Radial distributions of the axial density and the $B^{*\prime}B\pi$ coupling

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Introduction

- Radial distributions of the axial density $(\overline{q}\gamma_\mu\gamma_5 q)$ in a heavy-light meson $(Q\overline{q})$
- We work in the static limit of HQET: the heavy quark (Q) is static
- Ground states $(B,\ B^*)$ and excited states $(B',\ B^{*\prime})$ analysis
- Motivations

  $\rightarrow$ the radial distributions are related to the form factors associated to the $g_{B^*B\pi}$ and $g_{B^{*\prime}B\pi}$ (via a Fourier transform) couplings

  $\rightarrow$ understand the hierarchy of the couplings (in particular why $|g_{12}| < g_{11}$) where $g_{mn} = \langle B^0_m(\vec{0})|A_k(0)|B^{*+}_n(\vec{0},\lambda)\rangle$

  $\rightarrow$ insights on volume effects

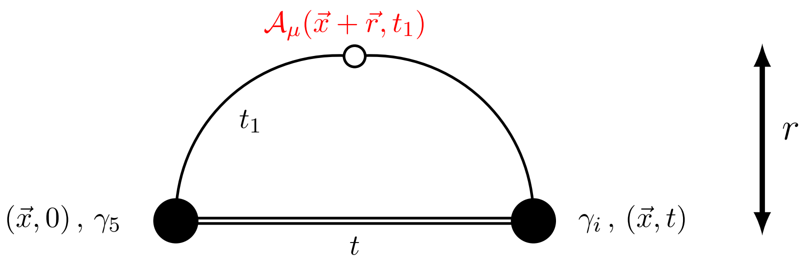
  $\rightarrow$ comparison with quark models
- Lattice setup:

> - $N_f = 2\ O(a)$ improved Wilson-Clover Fermions
>
> - HYP2 discretization for the static quark action
>
> - 3 lattice spacings a :
>
>   (0.048, 0.065, 0.075) < 0.1 fm
>
> - pion masses in the range [280 MeV, 440 MeV]

Definition

- The radial distributions of the axial current are defined by $(n, m = 1, 2, ...)$:

$$f^{(mn)}_{\gamma_\mu\gamma_5}(\vec{r}) = \langle\, B_m | A_\mu(\vec{r}) | B^*_n(\lambda)\,\rangle \quad , \quad B_n = n^{\mathrm{th}} \text{ radial excitation}$$



The axial current $A_\mu = \overline{q}\gamma_\mu\gamma_5 q$ (probe) is inserted at a distance r from the heavy quark Q

- We need to compute the following matrices of two and three-point correlation functions

$$C^{(3)}_{\gamma_\mu\gamma_5,ij}(t, t_1; \vec{r}) \;=\; \langle\, \mathcal{P}^{(j)}(t; \vec{x})\, \mathcal{A}_\mu(t_1; \vec{x} + \vec{r})\, \mathcal{V}^{(i)\dagger}_k(0; \vec{x})\,\rangle\,,$$

$$C^{(2)}_{\mathcal{P},ij}(t) = \langle\, \mathcal{P}^{(i)}(t)\, \mathcal{P}^{(j)\dagger}(0)\,\rangle \quad , \quad C^{(2)}_{\mathcal{V},ij}(t) = \langle\, \mathcal{V}^{(i)}(t)\, \mathcal{V}^{(j)\dagger}(0)\,\rangle$$

$\longrightarrow$ The pseudoscalar $\mathcal{P}^{(i)}$ and vector $\mathcal{V}^{(i)}$ interpolators correspond to different levels of Gaussian smearing

$$\mathcal{R}_{\gamma_\mu\gamma_5}(t, t_1, \vec{r}) = \frac{C^{(3)}_{\gamma_\mu\gamma_5,ij}(t, t_1; \vec{r})}{\left(C^{(2)}_{\mathcal{P},ii}(t)\; C^{(2)}_{\mathcal{V},jj}(t)\right)^{1/2}} \xrightarrow[t \gg t_1 \gg 1]{} f^{(11)}_{\gamma_\mu\gamma_5}(\vec{r}) = \langle\, B | A_\mu(\vec{r}) | B^*(\lambda)\,\rangle$$

Method: lattice computation

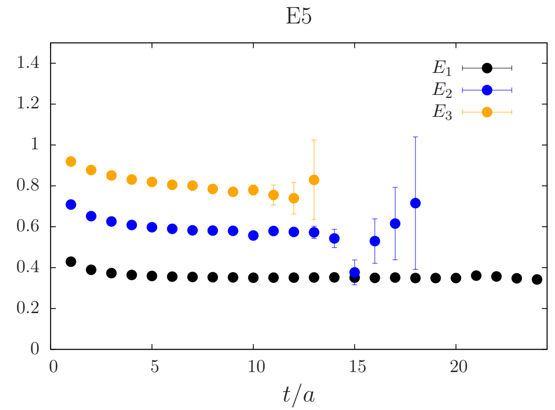
- To isolate the contribution of excited states :

  $\longrightarrow$ solve the Generalized Eigenvalue Problem

  $\longrightarrow$ basis: $N = 3$ interpolating operators

$$C_{\mathcal{P}}^{(2)}(t)\, v_n(t,t_0) = \lambda_n(t,t_0)\, C_{\mathcal{P}}^{(2)}(t_0)\, v_n(t,t_0)$$

$$C_{\mathcal{V}}^{(2)}(t)\, w_n(t,t_0) = \widetilde{\lambda}_n(t,t_0)\, C_{\mathcal{V}}^{(2)}(t_0)\, w_n(t,t_0)$$



- GEVP estimator [Bulava et. al, '11]

$$\mathcal{R}_{mn}^{\mathrm{GEVP}}(t,t_1;\vec{r}) = \Big(v_m(t_2), C^{(3)}_{\gamma_\mu\gamma_5}(t_1+t_2,t_1;\vec{r})w_n(t_1)\Big) \times \frac{\lambda_m(t_2+1)^{-t_2/2}\,\widetilde{\lambda}_n(t_1+1)^{-t_1/2}}{\Big(v_m(t_2), C_{\mathcal{P}}^{(2)}(t_2)v_m(t_2)\Big)^{1/2}\Big(w_n(t_1), C_{\mathcal{V}}^{(2)}(t_1)w_n(t_1)\Big)^{1/2}}$$

$\longrightarrow$ This method allows us to extract the radial distributions involving excited states

$$\mathcal{R}_{mn}^{\mathrm{GEVP}}(t,t_1;\vec{r}) = f^{(mn)}_{\gamma_\mu\gamma_5}(\vec{r}) + \mathcal{O}\left(e^{-\Delta_{N+1,m}t_2}, e^{-\Delta_{N+1,n}t_1}\right) \quad , \quad \Delta_{N+1,n} = E_{N+1} - E_n$$

$\longrightarrow$ The three point correlation function is evaluated at $t = t_1 + t_2$ which may be difficult (statistical noise)

Method: lattice computation

- To isolate the contribution of excited states :
  - ⟶ solve the Generalized Eigenvalue Problem
  - ⟶ basis: $N = 3$ interpolating operators

$$C_{\mathcal{P}}^{(2)}(t)\, v_n(t,t_0) = \lambda_n(t,t_0)\, C_{\mathcal{P}}^{(2)}(t_0)\, v_n(t,t_0)$$

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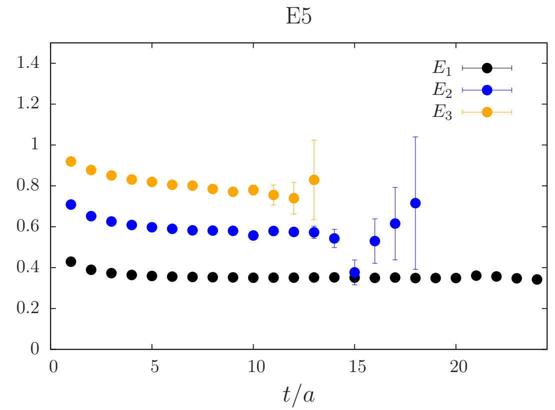


- Improved sGEVP estimators [Bulava et. al, '11] (the 3-pt correlation function is summed over the insertion time t_1)

$$\mathcal{R}_{mn}^{\mathrm{sGEVP}}(t,t_0;\vec{r}) = -\partial_t \left(\frac{\left| \left(v_m(t,t_0), \left[K(t,t_0;\vec{r})/\widetilde{\lambda}_n(t,t_0) - K(t_0,t_0;\vec{r}) \right] w_n(t,t_0) \right) \right|}{\left(v_m(t,t_0), C_{\mathcal{P}}^{(2)}(t_0) v_m(t,t_0) \right)^{1/2} \left(w_n(t,t_0), C_{\mathcal{V}}^{(2)}(t_0) w_n(t,t_0) \right)^{1/2}} e^{\Sigma_{mn}(t_0,t_0)t_0/2} \right)$$

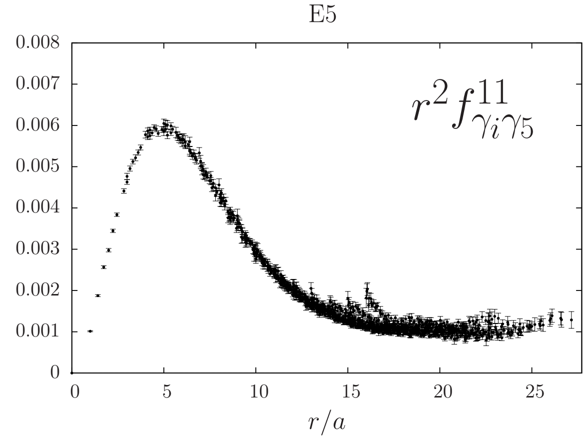
where $K_{ij}(t,t_0;\vec{r}) = \sum_{t_1} e^{-(t-t_1)\Sigma(t,t_0)} C_{ij}^{(3)}(t,t_1;\vec{r})$, $\Sigma_{mn}(t,t_0) = E_n(t,t_0) - E_m(t,t_0)$

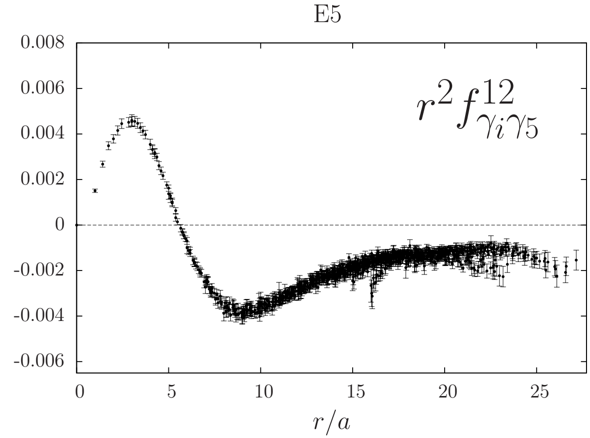
⟶ Faster suppression of higher excited states contribution [Blossier et. al, '13]

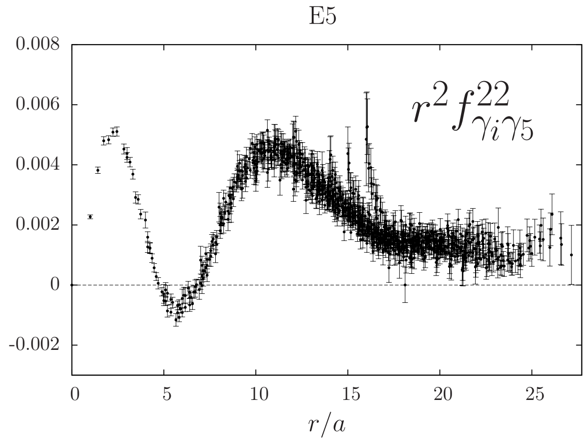
$$\mathcal{R}_{mn}^{\mathrm{sGEVP}}(t,t_0;\vec{r}) = f_{\gamma_\mu\gamma_5}^{(mn)}(\vec{r}) + \mathcal{O}\left(e^{-\Delta_{N+1,n}t}\right) \qquad m < n$$

⟶ Need to evaluate the three-point correlation function up to t only: better signal

Spatial component of the radial distributions: raw data







$$f^{(mn)}_{\gamma_i\gamma_5}(\vec{r}) = \langle\, B_m | A_i(\vec{r}) | B^*_n(\lambda)\,\rangle$$

- E5 : $a = 0.065$ fm and $m_\pi = 440$ MeV
- $\#r = 969 - 2925$ for $L/a = 32 - 48$ respectively
- node for excited states
- exponential fall-off
- "fishbone" structure at large radii
- preliminary results

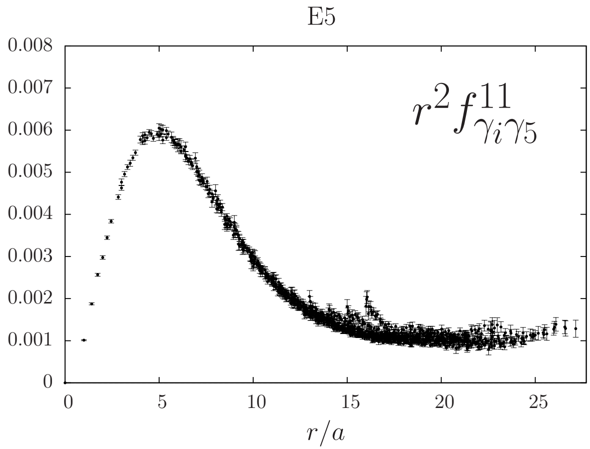
Volume effects

Lattice with periodic boundary conditions in space directions [Negele, '94]

$$a^3 f^{\rm lat}_{\gamma_i\gamma_5}(\vec{r}) = \sum_{\vec{n}} a^3 \widetilde{f}_{\gamma_i\gamma_5}(\vec{r}+\vec{n}L) \quad , \quad n_i \in \mathbb{Z} \, ,$$

Two kind of volume effects are expected:

- $f^{\rm lat}_{\gamma_i\gamma_5}(\vec{r})$ is the sum of all periodic images contributions
- $\widetilde{f}_{\gamma_i\gamma_5}(\vec{r})$ can still differ from the infinite volume distribution $f_{\gamma_i\gamma_5}(\vec{r})$ due to interactions with periodic images



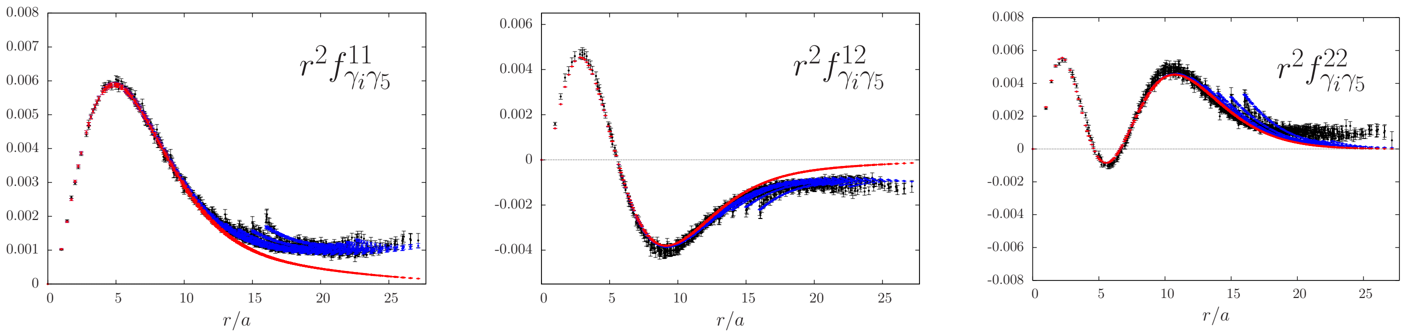
- $L/a = 32$
- $r^2 f^{11}_{\gamma_i\gamma_5}(\vec{r}) \neq 0$ for $r = L/2$ $\Rightarrow$ overlap of the tails
  $\Rightarrow$ "fishbone" structure
- We neglect interactions with periodic images
  $\Rightarrow \widetilde{f}_{\gamma_i\gamma_5}(\vec{r}) \approx f_{\gamma_i\gamma_5}(\vec{r})$, even in the overlap region

To remove these volume effects, we assume a functional form and fit the data with

$$f^{(mn)}_{\gamma_i\gamma_5}(\vec{r}) = P_{mn}(r)\, r^{\beta}\, \exp\left(-r/r_0\right) \, ,$$

where $P_{mn}(r)$ is a polynomial function

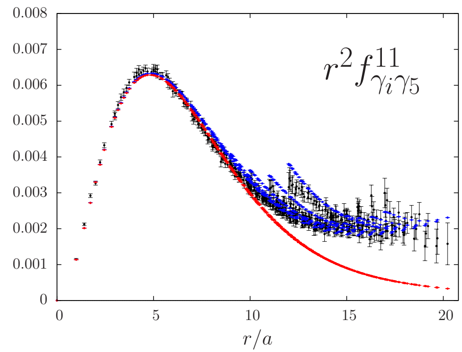
Volume effects: results for E5 ($a = 0.065$ fm and $m_\pi = 440$ MeV)



- Do not affect the computation of the couplings g_{mn} or form factors (as long as the distribution vanishes for $r > L$)
  $\rightarrow$ the contribution coming from periodic images compensates exactly the missing part of the tail for $r > L/2$.
- However, it affects quantities like $\langle r^2 \rangle_A$
- We have made the assumption that the tail of the distribution is not distorted by interactions:

<u>Test volume effects on a new ensemble:</u>

- Same β ($a = 0.065$ fm)
- Same pion mass ($m_\pi = 440$ MeV)
- Smaller volume :
  $L/a = 24$ instead of 32



fit the smaller volume ensemble ($L/a = 24$) using the same fit parameters ($L/a = 32$)

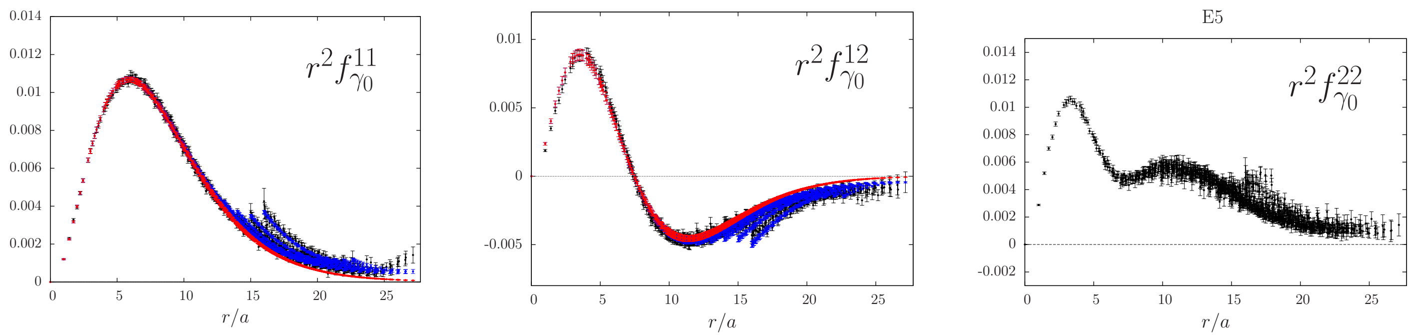
$\longrightarrow$ the deformation of the tail is negligible at our level of precision

Vector distributions and extraction of Z_V

- The vector (or charge) radial distributions are defined similarly by $(\gamma_\mu\gamma_5 \leftrightarrow \gamma_0)$

$$f_{\gamma_0}^{(mn)}(\vec{r}) = \langle B(\vec{p})|\left(\overline{\psi}_l\gamma_0\psi_l\right)(\vec{r})|B(\vec{p}\,')\rangle\,,$$

- Results for E5 ($m_\pi = 440$ MeV, $a = 0.065$ fm)



- Sum the vector radial distributions over all values of $\vec{r}$:

$$c_{mn} = \sum_{\vec{r}} f_{\gamma_0}^{(mn)}(\vec{r})$$

mn	11	22	12	23
c_{mn}	1.311(17)	1.212(52)	0.015(32)	−0.010(35)

→ Charge (vector current) conservation

$$Z_V\, c_{11} = 1 \quad \Rightarrow \quad Z_V = 0.763(12) \quad \text{at} \quad \beta = 5.3$$

→ Close to the non-perturbative estimate $Z_V = 0.750(5)$ from the ALPHA Collaboration [DellaMorte, '05] [Fritzsch, '12]

→ c_{12} and c_{23} are compatible with zero: the GEVP safely isolates the ground state and first excited state

Sum rules : g_{11}, g_{12} and g_{22}

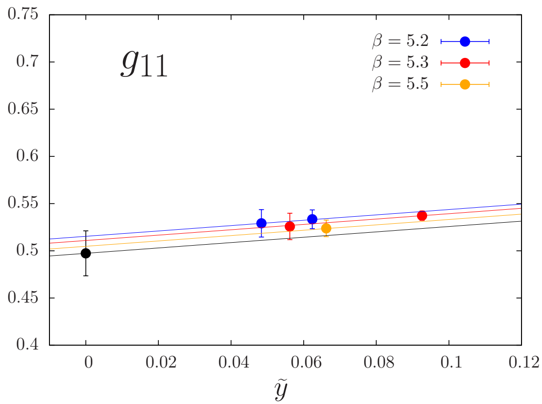
The sum over r of the spatial component of the radial distributions gives the form factor at $q^2 = q^2_{\max} = m_{B^*_m} - m_{B_n}$

$$g_{mn} = \sum_{\vec{r}} f^{(mn)}_{\gamma_i\gamma_5}(\vec{r}) = \langle B^0_m(\vec{0})|A_k(0)|B^{*+}_n(\vec{0},\lambda)\rangle$$

The renormalized $\mathcal{O}(a)$-improved couplings are then given by

$$\overline{g}_{mn} = Z_A(1 + b_A a m_q)\, g_{mn}$$

$\rightarrow$ Z_A is the light axial vector current renormalisation constant [DellaMorte et. al, '08] [Fritzsch et. al, '12]
$\rightarrow$ b_A is an improvement coefficient



$\tilde{y} = m_\pi^2/(8\pi^2 f_\pi^2)$

Extrapolations to the physical point :

$\rightarrow$ $\overline{g}_{11} = 0.499(24)$(?)

$\rightarrow$ $\overline{g}_{12} = -0.161(45)$(?)

$\rightarrow$ $\overline{g}_{22} = 0.363(38)$(?)

(preliminary, only naive extrapolations)

- The results are perfectly compatible with previous lattice calculation [Bernardoni et. al, '14] [Blossier et. al, '13]
- $g_{11} = \hat{g}$ is related to the $g_{B^*B\pi}$ coupling in the static limit ($q^2_{\max} \approx 0$)
- For g_{12} we have $q^2_{\max} = m_{B^{*\prime}} - m_B \neq 0$

Properties of the radial distributions

- First moment of the ground state radial distributions (square radius)

$$\langle r^2 \rangle_\Gamma = \frac{\displaystyle\int_0^\infty \mathrm{d}r\, r^4\, f_\Gamma^{(11)}(r)}{\displaystyle\int_0^\infty \mathrm{d}r\, r^2\, f_\Gamma^{(11)}(r)}$$

$\Gamma = 1$: $\langle r^2 \rangle_M = 0.215(9)\ \mathrm{fm}^2$

$\Gamma = \gamma_0$: $\langle r^2 \rangle_C = 0.345(6)\ \mathrm{fm}^2$ (preliminary)

$\Gamma = \gamma_i\gamma_5$: $\langle r^2 \rangle_A = 0.254(6)\ \mathrm{fm}^2$

$$\langle r^2 \rangle_M < \langle r^2 \rangle_A < \langle r^2 \rangle_C$$

- $g_{12} \ll g_{11} = \hat{g}$ can be understood by the presence of a node for the excited state

- Position of the node for $f_{\gamma_i\gamma_5}^{(12)}(\vec{r}) = \langle\, B | A_i(\vec{r}) | B^{*\prime}(\lambda)\, \rangle$

	$a = 0.075$ fm		$a = 0.065$ fm		$a = 0.048$ fm
m_π	330 MeV	280 MeV	440 MeV	310 MeV	340 MeV
r_n [fm]	0.371(6)	0.369(6)	0.369(4)	0.371(3)	0.358(4)

$\rightarrow$ no dependance on the lattice spacing / pion mass at our level of precision

$\rightarrow$ indication that our results for excited state $\left(f_{\gamma_i\gamma_5}^{(12)}(\vec{r})\right)$ are safe from multi-hadron thresholds $(B_1^*\pi)$

The $g_{B^{*\prime}B\pi}$ coupling

- The coupling is defined by the following on-shell matrix element

$$\left\langle B(p')\,\pi^+(q)\,|\,B^{*\prime}(p',\epsilon^{(\lambda)})\right\rangle = -g_{B^{*\prime}B\pi} \times q_\mu\,\epsilon^\mu(p')$$

Pseudoscalar B meson — Radially excited vector $B^{*\prime}$ meson

- Using the LSZ reduction and the PCAC relation, we are left with the following matrix element which can be computed on the lattice:

$$q^\mu\,\langle B^0(p)|A_\mu(0)|B^{*\prime+}(p+q)\rangle = g_{B^{*\prime}B\pi}\,(\epsilon\cdot q)\times\frac{f_\pi m_\pi^2}{m_\pi^2-q^2} + \ldots$$

- On the lattice with static heavy quarks: zero recoil kinematic configuration ($\vec{p}=\vec{p}\,'=\vec{0}$)
  - → Simulations correspond to $q^2=q^2_{\mathrm{max}}=(m_{B^{*\prime}}-m_B)^2\neq 0$ (far from the chiral limit)
  - → One should extrapolate the form factor to $q^2=0$ by taking the Fourier transform of the radial distribution [Becirevic et al. (2012)]
  - → Requires the knowledge of the spatial component $f^{(12)}_{\gamma_i\gamma_5}(\vec{r})$ but also of the time component $f^{(12)}_{\gamma_0\gamma_5}(\vec{r})$
  - → We are now computing the time component $f^{(12)}_{\gamma_0\gamma_5}(\vec{r})$

Conclusion

- We have computed the radial distributions of the axial vector density for the ground state and first excited state

  $\longrightarrow$ We use the GEVP and improved estimators to reduce the statistical noise

  $\longrightarrow$ We have checked the GEVP results on the radial distributions of the vector density (Z_V)

  $\longrightarrow$ We have five lattice ensembles to study discretization and quark mass effects

  $\longrightarrow$ Volume effects seems negligible at our level of precision

- Results are still preliminary

- The next step is to compute the time component of the axial vector distribution: $f^{(mn)}_{\gamma_0\gamma_5}(\vec{r})$

  $\longrightarrow$ $g_{B^{*\prime}B\pi}$ coupling at $q^2 = 0$ (Fourier transform of the distribution)

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Thank you

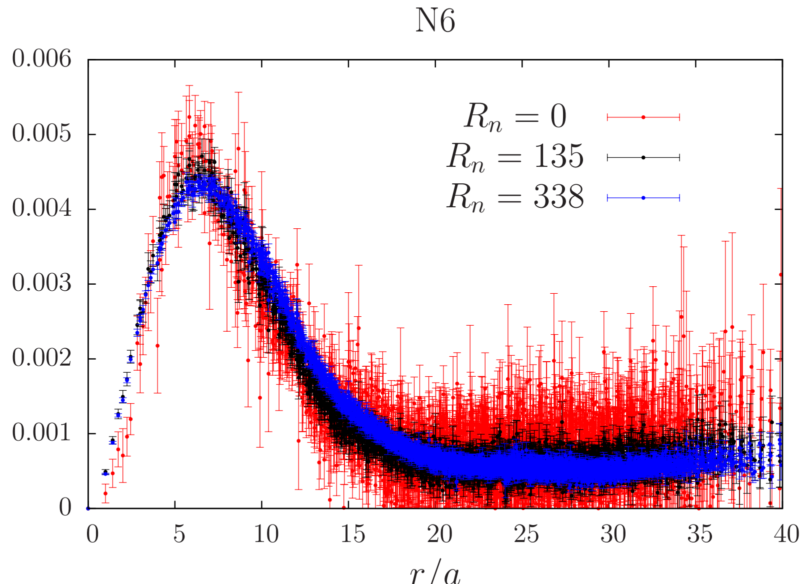
Smearing

- Gaussian smearing on the light quark field ψ_l is used to reduce the contamination of excited states:

$$\psi_l^{(i)}(x) = \left(1 + \kappa_G a^2 \Delta\right)^{R_i} \psi_l(x)$$

- It is applied on the contraction with the heavy-quark propagator but not on the probe which must stay local.
- The time t is chosen such that the radial distributions has reached a plateau:

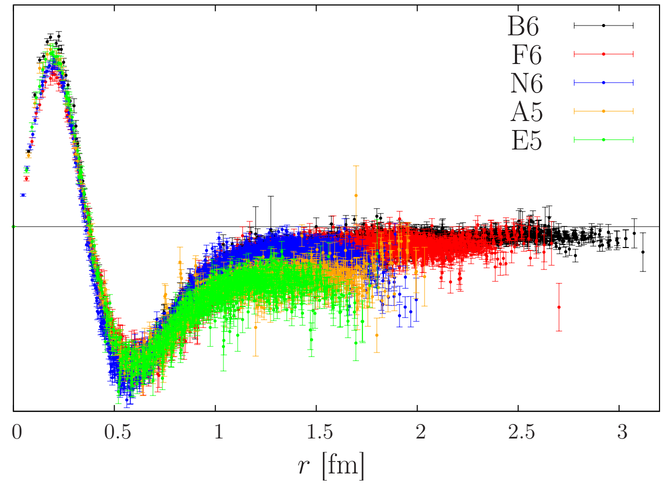
$$\mathcal{R}_{\gamma_\mu\gamma_5}(t, t_1, \vec{r}) = \frac{C^{(3)}_{\gamma_\mu\gamma_5,ij}(t, t_1; \vec{r})}{\left(C^{(2)}_{\mathcal{P},ii}(t)\ C^{(2)}_{\mathcal{V},jj}(t)\right)^{1/2}} \xrightarrow[t \gg t_1 \gg 1]{} f^{(11)}_{\gamma_\mu\gamma_5}(\vec{r}) = \langle\, B | A_\mu(\vec{r}) | B^*(\lambda) \,\rangle\,,$$



Multi-hadron threshold

- Within our lattice setup, the radial excitation of the vector B meson lies near the multi-particles threshold $B_1^*\pi$
- The mass of the axial B meson in the static limit is extracted from our previous study [Blossier et al. (2014)]
- We assume that the mass of the two particle state is simply given by $E_{B_1^*\pi} = m_{B_1^*} + m_\pi$

CLS	$am_{B^{*\prime}} - am_B$	$(am_{B_1^*} - am_B) + am_\pi$
A5	0.253(7)	0.281(4)
B6	0.235(8)	0.248(4)
E5	0.225(10)	0.278(6)
F6	0.213(11)	0.233(3)
N6	0.166(9)	0.176(3)



$\longrightarrow$ All lattice ensembles are near but bellow threshold

$\longrightarrow$ The position of the node is remarkably stable and does not depend on the pion mass contrary to what would be expected in the case of a mixing with the multi-particle state

$\longrightarrow$ Indication that our results are safe from multi-hadron thresholds