Large volume calculation of pion-pion scattering phase shifts with the stochastic LapH method

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People Involved

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Motivation

- Pi-pi scattering phase shifts are being calculated by several groups

- Can we push to lighter pion masses and larger volumes?
  - Larger volumes => improved resolution
  - Lighter pions => four pion thresholds are relevant

- A first exploratory study:
  - $N_f = 2 + 1$ anisotropic Wilson clover
  - $a_s / a_t \approx 3.5$ large volume, but good temporal resolution
  - $32^3 \times 256, \; m_\pi \approx 240\text{MeV}, \; a_s \approx 0.12\text{fm}, \; L \approx 4\text{fm}$
  - $m_\pi T \approx 10$ => safe from thermal effects

Hadron Spectrum Collaboration `09

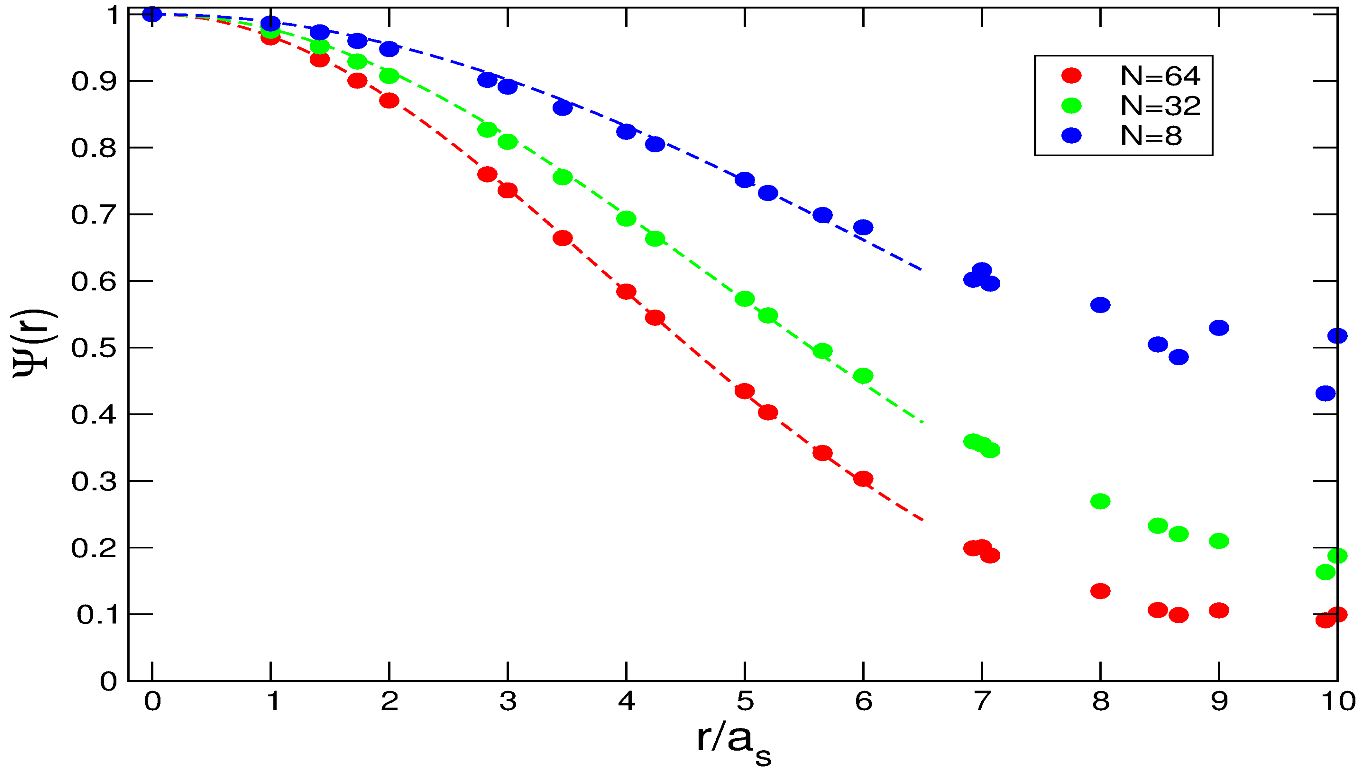
Disconnected diagrams require 'all-to-all' propagators

- 'Distillation': important physics is captured by a low-dimensional subspace.

$$\nabla^2 v_n = \lambda_n v_n$$

M. Peardon, et al. `08

- Subspace spanned by $N_{ev} << 12L^3 \times T$ eigenvectors of the (covariant) 3-D Laplace operator.
- Projector acts like a 'smearing' operator (think: Lossy compression)

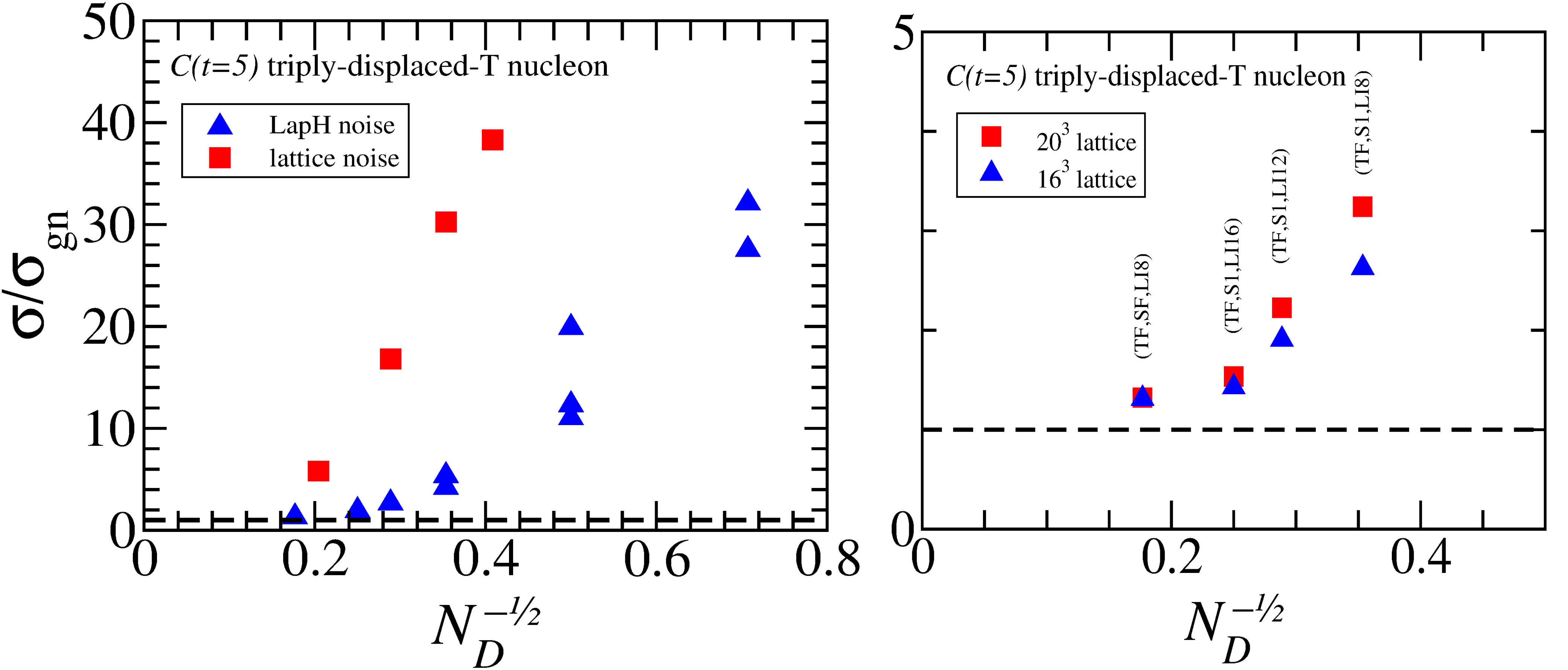


- Unfortunately, number of modes becomes prohibitively large in large volume: $N_{ev} \propto V$
- Use stochastic estimation in the low-dimensional subspace:

Morningstar, et al `11

$$\psi^r_{a\alpha}(x) = \rho^r_{\alpha i} v_{ia}(x), \quad \rho^r_{\alpha i} \in U(1)$$

- Locally coherent nature modes => good stochastic estimators with favorable volume scaling



Another advantage: Facilitates correlation matrix construction

- In order to extract excited energy levels, we require a matrix of temporal correlators between all operators in a basis.

$$C_{ij}(t) = \sum_n \langle 0|\hat{\mathcal{O}}_i|n\rangle\langle n|\hat{\mathcal{O}}_j^\dagger|0\rangle \mathrm{e}^{-E_n t}$$

- Complete separation of Dirac matrix simplifies this. $\psi^k = M^{-1}\eta^k$

$$\omega^i_{k\ell}(t) = \sum_{\vec{x}} \psi^{k\dagger}(x)\Gamma_i\psi^\ell(x) \qquad \kappa^j_{k\ell}(t_0) = \sum_{\vec{x}_0} \eta^{k\dagger}(x_0)\Gamma_j\eta^\ell(x_0)$$

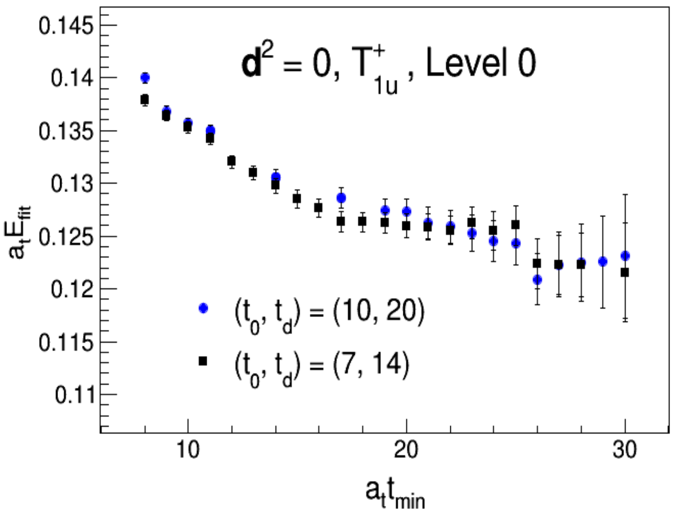
$$C^\pi_{ij}(t-t_0) = \omega^i_{k\ell}(t)\kappa^{j*}_{\ell k}(t_0)$$

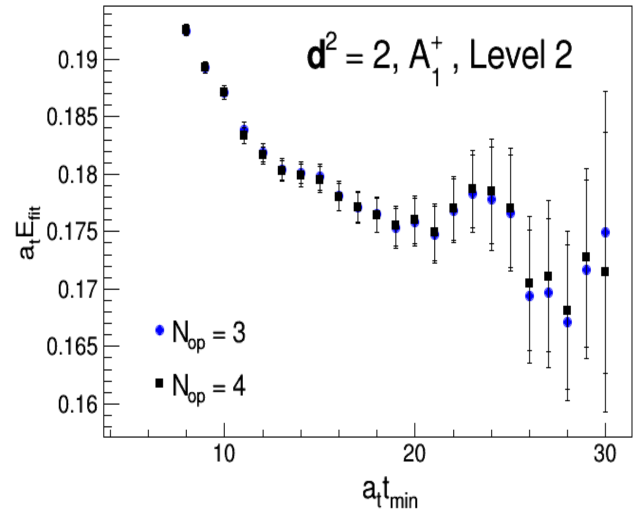
$$C_B^{I=2,\,2\pi}(t-t_0) = \omega^i_{k\ell}(t)\kappa^{j*}_{\ell m}(t_0)\omega^i_{mn}(t)\kappa^{j*}_{nk}(t_0)$$

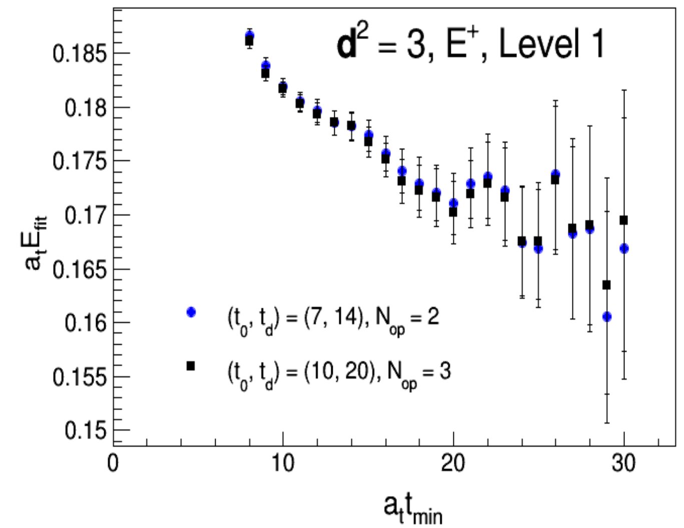
Using the Lüscher quantization condition: $\det[1 + F(S-1)] = 0$

$$\mathbf{P}_{tot} = \frac{2\pi}{L}\mathbf{d}, \quad \mathbf{p} = \frac{2\pi}{L}\mathbf{u}, \ \gamma = \frac{E}{E_{cm}}, \quad w_{\ell m} = \frac{8 Z_{\ell m}(\mathbf{d}, \gamma, \mathbf{u}^2)}{\gamma \pi^{-3/2} L^3 u^{\ell-2}}$$

$\mathbf{d}$	Λ	$\mathbf{p}^3 \cot\ \delta_1$
$(0,0,0)$	T_{1u}^+	w_{00}
$(0,0,n)$	A_1^+	$w_{00} + \frac{2}{\sqrt{5}} w_{20}$
	E^+	$w_{00} - \frac{1}{\sqrt{5}} w_{20}$
$(0,n,\pm n)$	A_1^+	$w_{00} + \frac{1}{2\sqrt{5}} w_{20} - \sqrt{\frac{6}{5}} i w_{21} - \sqrt{\frac{3}{10}} w_{22}$
	B_1^+	$w_{00} - \frac{1}{\sqrt{5}} w_{20} + \sqrt{\frac{6}{5}} w_{22}$
	B_2^+	$w_{00} + \frac{1}{2\sqrt{5}} w_{20} + \sqrt{\frac{6}{5}} i w_{21} - \sqrt{\frac{3}{10}} w_{22}$
$(n,\pm n,\pm n)$	A_1^+	$w_{00} + 2\sqrt{\frac{6}{5}} i w_{22}$
	E^+	$w_{00} - \sqrt{\frac{6}{5}} i w_{22}$





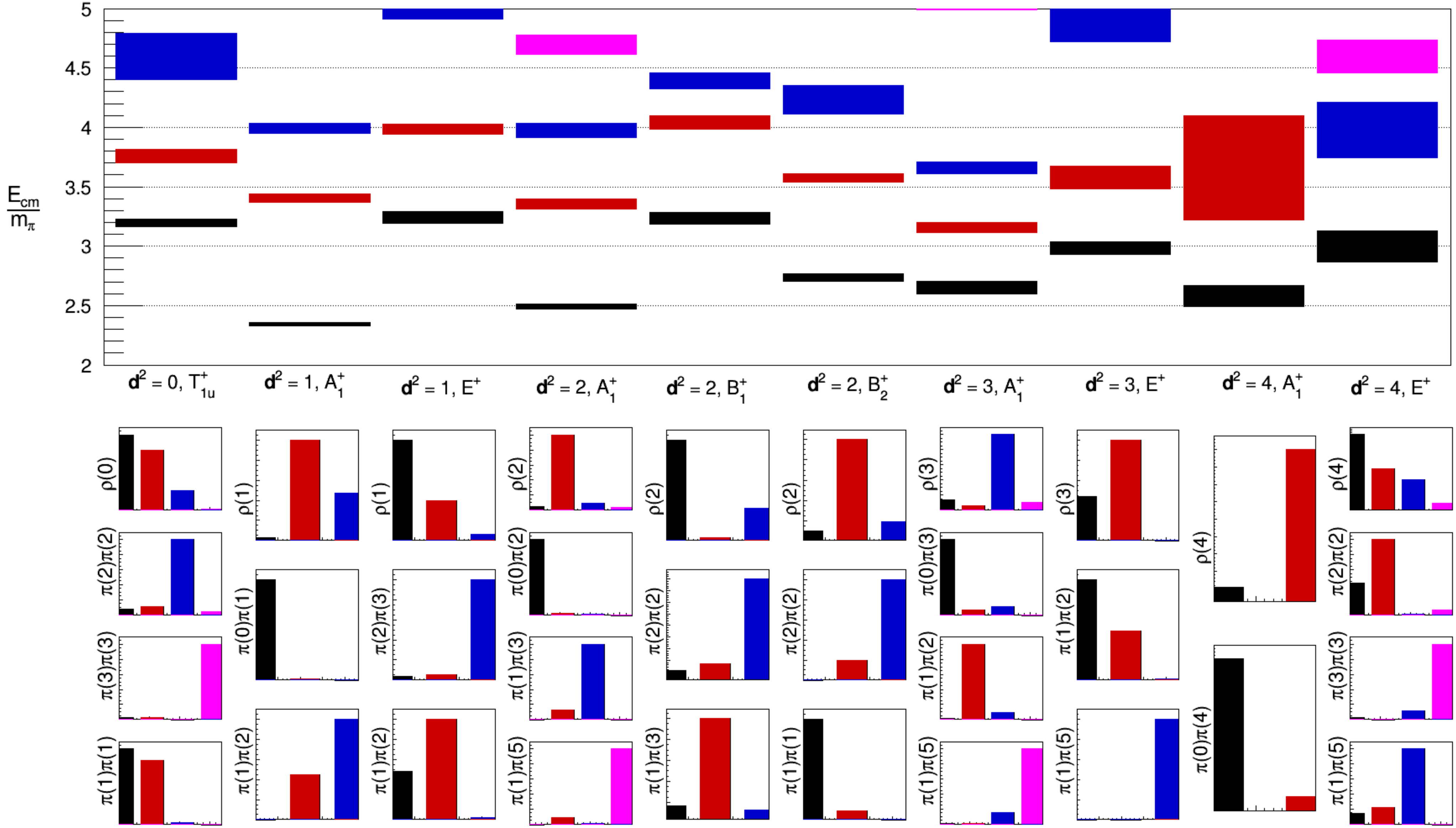


We solve the GEVP (on the mean) at (t_0, t_*) and form

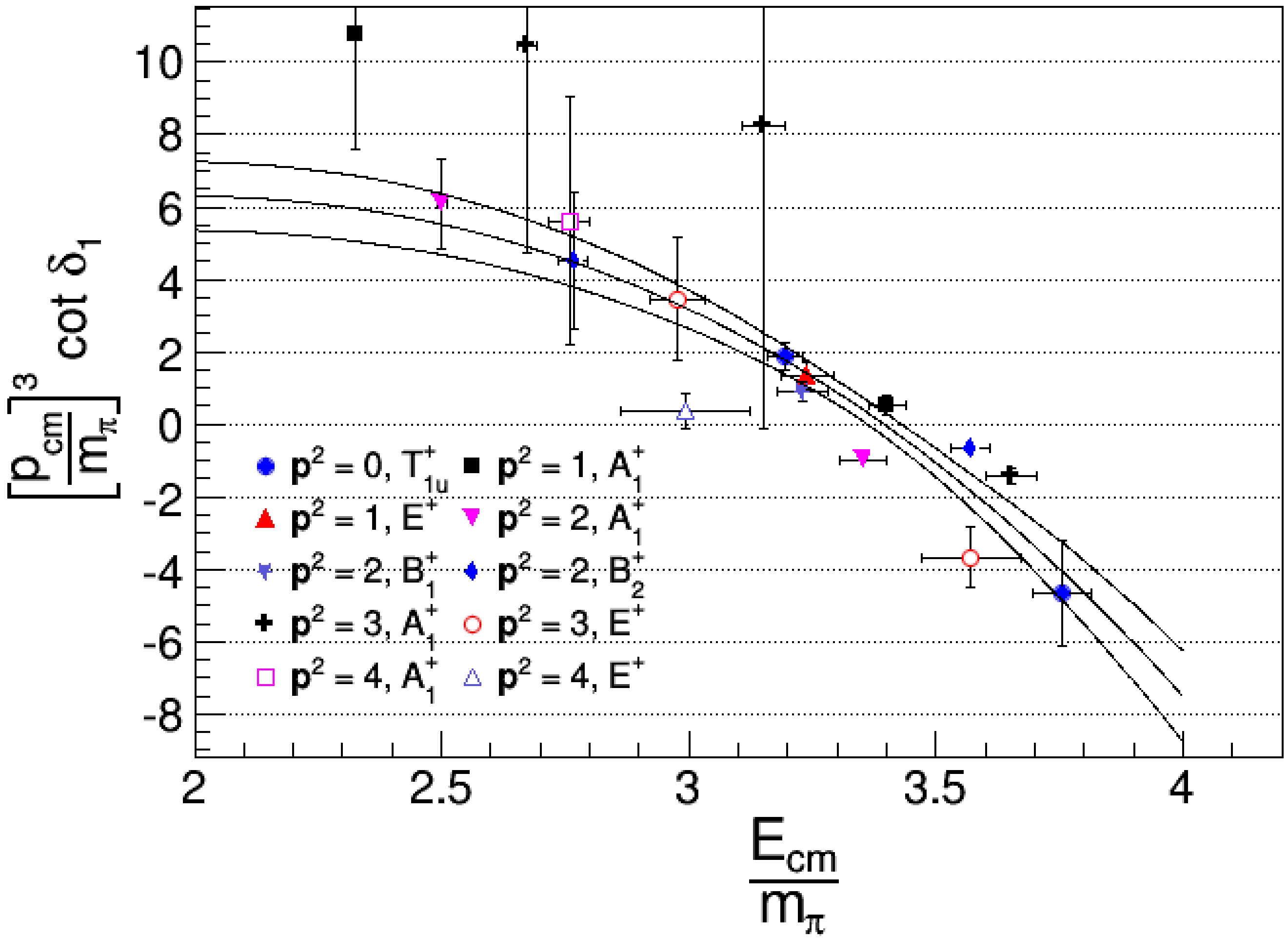
$$\hat{C}_{mn}(t) = (v_n, C(t) v_m)$$

- Fit the diagonal elements to a single exponential
- Consistent with fits to the full rotated matrix to the ansatz

$$\hat{C}_{mn}(t) = \sum_{i=1}^{n_{fit}} A_{im} A^*_{in} \, \mathrm{e}^{-E_i t}$$



Center-of-mass energies and overlaps: $Z_{in} = |\langle 0|\hat{\mathcal{O}}_i|n\rangle|^2$

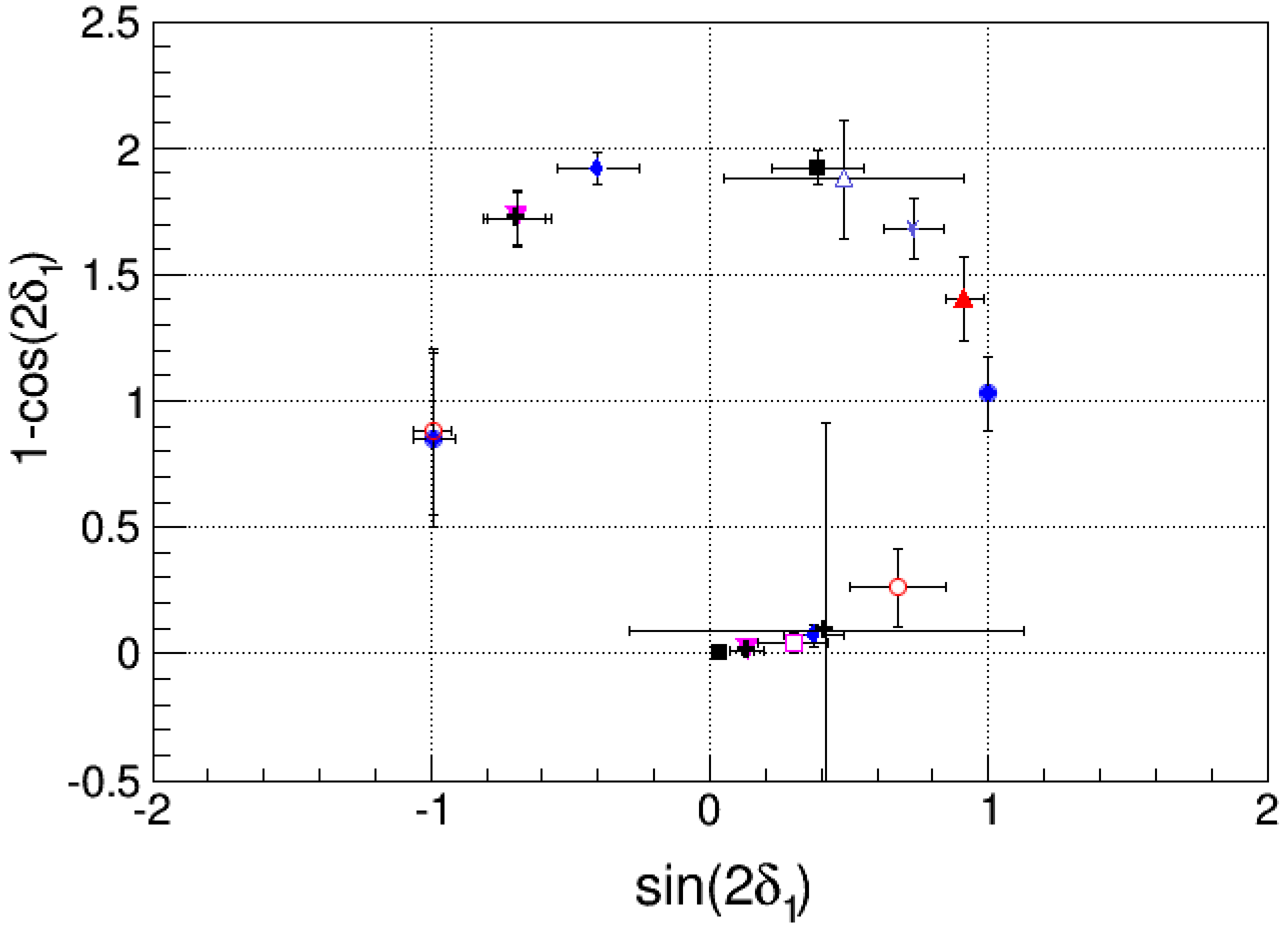


Breit-Wigner fit (consistent with effective range):

$$p^3 \cot\ \delta_1 = (m_R^2 - s)\frac{6\pi\sqrt{s}}{g_R^2}$$

$$\frac{m_\rho}{m_\pi} = 3.395(33), \qquad g_{\rho\pi\pi} = 6.71(49), \qquad \frac{\chi^2}{d.o.f} = 1.36$$

Argand plot shows characteristic phase motion:



Recent results from the JLab group on the same ensemble:

Wilson, et al `15

$$a_t m_\rho = 0.13175(35), \quad g_{\rho\pi\pi} = 5.688(75)$$

This work:

$$a_t m_\rho = 0.1337(11), \quad g_{\rho\pi\pi} = 6.71(49)$$

Dirac matrix inversions per configuration:

- JLab uses exact distillation

$$N_{inv} = 4 N_{ev} N_t = 4 \times 384 \times 256 = 393,216$$

- Our stochastic Laph dilution scheme requires

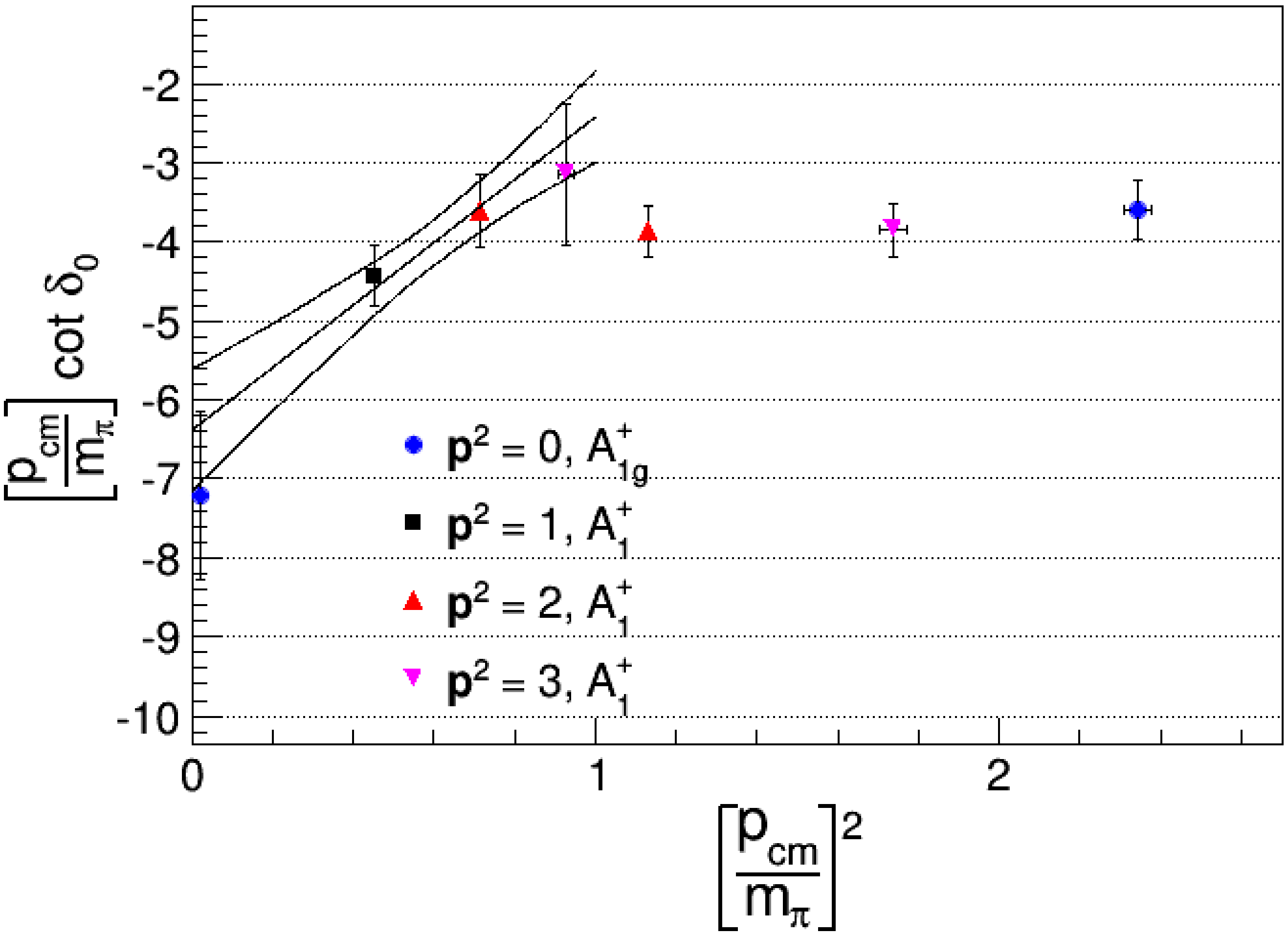
$$N_{inv} = 4 N_{dil}^{conn} N_{t_0} + N_{dil}^{disc} = 4 \times 32 \times 8 + 512 = 1536$$

Preliminary I = 2 results:

$$\mathbf{P}_{tot} = \frac{2\pi}{L}\mathbf{d}, \quad \mathbf{p} = \frac{2\pi}{L}\mathbf{u}, \; \gamma = \frac{E}{E_{cm}}, \quad w_{\ell m} = \frac{2Z_{\ell m}(\mathbf{d}, \gamma, \mathbf{u}^2)}{\gamma \pi^{-1/2} L u^{\ell}}$$

$\mathbf{d}$	Λ	$\lvert\mathbf{p}\rvert \cot\, \delta_0$
$(0,0,0)$	A_{1g}^{+}	w_{00}
$(0,0,n)$	A_{1}^{+}	w_{00}
$(0,n,\pm n)$	A_{1}^{+}	w_{00}
$(n,\pm n,\pm n)$	A_{1}^{+}	w_{00}

- We ignore higher partial waves $\ell \geq 2$



Effective range fit:

$$p \cot\ \delta_0 = \frac{1}{a_0} + \frac{1}{2} r p^2$$

$$m_\pi a_0 = -0.157(19), \quad m_\pi r = 7.9(2.4), \quad \frac{\chi^2}{d.o.f} = 0.61$$

- Future plans: The $I = 0,\ \pi\pi$ (vacuum) channel.

Morningstar, et al `11

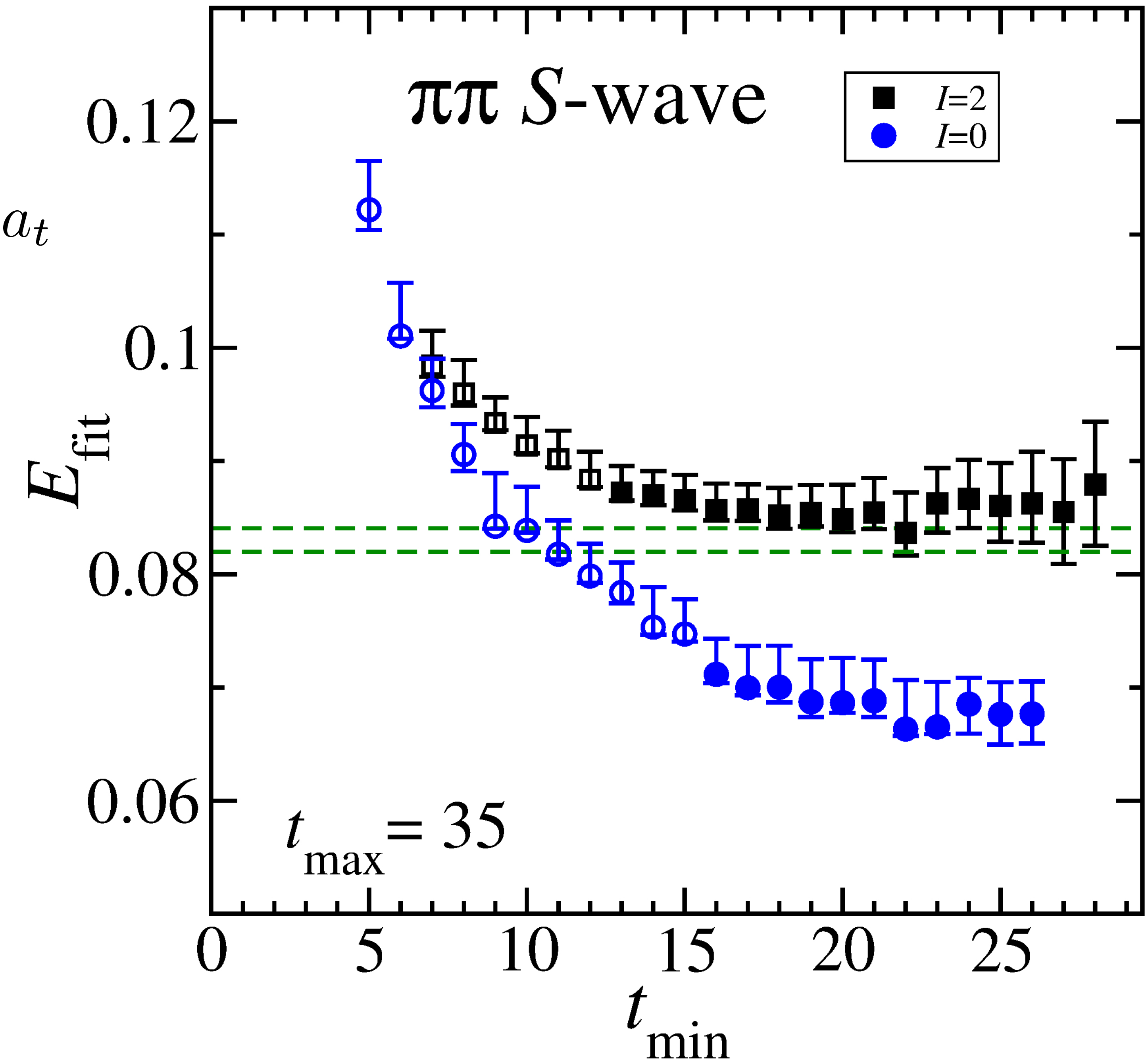
$N_f = 2 + 1$

$L/a = 24$

$a_s \approx 0.12\text{fm} = 3.5 a_t$

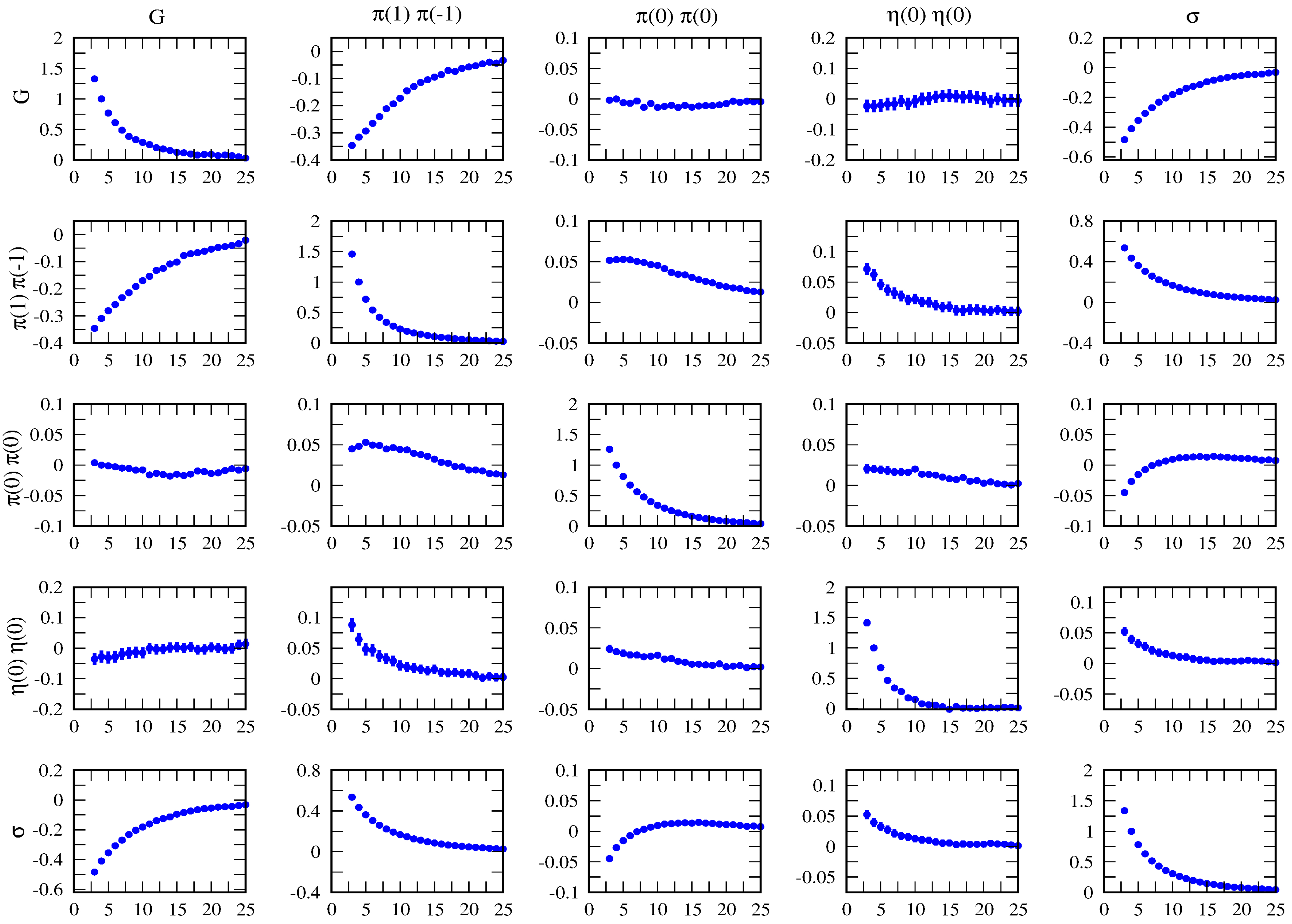
$m_\pi = 240\text{MeV}$

- Single $\pi\pi$ operator
- I=0 fit ignores thermal effects.



- Now a full correlation matrix: $\sigma \quad \pi\pi \quad \eta\eta \quad G$

Morningstar, et al `13



Conclusions

- Stochastic LapH seems to scale to larger volumes and lighter pion masses
- Systematics left to address
  - Chiral behavior: complicated by multi-hadron thresholds
  - (exponential) finite volume effects
  - Cutoff effects
- Future Plans: Resonance Form factors