Asymptotically free lattice gauge theory in five dimensions

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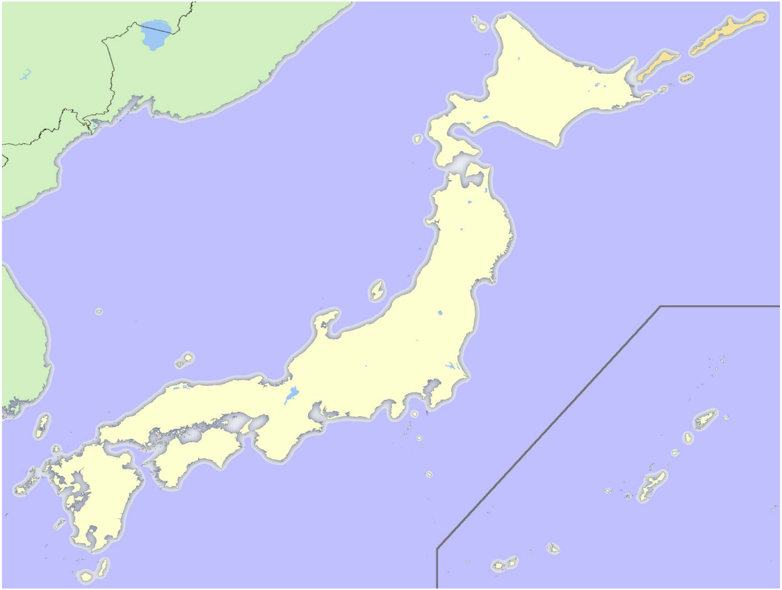
@Lattice 2015, Kobe, July 18, 2015




We are here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I'm from here
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Background I

Why 5D?

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Why 5D?

- Solution to the hierarchy problem
  - Gauge-Higgs unification
    Fairlie, Manton, Hosotani, ...
  - Brane world
    cf. Layered phase — Fu and Nielsen '84
- Holography
- D-theory/ Quantum link model
  Chandrasekharan and Wiese '97, Brower *et al.* '04
- Stochastic quantization — Parisi and Wu '81
- Gradient (Wilson) flow — Lüscher '10

⋮

Background II

Isotropic 5D Yang-Mills on the lattice:

- Confined (small β) $\leftrightarrow$ Deconfined (large β), separated by a first-order phase transition

  Creutz '79

- Perturbatively non-renormalizable

  $\rightarrow$ Valid only up to a cutoff scale

- Many simulations

  Irges *et al.* '09, '10, de Forcrand *et al.* '10, Farakos *et al.* '10, Knechtli *et al.* '12, Del Debbio '13, Itou *et al.* '14, ...

Lifshitz criticality

UV divergence is tamed for anisotropic propagators, e.g.,

$$\omega \sim \mathbf{p}_{\parallel}^{4} + \mathbf{p}_{\perp}^{2}$$

found in real materials like MnP

Hornreich, Luban, and Shtrikman: PRL 35 (1975) 1678

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↓

Similar ideas in high-energy physics

- Renormalizable gauge theories without Lorentz symmetry — Anselmi *et al.* '08∼
- UV-completion of quantum gravity — Horava '09

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1. Construction of a lattice-regularized action for non-Abelian Lifshitz-type gauge theories

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1. Construction of a lattice-regularized action for non-Abelian Lifshitz-type gauge theories
2. First non-perturbative simulation on the lattice $\rightarrow$ check the existence of the continuum limit

Action (continuum)

We consider an SU(N) gauge theory Horava '08

$$S = \frac{1}{2}\int \mathrm{d}x_0 \mathrm{d}^D x \left[\frac{1}{e^2}\,\mathrm{Tr}(F_{0i}F_{0i}) + \frac{1}{g^2}\,\mathrm{Tr}\left\{(D_i F_{ik})(D_j F_{jk})\right\}\right]$$

defined in $\mathbb{R}^{D+1}$ and $i = 1, 2, \ldots, D$.

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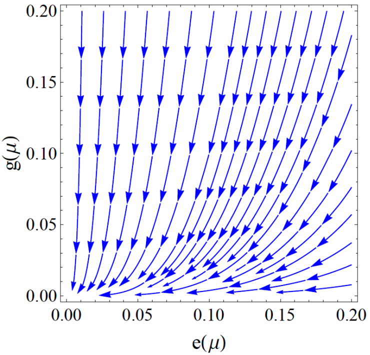
- Weighted power counting ($[x_0] = -2,\ [x_i] = -1$)
  $$\rightarrow \quad [e^2] = [g^2] = 4 - D.$$
- Critical dimensionality: 1+4 dimensions

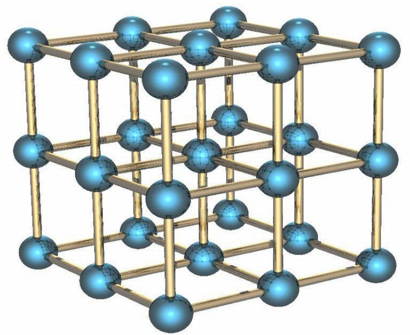
RG flow in $4 + 1$ d



- ✓ $(e, g) = (0, 0)$ is a stable UV fixed point
  (Asymptotic freedom)
- ✓ No other interactions are generated
  (Renormalizability) Zinn-Justin '86, Okano '86

Lattice action I

Strongly coupled in IR $\rightarrow$ lattice regularization!



Lattice action I

Strongly coupled in IR → lattice regularization!

- **Electric part:** (as usual)

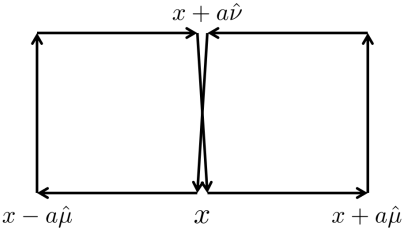
$$\frac{1}{e^2}\,\mathrm{Tr}\,F_{0i}^2 \longrightarrow \frac{1}{e_{\mathrm{lat}}^2}\sum_x \sum_{i=1}^{D} \mathrm{Re}\,\mathrm{Tr}\left\{\mathbb{1} - P_{0i}(x)\right\}$$

with a temporal Wilson plaquette P_{0i} .

Lattice action II

- **Magnetic part:**
  Use a twisted 2×1 Wilson loop

$T_{\mu\nu}(x) \equiv$



$$= \exp\left(ia^3 D_i F_{ij}(x) + \mathcal{O}(a^4)\right).$$

Lattice action II

- **Magnetic part:**
  In the naive continuum limit,

$$\sum_x \sum_{j=1}^{D} \mathrm{Re}\,\mathrm{Tr}\left\{ \mathbb{1} - \prod_{\substack{i=1 \\ i \neq j}}^{D} T_{ij}(x) \right\}$$

$$= \sum_x \sum_{j=1}^{D} \mathrm{Re}\,\mathrm{Tr}\left\{ \mathbb{1} - \exp\left(i a^3 \sum_{i=1}^{D} D_i F_{ij}(x) + \mathcal{O}(a^4) \right) \right\}$$

$$= \frac{a^6}{2} \sum_x \sum_{j=1}^{D} \mathrm{Tr}\left\{ \left(\sum_{i=1}^{D} D_i F_{ij}(x) \right)^2 \right\} + \mathcal{O}(a^7).$$

Lattice action III

Total lattice action in $(1+D)$d :

$$S_{\text{lat}} = \frac{1}{2N}\left[\beta_e \sum_x \sum_{i=1}^{D} \mathrm{Re}\,\mathrm{Tr}\left\{\mathbb{1} - P_{0i}(x)\right\} + \beta_g \sum_x \sum_{j=1}^{D} \mathrm{Re}\,\mathrm{Tr}\left\{\mathbb{1} - \prod_{\substack{i=1 \\ i\neq j}}^{D} T_{ij}(x)\right\}\right],$$

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Continuum limit: $\boldsymbol{\beta}_{\mathbf{e}}$ and $\boldsymbol{\beta}_{\mathbf{g}} \longrightarrow \infty$ for $D \leq 4$

Asymptotics

Action density s for $\beta \equiv \beta_e = \beta_g$:

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- Strong-coupling expansion

$$s = D\beta + \mathcal{O}(\beta^2) \qquad \text{as} \quad \beta \to 0$$

Asymptotics

Action density s for $\beta \equiv \beta_e = \beta_g$:

- Strong-coupling expansion

$$s = D\beta + \mathcal{O}(\beta^2) \qquad \text{as} \quad \beta \to 0$$

- Weak-coupling expansion

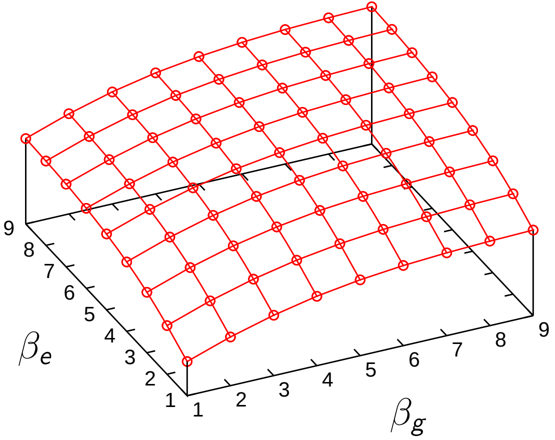
$$s = \frac{(N^2-1)D}{2} + \mathcal{O}\left(\frac{1}{\beta}\right) \quad \text{as} \quad \beta \to \infty$$

Simulation

- SU(3) gauge group
- Hybrid Monte Carlo for gauge fields
- Volume: 6^5 and 10^5
- Couplings: $1 \leq \beta_e, \beta_g \leq 9$ scanned
- Observables:
  action density and a heavy-quark potential (extracted from temporal Wilson loops)
- Also simulated isotropic YM for comparison

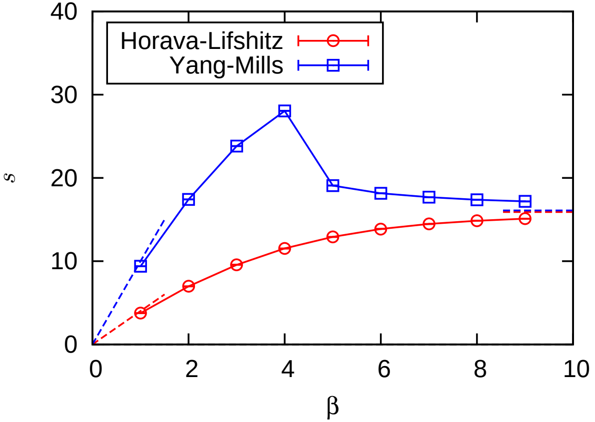
Numerical results I

Action density on the 6^5 lattice



Numerical results II

Action density for $\beta_e = \beta_g \equiv \beta$ juxtaposed with isotropic YM:



Numerical results II

- ✓ Weak and strong limits are well reproduced
- ✓ 1st order transition in isotropic YM at $\beta \sim 4.5$ (consistent with Itou *et al.*, 1403.6277)
- ✓ No singularity in the Lifshitz-type theory
- ✓ Action densities for the electric/magnetic part are roughly of equal magnitude

$\uparrow$

$\mathrm{tr}(D_i F_{ij})^2$ is the *same* order as $\mathrm{tr}\, F_{0i}^2$ in the present anisotropic scaling

Numerical results III

Potential $$V(x) = -\lim_{t\to\infty} \frac{1}{t} \ln\langle W_{0i}(t,x)\rangle$$

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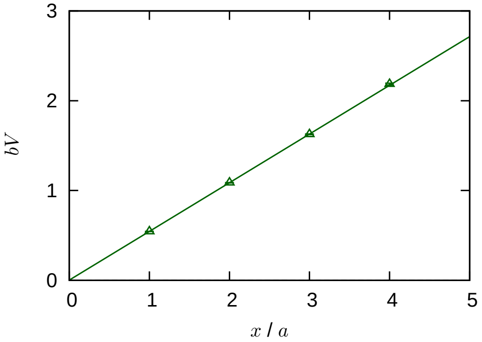

10^5, $\beta_e = \beta_g = 9$

Linear potential

Conclusion

- ✓ SU(3) Lifshitz gauge theory is simulated on the lattice for the first time
- ✓ Continuum limit looks smoothly approached

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Future directions

- Simulations at larger β/ larger volume
- Simulations for other gauge groups /at nonzero temperature /in other dimensions, etc.
- Check if 1st order transition returns when spatial plaquettes are added to the action
- Inclusion of matter fields