Nonperturbative renormalization in the RI-SMOM scheme and Gribov uncertainty in the RI-MOM scheme for staggered bilinears

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Indirect CP violation: ϵ_K

SWME collaboration has been devoted to calculate the indirect CP violation parameter ϵ_K and V_{cb} on the lattice, which is related to

- Hadronic matrix element
  - Standard model (SM) : B_K
  - Beyond the standard model : $B_2, \cdots, B_5$
- CKM matrix element: V_{cb}

Lattice QCD calculation can provide a high precision test of the standard model.

Renormalization of B_K

Hadronic matrix element: B_K

$$O_{\Delta S=2} = \sum_{\nu} [\bar{s}\gamma_\nu(1-\gamma_5)d][\bar{s}\gamma_\nu(1-\gamma_5)d]$$

Goal: Reducing the error from matching factor calculation for improved staggered fermions

- $\approx 4.4\%$ error from one-loop perturbation theory [J.J.Kim et al. PRD 81 (2010) 114503, PRD 83 (2011) 094503]
- $\rightarrow \approx 2\%$ error by non-perturbative renormalization (NPR) [J.H.Kim et al. Lattice 2014]
    - Regularization independent momentum subtraction (RI-MOM) scheme
    - renormalization condition on a correlation function with *exceptional* momenta as a subtraction point

Contents

1. NPR for staggered bilinears with regularization independent symmetric momentum scheme (RI-SMOM scheme)
2. Gribov uncertainty of Z_q in the staggered NPR with exceptional subtraction point (RI-MOM scheme)

Part I

NPR in RI-SMOM scheme for staggered bilinears

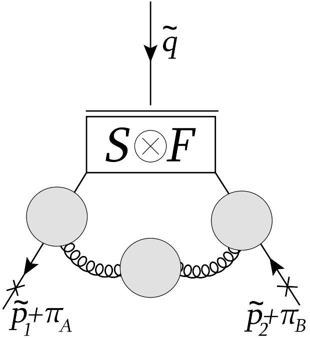
Bilinear operator

Staggered bilinear Operator

$$O_i^{S\otimes F}(y) = \sum_{AB} \overline{\chi}_i(y_A)\overline{(\gamma_S \otimes \xi_F)}_{AB} U_{i;AB}(y)\chi_i(y_B)$$

- i : gauge configuration index
- y : hypercube coordinates on the lattice with its spacing $2a$, $y_A = 2y + A$
- A, B : hypercube vectors. Each element is 0 or 1. ex) $B = (0, 0, 1, 1)$
- $\overline{(\gamma_S \otimes \xi_F)}_{CD} = \frac{1}{4}\mathrm{tr}[\gamma_C^\dagger \gamma_S \gamma_D \gamma_F^\dagger]$, where $\gamma_D = \gamma_1^{D_1}\gamma_2^{D_2}\gamma_3^{D_3}\gamma_4^{D_4}$

Amputated Green's function



Amputated Green's function in the momentum space

$$\widetilde{\Lambda}^{S \otimes F}_{c_1 c_2}(\widetilde{p}_1 + \pi_A, \widetilde{p}_2 + \pi_B)$$

- $\widetilde{p}_i$ is the momentum in reduced Brillouin zone.

$$p \in (-\frac{\pi}{a}, \frac{\pi}{a}]^4, \qquad \tilde{p} \in (-\frac{\pi}{2a}, \frac{\pi}{2a}]^4$$

  where $p = \tilde{p} + \pi_B$ and $\pi_B (\equiv \frac{\pi}{a} B)$ is cut-off momentum in hypercube.
- c_1, c_2 : color index

Projected amputated Green's function

Projected amputated Green's function

$$\Gamma^{\alpha\beta}(\widetilde{p}_1,\widetilde{p}_2) = \sum_{AB}\sum_{c_1 c_2}[\widetilde{\Lambda}^{\alpha}_{c_1 c_2}(\widetilde{p}_1+\pi_A,\widetilde{p}_2+\pi_B)\hat{\mathbb{P}}^{\beta}_{BA;c_2 c_1}]$$

The projection operator is

$$\hat{\mathbb{P}}^{\beta}_{BA;c_2 c_1} = \frac{1}{48}\overline{\overline{(\gamma^{\dagger}_{S'}\otimes\xi^{\dagger}_{F'})}}_{BA}\delta_{c_2 c_1}$$

- $\alpha = (\gamma_S \otimes \xi_F)$, $\beta = (\gamma_{S'} \otimes \xi_{F'})$
- $\overline{\overline{(\gamma_S\otimes\xi_F)}}_{AB} = \frac{1}{16}\sum_{CD}(-1)^{A\cdot C}\overline{(\gamma_S\otimes\xi_F)}_{CD}(-1)^{D\cdot B}$

Scheme condition for $S \otimes S$

sym implies the condition $\tilde{p}_1^2 = \tilde{p}_2^2 = \tilde{q}^2$ where $\tilde{q} = \tilde{p}_1 - \tilde{p}_2$

$$\begin{aligned}\Gamma^{S\otimes S}(\widetilde{p_1}, \widetilde{p_2})|_{sym} &= 1 \quad \text{(tree level)} \\ &= \frac{1}{48}\sum_{AB}\sum_{c_1 c_2}[\widetilde{\Lambda}^{S\otimes S}_{c_1 c_2}(\widetilde{p_1} + \pi_A, \widetilde{p_2} + \pi_B)\overline{\overline{(1 \otimes 1)}}_{BA}\delta_{c_2 c_1}]_{sym}\end{aligned}$$

Renormalization factor of the scalar operator $Z_{S\otimes S} = \frac{1}{Z_m}$ and,
$\Gamma^{S\otimes S} = Z_q^{-1} Z_{S\otimes S} \Gamma_B^{S\otimes S} = 1$ determines the mass renormalization Z_m.

$$\Gamma_B^{S\otimes S} = Z_m Z_q$$

Scheme conditions for $V \otimes S$

$\mathrm{RI\text{-}SMOM}_{\gamma_\mu}$ scheme uses projectors the same as RI-MOM scheme.

$$\begin{aligned}
&\Gamma^{V\otimes S}(\widetilde{p}_1, \widetilde{p}_2)|_{sym} = 1 \quad \text{(tree level)} \\
&= \frac{1}{48\cdot 4}\sum_{\mu}\sum_{AB}\sum_{c_1c_2}[\widetilde{\Lambda}^{V_\mu\otimes S}_{c_1c_2}(\widetilde{p}_1+\pi_A, \widetilde{p}_2+\pi_B)\overline{\overline{(\gamma_\mu^\dagger\otimes 1)}}_{BA}\delta_{c_2c_1}]_{sym}
\end{aligned}$$

RI-SMOM scheme condition for the vector operator

$$\begin{aligned}
&\Gamma^{V\otimes S}(\widetilde{p}_1, \widetilde{p}_2)|_{sym} = 1 \quad \text{(tree level)} \\
&= \frac{1}{48\cdot \tilde{q}^2}\sum_{\mu}\sum_{AB}\sum_{c_1c_2}[\tilde{q}_\mu\widetilde{\Lambda}^{V_\mu\otimes S}_{c_1c_2}(\widetilde{p}_1+\pi_A, \widetilde{p}_2+\pi_B)\sum_{\nu}\tilde{q}_\nu\overline{\overline{(\gamma_\nu^\dagger\otimes 1)}}_{BA}\delta_{c_2c_1}]_{sym}
\end{aligned}$$

and we also tried the substitution $\widetilde{q}a \to \sin(\widetilde{q}a)$. Renormalization factor of conserved vector current $Z_V = 1$ and, $\Gamma^{V\otimes S} = Z_q^{-1}\Gamma_B^{V\otimes S} = 1$ determines the wave function renormalization Z_q.

$$\Gamma_B^{V\otimes S} = Z_q$$

Simulation details

- $20^3 \times 64$ MILC asqtad lattice ($a \approx 0.12 fm$, $am_l/am_s = 0.01/0.05$)
- Landau gauge fixed, HYP smearing, tadpole improved
- 10 number of configurations
- 1 valence quark mass (0.05)
- 5 sets of *simple* symmetric external momenta in the units of $(\frac{2\pi}{L_s}, \frac{2\pi}{L_s}, \frac{2\pi}{L_s}, \frac{2\pi}{L_t})$

$\tilde{p}_1$	$\tilde{p}_2$	$(a\tilde{p})^2$	$(a\tilde{p})^4$	GeV
$(1,1,0,0)$	$(1,0,1,0)$	0.1974	0.0195	0.7363
$(2,2,0,0)$	$(2,0,2,0)$	0.7896	0.3117	1.4727
$(3,3,0,0)$	$(3,0,3,0)$	1.7765	1.5780	2.2090
$(4,4,0,0)$	$(4,0,4,0)$	3.1583	4.9873	2.9454
$(5,5,0,0)$	$(5,0,5,0)$	4.9348	12.1761	3.6817

- 3 sets of *complicated* symmetric momenta

$\tilde{p}_1$	$\tilde{p}_1$	$\tilde{q}$	$(a\widetilde{p})^2$	$(a\widetilde{p})^4$	GeV
$(1,2,3,0)$	$(-2,3,1,0)$	$(3,-1,2,0)$	1.3817	0.9546	1.9482
$(2,4,2,0)$	$(-2,2,4,0)$	$(4,2,-2,0)$	2.3687	2.8054	2.5508
$(1,3,4,0)$	$(-3,4,1,0)$	$(4,-1,3,0)$	2.5661	3.2924	2.6549

Z_q

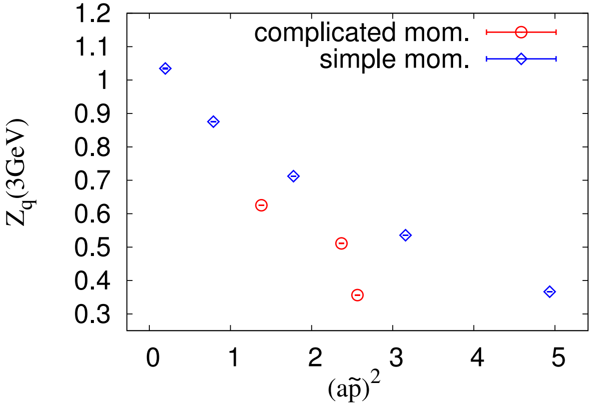


Figure : Z_q from conserved vector ($\gamma_\mu \otimes 1$) operator in the $\mathrm{RI\text{-}SMOM}_{\gamma_\mu}$ scheme

Z_q from various schemes

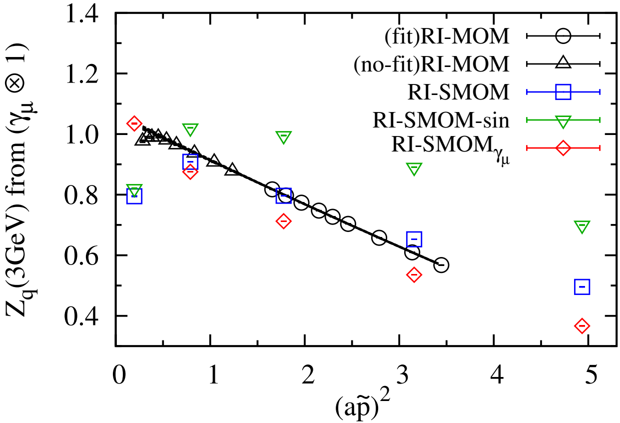


Figure : Z_q from conserved vector ($\gamma_\mu \otimes 1$) operator at 3GeV in the various renormalization schemes

Z_m from various schemes

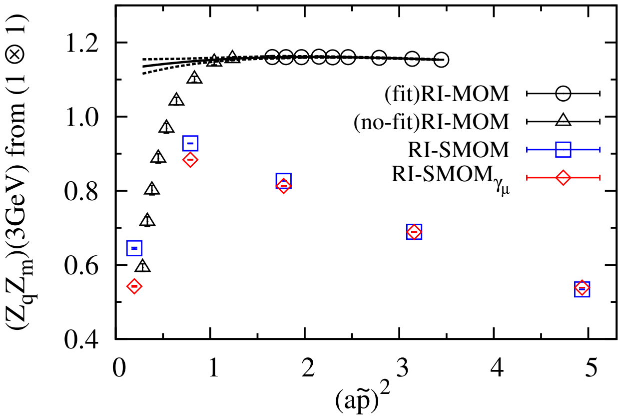


Figure : $Z_q \cdot Z_m$ from scalar $(1 \otimes 1)$ operator at 3GeV in the various renormalization schemes

Part II

Gribov error for Z_q of staggered NPR in RI-MOM scheme

Gauge fixing functional

On the lattice, the gauge fixing functional F in terms of link variable $U_\mu(x)$,

$$F = \frac{1}{2N_C}\frac{1}{4V}\sum_{\mu,x}\mathrm{Tr}[U_\mu(x) + U_\mu(x)^\dagger].$$

When the gauge is fixed, its variation w.r.t. gauge transformation $G(x) = e^{i\omega^a(x)T^a} \in SU(N_C)$,

$$\Delta(x) \equiv \frac{\delta F}{\delta\omega^a(x)}T^a \to 0.$$

In other words, the Landau gauge fixing condition is

$$\theta \equiv \frac{1}{N_C V}\sum_x \mathrm{Tr}[\Delta(x)\Delta^\dagger(x)] = 0.$$

Gribov copy

- Gribov (1978) discovered that for non-abelian gauge theories, usual linear gauge conditions does not fix the gauge fields in a unique way.
- There can be two different configurations that both satisfy the gauge fixing condition, but related by nontrivial gauge transformation. Simply,

$$\{U\} \to \{U^{g1}\}, \{U^{g2}\} \tag{1}$$

  such that $\theta[U^{g1}] = \theta[U^{g2}] = 0$, but $F[U^{g1}] \neq F[U^{g2}]$.
- Gribov ambiguity can appear in the matrix elements between quark states:
  - Need gauge fixing.
  - Additional Gribov copy degree exists and that may appear in the result with statistical uncertainty.

Simulation detail

- $20^3 \times 64$ MILC asqtad lattice ($a \approx 0.12fm$, $am_l/am_s = 0.01/0.05$)
- Landau gauge fixed, HYP smearing, tadpole improved
- 10 number of configurations
- 5 valence quark mass (0.01$\sim$0.05)
- Use exceptional momenta $\tilde{q} = \tilde{p}_1 - \tilde{p}_2 = 0$ for RI-MOM scheme

Gribov effect of nonperturbative $Z_q^{\mathrm{RI\text{-}MOM}}$

$$F = \frac{1}{2N_C}\frac{1}{4V}\sum_{\mu,x}\mathrm{Tr}[U_\mu(x) + U_\mu(x)^\dagger].$$

For each configuration (mother, with F_m),

1. Generate 100 randomly transformed configuration.
2. For all of them, run the gauge fixing (multi-GPU implemented Fourier accelerated steepest descent method algorithm) and calculate F.
3. Pick the 1 configuration (daughter, one of the gribov copy with F_d) which maximizes $\delta F = |F_m - F_d|$.
4. Calculate $\widetilde{Z}_q^{\mathrm{RI\text{-}MOM}}$ from the daughter configuration.
5. Compare it to $Z_q^{\mathrm{RI\text{-}MOM}}$ from the mother configuration.

Distribution of gauge fixed functional F

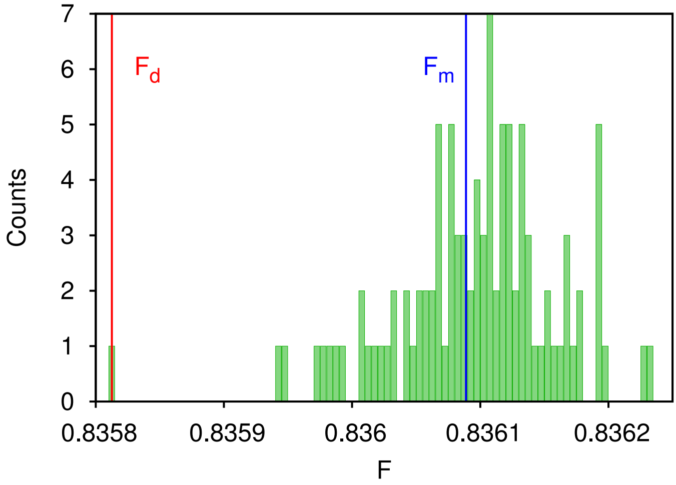


Figure : Histogram of 100 confs. generated from a single mother conf. of MILC asqtad coarse $20^3 \times 64$, $\beta = 6.76$ lattice.

Distribution of the daughter configurations

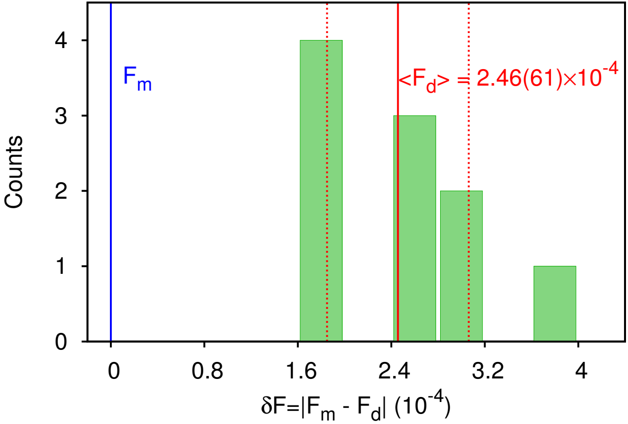
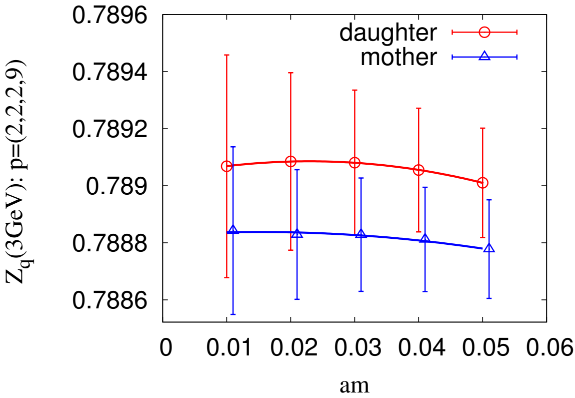


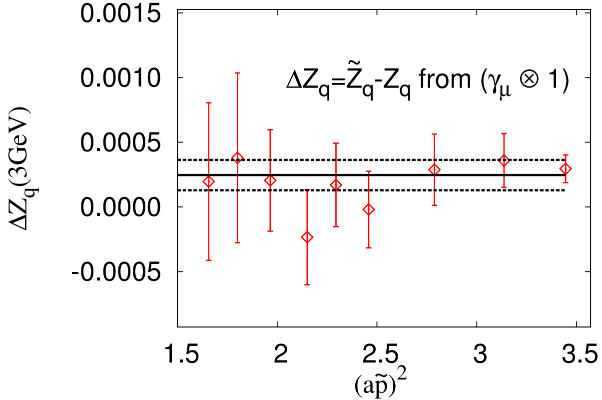
Figure : The distance between mother and daughter for the set of 10 configurations

Z_q chiral extrapolation in RI-MOM

$f = c_1 + c_2 am + c_3 (am)^2$	c_1	c_2	c_3	$\chi^2/d.o.f$
daughter	0.78903(49)	0.005(13)	-0.10(13)	0.00022(70)
mother	0.78883(37)	0.001(12)	-0.05(13)	0.0012(38)
difference	0.33σ			



$\Delta Z_q \equiv \widetilde{Z}_q - Z_q$ momentum fit in RI-MOM



- Fitting function: $f(m, a, \tilde{p}) = c_1 = 0.25(12) \times 10^{-3}$
- Fitting quality: $\chi^2/d.o.f = 0.38(42)$

Conclusion

- We studied NPR of the staggered bilinears in the RI-SMOM scheme. We plot the data points from the single valence quark mass. We will do the measurement for additional valence quark masses to do the chiral extrapolation.
- We observed that the Gribov error (ΔZ_q^{Gribov}) of staggered NPR with RI-MOM scheme is about 0.02%, and it is $\lesssim 1/20$ of the statistical error of the Z_q from NPR.

$Z_q(\text{3GeV}, \overline{\text{MS}})$	sys. error	stat. error	$\Delta Z_q^{Gribov}(\text{3GeV}, \overline{\text{MS}})$
1.0429	0.0243	0.0058	0.00024(12)

- Hence we may neglect the systematic error due to Gribov ambiguity.