Determination of $\Lambda_{\rm QCD}^{N_{\rm f}=3}$ by the ALPHA-collaboration; a status update

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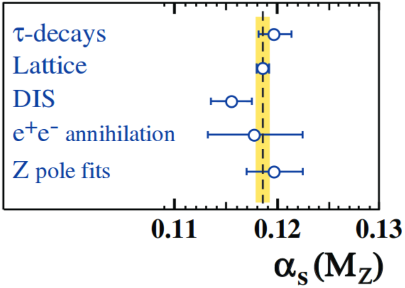
ALPHA Collaboration

Lattice 2015

Kobe, 14 July 2015

Currently quoted results for $\alpha_s(m_Z)$

World average [PDG 2014]: $\alpha_s(m_Z) = 0.1185(6)$



- PDG error estimate determined by lattice results!
  How realistic are these small errors?
- FLAG group average: $\alpha_s(m_Z)|_{\text{lattice}} = 0.1184(12)$
  [arXiv:1310.8555v2]

The QCD Λ-parameter and $\alpha_s(\mu) = \bar{g}^2(\mu)/4\pi$

$$\frac{\Lambda}{\mu} = \left[b_0\bar{g}^2(\mu)\right]^{-\frac{b_1}{2b_0^2}} \mathrm{e}^{-\frac{1}{2b_0\bar{g}^2(\mu)}} \exp\left\{-\int_0^{\bar{g}(\mu)} dg \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]\right\}$$

- Assume: coupling $\bar{g}(\mu)$ non-perturbatively defined, N_f massless quarks
- $\beta(g)$ has expansion $\beta(g) = -b_0 g^3 - b_1 g^5 + ..$

$$b_0 = (11 - \tfrac{2}{3}N_\mathrm{f})/(4\pi)^2, \qquad b_1 = (102 - \tfrac{38}{3}N_\mathrm{f})/(4\pi)^4, \quad \ldots$$

- For $\alpha_s(m_Z)$ want $\Lambda^{N_\mathrm{f}=5}$; Given $\Lambda^{N_\mathrm{f}=3}$ one still needs to match across charm and bottom thresholds!
- Scheme dependence of Λ almost trivial:

$$g_\mathrm{X}^2(\mu) = g_\mathrm{Y}^2(\mu) + c_\mathrm{XY} g_\mathrm{Y}^4(\mu) + \ldots \quad \Rightarrow \quad \frac{\Lambda_\mathrm{X}}{\Lambda_\mathrm{Y}} = \mathrm{e}^{c_\mathrm{XY}/2b_0}$$

$\Rightarrow$ use $\Lambda_{\overline{\mathrm{MS}}} = \Lambda_\mathrm{QCD}$ as reference!

The QCD Λ-parameter and $\alpha_s(\mu) = \bar{g}^2(\mu)/4\pi$

$$\frac{\Lambda}{\mu} = \left[b_0\bar{g}^2(\mu)\right]^{-\frac{b_1}{2b_0^2}} \mathrm{e}^{-\frac{1}{2b_0\bar{g}^2(\mu)}} \exp\left\{-\int_0^{\bar{g}(\mu)} dg \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right]\right\}$$

- Relation is exact at any scale μ.
    - require large μ to evaluate integral perturbatively
    - require small μ to match hadronic scale

⇒ use step-scaling method to bridge large scale difference [Lüscher, Weisz, Wolff '91]

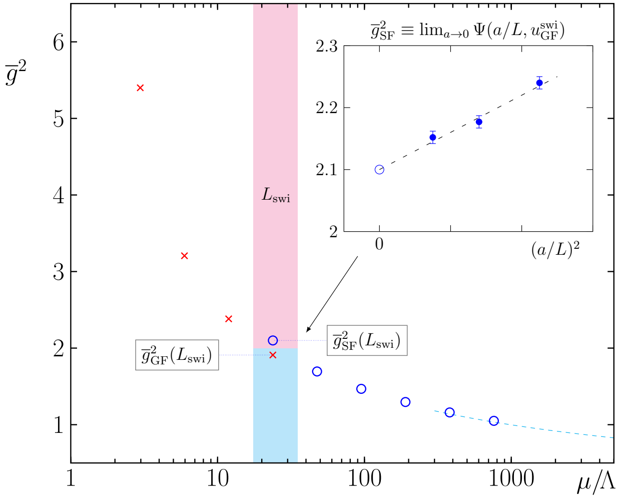
- Consider 2 renormalized finite volume couplings ($L = 1/\mu$):
    - $g_{\mathrm{SF}}(L)$: from Schrödinger functional (SF) with Abelian background field [Lüscher et al. '92]
    - $g_{\mathrm{GF}}(L)$: from gradient flow observable $\langle E(t,x)\rangle$ in finite volume, SF boundary conditions [Fritzsch & Ramos '13]

Overview of the strategy

- Obtain F_K calculated on $N_\mathrm{f} = 2+1$ CLS configurations:
  - $\mathrm{O}(a)$ improved Wilson quarks [Bulava & Schaefer '13]
  - Tree-level $\mathrm{O}(a^2)$ improved Lüscher-Weisz action,
  - Open boundary conditions, openQCD code [Lüscher & Schaefer '10–'14];
  - Very precise Z_A [talk by M. Dalla Brida]

  [JHEP 1502 (2015) 043, & talk by S. Schaefer];
- Match F_K to $L_\mathrm{max} \approx 0.5\,\mathrm{fm}$ (defined through GF-coupling)
- Step scaling (2-3 steps) for $g_\mathrm{GF}(L)$ from 0.5 fm to $L_\mathrm{swi} \approx 0.05\,\mathrm{fm}$
- At scale L_swi switch from GF to SF scheme; also change from Lüscher-Weisz to Wilson gauge action
- Step-scaling for $g_\mathrm{SF}(L)$, extract $L_\mathrm{swi}\Lambda_\mathrm{QCD}$
- Combine results to obtain Λ_QCD/F_K

Overview of the strategy



Why not just a single coupling?

- Need 1-loop matching to $g^2_{\overline{\rm MS}}$ to obtain $\Lambda/\Lambda_{\rm QCD}$.
- Precision: 3-5% for $\Lambda \Leftrightarrow$ 0.5-1% for $\alpha_s(m_Z)$.
- Difficult to reach without 3-loop result for $\beta(g)$:

$$I(\bar{g}(\mu)) = \int_0^{\bar{g}(\mu)} dg \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g} \right] \Bigg\} = \left(\frac{b_2}{2b_0^2} - \frac{b_1^2}{2b_0^3} \right) \bar{g}^2(\mu) + \ldots$$

$\Rightarrow$ approximate $\exp\left(-I(\bar{g})\right) = 1$ unless b_2 is known!
To be within 2% (1%) of the 3- or 4-loop value need to reach:

$$\bar{g}^2_{\overline{\rm MS}}(\mu) < 2.5\,(1.3) \qquad \bar{g}^2_{\rm SF}(L) < 0.87\,(0.44)$$

N.B.: only known a posteriori for a new scheme (e.g. GF)!

Comparison g_{GF} vs. g_{SF}

SF-coupling:

- 3-loop β-function (i.e. b_2) is known [Bode, Weisz, Wolff '99]
- 2-loop c_t known: O(a) boundary effects highly suppressed
- $\Delta(1/\bar{g}^2) \propto (\Delta L)/L$, roughly independent of $\bar{g}$.
- requires very large statistics; variance increases with L/a.
- large physical volumes very difficult (N.B. coupling defined by variation of b.c.'s).

GF-coupling (finite volume, SF b.c.'s)

- high statistical precision
- can be measured in large physical volumes; ideal to match hadronic physics!
- $\Lambda_{\mathrm{GF}}/\Lambda_{\mathrm{QCD}}$ not yet known; only universal b_0, b_1 can be used.
- $\Delta(1/\bar{g}^2) \propto 1/\bar{g}^2$: more expensive as $\bar{g}$ decreases.
- Relatively large O(a^2) effects; can we reduce these?

Strategy to calculate $L_{\rm swi}\Lambda_{\rm QCD}$

- Define $L_{\rm swi}$ implicitly by $\bar{g}^2_{\rm SF}(L_{\rm swi}) = 2.012$
- Obtain <u>continuum</u> step scaling function (SSF) by fit ansatz for continuum & cutoff effects

$$\sigma(u) = \bar{g}^2_{\rm SF}(2L)|_{u=\bar{g}^2_{\rm SF}(L)}$$

  for a range of u-values, $u \in [1.10891, 2.0120]$
- Given $\sigma(u)$ start with $u_0 = 2.012$ and find $u_1,\ u_2,\ldots,u_5$.

$$u_{n-1} = \sigma(u_n), \quad n = 1, \ldots, 5, \qquad \Rightarrow \quad u_n = \bar{g}^2_{\rm SF}\left(2^{-n} L_{\rm swi}\right)$$

- At scale $2^{-n}L_{\rm swi}$ evaluate $I(\bar{g})$ and obtain $\Lambda_{\rm SF}$
- Connect to $\overline{\rm MS}$ scheme $\Lambda^{N_{\rm f}=3}_{\rm SF}/\Lambda^{N_{\rm f}=3}_{\rm QCD} = 0.382863(1)$

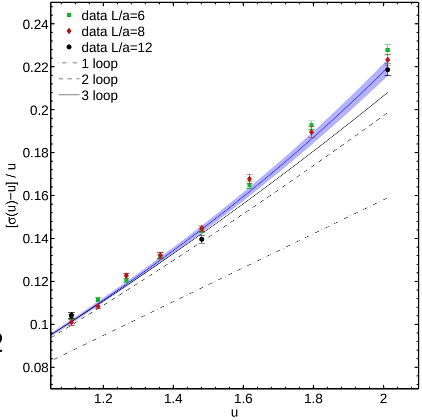
<u>Simulations</u>:

- On lattices with sizes $L/a = 4, 6, 8, 12$ measure $u = \bar{g}^2(L)$.
- requires precise knowledge of massless limit, i.e. $\kappa_{\rm cr}(g_0, L/a)$
- Double lattice size and measure $\Sigma(u, a/L) = \bar{g}^2(2L)$
- use $\Sigma(u, a/L)$ or reduce cutoff effects perturbatively up to 2-loop order $\to \Sigma'(u, a/L)$.

Obtaining the SSF in the continuum

Example for global fit ansatz:

$$\begin{aligned}\Sigma'(u, a/L) &= u + s_0 u^2 + s_1 u^3 \\ &+ c_1 u^4 + c_2 u^5 \\ &+ \rho_1 u^4 \frac{a^2}{L^2}\end{aligned}$$



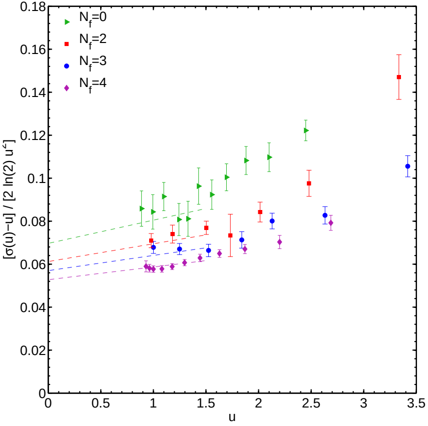
- s_0, s_1 fixed to perturbative values:

  $s_0 = 2b_0 \ln 2, \quad s_1 = s_0^2 + 2b_1 \ln 2$

- 3 parameters: c_1, c_2, ρ_1; 19 data points,

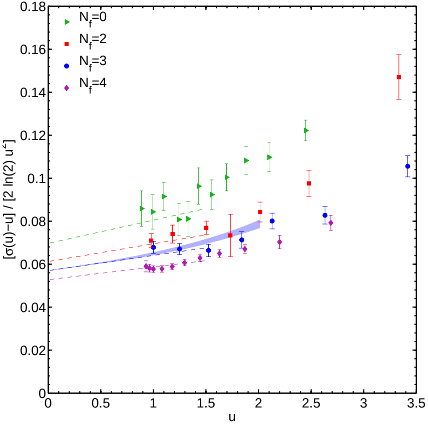
  $\chi^2/\text{d.o.f.} = 1.099$

Precision compared to earlier results for $N_\mathrm{f} = 0, 2, 3, 4$



- $N_\mathrm{f} = 0, 2, 4$ [ALPHA,'92–'12]
- $N_\mathrm{f} = 3$ [PACS-CS '09]

Precision compared to earlier results for $N_f = 0, 2, 3, 4$



- $N_f = 0, 2, 3, 4$ [ALPHA,'92–'15]
- $N_f = 3$ [PACS-CS '09]
- Various fits (3-5 parameters, perturbatively improved & unimproved data), find stability after $n = 2, 3$ step-scaling steps

$$\Rightarrow L_{\rm swi}\Lambda_{\overline{\rm MS}}^{N_f=3} = 0.0802(16)$$

(preliminary)

On the definition of g_{GF}^2

- Choose same bare action as CLS in large volume;
- SF boundaries: use variant B by [Aoki, Frezzotti, Weisz, '98]
- Boundary O(a) improvement: $c_{\mathrm{t}}, \tilde{c}_{\mathrm{t}}$ to 1-loop order [Aoki, Ide, Takeda '03; Vilaseca '15]
- Study of O(a) boundary effects (PT and quenched):
  1. $T = L$, $c = \sqrt{8t}/L = 0.3$ seems OK;
  2. advantageous to restrict to magnetic components at $x_0 = T/2$:

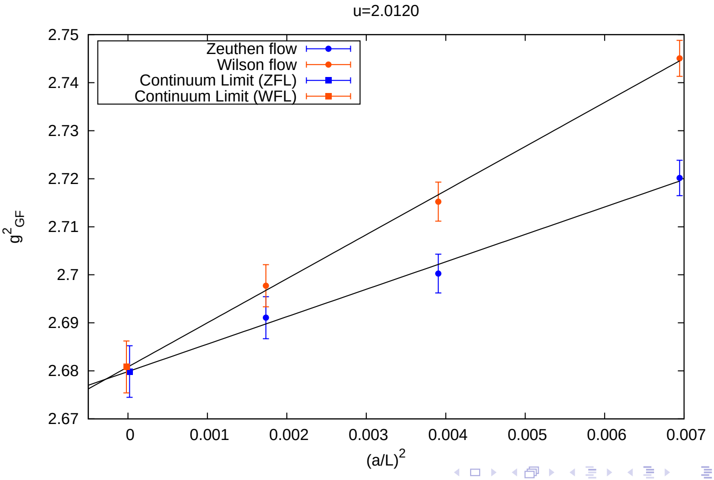
$$-\tfrac{1}{2}\langle \,\mathrm{tr}\,\{G_{kl}(t,x)G_{kl}(t,x)\}\rangle\big|_{x_0=T/2,T=L,m_{\mathrm{q}}=0} = \mathcal{N}(c,a/L)\, g_{\mathrm{GF}}^2(L)$$

- Use $\mathcal{N}(c,a/L)$ for given $L/a \Rightarrow g_{\mathrm{GF}}^2 = g_0^2$ exact at tree-level.
- Wilson flow & O(a^2) improved Zeuthen flow
- Clover & O(a^2) improved observable
- topology freezing: use projection on $Q = 0$ sector [Fritzsch, Ramos, Stollenwerk '13]; becomes relevant for $L > 0.25\,\mathrm{fm}$

Matching at the switching scale L_{swi} (Wilson action)

$$\bar{g}^2_{\mathrm{SF}}(L_{\mathrm{swi}}) = 2.012 \Rightarrow (\beta, L/a) \rightarrow (\beta, 2L/a) \Rightarrow \bar{g}^2_{\mathrm{GF}}(2L_{\mathrm{swi}}) = 2.6808(54)$$

(preliminary)



Summary status: $\Lambda_{\rm QCD}^{N_f=3}$ with target error < 4-5%

- SF coupling for $L < L_{\rm swi} \approx 0.05\,{\rm fm}$;
  unprecedented precision (high statistics & precise tuning of κ):

$$\bar{g}^2_{\rm SF}(L_{\rm swi}) = 2.012 \quad \Rightarrow \quad L_{\rm swi}\Lambda_{\overline{\rm MS}}^{N_f=3} = 0.0802(16) \quad (\text{preliminary})$$

- Definition of gradient flow coupling $\bar{g}^2_{\rm GF}(L)$ settled:
  - reduced boundary O(a) effects by restricting $E(t,x)$ to magnetic components;
  - Symanzik O(a^2) improvement: Zeuthen flow and observable.
- Matching at switching scale $L_{\rm swi}$

$$\bar{g}^2_{\rm GF}(2L_{\rm swi}) = 2.6808(54) \quad (\text{preliminary})$$

- Running of $\bar{g}^2_{\rm GF}(L)$ from $0.05 - 0.1\,{\rm fm}$ to $0.5\,{\rm fm}$:
  - precision tuning of κ finished;
  - simulations for step scaling function underway.
- Connect to F_K on CLS config's: details to be defined.