NPR determination of quark masses from the HISQ action

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Intro & Motivation

- Quark masses – fundamental parameters of the Standard Model.
- Many applications to phenomenology and BSM physics. Example: Higgs partial widths.
  - Couplings proportional to quark masses.
  - Main source of uncertainty in partial widths from m_b, m_c, α_s. [1404.0319]
- Would like to reduce uncertainties via:
  - Determinations from multiple formulations.
  - Multiple determinations in a given formulation.

Outline

- Theory background.
- HPQCD approaches.
  - $\langle JJ \rangle$ time moments.
  - Mass ratios.
  - NPR.
- NPR with staggered (HISQ) fermions.
- NPR results.
- Conclusion.

Theory Background

Bare quark masses are input parameters to lattice simulation. These parameters are tuned to reproduce physical quantities, e.g.

- $m_{ud0} \to m_\pi^2$
- $m_{s0} \to m_K^2$
- $m_{c0} \to m_{\eta_c}$

Need a way to convert between lattice input parameters and renormalized quark masses:

$$m_q^{\overline{\mathrm{MS}}}(\mu) = Z_m^{\overline{\mathrm{MS}}}(\mu, 1/a)\, m_{q0}$$

Strategies

HPQCD is pursuing several strategies for accurate determination of quark masses, including

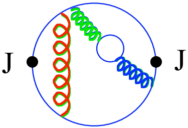
- Time moments of current correlators (charm and bottom, also w/ NRQCD).
- Quark mass ratios.
- Non-perturbative renormalization method (NPR).

This is important to assess systematics and quantify uncertainties.

Current-current correlators

Calculate time-moments of $J_5 \equiv \bar{\psi}_h \gamma_5 \psi_h$ correlators:

$$G(t) = a^6 \sum_{\mathbf{x}} (am_{0h})^2 \langle J_5(t, \mathbf{x}) J_5(0,0) \rangle$$



- Currents are absolutely normalized (no Zs required).
- $G(t)$ is UV finite $\rightarrow G(t)_{\mathrm{cont}} = G(t)_{\mathrm{latt}} + \mathcal{O}(a^2)$.

Moments

The time-moments $G_n = \sum_t (t/a)^n G(t)$ can be computed in perturbation theory. For $n \geq 4$,

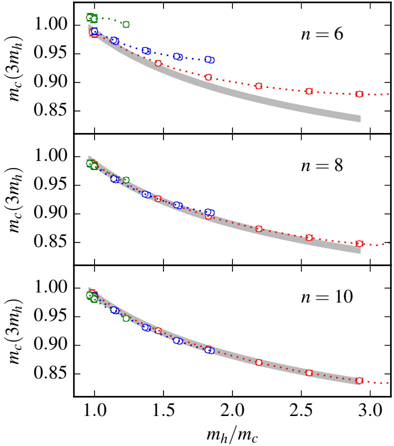
$$G_n = \frac{g_n(\alpha_{\overline{\mathrm{MS}}}, \mu)}{am_h(\mu)^{n-4}} \,.$$

Basic strategy:

1. Calculate $G_{n,\mathrm{latt}}$ for a variety of lattice spacings and m_{h0}.
2. Compare continuum limit $G_{n,\mathrm{cont}}$ with $G_{n,\mathrm{pert}}$ (at reference scale $\mu = m_h$, say).
3. Determine best-fit values for $\alpha_{\overline{\mathrm{MS}}}(m_h), m_h(m_h)$.

Results for $n_f = 4$

[1408.4169]



$$m_c(3m_h) = \frac{r_n(\alpha_{\overline{\mathrm{MS}}}, \mu = 3m_h)}{R_n}$$

- Discretization effects grow with am_h and decrease with n.
- Grey band shows best-fit $m_c(3m_c)$ evolved perturbatively.

$$\boxed{m_c^{\overline{\mathrm{MS}}}(3\ \mathrm{GeV}) = 0.9851(63)\ \mathrm{GeV}}$$

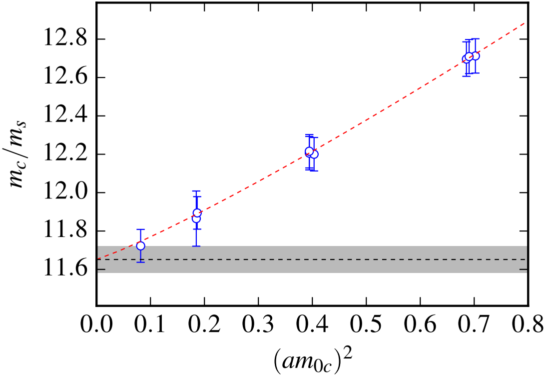
Mass ratios

$$\frac{m1_0}{m2_0} = \frac{m1^{\overline{\mathrm{MS}}}(\mu)}{m2^{\overline{\mathrm{MS}}}(\mu)} + \mathcal{O}(a^2)$$

- Tuning of simulation $\rightarrow$ accurate determination of bare ratios.
- Precise determination of one renormalized mass can be translated to other masses.

m_c/m_s ($n_f = 4$)

[1408.4169]



$$m_c/m_s = 11.652(65) \to m_s^{\overline{\mathrm{MS}}}(2\ \mathrm{GeV}) = 93.6(8)\ \mathrm{MeV}.$$

NPR method

General idea is to renormalize operators using a scheme that is well-defined both in the continuum and on the lattice, e.g. the RI schemes:

Calculate off-shell Green's functions of operator-of-interest with external quark states.

$$G_{\Gamma}^{ij}(p) = \langle q^i(p) \left(\sum_x \bar{q}(x) \Gamma q(x) \right) \bar{q}^j(-p) \rangle_{\mathrm{amp}}$$

Require that the trace of the renormalized operator takes its tree-level value:

$$\Lambda_{\Gamma}(p) \equiv \frac{1}{12} \mathrm{Tr} \left[\Gamma \, G_{\Gamma}(p) \right] \simeq \frac{Z_q(p)}{Z_{\Gamma}(p)}$$

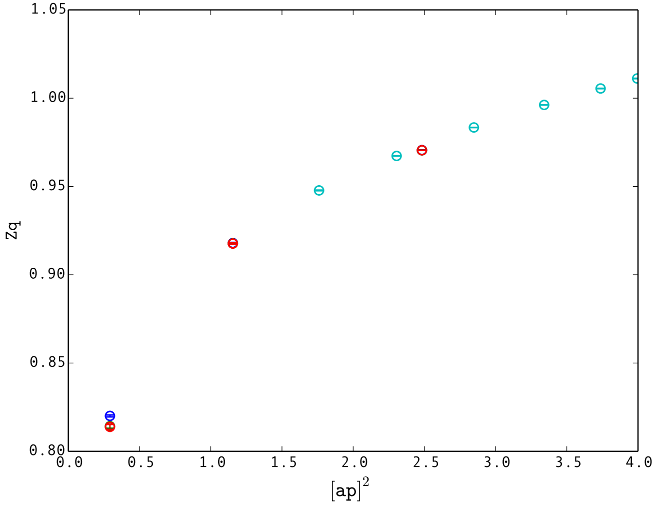
NPR method (cont.)

The RI (and $\overline{\mathrm{MS}}$) schemes satisfy $Z_m = Z_S^{-1} = Z_P^{-1}$. Z_m can be extracted from the scalar correlator provided

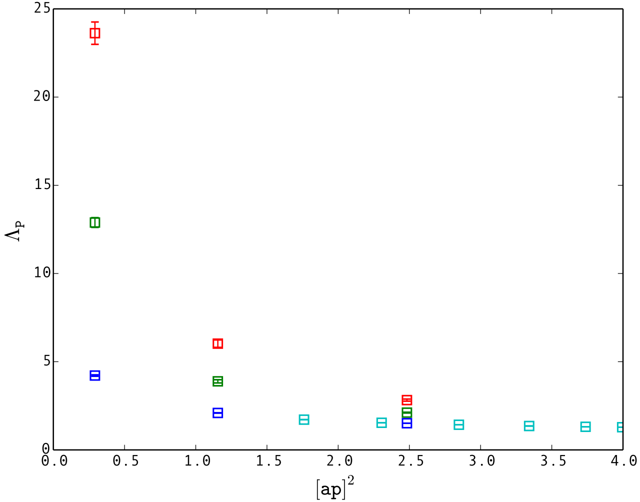
$$\Lambda_{\mathrm{QCD}} \ll |p| \ll \pi/a$$

After determining $Z_m^{RI}(p)$, a perturbative calculation can be used to convert $Z^{\overline{\mathrm{MS}}}(p) = C^{\overline{\mathrm{MS}}\leftarrow RI}(p)\, Z_m^{RI}(p)$.

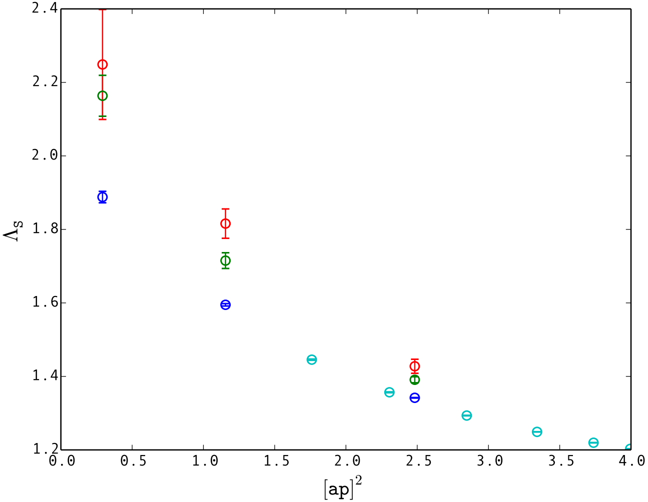
Results – Z_q



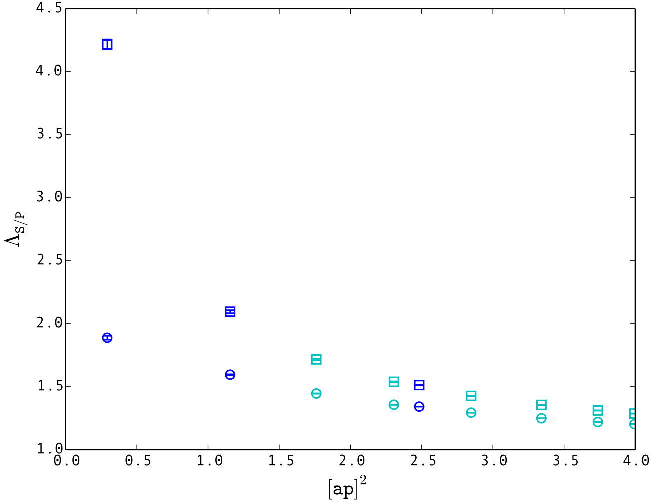
Results – Λ_P



Results – Λ_S



Results – $\Lambda_{S/P}$



Conclusions & Future Work

- Accurate determinations of quark masses are of fundamental importance for (B)SM physics.

- LQCD simulations provide an effective and controlled way to determine quark masses.
  - Systematically improveable.
  - Multiple complementary approaches $\rightarrow$ assess systematics, check consistency.
  - Control of input parameters.

- I have discussed recent work determining Z_m via NPR methods. This is complementary to other approaches.

Thank you!