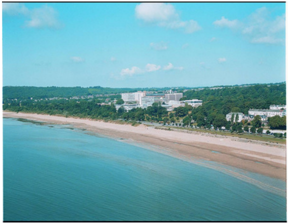
Nucleons and parity doubling across the deconfinement transition

Chris Allton

Swansea University, U.K.



Lattice 2015, Kobe, July 2015

arXiv:1502.03603

FASTSUM Collaboration

Gert Aarts[1], CRA[1], Alessandro Amato[1,2], Davide de Boni[1], Wynne Evans[1,3], Pietro Giudice[4], Simon Hands[1], Benjamin Jäger[1], Aoife Kelly[5], Seyong Kim[6], Maria-Paola Lombardo[7], Dhagash Mehta[8], Bugra Oktay[9], Chrisanthi Praki[1], Sinead Ryan[10], Jon-Ivar Skullerud[5], Tim Harris[10,11]

[1] Swansea University

[2] University of Helsinki

[3] University of Bern

[4] Münster University

[5] Maynooth University

[6] Sejong University

[7] Frascati, INFN

[8] North Carolina State University

[9] University of Utah

[10] Trinity College Dublin

[11] University of Mainz

Setting the scene

- anisotropic lattices $a_\tau < a_s$
  - allowing better resolution, particularly at finite temperatures (since $T = (N_\tau a_\tau)^{-1}$)
- "2nd" generation lattice ensembles
  - moving towards continuum, infinite volume, realistic light quark masses

Physics/lattice parameters

2nd Generation

2+1 flavours

larger volume: $(3\text{fm})^3 - (4\text{fm})^3$
finer lattices: $a_s = 0.123$ fm
quark mass: $M_\pi / M_\rho \sim 0.45$
temporal cut-off: $a_\tau \sim 5.6$ GeV

N_s	N_τ	T(MeV)	T/T_c
24, 32	16	352	1.90
24	20	281	1.52
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24, 32	28	201	1.09
24, 32	32	176	0.95
24	36	156	0.84
24	40	141	0.76
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16	128	44	0.24

Gauge Action:

Symanzik-improved, tree-level tadpole

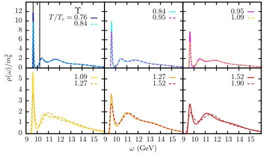
Fermion Action:

clover, stout-links, tree-level tadpole

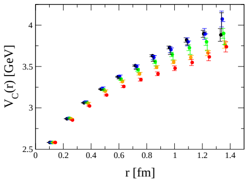
(Hadron Spectrum Collaboration)

Previous Physics Results

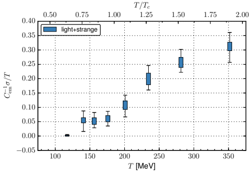
- Bottomonium and Charmonium Spectral Functions



- Charmonium Potential



- Conductivity, Susceptibility and Diffusion Coefficient



Baryons at Finite Temperature

- little work on Baryons @ $T \neq 0$
  - DeTar and Kogut (1987) screening masses
  - QCD-TARO (2005) $\mu \neq 0$
  - Datta et al (2013) quenched

We use a standard baryon operator:

$$O_N(\mathbf{x},\tau) = \epsilon_{abc} u_a(\mathbf{x},\tau) \left[u_b^T(\mathbf{x},\tau) \mathcal{C}\gamma_5 d_c(\mathbf{x},\tau) \right]$$

and parity project it:

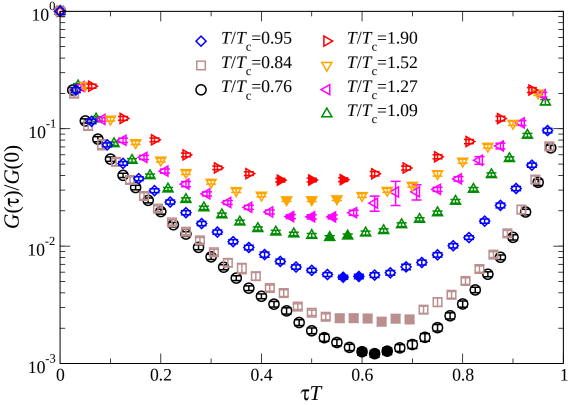
$$O_{N_\pm}(\mathbf{x},\tau) = P_\pm O_{N_\pm}(\mathbf{x},\tau)$$

Forward (+ve) and backward (−ve) parity states in correlator [Praki's talk]:

$$\begin{aligned} G_+(\tau) &= \int d^3x \, \langle O_{N_+}(\mathbf{x},\tau) \overline{O}_{N_+}(\mathbf{0},0) \rangle \\ &= \int_0^\infty \frac{d\omega}{2\pi} \left[\frac{e^{-\omega\tau}}{1+e^{-\omega/T}} \, \rho_+(\omega) - \frac{e^{-\omega(1/T-\tau)}}{1+e^{-\omega/T}} \, \rho_-(\omega) \right] \end{aligned}$$

Baryon Correlators

(Using Gaussian smeared baryon operators)



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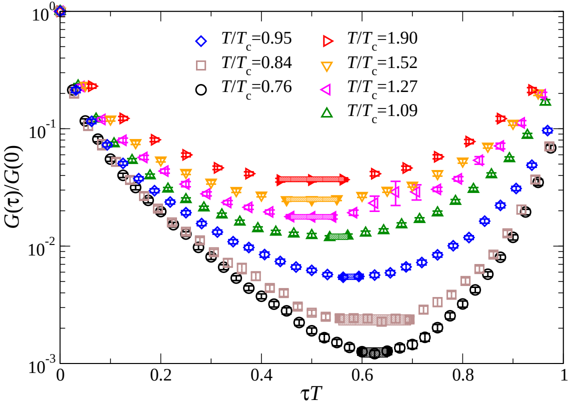
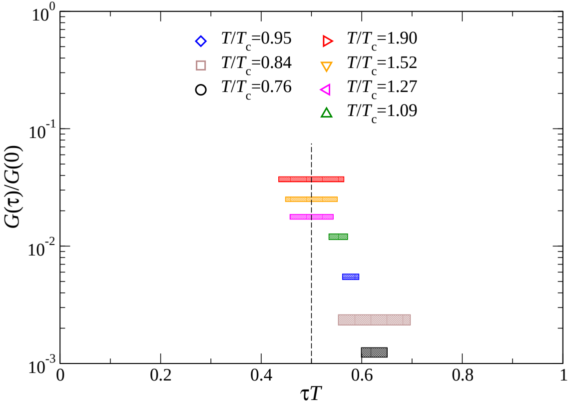


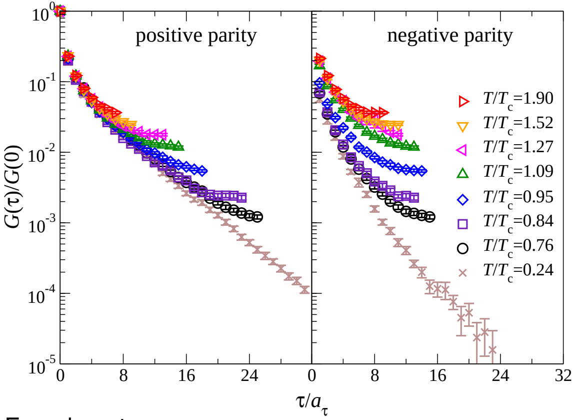
Baryon Correlators

(Using Gaussian smeared baryon operators)

$\longrightarrow$ parity doubling for $T \gtrsim T_C$ observed at correlator level



Correlators - Parity Comparison



Experiment:

$+$ve parity: $M_N = 939$ MeV $\quad -$ve parity: $M_{N*} = 1535$ MeV

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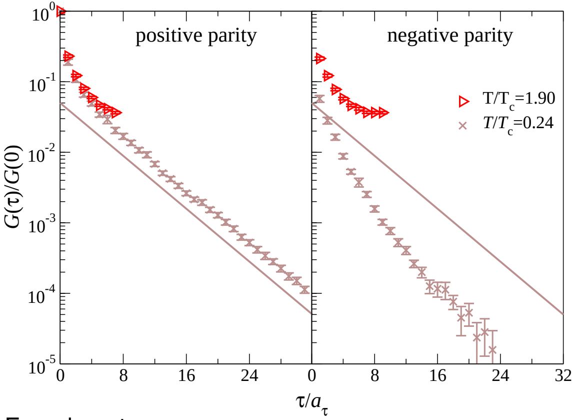


Experiment:

+ve parity: $M_N = 939$ MeV   −ve parity: $M_{N*} = 1535$ MeV

Naive Exponential Fits

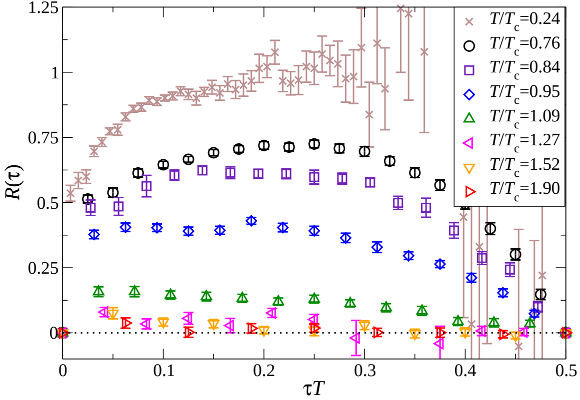
T/T_c	m_+ [GeV]	m_- [GeV]	$m_+ - m_-$ [MeV]	
0.24	1.20(3)	1.9(3)	$\sim$700	*cf* expt: $\sim$600
0.76	1.18(9)	1.6(2)		
0.84	1.08(9)	1.6(1)		
0.95	1.12(14)	1.3(2)		

Parity Comparison

Define $R(t) = \dfrac{G(\tau) - G(N_\tau - \tau)}{G(\tau) + G(N_\tau - \tau)}$ Datta et al, arXiv:1212.2927

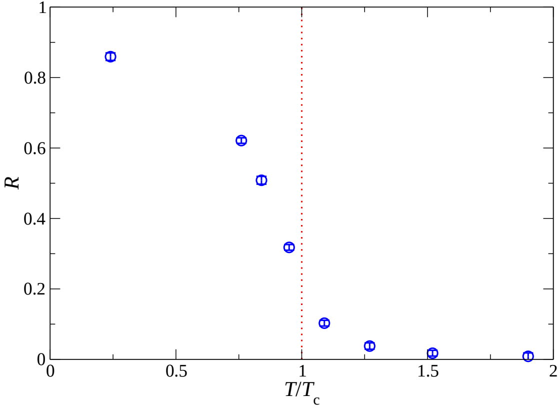
Note: $R(1/2T) \equiv 0$

with: $R(\tau) \equiv 0$ for parity symmetry



Parity Restoration

Define $$R = \frac{\sum_{\tau=1}^{N_\tau/2-1} R(\tau)/\sigma^2(\tau)}{\sum_{\tau=1}^{N_\tau/2-1} 1/\sigma^2(\tau)}$$



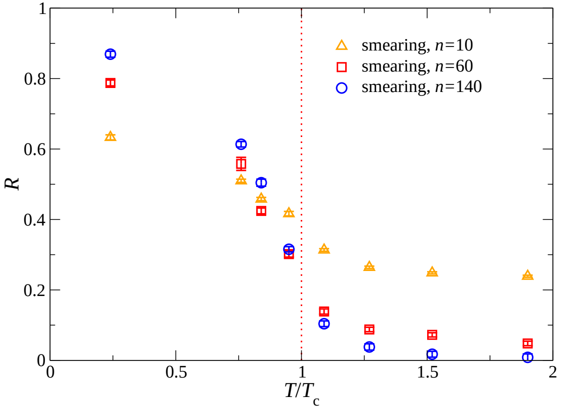
Effects of Smearing

Above results used Gaussian smearing with sources/sinks, η smeared with:

$\eta' = C(1 + \kappa H)^n \eta$ using $\kappa = 8.7$ and $n = 140$ Capitani et al [arXiv:1205.0180]

Systematics checks of smearing

- vary n
- vary τ-range



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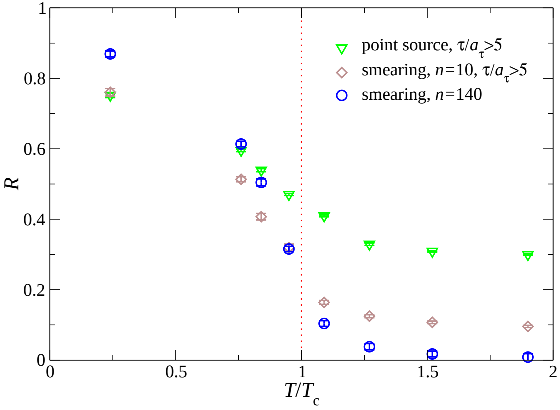

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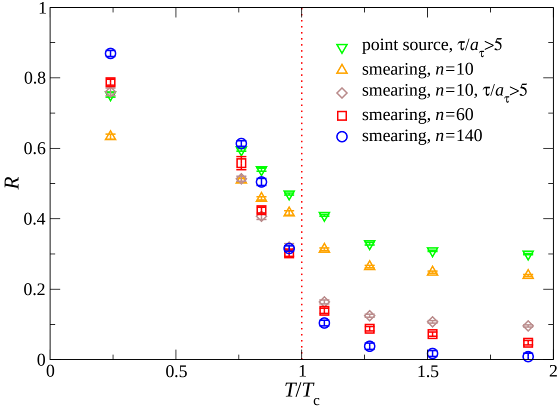
- vary τ-range

Implies parity doubling is:

- ground state feature (*recall* Wilson term breaks chiral symmetry)
- not an artefact of smearing



Maximum Entropy Method (MEM)

Cont: $G(\tau) = \int K(\tau,\omega)\rho(\omega)d\omega$ Lat: $G(\tau_i) = \sum_j K(\tau_i,\omega_j)\,\rho(\omega_j)$

Input data: $\tau_i,\ i = \{1,\ldots,\mathcal{O}(10)\}$ Output data : $\omega_j,\ j = \{1,\ldots,\mathcal{O}(10^3)\}$

$\longrightarrow$ ill-posed

Bayes Th'm: $P[\rho|DH] = \frac{P[D|\rho H]P[\rho|H]}{P[D|H]} \propto \exp(-\chi^2 + \alpha S)$

H = prior knowledge D = data

Shannon-Jaynes entropy: $S = \int_0^\infty \frac{d\omega}{2\pi}\left[\rho(\omega) - m(\omega) - \rho(\omega)\ln\frac{\rho(\omega)}{m(\omega)}\right]$

Competition between minimising χ^2 and maximising S

Asakawa, Hatsuda, Nakahara, Prog.Part.Nucl.Phys. 46 (2001) 459

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MEM for finite T baryons

Recall: $$G_+(\tau) = \int d^3x \, \langle O_{N_+}(\mathbf{x},\tau)\overline{O}_{N_+}(\mathbf{0},0)\rangle$$

Praki's talk: $$= \int_0^\infty \frac{d\omega}{2\pi}\left[\frac{e^{-\omega\tau}}{1+e^{-\omega/T}}\,\rho_+(\omega) - \frac{e^{-\omega(1/T-\tau)}}{1+e^{-\omega/T}}\,\rho_-(\omega)\right]$$

So can define: $$K(\tau,\omega) = \frac{e^{-\omega\tau}}{1+e^{-\omega/T}} \qquad \omega > 0$$
$$= \frac{e^{+\omega(1/T-\tau)}}{1+e^{+\omega/T}} \qquad \omega < 0$$

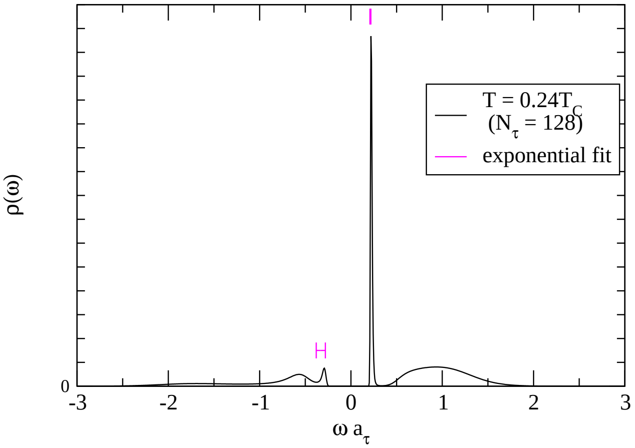
and use MEM with $$G_+(\tau) \equiv \int_{-\infty}^{+\infty} K(\tau,\omega)\rho(\omega)d\omega$$

giving: $$\rho_+(\omega) \equiv \rho(\omega) \qquad \omega > 0$$
$$\rho_-(-\omega) \equiv -\rho(\omega) \qquad \omega < 0$$

BUT need to assume $\rho(\omega)$ is positive definite for MEM to work

Baryonic Spectral Functions

Preliminary



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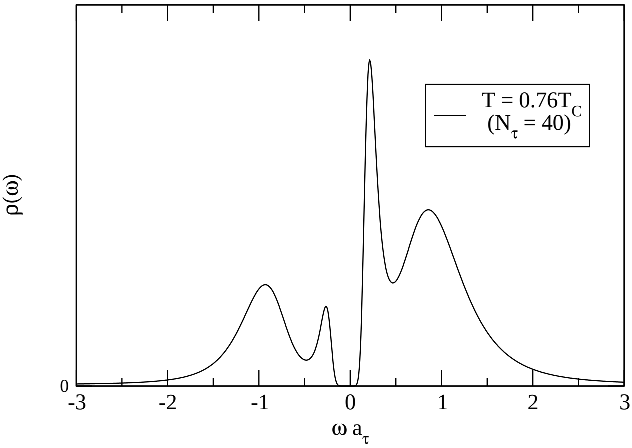

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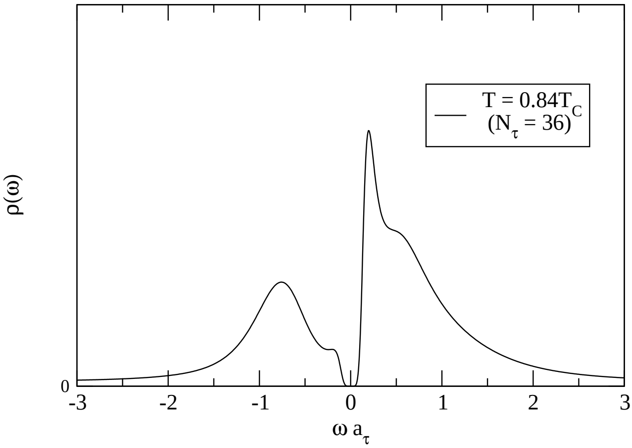

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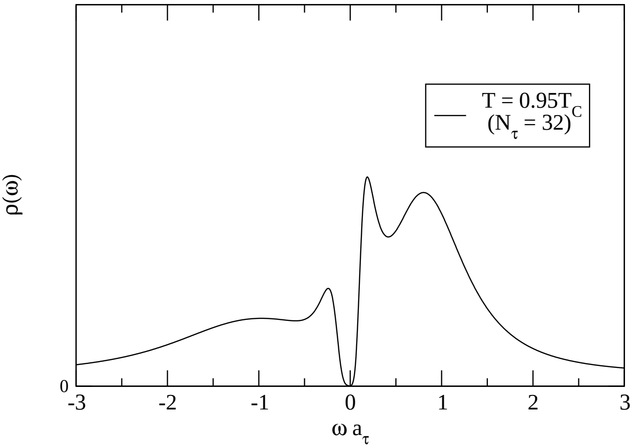

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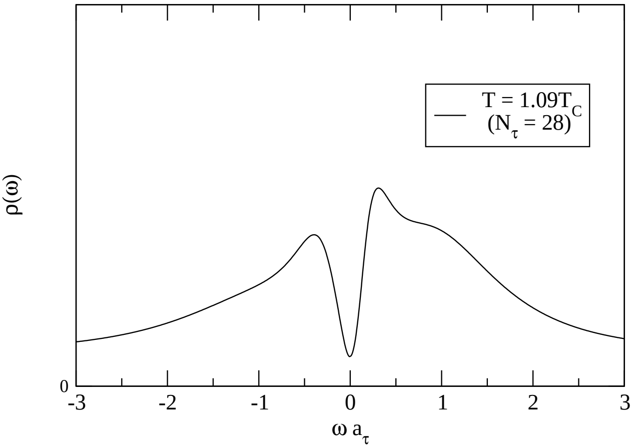

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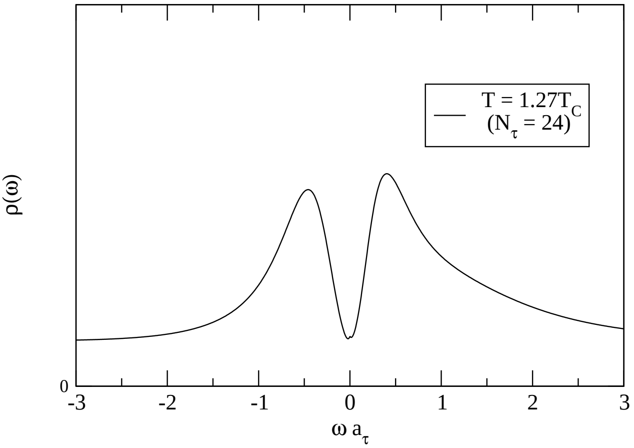

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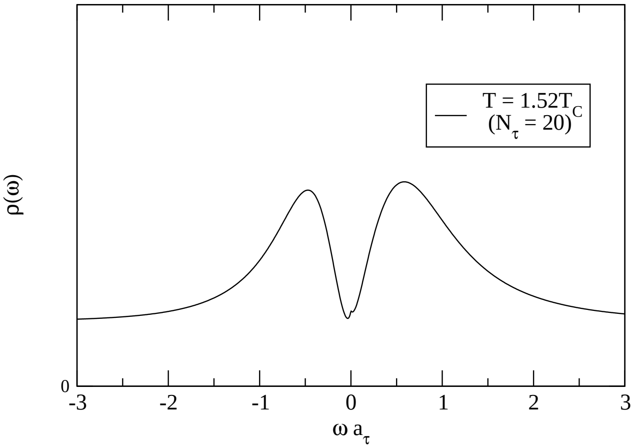

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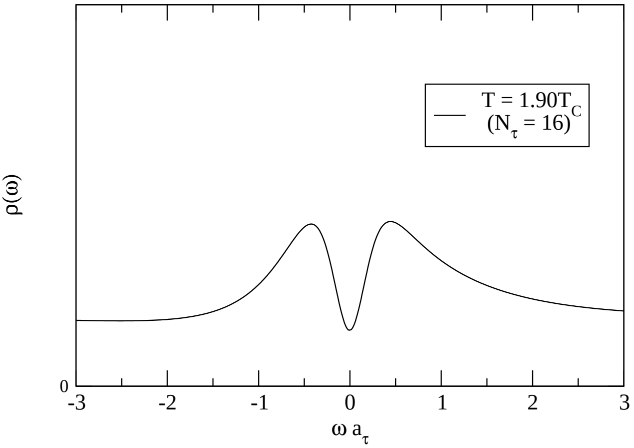

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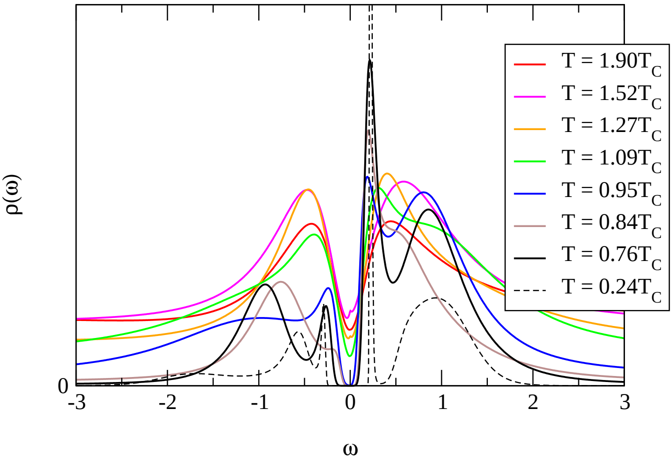
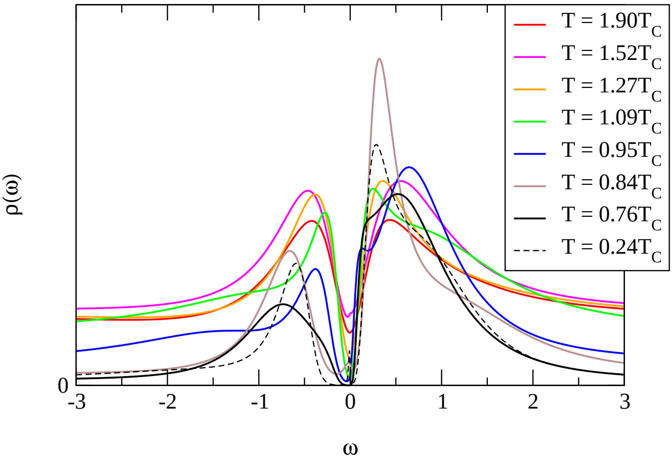


Baryonic Spectral Functions - Systematic Checks

Checking systematics by using MEM on fixed τ windows:

$$\tau = 1, 2, \ldots 7, N_\tau - 7, N_\tau - 6, \ldots, N_\tau - 1$$

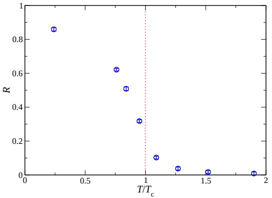
Preliminary

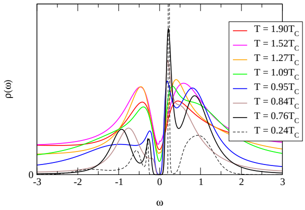


Summary

Baryonic Parity Restoration:

- Signicant thermal effects in $-$ve parity nucleon
- No observed thermal modification of $+$ve parity mass below T_C
- Degeneracy in ground state of baryonic parity partners above T_c
- Finite temperature baryonic spectral functions determined





Future work:

- strange baryons
- chiral fermions
- finer lattices

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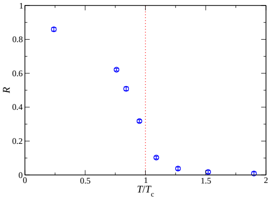

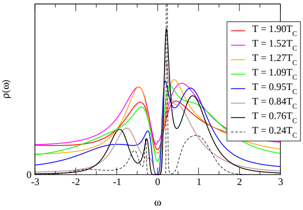

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Physics/lattice parameters

2nd Generation

2+1 flavours

larger volume: $(3\text{fm})^3$ – $(4\text{fm})^3$
finer lattices: $a_s = 0.123$ fm
quark mass: $M_\pi/M_\rho \sim 0.45$
temporal cut-off: $a_\tau \sim 5.6$ GeV

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24, 32	16	352	1.90
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3rd Generation

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quark mass: $M_\pi/M_\rho \sim 0.45$
temporal cut-off: $a_\tau \sim 11.2$ GeV

N_s	N_τ	T(MeV)	T/T_c
24, 32	32	352	1.90
24	40	281	1.52
24, 32	48	235	1.27
24, 32	56	201	1.09
24, 32	64	176	0.95
24	72	156	0.84
24	80	141	0.76
32	96	117	0.63
16	256	44	0.24

Particle Data Book

$\sim 1,500$ pages

zero pages on Quark-Gluon Plasma...

SLIDES TO HELP ME ANSWER DUMB QUESTIONS

SLIDES TO HELP ME ANSWER TRICKY QUESTIONS

Physics/lattice parameters

1st Generation

2 flavours

smaller volume: $(2\text{fm})^3$
coarser lattices: $a_s = 0.167$ fm
quark mass: $M_\pi/M_\rho \sim 0.55$
temporal cut-off: $a_\tau \sim 7.4$ GeV

N_s	N_τ	T(MeV)	T/T_c
12	16	460	2.09
12	18	409	1.86
12	20	368	1.68
12	24	306	1.40
12	28	263	1.20
12	32	230	1.05
12	80	90	0.42

2nd Generation

2+1 flavours

larger volume: $(3\text{fm})^3 - (4\text{fm})^3$
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MEM systematics

- default model
- time range
- energy discretisation: $\omega = \{\omega_{\text{min}}, \omega_{\text{min}} + \Delta\omega \ \ldots \omega_{\text{max}}\}$
- number of configs
- numerical precision

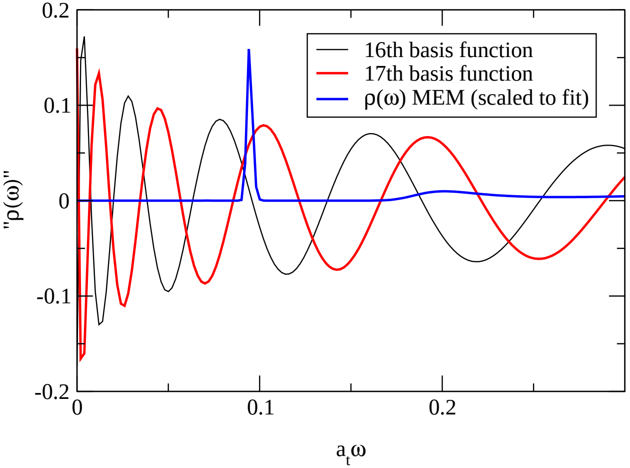
(All true also for BR)

Recall $\mathcal{I}(\rho) \leq N_t$ for MEM

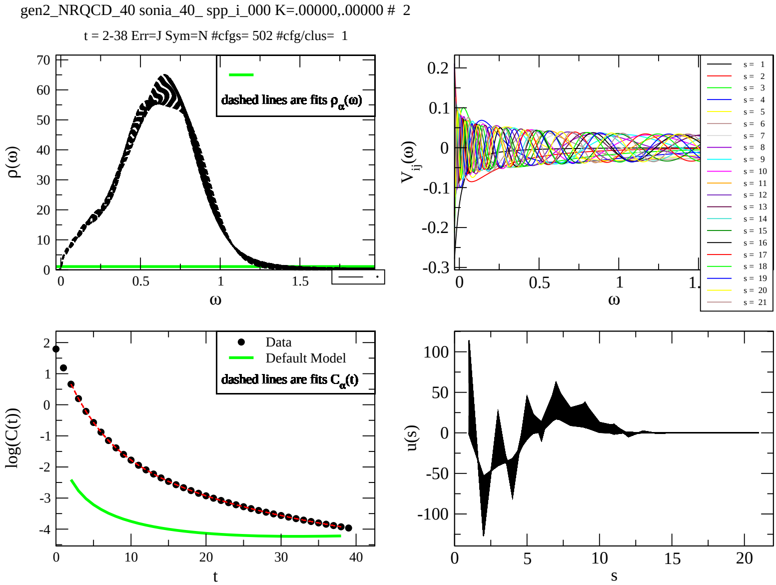
Can vary this in free case by varying N_t

Feature Resolution

MEM can reproduce features smaller than the characteristic size of its basis functions:



MEM: more than you ever wanted to know



The Task

Given data D

Find fit F by maximising $P(F|D)$

Bayes Theorem

Need to maximise $P(F|D)$

Bayes Theorem:

$$P(F|D)P(D) = P(D|F)P(F)$$

$$\text{i.e.} \quad P(F|D) = \frac{P(D|F)P(F)}{P(D)}$$

But $P(D|F) \sim e^{-\chi^2}$ $\longrightarrow$ minimising χ^2 $\neq$ maximising $P(F|D)$

$\longrightarrow$ *Maximum Likelihood Method* wrong??

No! Since for simple $F(t) = Ze^{-Mt}$, $P(F) = P(Z, M) \sim \text{const}$

Bayes Theorem

Need to maximise $P(F|D)$

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No! Since for simple $F(t) = Ze^{-Mt}$, $P(F) = P(Z, M) \sim \text{const}$

Priors

Actually $P(F = \text{elephant}) \equiv 0$

$\longrightarrow$ "priors" which encode any additional information

(a.k.a. predisposition, prejudices, impartialities, biases, prediliction, subjectivity, . . .)

E.g. in L.G.T. $P(M < 0) \equiv 0$

Maximum Likelihood Method applies this prior implicitly

Can encode prior information with "entropy"$= S$ (dis-information)

Define $\mathcal{I}(F)$ = "Information content" of F

"Bland" F has $\mathcal{I}(F) \sim 0$ and $S \gg 0$

"Spiky" F has $\mathcal{I}(F) \gg 0$ and $S \equiv 0$

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"Spiky" F has $\mathcal{I}(F) \gg 0$ and $S \equiv 0$

Entropy

	No Data	Data
No Prior	$\mathcal{I}(F) \equiv 0$	F from min χ^2
Prior	$F \equiv$ prior	F from max $P(F\|D)$

$$P(F) = e^{-S}$$