The Strange Quark Spin in Nucleon from Anomalous Ward Indentity

Ming Gong, Yibo Yang, Keh-Fei Liu, Mingyang Sun

University of Kentucky

Institute of High Energy Physics, Chinese Academy of Sciences

Abstract

We report the quark spin contribution from strange quark in nucleon calculation from the anomalous Ward identity with overlap fermions. Such a formulation decomposes the divergence of the flavor-singlet axial-vector current into a quark pseudoscalar term and a triangle anomaly term. We use the overlap fermion for the valence and the quark loop so that the renormalization constants Z_m and Z_p cancel in the pseudoscalar operator $2mP$. In addition, the overlap operator is used to calculate the local topological charge in the anomaly so that there is no renormalization for the anomaly term either. We also make a global fit among different momenta to disentangle h_A from g_A.

We observed that the contribution from the anomaly term is negative and the contribution from the pseudoscalar term is positive, and the net contribution of strange quark is negative. A preliminary analysis shows that the strange quark contribution to flavor-singlet $g_A(0)$ is $-0.068(8)$ on one ensemble with $m_\pi = 330MeV$.

Introduction

Apportioning the spin of nucleon among its constituents of quarks and gluon is one of the most challenging issues in QCD both experimentally and theoretically. It is shown by X. Ji (1997) that there is a gauge-invariant separation of the nucleon spin operator into the quark spin, quark orbital angular momentum, and glue angular momentum operators. The recent lattice studies (G. S. Bali et al., 2012; M. Engelhardt, 2012; A. Abdel-Rehim et al., 2013; R. Babich et al., 2012) with light dynamical fermions found that the strange quark spin (Δ_s) is in the range from -0.02 to -0.03 which is several time smaller than that from a global fit of DIS and semi-inclusive DIS (D. de Florian et al., 2009) which gives $\Delta_s \approx -0.11$. Such a discrepancy between the global fit of experiments and the lattice calculation of the quark spin from the axial-vector current has raised a concern that the renormalization constant for the flavor-singlet axial-vector current could be substantially different from that of the isovector axial-vector current since the latter is commonly used for the lattice calculations of the flavor-singlet axial-vector current for the quark spin. To alleviate this concern, we use the anomalous Ward identity (AWI) together with the flavor-singlet axial-vector current to calculate the quark spin.

The anomalous Ward identity includes the triangle anomaly in the divergence of the flavor-singlet axial-vector current

$$\partial^\mu A^0_\mu = 2\sum_{f=1}^{N_f} m_f \bar{q}_f i\gamma_5 q_f + 2iN_f q, \quad (1)$$

where q is the local topological charge operator and is equal to $\frac{1}{16\pi^2} tr_c G_{\mu\nu}\tilde{G}_{\mu\nu}$ in the continuum. We put this identity between the nucleon states and calculate the matrix element on the right-hand side with a momentum transfer $\vec{q}$

$$\langle p's\,|A_\mu|\,ps\rangle s_\mu = \lim_{\vec{q}\to 0} \frac{i|\vec{s}|}{\vec{q}\cdot\vec{s}} \langle p', s|2\sum_{f=1}^{N_f} m_f \bar{q}_f i\gamma_5 q_f + 2iN_f q\,|p, s\rangle. \quad (2)$$

The relation of the form factors of the anomalous Ward identity is

$$2m_q g_P(Q^2) + 2g_{G\tilde{G}} = 2m g_A^R(Q^2) - Q^2 h_A^R(Q^2) \quad (3)$$

We calculate the form factors from nucleon three-point functions:

$$\lim_{t_f - t, t\to\infty} \frac{\Gamma^i C_3^{A_j}(t_f, \vec{0}; t, \vec{q})}{C_2(t_f, \vec{0})} e^{(E_q - m)t} = \frac{m + E_q}{2E_q}\delta_{ij} g_A^b(Q^2) - \frac{q_i q_j}{2E_q} h_A^b(Q^2) \quad (4)$$

and

$$\lim_{t_f - t, t\to\infty} \frac{\Gamma^i C_3^{mP+G\tilde{G}}(t_f, \vec{0}; t, \vec{q})}{C_2(t_f, \vec{0})} e^{(E_q - m)t} = \frac{q_i}{2E_q}\left(m_q g_P(Q^2) + g_{G\tilde{G}}(Q^2)\right) \quad (5)$$

After all, we do a global fit to extract renormalized $g_A^R(Q^2)$ and $h_A^R(Q^2)$.

Numerical details

We adopt the overlap fermion as the valence quark on 2+1 flavor domain-wall fermion (DWF) sea on 203 configurations from RBC-UKQCD collaboration. The lattice size is $24^3 \times 64$ with the spacing $a^{-1} = 1.77GeV$ and pion mass at $m_\pi = 330MeV$.

The overlap propagators are calculated with the deflation algorithm on hyp-smeared gauge field to improve the efficiency (Li et al., 2010). The lowest 200 pairs of overlap eigenvectors of each configuration are prepared for the deflation algorithm.

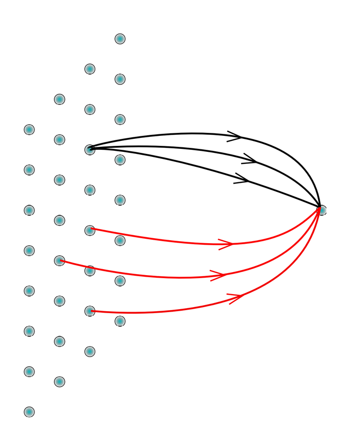

Figure 1: Diagram to illustrate the signal and noise of the nucleon correlation function with Z_3 noise grid source on a time slice. The upper part with three quarks originating from the same spatial site is an example of the gauge-invariant signal and the lower one is an example of the gauge-noninvariant noise which will be suppressed by gauge average and noise average.

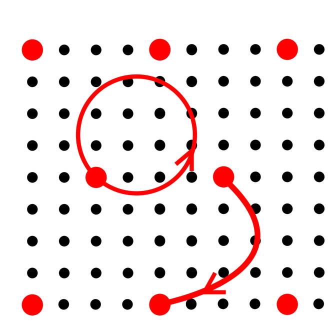

Figure 2: Diagram to show the finite-noise contribution of the loops. The red dots are the grid sources on even sites. The closed loop shown is an example of the signal and the open-jawed curve is an example of a noise contribution which is suppressed by gauge and noise averaging and by the finite distance separation of the quark propagator.

We calculate the contractions of the nucleon correlation functions and the quark loops with the low-mode-substitution (LMS) and low-mode-averaging (LMA) techniques respectively (Gong et al., 2013). With these techniques, the low-mode and the high-mode contributions are treated separately. We calculate the quark propagators with gauge-invariant smeared stochastic grid sources, which increases the statistics but introduces the stochastic noises. Therefore, we construct the low-mode part of the correlation functions with the overlap eigenvectors exactly from each site of the source grid while using the grid propagators to estimate the high-mode part. This LMS technique increases the nucleon signal-noise ratio by 7 times. The LMA technique for the quark loops is similar to the LMS technique for the nucleon, except for that the low-mode parts of the loops from all space-time sites are considered and averaged.

It is observed that the pseudoscalar quark loops are well saturated by the low modes. The lowest 20 pairs of the overlap eigenmodes would contribute more than 90% of the pseudoscalar loop with light quarks in configurations with zero modes (Gong et al., 2013). On the other hand, it is well-known that the contribution to the triangle anomaly comes mainly from the cut-off part of the regulator. Therefore, the pseudoscalar density and the topological density represent the low-frequency and high-frequency parts of the axial-vector quark loop respectively. This leads to very good signals of pseudoscalar density with the help of our LMA approach.

The topological term, however, needs another technique to improve its signal. The index theorem shows that the local version of the overlap Dirac operator gives the topological charge density operator in the continuum (Y. Kikukawa & A. Yamada, 1999; D. H. Adams, 2002; K. Fujikawa, 1999; H.Suzuki, 1999), i.e.

$$Tr\gamma_5(1 - \frac{1}{2}aD_{ov}(x,x)) = \frac{1}{16\pi^2} tr_c G_{\mu\nu}\tilde{G}_{\mu\nu}(x) + \mathcal{O}(a^2) \quad (6)$$

This operator is less noisy than the traditional plaquette operators, and it is not as expensive as the quark loops. We can use dilution on stochastic grids to increase the statistics.

For the 3pt functions, we loop over all the source time slices and average them. The current time slices between source and sink are summed over and a linear fitting on the sink time slices gives the matrix element by its slope. By carefully choosing the fitting ranges, we can avoid the contamination from the excited states.

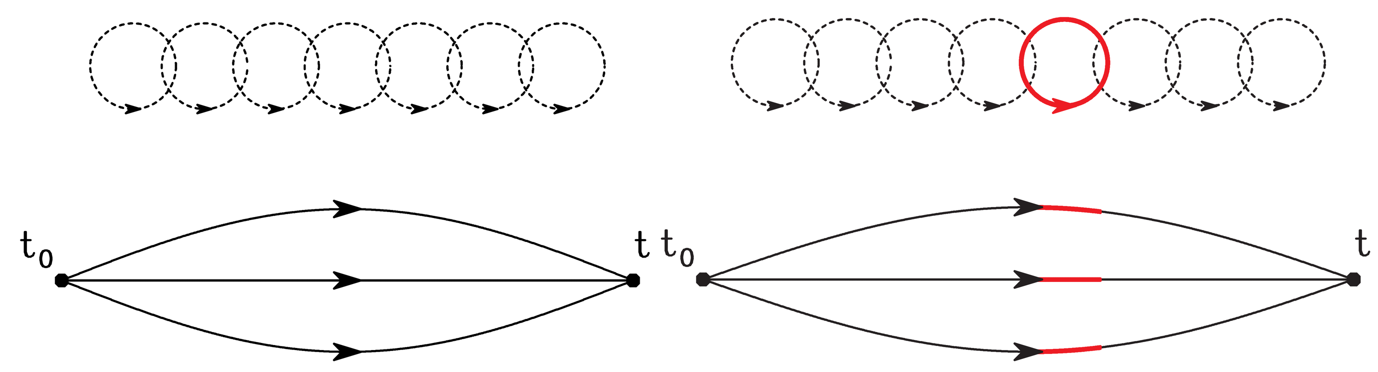

Figure 3: The cartoon shows the summed ratio method for the disconnected 3pt function. The red part is the additional contribution to the sum when the propagator length is increased, which is within the plateau region.

The axial-vector current provides the enough information to solve g_A^b and h_A^b, but the renormalization constant is unknown. However, the $2mP + G\tilde{G}$ current is free of renormalization factors and the signal is much better, but g_A^R and h_A^R could not be disentangled from it. So, we solve g_A^R and h_A^R from a global fit with both axial-vector and $2mP + G\tilde{G}$ matrix elements by assuming $g_A^R(Q^2) = Z_{g_A} g_A^b(Q^2)$ and $h_A^R(Q^2) = Z_{h_A} h_A^b(Q^2)$ where Z_{g_A} and Z_{h_A} are constants.

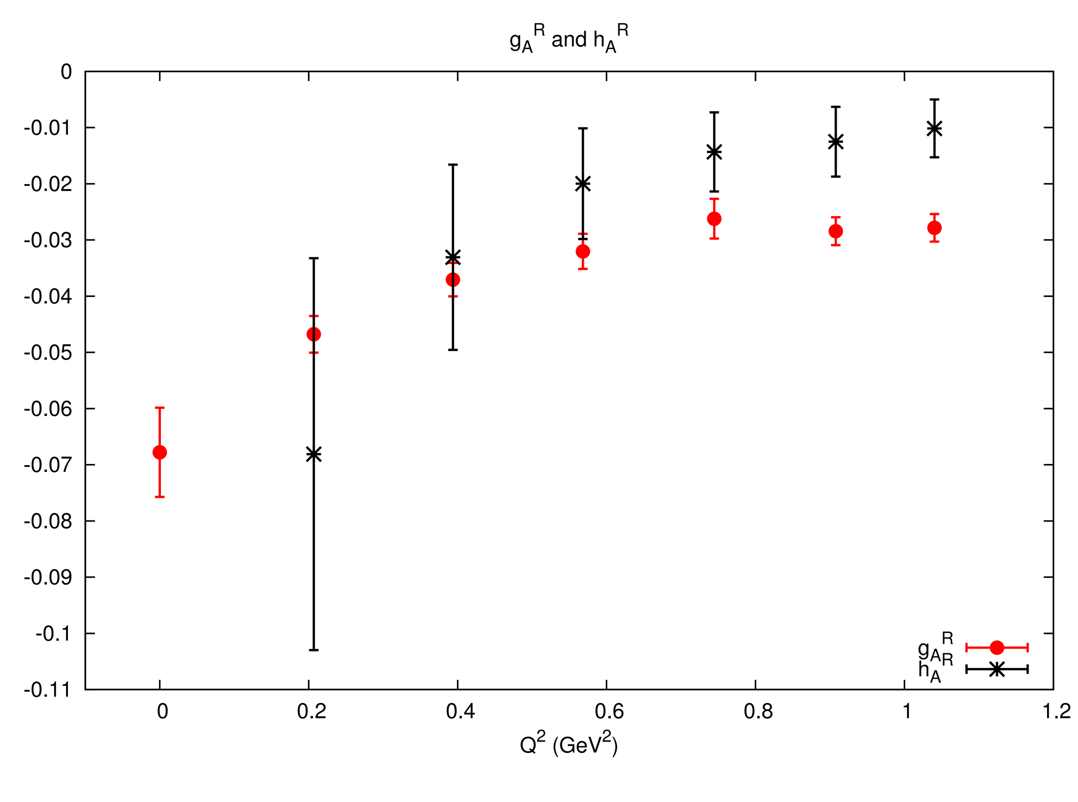

Figure 4: The plot shows the preliminary fitting results for $g_A^R(Q^2)$ and $h_A^R(Q^2)$.

From the fitting results we find that $h_A^R(Q^2)$ contributes less than 10% with the AWI approach considering the Q^2 suppression. The renormalization constants are found to be $Z_{g_A}^{-1} = 0.39(3)$ and $Z_{h_A}^{-1} = 12(6)$. Z_{h_A} is so small that it plays a noticeable roll in the axial-vector current with a finite momentum transfer.

Conclusions

- The strange quark spin of nucleon is calculated with AWI.
- A preliminary analysis show $g_A(0) = -0.068(8)$ on one lattice.
- The renormalization constants are also studied.
- The extrapolations to the physical quark mass, to the continuum limit and to the infinite volume will be carried out.