Nuclear Parity Violation from Lattice QCD

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Cal Lat

Motivation I

- GWS model of electroweak interaction is huge success
- **flavour changing charged current** is well understood from precision measurements in collider experiments:

$$J^{+}_{\mu} = \text{cos}\,\theta_C\,\bar{u}\gamma_\mu(1+\gamma_5)d + \text{sin}\,\theta_C\,\bar{u}\gamma_\mu(1+\gamma_5)s$$

$$H^{CC,\Delta S=0}_{\text{ew}-\text{eff}} = \frac{G_F}{\sqrt{2}} J^{+\dagger}_{\mu} J^{+\mu} + \text{h.c.}$$

  - vector boson d.o.f. are integrated out
  - effective Hamiltonian has isospin changing $\Delta I{=}0,1,2$ interactions
  - $\Delta I{=}1$ component is suppressed by $\sin^2\theta_C{\sim}0.04$
    $\Rightarrow \Delta I{=}0,2$ transitions strongly dominate EW CC interaction

- $\Delta I{=}1$ **interaction is good probe for parity violating neutral current/hadronic neutral current (HNC)**

Motivation II

- **flavour conserving neutral current** is given by

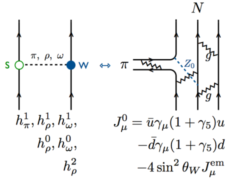
$$J^0_\mu = \bar{u}\gamma_\mu(1+\gamma_5)u - \bar{d}\gamma_\mu(1+\gamma_5)d - 4\,\mathrm{sin}^2\,\theta_W J^{\mathrm{em}}_\mu$$

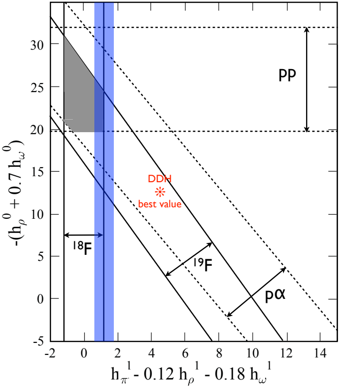
$$H^{NC}_{\mathrm{ew-eff}} = \frac{G_F}{2\sqrt{2}} J^{0\dagger}_\mu J^{0\mu} + \mathrm{h.c.}$$

  - Effective Hamiltonian generates $\Delta I{=}0,1,2$ interactions
  - no perturbative argument for enhancement/suppression of some components
- hard to measure in collider experiments because it allows no FC
- **HNC least constrained observable in the Standard Model**
- nuclear systems perfect testbed for studying HNC
- challenge: EW effects suppressed by $G_F F_\pi^2 {\sim} \mathcal{O}(10^{-7})$ w/ respect to strong interaction

Motivation III

- some systems alleviate that constraint due to nearly-degenerate energy levels w/ opposite parity, but those have large A
  $\Rightarrow$ hard to control systematic uncertainties due to nuclear ME
- small nuclear systems have better controlled systematics
- ongoing experimental effort by NPDGamma at SNS (ORNL), measuring asymmetry in $np \to d\gamma$ with predicted sensitivity of $\mathcal{O}(10^{-8})$
  (Alarcon, Balascuta [Hyperfine Interact. 214, 149])
- **good understanding of QCD corrections is required**





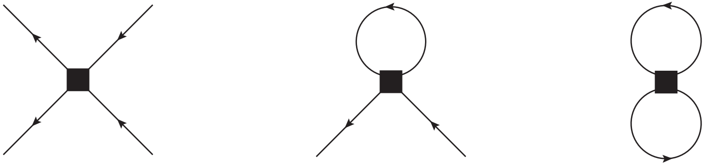
(Haxton, Holstein, Wasem)

Lattice Calculation of NPV

- focus on local isotensor operator

$$\mathcal{O}^{\Delta I=2}(\mathbf{p}=0) = \sum_{\mathbf{x},\mu} \left(\bar{q}\gamma_\mu\gamma_5\tau^+ q\right)(\mathbf{x}) \otimes \left(\bar{q}\gamma^\mu\tau^+ q\right)(\mathbf{x})$$

- ME can be related to coupling h_ρ^2
- why not $\Delta I=0, 1$?
  - **no disconnected** diagrams (isospin limit)



$\Delta I=0,\ 1,\ 2$ $\Delta I=0,\ 1$ $\Delta I=0$

  - **no mixing** under renormalization (in absence of QED)
    (Tiburzi [1207.4996])
- evaluate this operator in $nn \to pp$ channel
  - reduces number of diagrams significantly

Interpolating Operators I

- process $\langle pp|\mathcal{O}^{\Delta I=2}|nn\rangle$ is PV and thus **changes orbital angular momentum** $\Rightarrow$ need to compute $\langle pp(^3P_0)|\mathcal{O}^{\Delta I=2}|nn(^1S_0)\rangle$
- good operators for projecting onto S-wave (easy) and P-wave necessary (more involved) (Luu, Savage [1101.3347])
- a) create **non-local operators** with ℓ, m_ℓ, s, m_s QN

$$\begin{aligned}\langle \mathbf{x}_0|\ell, m_\ell; s, m_s\rangle \equiv& \left(\bar{N}\bar{N}\right)^{m_\ell, m_s}_{\ell, s}(\mathbf{x}_0)\\ =& \sum_{\{\Delta\mathbf{x}\},\alpha,\beta} Y_\ell^{m_\ell}\left(\widehat{\Delta\mathbf{x}}\right)\cdot \bar{N}_\alpha(\mathbf{x}_0+\Delta\mathbf{x})\bar{N}_\beta(\mathbf{x}_0)\cdot\Gamma^{s,m_s}_{\alpha\beta}\end{aligned}$$

- b) **project onto total angular momentum** using CG coefficients

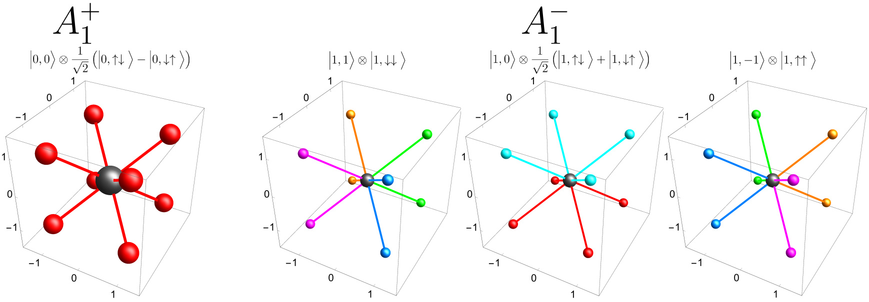
$$\langle \mathbf{x}_0|j, m_j\rangle = \sum_{\ell, m_\ell, s, m_s} \mathrm{CG}(j, m_j; \ell, m_\ell; s, m_s)\left(\bar{N}\bar{N}\right)^{m_\ell, m_s}_{\ell, s}(\mathbf{x}_0)$$

- c) **subduce** result onto cubic irreps (Dudek et al. [1004.4930])

$$\langle \mathbf{x}_0|\Lambda, \mu\rangle \equiv \left(\bar{N}\bar{N}\right)^{\mu}_{\Lambda}(\mathbf{x}_0) = \sum_{j, m_j}\mathrm{CG}(\Lambda, \mu; j, m_j)\langle \mathbf{x}_0|j, m_j\rangle$$

Interpolating Operators II

- use **local single-nucleon-interpolators** (Basak et al., [hep-lat/050801])
- **corner topology** ($\Delta\mathbf{x}^2 \propto 3$) for A_1^+ ($\sim {}^1S_0$) and A_1^- ($\sim {}^3P_0$)



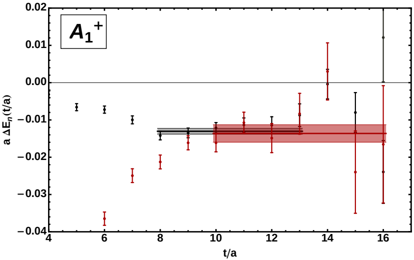
- successfully used in our higher PW nn-scattering calculation
  $\Rightarrow$ **Amy's talk, Wed. 07/15, 3 PM, Had. Spec. Int.**

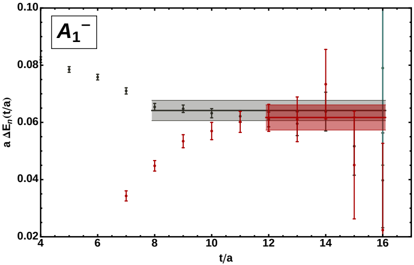
Interpolating Operators III

- optimal sources/sink defined in **p**-space $\Rightarrow$ would **require all-to-all propagators (or stochastic $\mathcal{O}^{\Delta I}$ projection)** $\Rightarrow$ **x**-space sources/sinks
- stochastic projection to zero cms momentum

$$(\bar{N}\bar{N})^{\mu}_{\Lambda}(\mathbf{P}{=}0) \approx \sum_{\{\mathbf{x}_0\}\in \text{QMC(latt)}} (\bar{N}\bar{N})^{\mu}_{\Lambda}(\mathbf{x}_0)$$

- **x**-space sources/sinks have overlaps with A_1^+ and A_1^- ground states

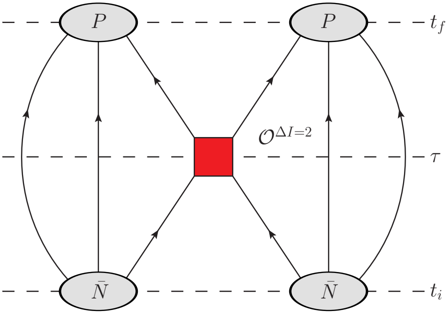




- **x**-space setup **reduces cost for contractions**

Contractions

- setup:
  - A_1^+ source at t_i
  - A_1^- sink at t_f
  - τ varies between t_i, t_f
- use unified contraction method at source and sink (Doi, Endres [1205.0585], Detmold, Orginos [1207.1452])
- factor out 4-quark-object and propagators connecting blocks and EW insertion ⇒ **skeleton-method**
- additionally: reverse process by swapping interpretation of source and sink



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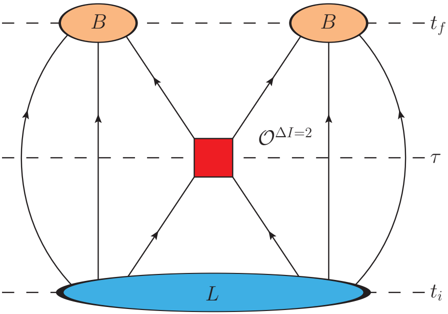

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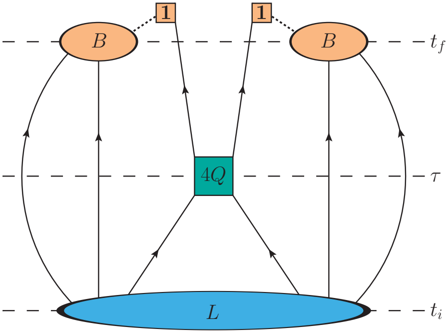

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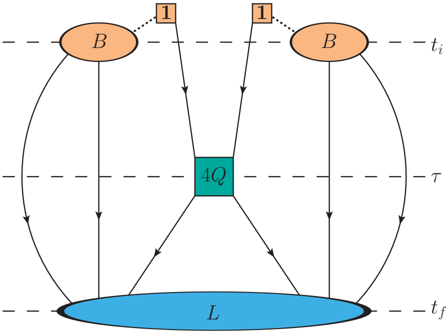


Calculation Details I

- calculations performed at ~ 800 MeV **pion mass**, to reduce noise in correlation functions
- ensemble overview: $a \sim 0.145$ fm, 6400 measurements on $24^3 \times 48$ lattice and 8×8 displacements per measurement with distance 6, i.e. $\Delta \mathbf{x} \propto (\pm 6, \pm 6, \pm 6)$
- no renormalization performed yet, but **can be done pertubatively at our requested level of preicsion** (Tiburzi [1207.4996])
- Lellouch-Luscher matching functions for relating finite volume ME to infinite volume counterpart has to be computed

$$\left\langle pp(^3P_1)\mathcal{O}^{\Delta I=2}pp(^1S_0)\right\rangle_{V=\infty} \equiv LL\left(\delta_{^1S_0}, \frac{\partial \delta_{^1S_0}}{\partial E}, \delta_{^3P_0}, \frac{\partial \delta_{^3P_0}}{\partial E}\right)\left\langle pp(^3P_1)\mathcal{O}^{\Delta I=2}pp(^1S_0)\right\rangle_V$$

- we computed phase shifts for nn-scattering in P and S-wave
- all results are **preliminary**

Calculation Details II

- The bare PV amplitude is time-dependent and contains vacuum overlaps (Z-factors) which depend on the interpolating operators
- for removing all of these, compute

$$
\begin{aligned}
C_{++}(t) &\sim \langle A_1^+(t) | A_1^+(0) \rangle, \\
C_{--}(t) &\sim \langle A_1^-(t) | A_1^-(0) \rangle, \\
C_{-+}(t_f, t, t_i) &\sim \langle A_1^-(t_f) | \mathcal{O}^{\Delta I=2}(t) | A_1^+(t_i) \rangle, \\
C_{+-}(t_f, t, t_i) &\sim \langle A_1^+(t_f) | \mathcal{O}^{\Delta I=2}(t) | A_1^-(t_i) \rangle
\end{aligned}
$$

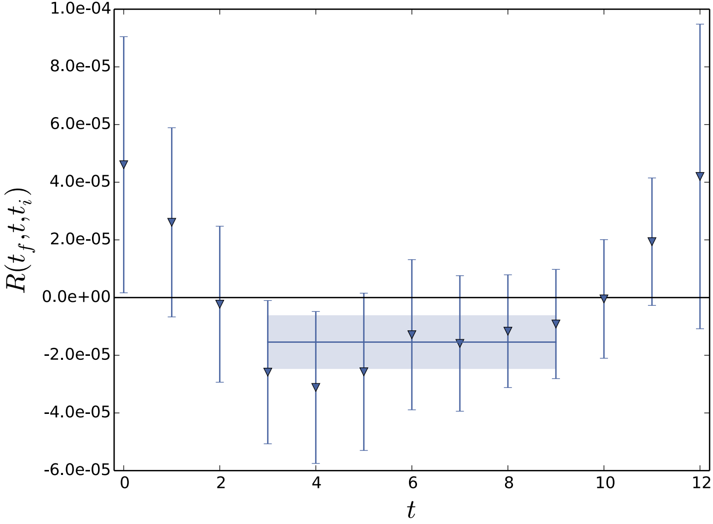
- compute ratio to **cancel overlap factors and energy dependence**

$$
R_{-+}(t_f, t, t_i) = \frac{C_{-+}(t_f, t, t_i)}{\sqrt{C_{--}(t_f - t_i) C_{++}(t_f - t_i)}} \sqrt{\frac{C_{--}(t_f - t) C_{++}(t - t_i)}{C_{++}(t_f - t) C_{--}(t - t_i)}}
$$

- use asymmetric subtraction to **remove energy injection by** $\mathcal{O}^{\Delta I=2}$

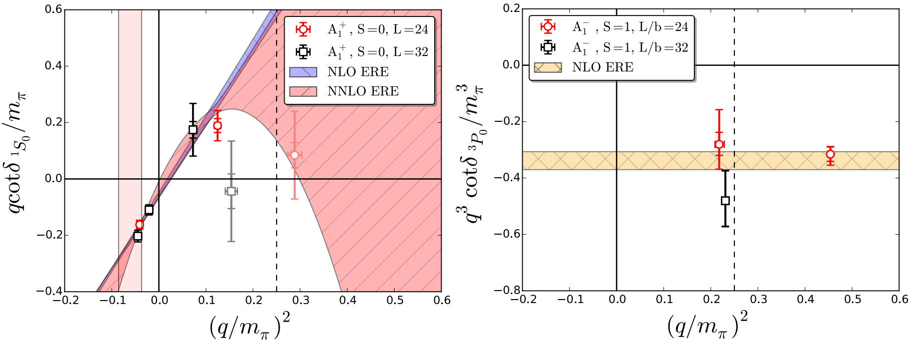
$$
R(t_f, t, t_i) \equiv \frac{1}{2} \left(R_{-+}(t_f, t, t_i) - R_{+-}(t_f, t, t_i) \right)
$$

Bare Matrix Element



- looks promising and **more statistics on it's way**

Phase Shifts



- energy dependence of $\delta_{^1S_0}$ determined
- need to augment statistics with different source topology for $\delta_{^3P_0}$
- need to estimate PW mixing in A_1^-

Summary

- hadronic neutral current least constrained observable of the SM
- NPDGamma is trying to improve that constraint
  $\Rightarrow$ Lattice QCD can help to improve systematic uncertainties
- we **built framework for and started calculation** of nuclear parity violation in Lattice QCD
- obtained a signal but more statistics needed
- use of non-local interpolating operators necessary
  $\Rightarrow$ **calculation is 160 times more expensive**
- **S- and P-wave strong scattering needs to be fully understood** before serious attempts for computing NPV can be made $\Rightarrow$ we are almost there

Outlook

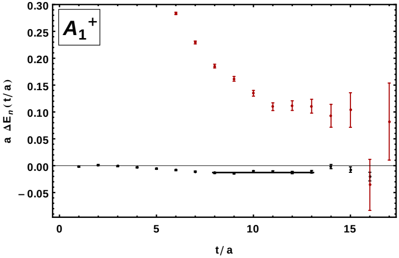
- increase statistics and finish calculation of h_ρ^2 at $m_\pi\sim 800$ MeV
- compute LL factor
- investigate possibilities to compute ME for $\Delta I=1$ (difficult) and $\Delta I=0$ (very difficult)
- stochastic estimation of disconnected diagrams fits into skeleton decomposition approach $\Rightarrow$ minor code changes necessary
- we started exploratory calculations at $m_\pi\sim 400$ MeV

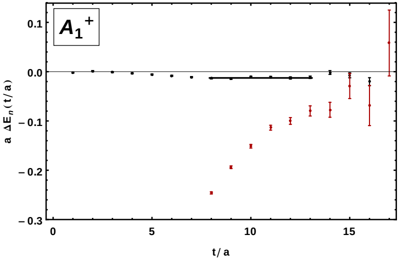
Thank You

Backup Slides

Local Interpolating Operators

- the use of local two-nucleon-operators would significantly reduce the cost for the calculation
- number of fundamental contractions **would reduce from 120 to 6**
- but: local A_1^+ operator has **almost no overlap** with A_1^+ ground state





- use of **non-local two-nucleon operators mandatory**
- momentum space sources and sinks would be optimal $\to$ requires multiple momentum space sources on quark level to sample Fourier modes $\to$ extremely expensive