

# Classifying the phases of gauge theories by spectral density of probing chiral quarks

Ivan Horváth

University of Kentucky, Lexington, KY

Andrei Alexandru

George Washington University, Washington, DC

Refs: [arXiv:1502.07732](#), 1405.2968, 1409.7094

Credits: [M. Sun](#)

Data Credits: [A. Hasenfratz](#), [D. Schaich](#), [Z. Fodor](#), [S. Borsanyi](#)

# The Plan:

- ◆ Phase and the Phase Structure

- ◆ Probing (“valence”) Quarks

- ◆ The Result

- ◆ Lattice Input

- Existence of anomalous phase in pure-gluon system
- Anomalous phase of heated quark-gluon matter (RHIC, LHC) !!!
- Anomalous phase without thermal effects (many flavors)

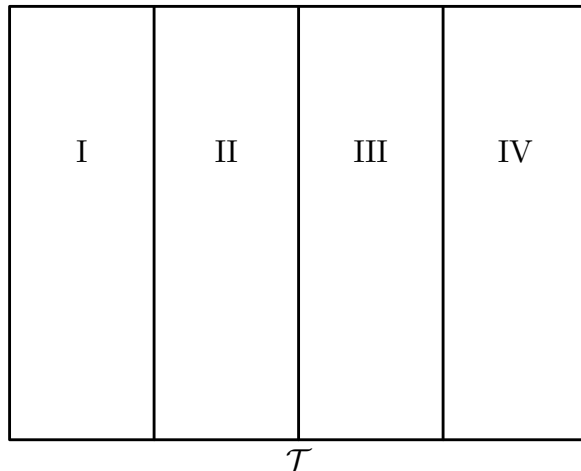
- ◆ Generalization and One Consequence

- Anomalous phase in massless quark dynamics?

- ◆ Summary

**Phase:** region of parameter space with particular dynamical feature(s)

**Phase Structure:** associated partition of the parameter space



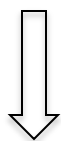
$\mathcal{T}$  : SU(3) theories w fundamental quarks  
arbitrary number of flavors  
arbitrary masses  
arbitrary temperature

- phase boundaries present by definition
- transitions mark well-defined change of dynamics but not necessarily associated with singularities
- **Which dynamical features are useful to focus on?**  
[ non-Pearson correlations provide one option: [arXiv:1009.4451](#), [1210.7849](#), [1405.2968](#) ]

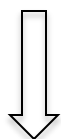
## Probing Quarks (aka valence quarks)

Let “good” external quarks live on the gauge vacua from  $\mathcal{T}$  and analyze their response!

pure glue ( $N_f=0$ )



infinitely heavy  
quark probe



Wilson loops  
[extended gauge objects]  
identifies confinement

generic element of  $\mathcal{T}$



very light  
quark probe



Dirac spectral density  
[extended gauge object]  
identifies confinement???

“good” external quarks = chirally clean = defined by overlap Dirac matrix

## Why Dirac spectral density?

- Dirac operator  $D=D[A]$  defines dynamics of probing quarks

$$\bar{\eta} (D + m_v) \eta$$

- Naturally incorporates scale

$$D\psi_\lambda = i \lambda \psi_\lambda$$

- Reduce  $D[A]$  to the simplest gauge invariant object incorporating scale:

$$\sigma(\lambda, m_f, V) \equiv \frac{1}{V} \left\langle \sum_{0 \leq \lambda_k < \lambda} 1 \right\rangle_{m_f, V}$$

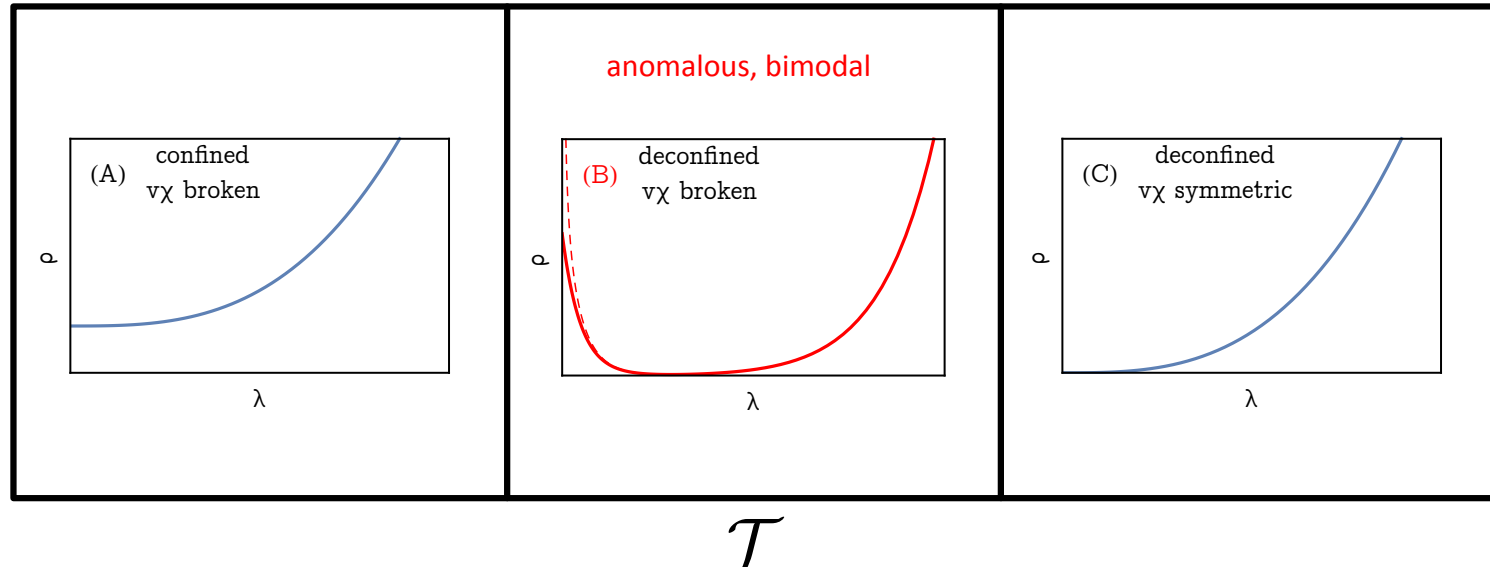
cumulative density

$$\rho(\lambda, m_f, V) \equiv \frac{\partial}{\partial \lambda} \sigma(\lambda, m_f, V)$$

spectral density

View Dirac spectral density as a vacuum object assigned to each theory in  $\mathcal{T}$ !

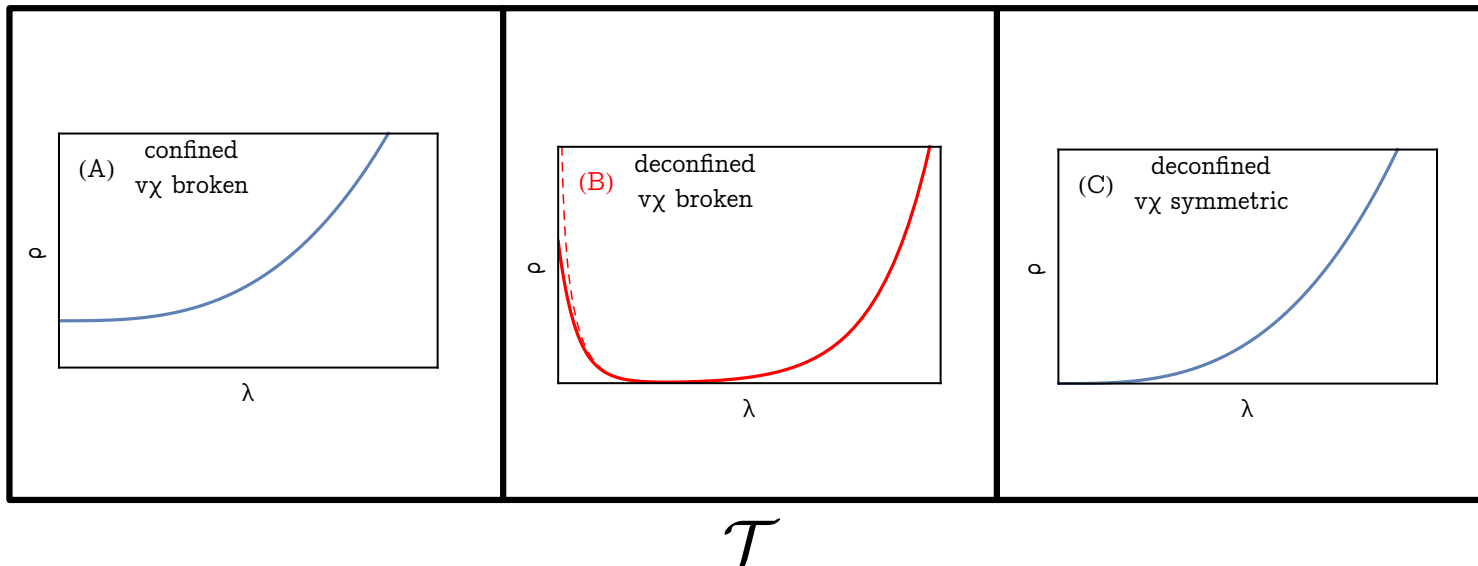
- $\rho(\lambda=0) > 0$  indicates breakdown of probing/valence chiral symmetry via Banks-Casher relation



- The relevant feature of (B) is infrared-ultraviolet bimodality (separation)
- Two types of  $v\chi$  broken phases: standard (A) and anomalous (B)
- One type of confined phase : (A) identifies confinement in  $\mathcal{T}$
- No confined and  $v\chi$  symmetric phase: agree with Casher
- Anomalous phase generic when crossing between (A) and (C)

## The Result... comments

- Probing quarks always defined by overlap
- Possible divergence of valence condensate due to mixed nature NOT RELEVANT
  - shape (bimodality) matters, divergence at fixed cutoff may in fact be crucial
  - EXAMINING THE VACUUM RATHER THAN CALCULATE PHYSICAL OBSERVABLE!**
- Other aspects also well distinguish the anomalous phase
  - non-Pearson correlation (for chiral polarization) [arXiv: 1405.2968](https://arxiv.org/abs/1405.2968)
  - spatial inhomogeneity of modes (not discussed here)



# Lattice Input

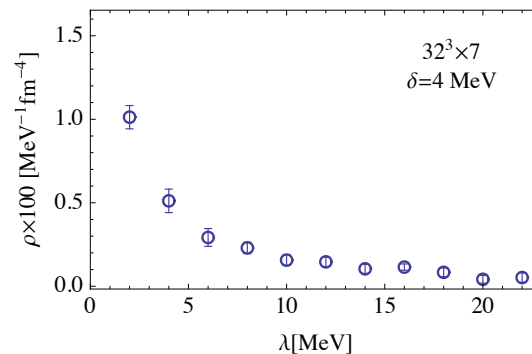
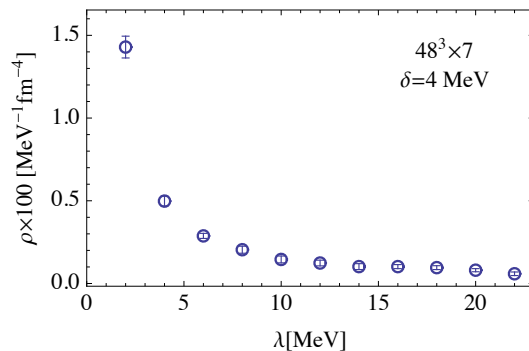
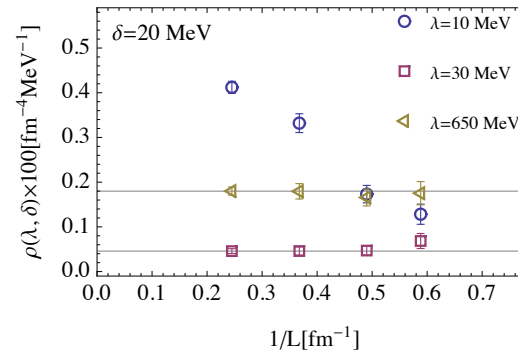
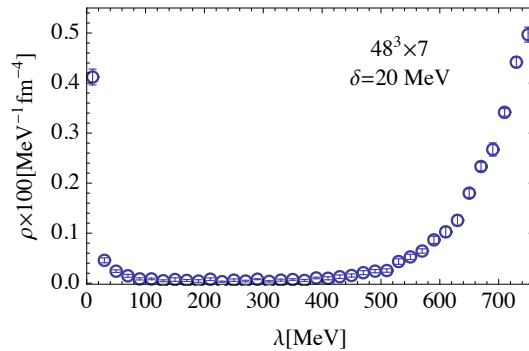
- i. Show that theory with anomalous spectral density exists anywhere in  $\mathcal{T}$

Do this carefully in thermal  $N_f=0$ !

anomalous peak first seen by  
Edwards, Heller, Kiskis, Narayanan, 1999

Fix  $T/T_c$  and check stability in both infrared and ultraviolet cutoffs

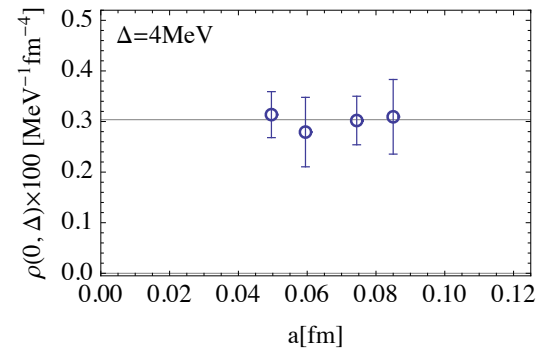
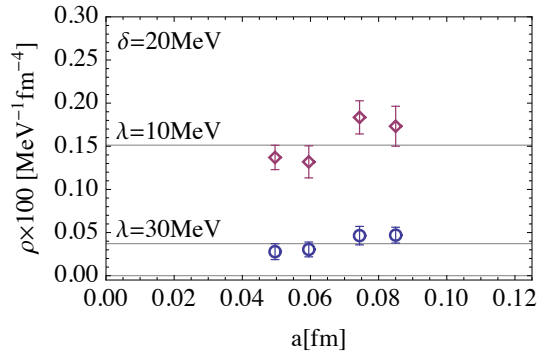
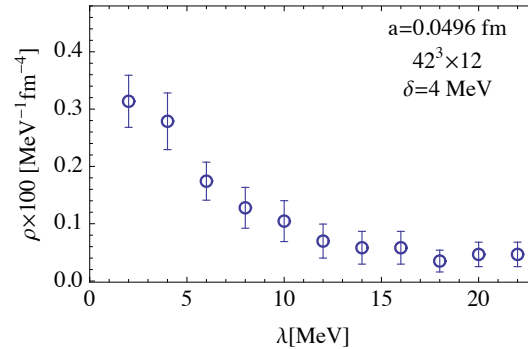
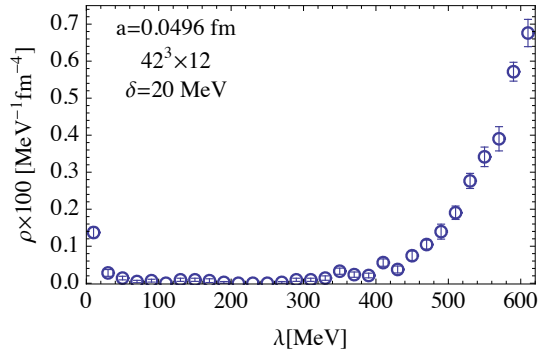
Wilson action,  $a=0.085$  fm,  $T = 1.12 T_c$ ,  $N^3 \times 7$ ,  $N = 16, 20, 24, 32, 48$





# Lattice Input... Thermal $N_f=0$ : ultraviolet cutoff

Wilson action ,  $T = 1.12 T_c$  fixed , Volume fixed:  $L=2$  fm , range of lattice spacings

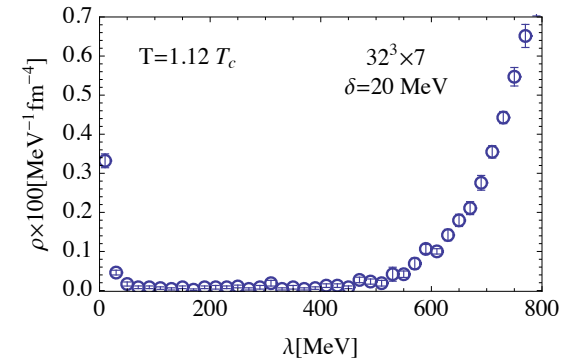
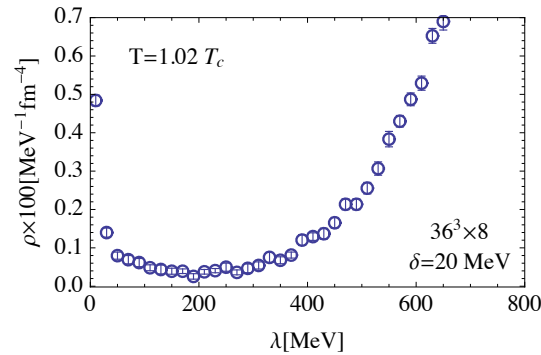
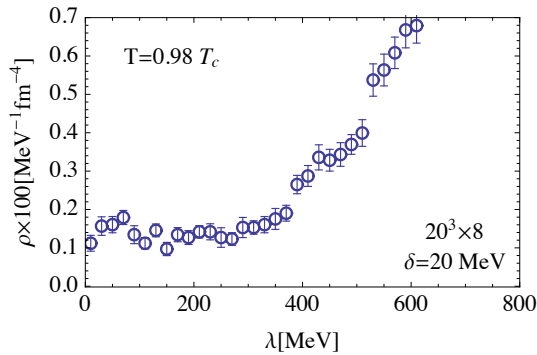


Anomalous spectral density, deconfinement and chiral symmetry breaking simultaneously exist in the continuum theory!

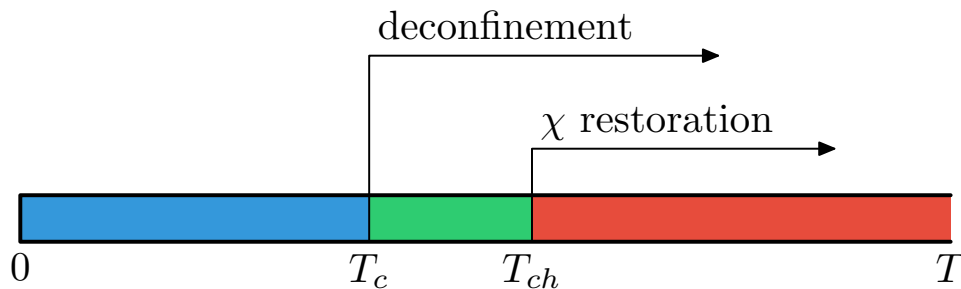
# Lattice Input...

ii. Show that thermal  $N_f=0$  follows the proposed behavior wrt deconfinement

Tune temperatures to “hug”  $Z_3$  transition:



Deconfinement transition and transition to anomalous density coincide!



**ANOMALOUS PHASE EXISTS!**

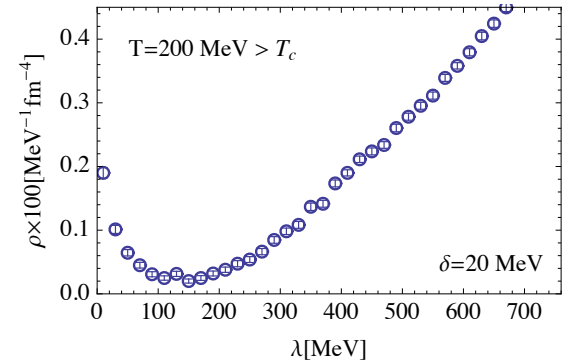
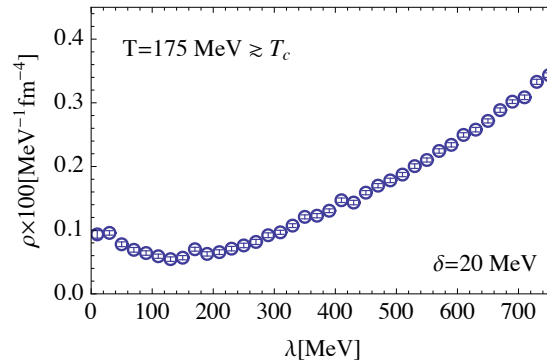
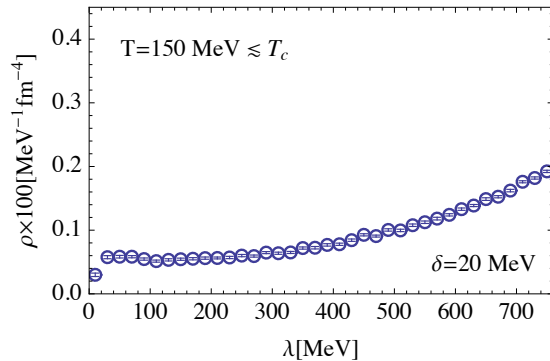
## Lattice Input...

iii. Show that thermal effects generically lead to anomalous phase

$N_f=2+1$  at physical point: “real world” QCD

Ensembles from [Borsanyi et al, 2010](#) : 32x8 staggered, checked carefully for artifacts used for determination of crossover temperatures

They report:  $T_c$  (Polyakov line) = 170 MeV       $T_c$ (quark condensate) = 155 MeV



(1) Anomalous phase in “real world” quark-gluon matter! (see also [Dick et al 1502.06190](#))

(2) Onset coincides with inflection point in L (“deconfinement”) to available precision!

(3) Dirac density defines its own  $T_c$

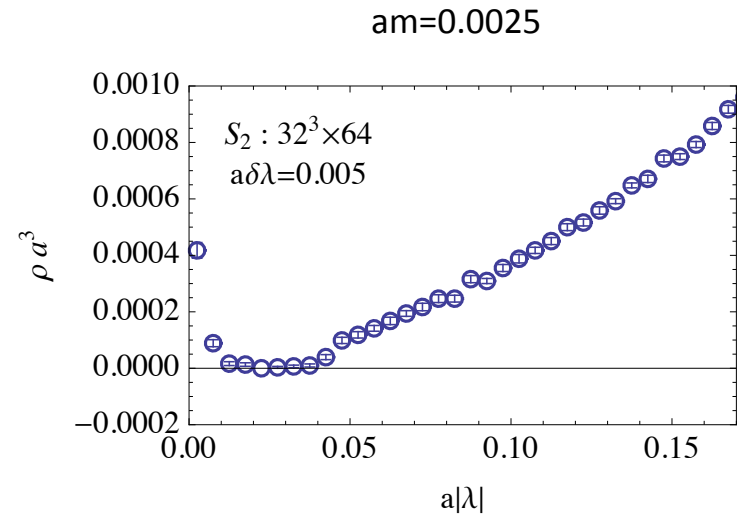
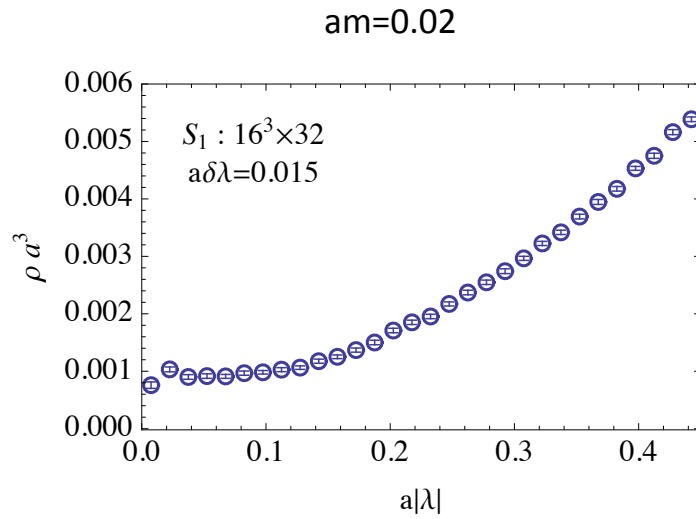
## Lattice Input...

iv. Show that light-quark effects alone also lead to anomalous phase

$N_f=12, T=0$

Ensembles: A. Hasenfratz et al, arXiv:1207.7162

staggered with nHYP,  $\beta_F=2.8, \beta_A/\beta_F=-0.25$



Light quarks generate anomalous phase without the help of temperature!

Anomalous range:  $0 \leq m_{ch} < m < m_c$

Conformality at  $m=0$  only if  $m_{ch} > 0$ ! Restoration hasn't been seen!

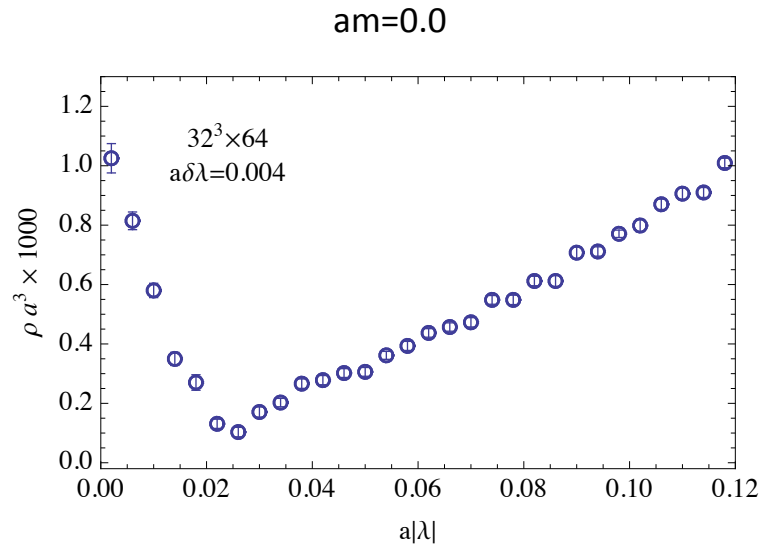
## Lattice Input...

iv. Show that light-quark effects alone also lead to anomalous phase...

$N_f=8, T=0$

Ensemble: A. Hasenfratz, D. Schaich

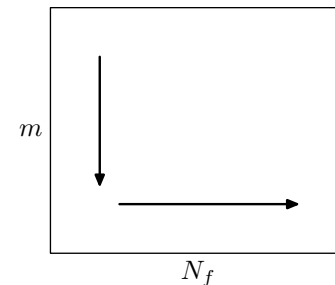
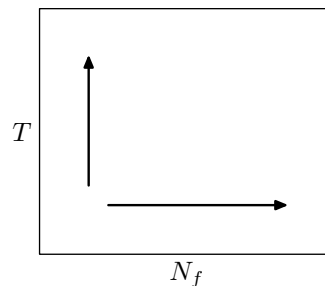
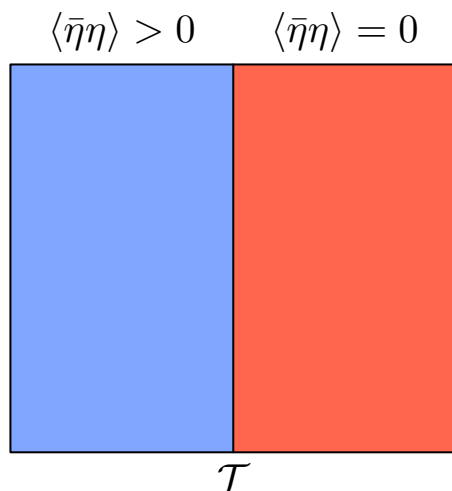
staggered with nHYP,  $\beta_F=4.8, \beta_A/\beta_F=-0.25$



not published

Anomalous phase present at  $N_f=8$ !

# Generalization

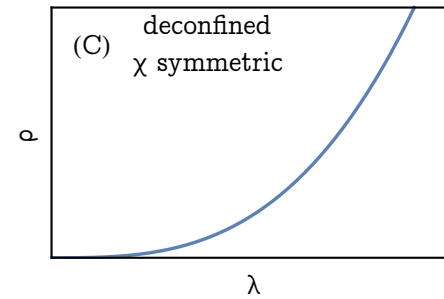
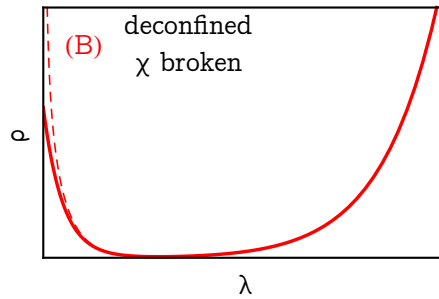
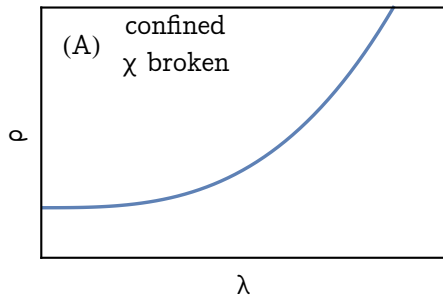
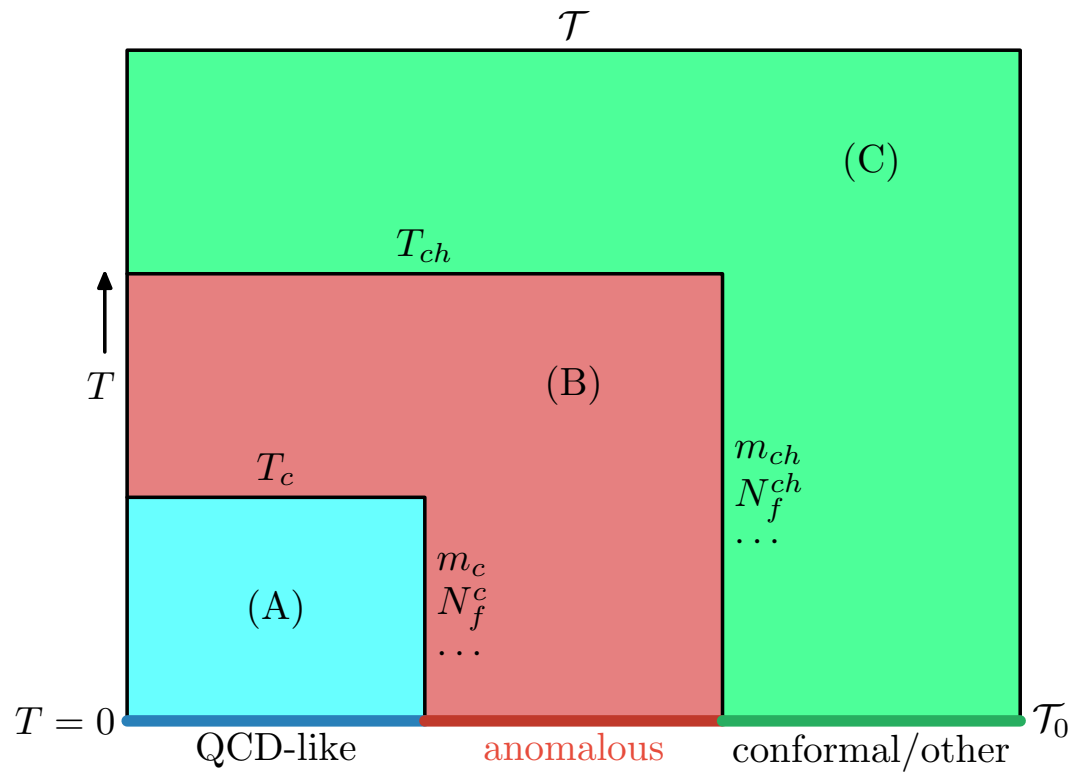


Direction of weakening condensate!

- (1) Temperature and dynamical quark effects the only mechanisms
- (2) Both generate anomalous phase

We conclude this happens on generic paths connecting broken and symmetric vacua in  $\mathcal{T}$ .

# Generalization...



## Interesting Consequence

$$T_c < T < T_{ch}$$

thermal anomalous phase

$$m_{ch} < m < m_c$$

mass anomalous phase

$$N_f^c < N_f < N_f^{ch} \quad (m=0)$$

anomalous window???

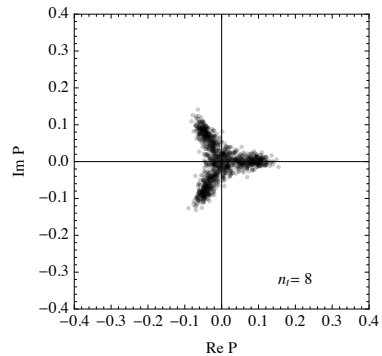
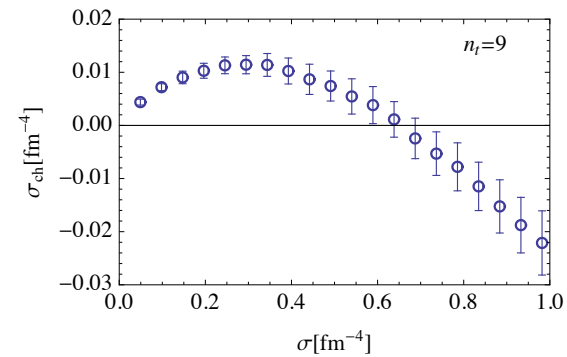
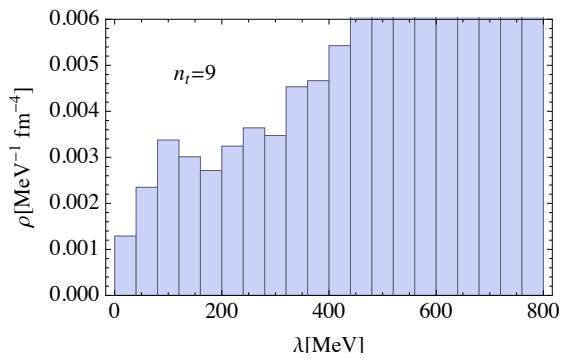
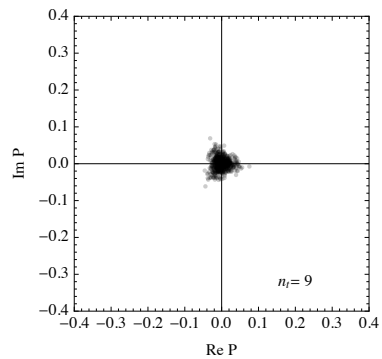
Here deconfinement precedes the onset of conformal window with massless flavors!

Data is consistent with this possibility so far!

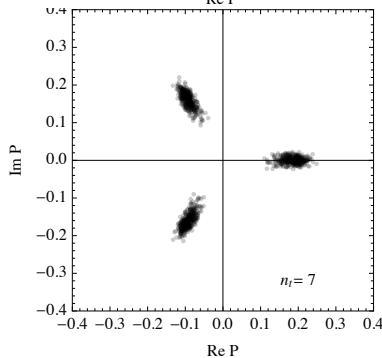
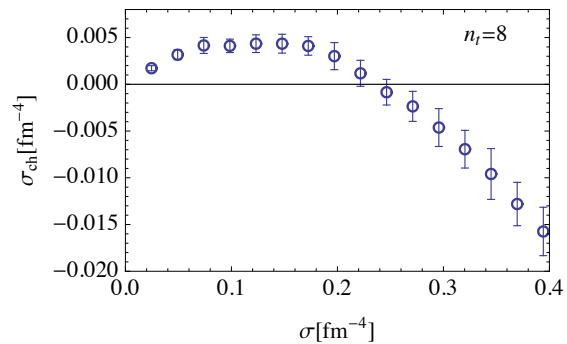
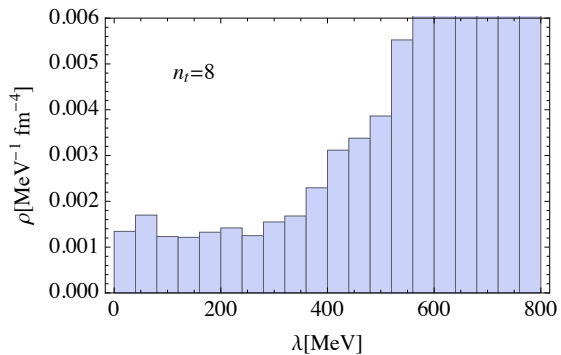


## Summary

1. Anomalous phase in  $\mathcal{T}$  exists and is generic!
2. We propose that it is a deconfined phase
3. Most likely a property of real-world thermal quarks and gluons (plasma state)
4. Changes global landscape of SU(3) theories with fundamentals
5. Its nature needs to be investigated in detail



$0.98 T_c$



$1.12 T_c$

