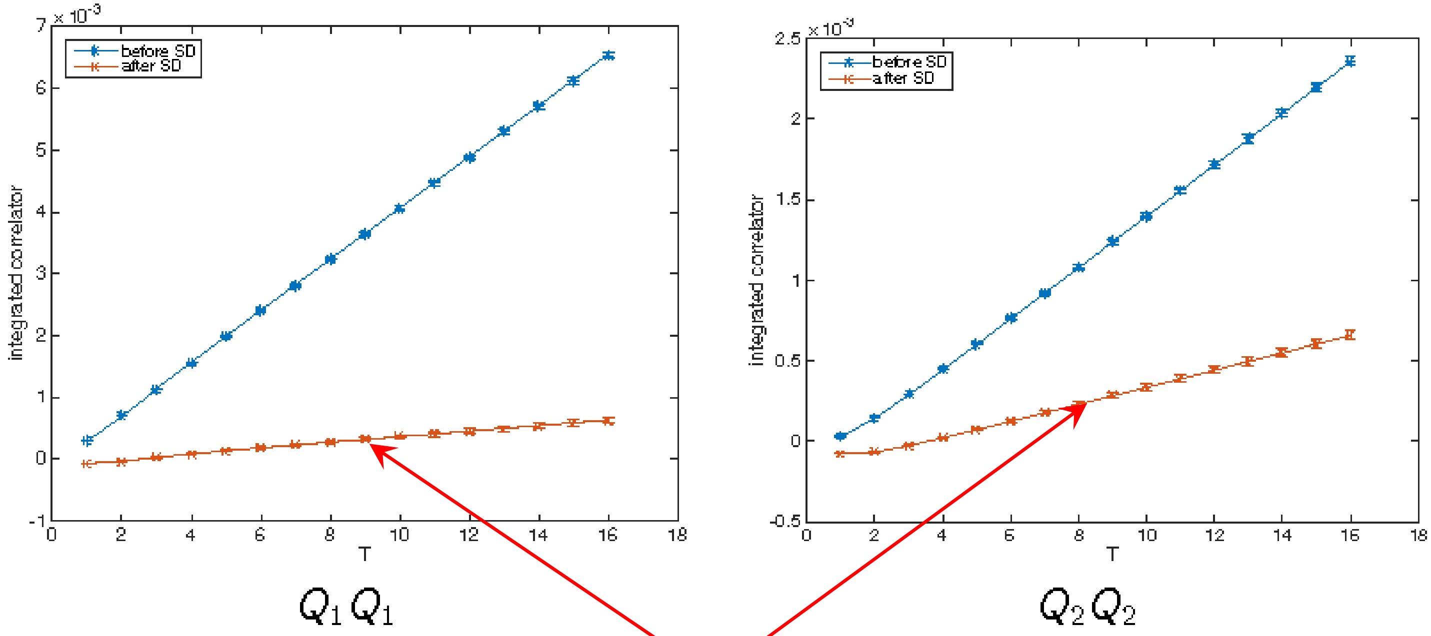
- Effect of a cutoff radius $|x - y| < R$ at $\mu = 1.93$ GeV

Cutoff	3	4	5	6	none
E_{11}^{lat}	0.1462	0.1501	0.1493	0.1489	0.1489
E_{22}^{lat}	0.0418	0.0427	0.0425	0.0425	0.0425

Integrated Correlators

Preliminary

$$\langle \overline{K^0} | \int_0^T dt_2 \int_0^T dt_1 T\left\{ Q_i(t_1) Q_j(t_2) \right\} | K^0 \rangle$$



After short-distance subtraction

Combine short(PT)- and long(lattice)-distance parts

Preliminary

- Use C. Lehner's ***PhySyHCAl*** to add back the correct perturbative short distance part at LO.

μ (GeV)	Im $M_{0\bar{0}}^{ut,ld}$ (10^{-15} MeV)	Im $M_{0\bar{0}}^{ut,cont}$ (10^{-15} MeV)	Im $M_{0\bar{0}}^{ut}$ (10^{-15} MeV)
1.54	-0.871(30)	-4.772(56)	-5.642(64)
1.92	-1.065(30)	-4.546(54)	-5.601(62)
2.11	-1.151(31)	-4.435(52)	-5.586(61)
2.31	-1.226(31)	-4.350(51)	-5.576(60)
2.56	-1.302(30)	-4.208(50)	-5.511(58)

- Result: ***tt*** ***ut*$_{sd}$** ***ut*$_{ld}$** **Im(A_0)**

$|\varepsilon_K| = (1.806 + 0.892 + 0.209(6) + 0.111) \times 10^{-3}$

0.097 naïve PT

$= 3.019(45) \times 10^{-3}$

($2.228(11) \times 10^{-3}$ expt.)

Outlook

- Long-distance component of ε_K will be important when $\boldsymbol{V_{cb}}$ becomes better known.
- Accessible to lattice QCD.
- Controlled, 15-20% errors should be possible in ~2-3 years, with $1/a \geq 3$ GeV ensembles.
- Perturbative calculation of short distance part needed. (Use Pauli-Villars rather than Rome-Southampton methods? Xu Feng)