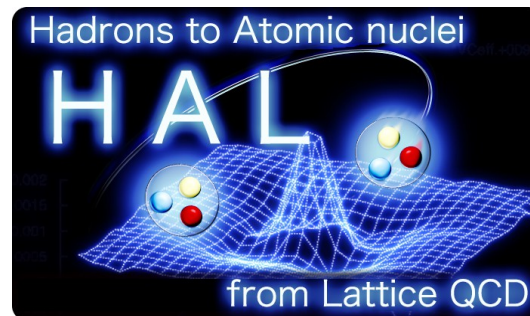


# First results of baryon interactions from lattice QCD with physical masses (3)

## --- Strangeness $S=-2$ two-baryon system ---

Kenji Sasaki (CCS, University of Tsukuba)

for HAL QCD Collaboration



### **HAL** (**H**adrons to **A**tomical nuclei from **L**attice) QCD Collaboration

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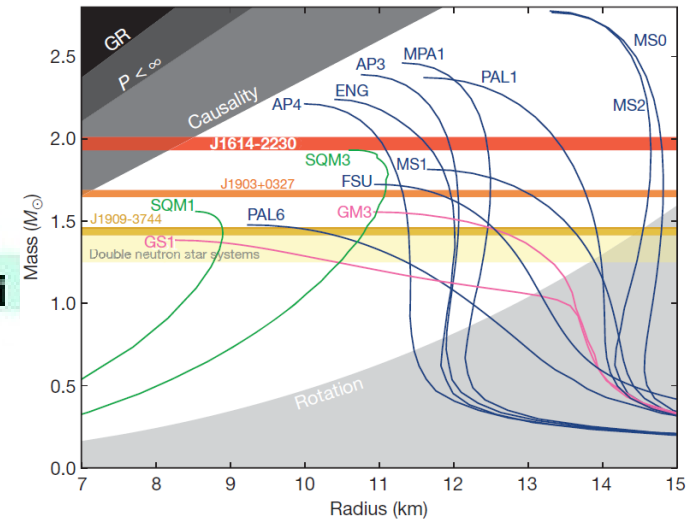
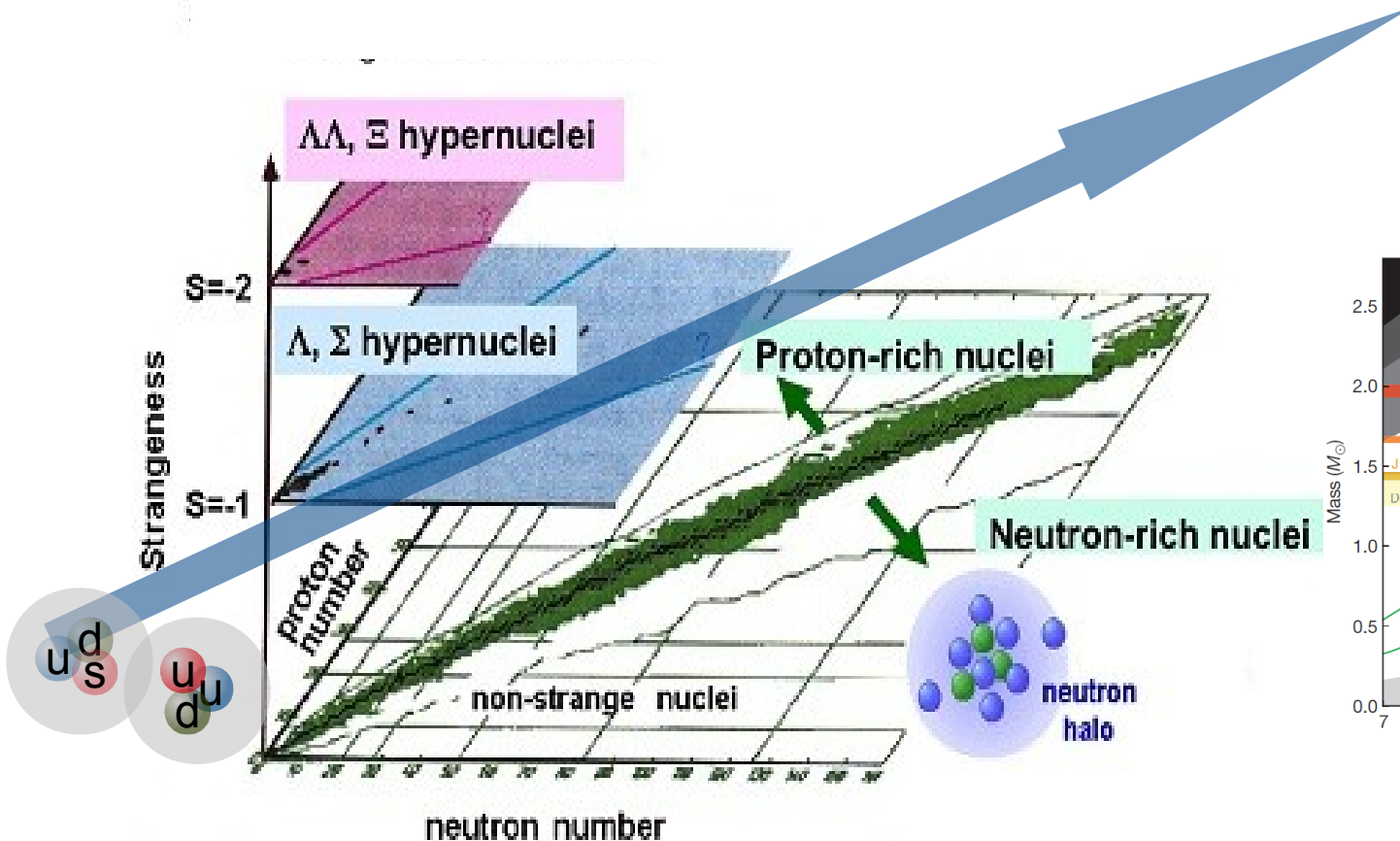
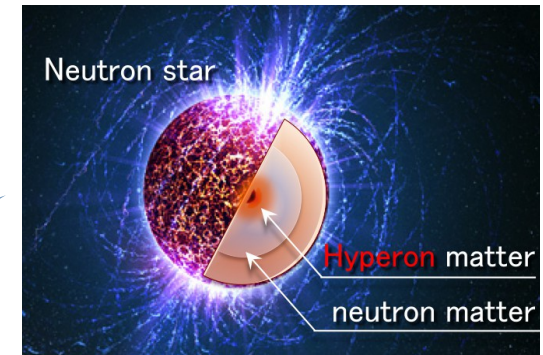
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# *Introduction*

# Introduction

BB interactions are inputs for nuclear structure, astrophysical phenomena

Once we obtain a “reliable” nuclear potential,  
we apply them to the structure of (hyper-) nucleus  
and neutron star calcration.



**We derive hadronic interactions from Lattice QCD.**

# Introduction

Study the hyperon-nucleon (YN) and hyperon-hyperon (YY) interactions

## Lattice QCD simulation

Difficult to calculate light quarks

Suffered from statistical noise



High performance  
for more data

Massively parallel super computer



Accessibility

Exp.

S=0

S=-1

S=-2

S=-3

S=-4

Lat.

## Experiment

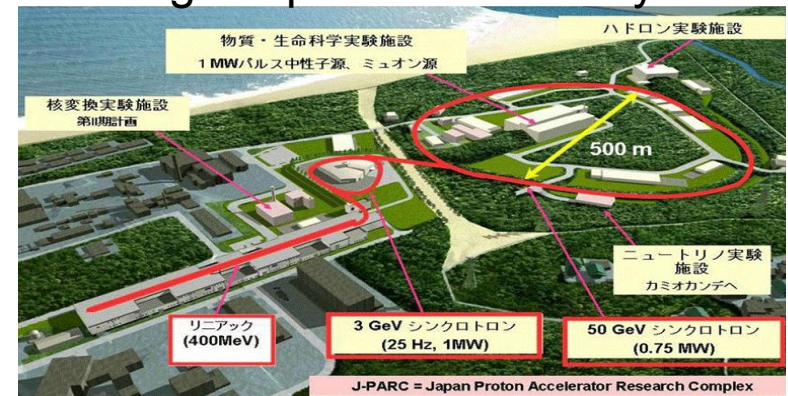
Difficult to perform collision experiment

Collision data are scarce



More intensity  
for more data

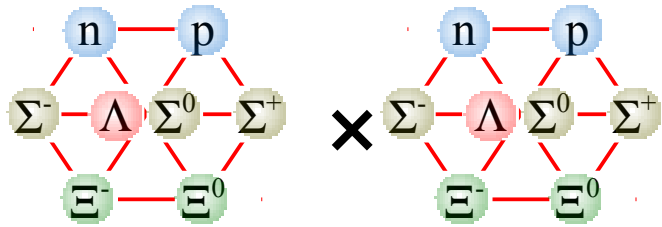
Huge experimental facility



**They would be complement each other  
to complete knowledge of generalized BB interaction.**

# $SU(3)$ feature of BB interaction

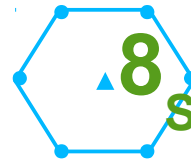
Three flavor (u,d,s) world



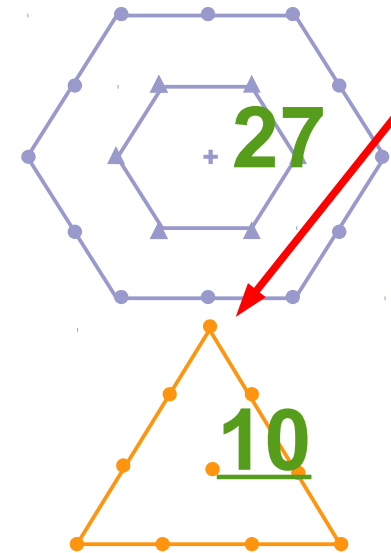
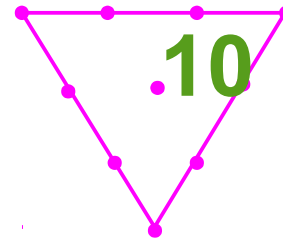
=

Flavor symmetric

• 1



Flavor anti-symmetric



NN sector

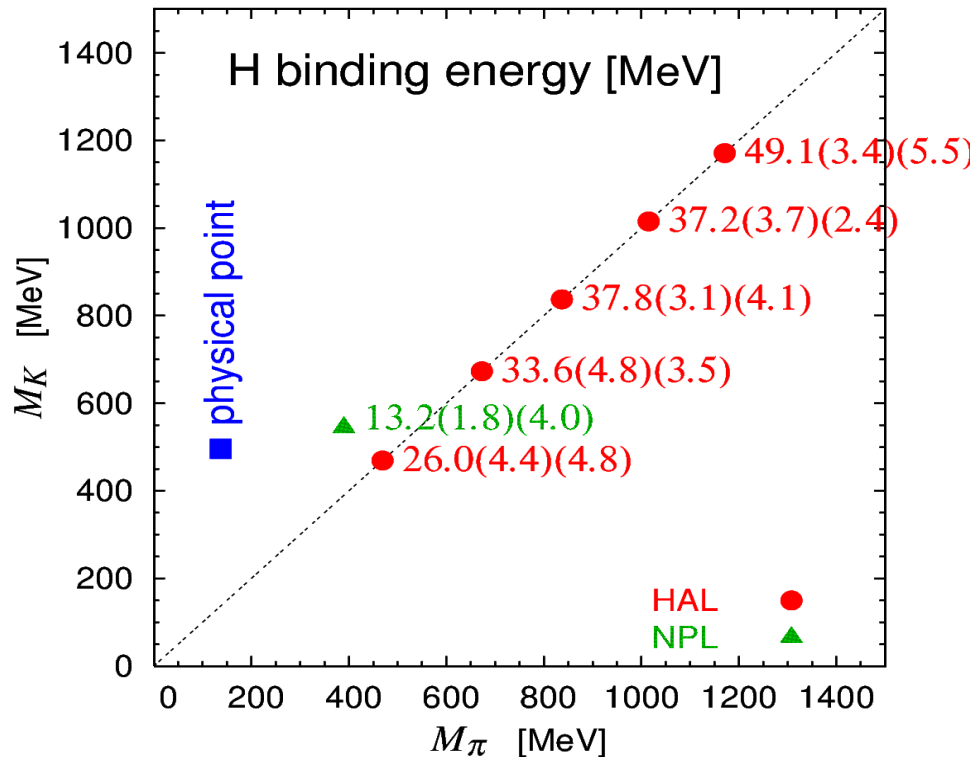
Strong attraction is expected.

In view of quark degrees of freedom

Oka, Shimizu and Yazaki NPA464 (1987)

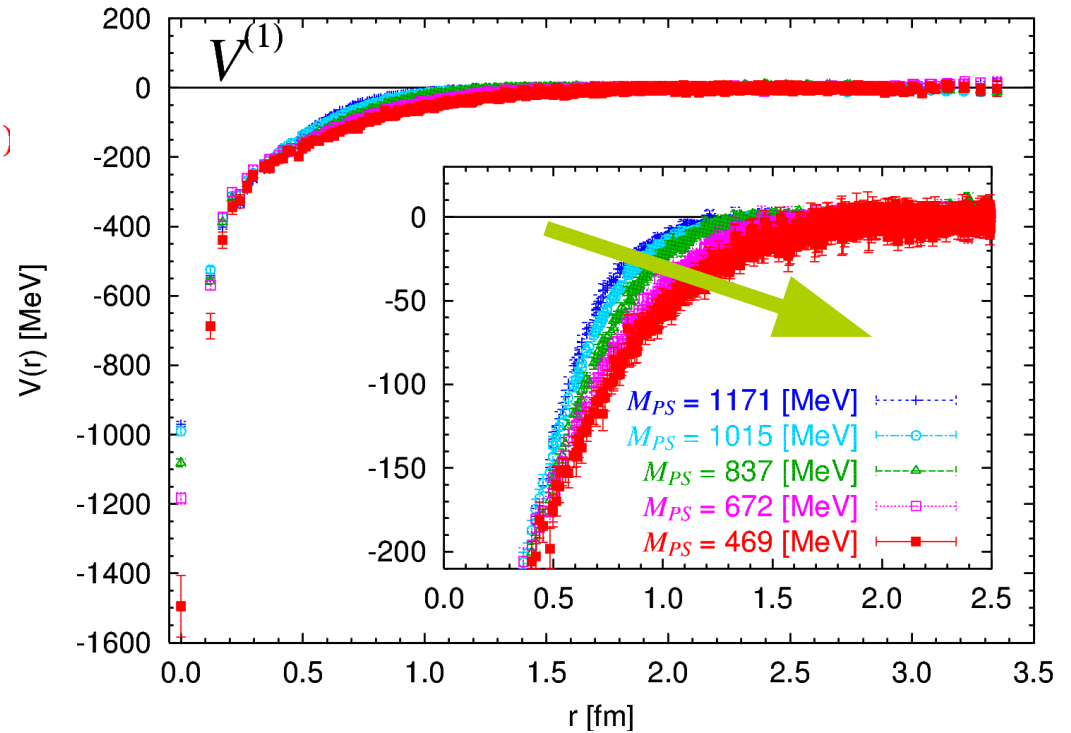
- Short range repulsion in BB interaction could be a result of Pauli principle and color-magnetic interaction for the quarks.
- Strengths of repulsive core in YN and YY interaction are largely depend on their flavor structures.
- For the s-wave BB system, **no repulsive core** is predicted in **flavor singlet state** which is known as **H-dibaryon** channel.

# H-dibaryon (theoretical studies)



HAL : PRL106(2011)162002

NPL : PRL106(2011)162001



- Both results shows the bound H-dibaryon state in heavy pion region.
- Potential in flavor singlet channel is getting more attractive as decreasing quark masses

Does the H-dibaryon state survive on the physical point?

# Interests of $S=-2$ multi-baryon system

## H-dibaryon

- The flavor singlet state with  $J=0$  predicted by R.L. Jaffe.
  - Strongly attractive color magnetic interaction.
  - No quark Pauli principle for flavor singlet state.

## Double- $\Lambda$ hypernucleus

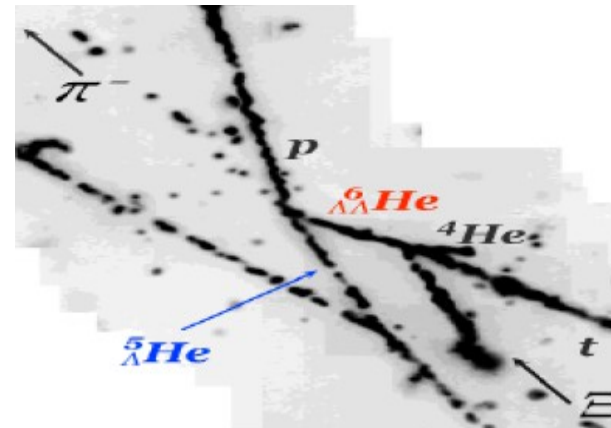
- Conclusions of the “NAGARA Event”

K.Nakazawa and KEK-E176 & E373 Collaborators

$\Lambda$ -N attraction

$\Lambda$ - $\Lambda$  weak attraction

$$m_H \geq 2m_\Lambda - 6.9\text{MeV}$$

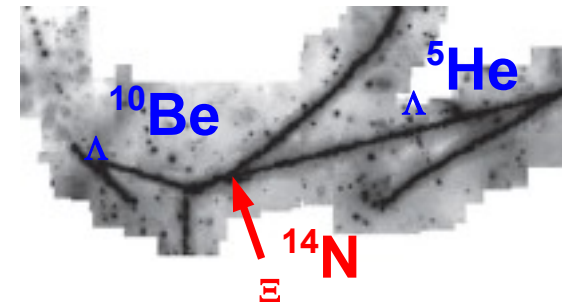


## $\Xi$ hypernucleus

- Conclusions of the “KISO Event”

K.Nakazawa and KEK-E373 Collaborators

$\Xi$ -N attraction

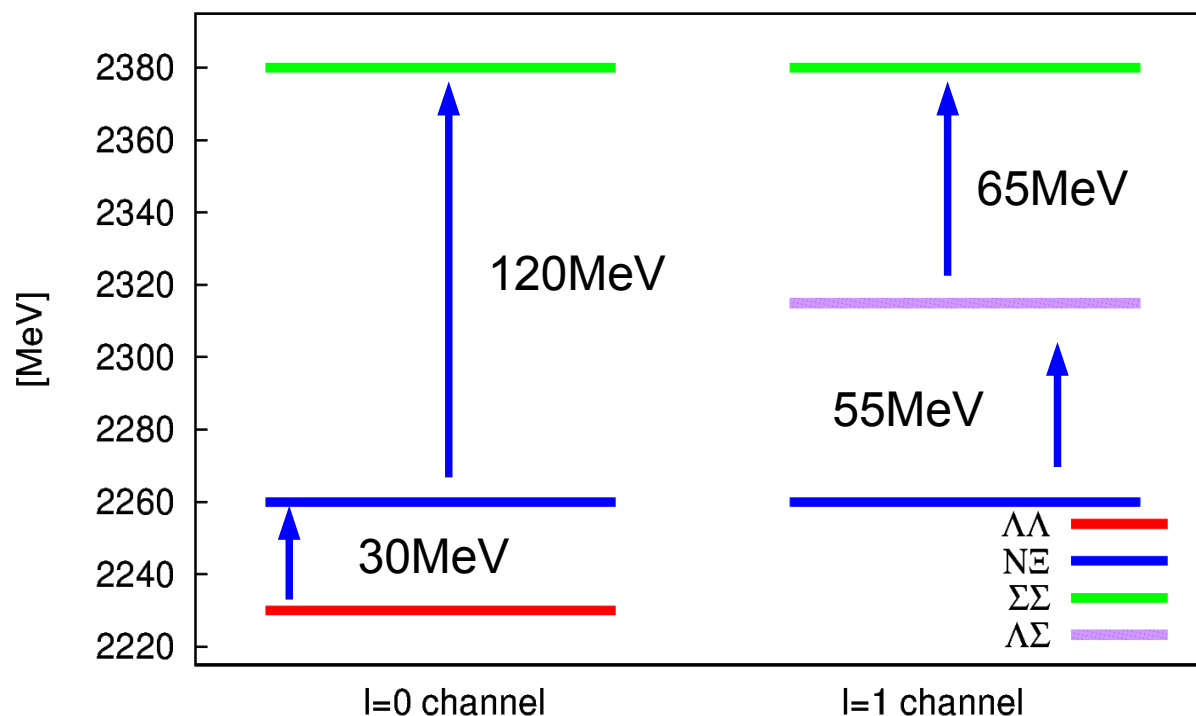


# Numerical setup

- ▶ **2+1 flavor** gauge configurations.
  - Iwasaki gauge action &  $O(a)$  improved Wilson quark action
  - $a = 0.086$  [fm],  $a^{-1} = 2.300$  GeV.
  - $96^3 \times 96$  lattice,  $L = 8$  [fm].
  - 200 confs x 12 sources x 4 rotations.
- ▶ **Flat wall source** is considered to produce S-wave B-B state.



Exp.	Mass [MeV]
$\pi$	140
$K$	495
$m_\pi / m_K$	0.28
$N$	940
$\Lambda$	1115
$\Sigma$	1190
$\Xi$	1320





# *Numerical results*

# Lists of channels

## I=0 states

Spin	BB channels			SU(3) representation		
$^1S_0$	$\Lambda\Lambda$	$N\Xi$	$\Sigma\Sigma$	1	8s	27
$^3S_1$	--	$N\Xi$	--	8a	--	--

Strong attraction  
(H-dibaryon)

## I=1 states

Spin	BB channels			SU(3) representation		
$^1S_0$	$N\Xi$	--	$\Lambda\Sigma$	--	8s	27
$^3S_1$	$N\Xi$	$\Sigma\Sigma$	$\Lambda\Sigma$	8a	10	10*

Attraction

Strong repulsion

Similar to  
The NN potential

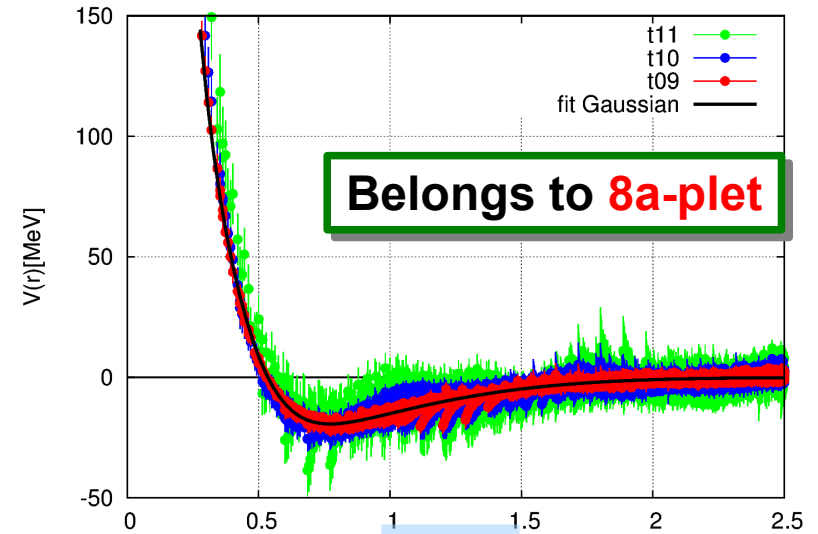
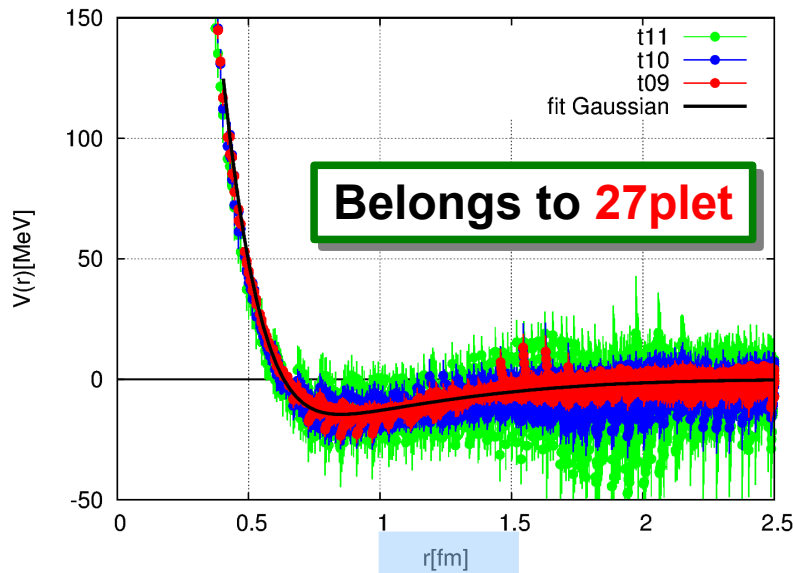
## I=2 states

Spin	BB channels			SU(3) representation		
$^1S_0$	$\Sigma\Sigma$			--	--	27
$^3S_1$						

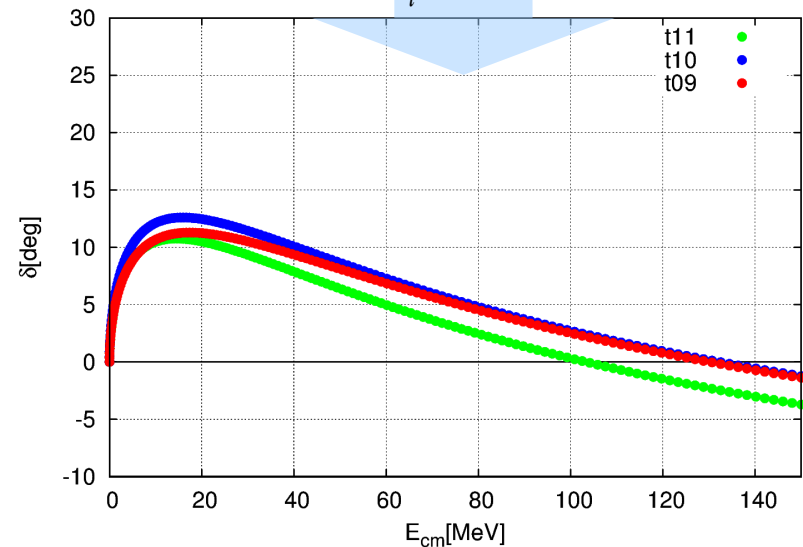
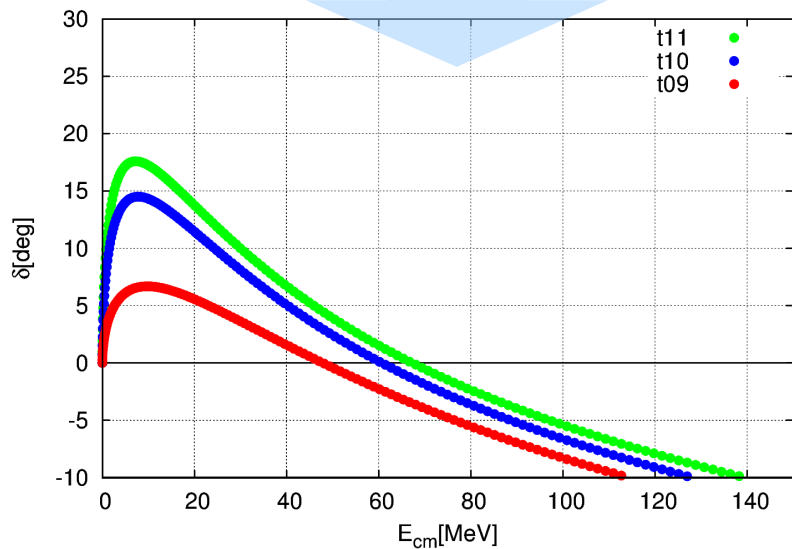
Repulsion

# $\Sigma\Sigma (I=2) ^1S_0$ channel

# $N\Xi (I=0) ^3S_1$ channel

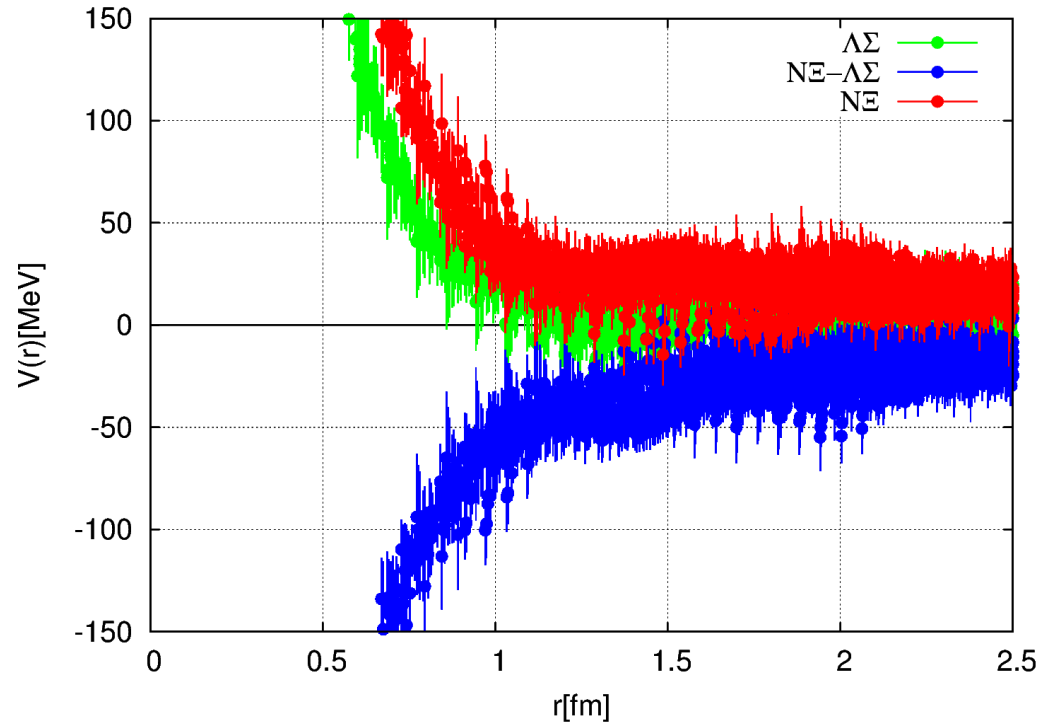


Potential is fitted by three-ranged Gaussian function.  $f(r) = \sum_i A_i \exp(-B_i r)$



Potential looks almost saturated at t=10.

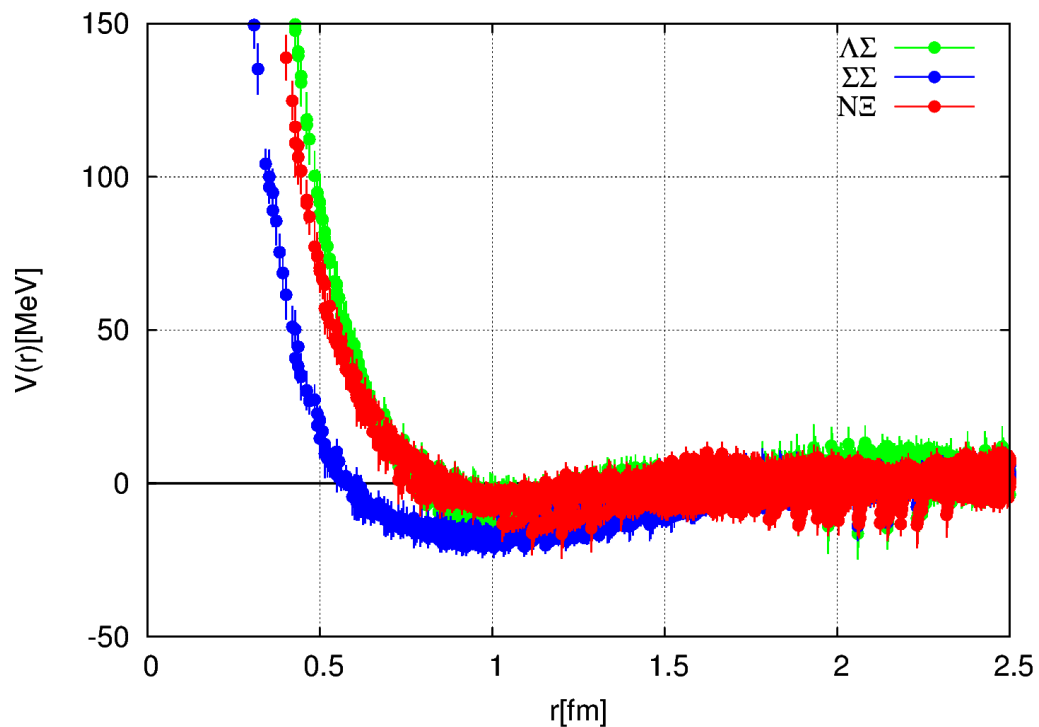
# $N\Xi, \Lambda\Sigma (l=1) ^1S_0$ channel



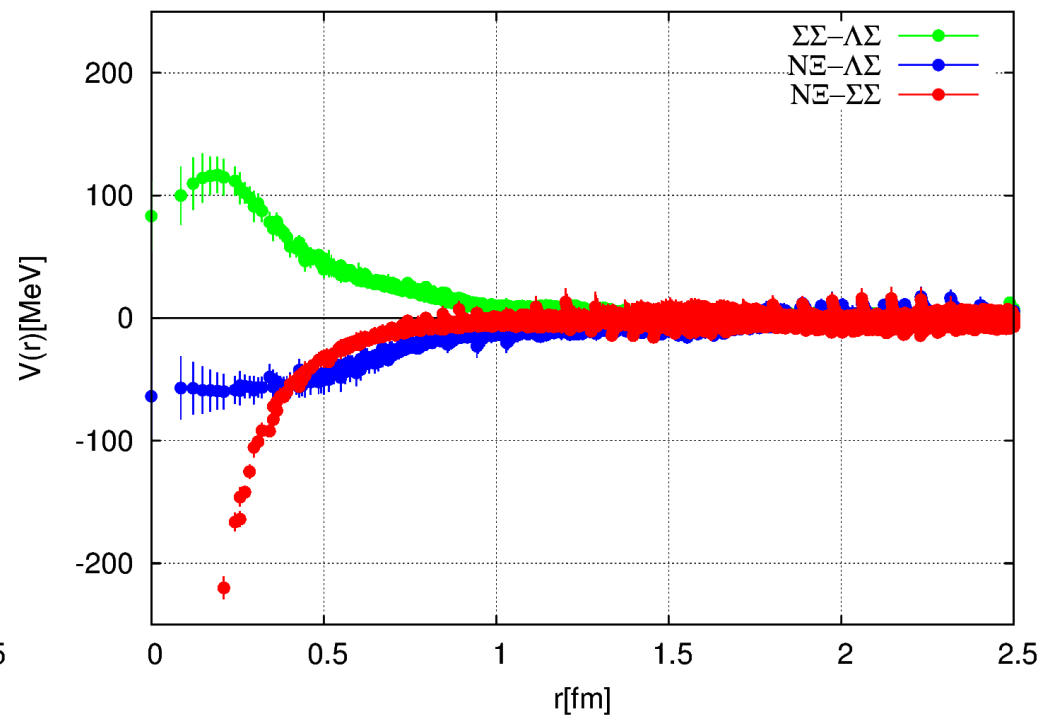
- All diagonal elements are totally repulsive in whole range.
- Diagonal  $N\Xi$  potential is strongly repulsive unlike the  $l=0$   $^3S_1$  case.  
It means that the  $N\Xi$  potential is strongly dependent on the channel.

# $N\Sigma, \Lambda\Sigma, \Sigma\Sigma (l=1) {}^3S_1$ channel

## Diagonal elements



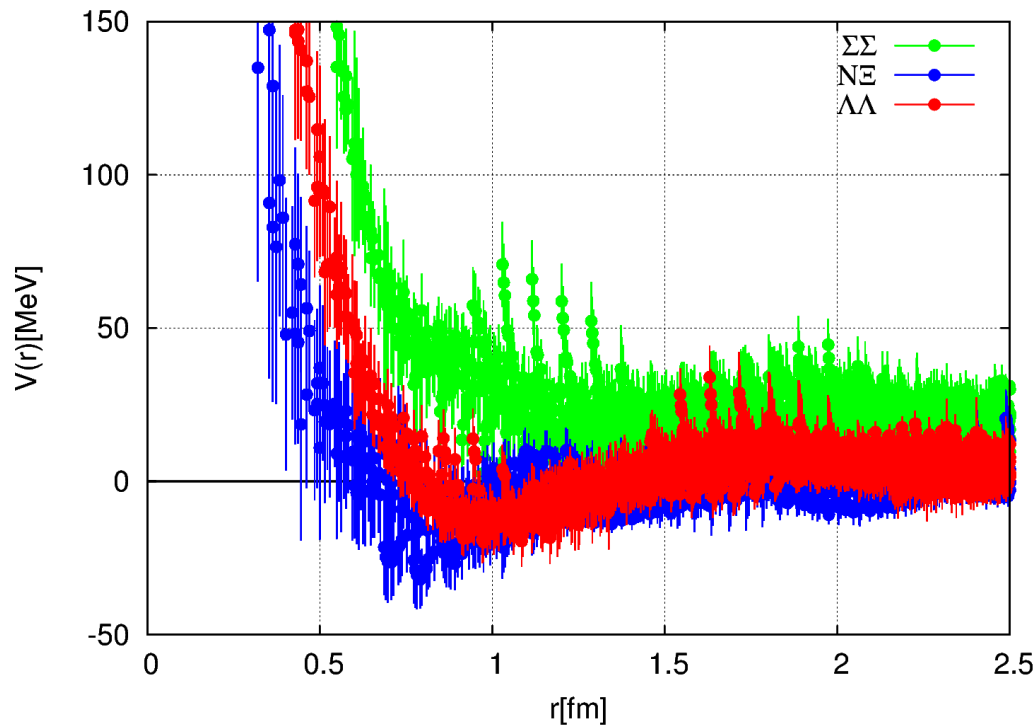
## Off-diagonal elements



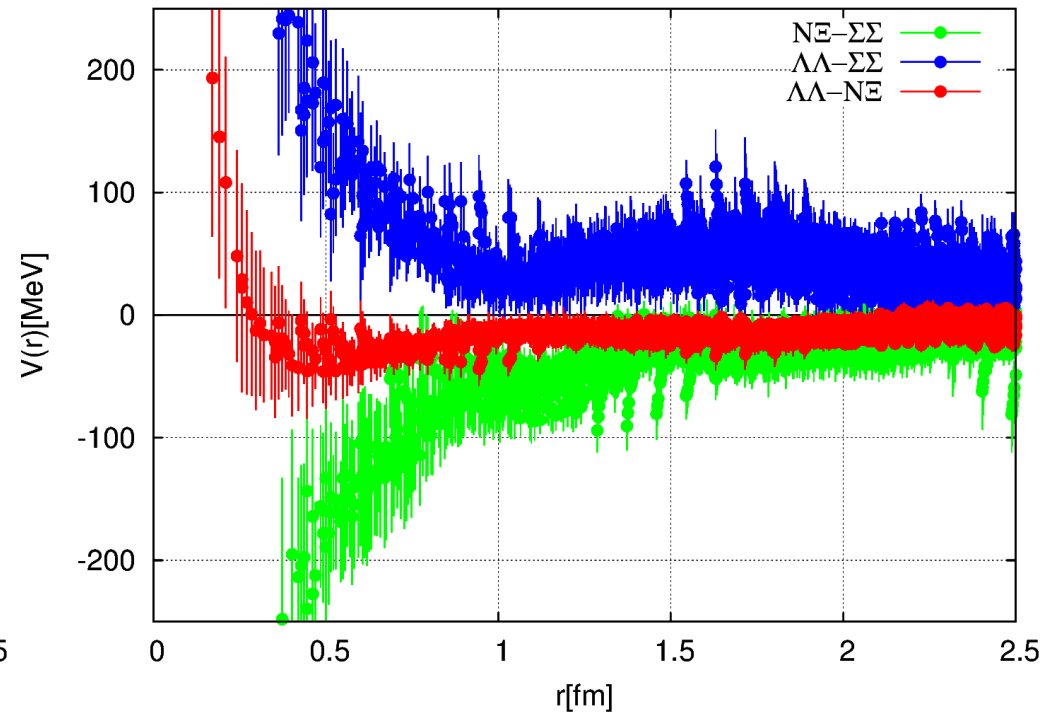
- All diagonal elements have a repulsive core and shallow attractive pocket.
- Diagonal  $\Sigma\Sigma$  potential is most attractive within them.
- We find that  $N\Sigma-\Sigma\Sigma$  transition potential is relatively strong comparing to the other transition potentials

# $\Lambda\Lambda, N\Xi, \Sigma\Sigma (I=0) ^1S_0$ channel

## Diagonal elements



## Off-diagonal elements



- All diagonal elements have a repulsive core  $\Sigma\Sigma-\Sigma\Sigma$  potential is strongly repulsive.
- Off-diagonal potentials are relatively strong except for  $\Lambda\Lambda-N\Xi$  transition
- We need more statistics to discuss physical observables through this potential.

# Comparison of potential matrices

Transformation of potentials

from the particle basis to the SU(3) irreducible representation (irrep) basis.

SU(3) Clebsh-Gordan coefficients

$$\begin{pmatrix} |1\rangle \\ |8\rangle \\ |27\rangle \end{pmatrix} = U \begin{pmatrix} |\Lambda\Lambda\rangle \\ |N\Xi\rangle \\ |\Sigma\Sigma\rangle \end{pmatrix}, \quad U \begin{pmatrix} V^{\Lambda\Lambda} & V^{\Lambda\Lambda}_{N\Xi} & V^{\Lambda\Lambda}_{\Sigma\Sigma} \\ V^{N\Xi}_{\Lambda\Lambda} & V^{N\Xi} & V^{N\Xi}_{\Sigma\Sigma} \\ V^{\Sigma\Sigma}_{\Lambda\Lambda} & V^{\Sigma\Sigma}_{N\Xi} & V^{\Sigma\Sigma} \end{pmatrix} U^t \rightarrow \begin{pmatrix} V_1 & & \\ & V_8 & \\ & & V_{27} \end{pmatrix}$$

In the SU(3) irreducible representation basis,

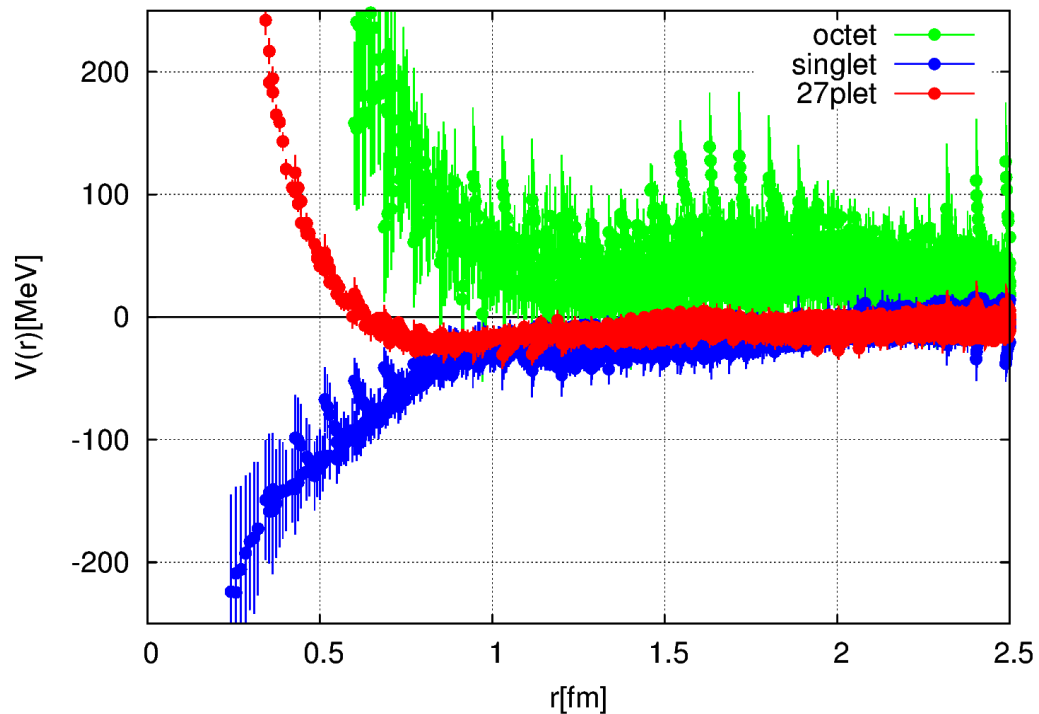
the potential matrix should be diagonal in the SU(3) symmetric configuration.



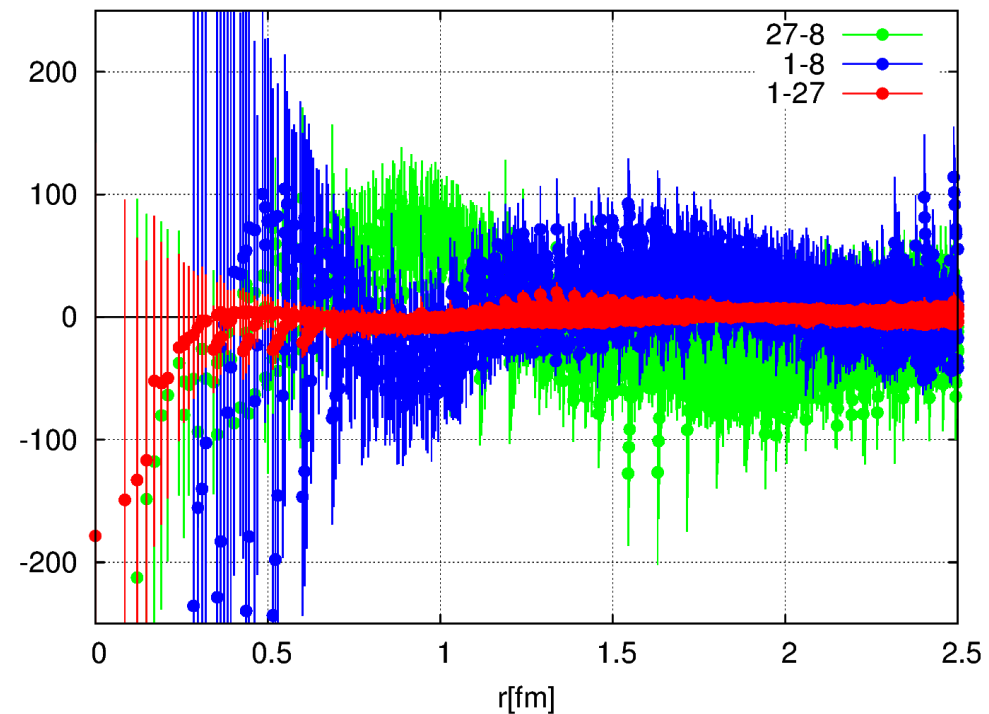
Off-diagonal part of the potential matrix in the SU(3) irrep basis would be an effectual measure of the SU(3) breaking effect.

# $1, 8s, 27plet \ ^1S_0$ channel

## Diagonal elements



## Off-diagonal elements



- Potential of flavor singlet channel does not have a repulsive core
- Potential of flavor octet channel is strongly repulsive which reflect a Pauli effect.
- Off-diagonal potentials are visible only in  $r < 1$  fm region.



# Summary and outlook

- ▶ We have investigated  $S=-2$  BB interactions from lattice QCD near the physical point.
- ▶ We find that
  - Potential in  $\Lambda\Lambda$   $^1S_0$  channel is weakly attractive.
  - NE potential is largely depends on its channel.
  - Potential in flavor singlet  $^1S_0$  channel is strongly attractive.
- ▶ It is not enough statistics to calculate several observables and to discuss the fate of H-dibaryon.
- ▶ Further investigation will be performed with high statistics data.



*Backup slides*

# Coupled channel Schrödinger equation

Preparation for the NBS wave function

$$\Psi^\alpha(E, t, \vec{r}) = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^\alpha(t, \vec{r}) | E \rangle$$

$$\Psi^\beta(E, t, \vec{r}) = \sum_{\vec{x}} \langle 0 | (B_1 B_2)^\beta(t, \vec{x}) | E \rangle$$

Two-channel coupling case

The same "in" state

Inside the interaction range

In the *leading order of velocity expansion* of non-local potential,

Coupled channel Schrödinger equation.

$$\left( \frac{p_\alpha^2}{2\mu_\alpha} + \frac{\nabla^2}{2\mu_\alpha} \right) \psi^\alpha(\vec{x}, E) = V_\alpha^\alpha(\vec{x}) \psi^\alpha(\vec{x}, E) + V_\alpha^\beta(\vec{x}) \psi^\beta(\vec{x}, E)$$

Factorization of interaction kernel

$\mu_\alpha$  : reduced mass

$p_\alpha$  : asymptotic momentum.

Asymptotic momentum are replaced by the time-derivative of  $R$ .

$$R_I^{B_1 B_2}(t, \vec{r}) = \sum_{\vec{x}} \langle 0 | B_1(t, \vec{x} + \vec{r}) B_2(t, \vec{x}) \bar{I}(0) | 0 \rangle e^{(m_1 + m_2)t}$$

$$\begin{pmatrix} V_\alpha^\alpha(\vec{r}) & V_\beta^\alpha(\vec{r})x \\ V_\alpha^\beta(\vec{r})x^{-1} & V_\beta^\beta(\vec{r}) \end{pmatrix} = \begin{pmatrix} \left( \frac{\nabla^2}{2\mu_\alpha} - \frac{\partial}{\partial t} \right) R_{I1}^\alpha(\vec{r}, E) & \left( \frac{\nabla^2}{2\mu_\beta} - \frac{\partial}{\partial t} \right) R_{I2}^\beta(\vec{r}, E) \\ \left( \frac{\nabla^2}{2\mu_\alpha} - \frac{\partial}{\partial t} \right) R_{I1}^\alpha(\vec{r}, E) & \left( \frac{\nabla^2}{2\mu_\beta} - \frac{\partial}{\partial t} \right) R_{I2}^\beta(\vec{r}, E) \end{pmatrix} \begin{pmatrix} R_{I1}^\alpha(\vec{r}, E) & R_{I1}^\beta(\vec{r}, E) \\ R_{I2}^\alpha(\vec{r}, E) & R_{I2}^\beta(\vec{r}, E) \end{pmatrix}^{-1}$$

$$x = \frac{\exp(-(m_{\alpha_1} + m_{\alpha_2})t)}{\exp(-(m_{\beta_1} + m_{\beta_2})t)}$$