Boundary effects on the chiral condensate from Lattice QCD

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Lattice 2015 --Kobe

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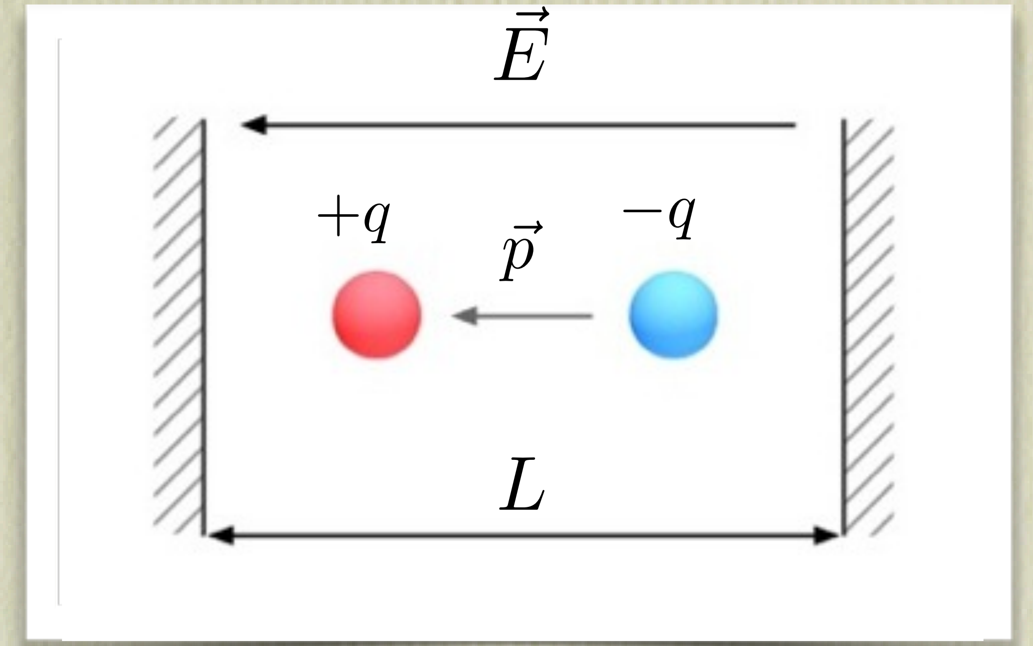
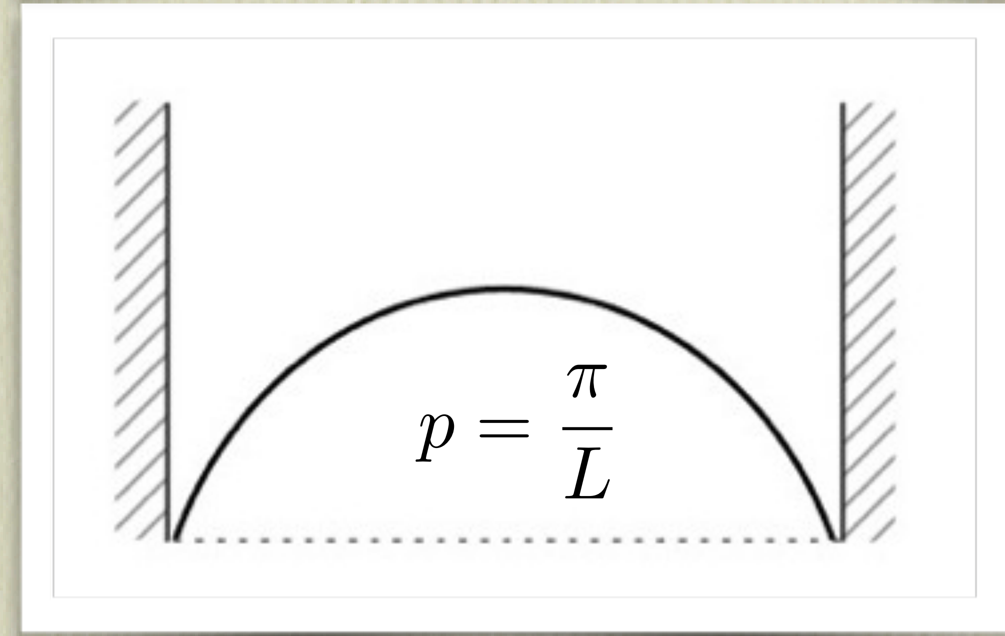
Outline

- Motivation
- Chiral condensate evaluation
- Numerical results with nHYP fermions
- Numerical results with overlap fermions
- Conclusions

Motivation

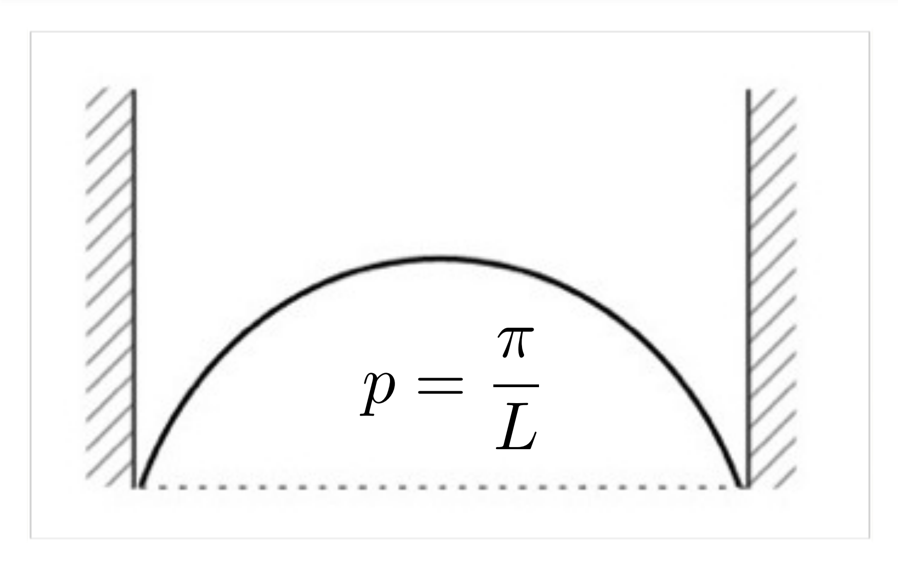
- To compute electric polarizabilities we need to compute hadron energies in the presence of a constant electric field.
- Euclidean formulation requires that the Hamiltonian of the system is bounded from below, i.e. there is a vacuum state of lowest energy.
- In the presence of a real electric field, the vacuum is no longer stable -- Schwinger instability against pair creation.
- In a finite volume box we can make the system stable by limiting the maximal distance between charges.
- We use Dirichlet boundary conditions in space to stabilize the system.
- Note that this instability exists even in a finite volume box if we use periodic boundary conditions.

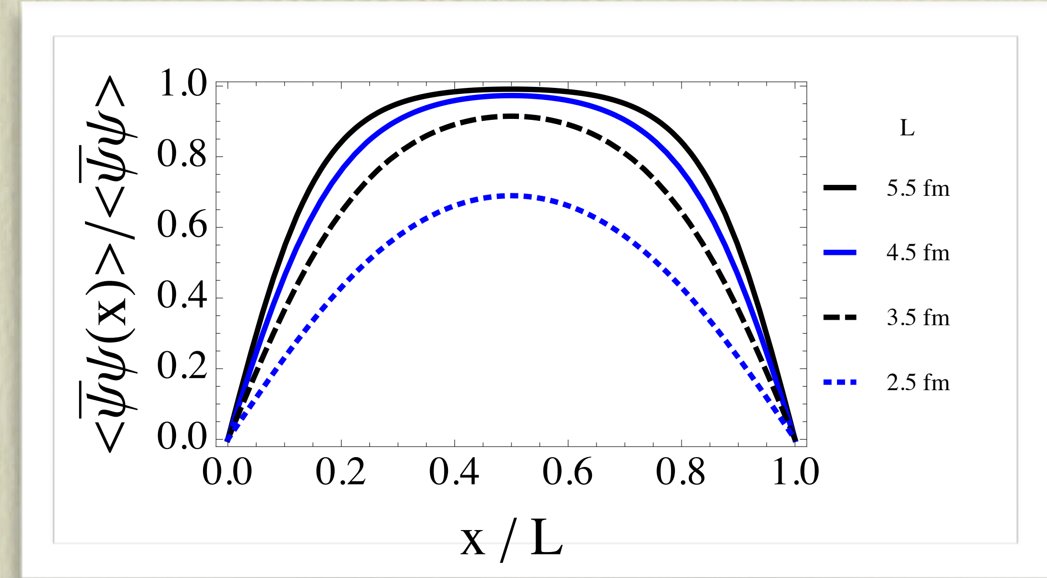




Dirichlet boundary conditions

- Dirichlet boundary conditions are equivalent to a hard wall in the direction of the electric field.
- The lowest energy for one-particle states corresponds to a non-zero momentum.
- The chiral condensate also vanishes on the boundary, but it is expected to get restored to its bulk value away from the wall.
- A sigma-model calculation estimated the thickness of the region where the condensate is perturbed to be sizable.
- Assuming that a sigma particle of mass 440 MeV saturates the scalar channel, the condensate get restored to 90% of its bulk value about 1.3 fm away from the wall.





B. Tiburzi, Phys.Rev. D88 (2013) 034027

Chiral condensate

- Chiral condensate $\langle \bar{q}q(x) \rangle = -\mathrm{Tr}\, M^{-1}_{x,x}$

- For periodic boundary conditions we have translational invariance

$$\langle \bar{q}q(x) \rangle = -\left\langle \mathrm{Tr}_{s,c}\, M^{-1}_{x,x} \right\rangle = -\frac{1}{V} \left\langle \mathrm{Tr}\, M^{-1} \right\rangle$$

- For Dirichlet boundary conditions we have translational invariance only in the directions parallel to the boundary

$$\langle \bar{q}q(x) \rangle = -\left\langle \mathrm{Tr}_{s,c}\, M^{-1}_{x,x} \right\rangle = -\frac{1}{V_3} \sum_{y, y_1 = x_1} \left\langle \mathrm{Tr}_{s,c}\, M^{-1}_{y,y} \right\rangle$$

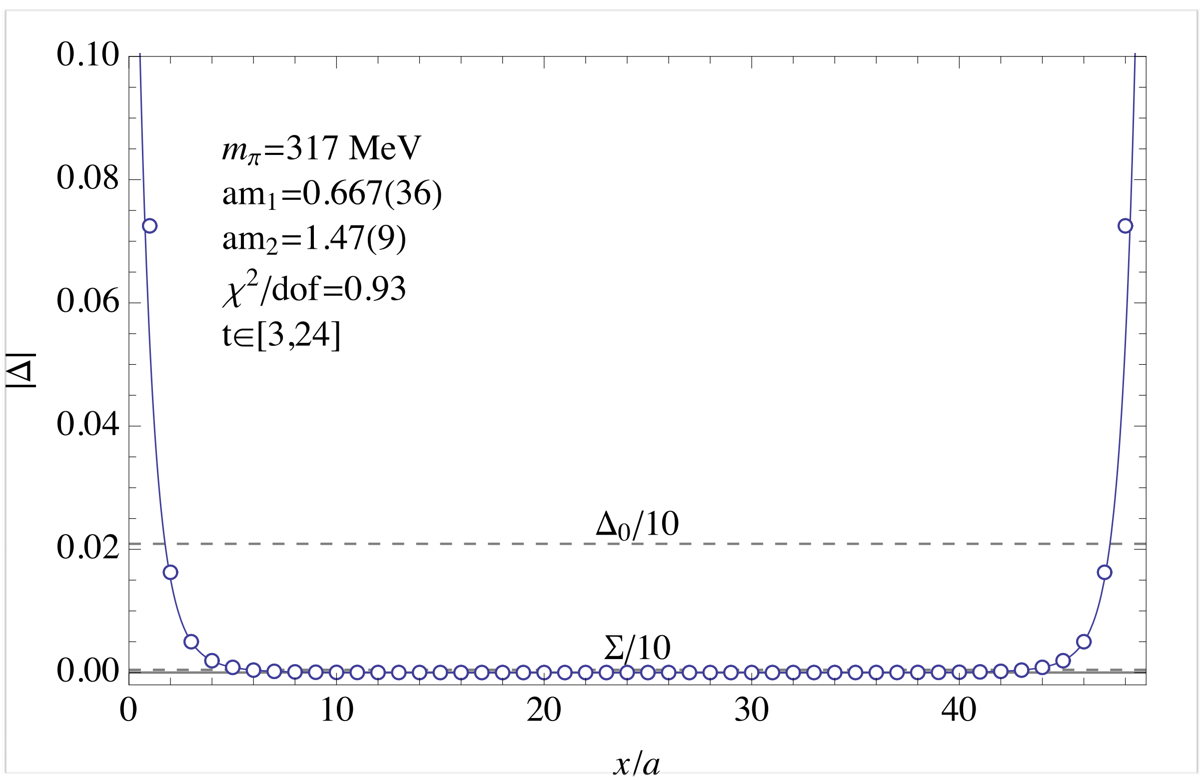
- The chiral condensate is defined in the massless limit

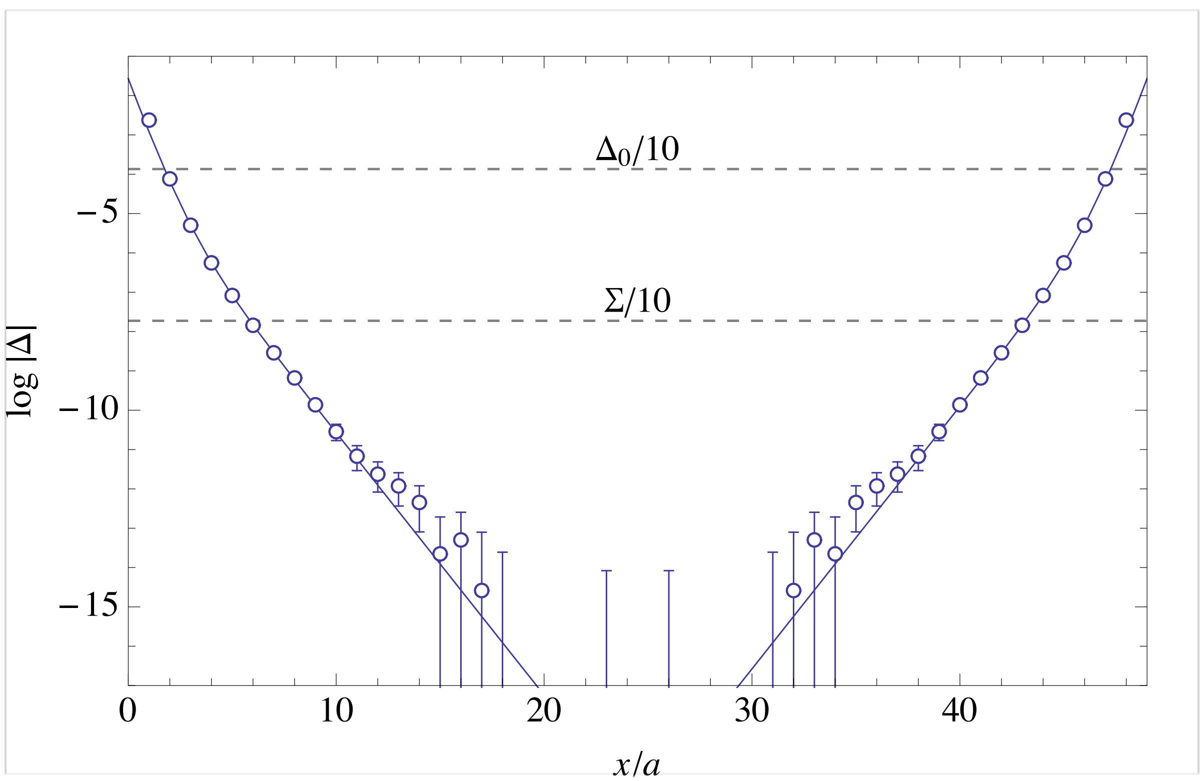
$$\langle \bar{q}q \rangle (m) = \langle \bar{q}q \rangle + c_0(x) a^{-3} + c_1(x) m a^{-2} + c_2(x) m^2 a^{-1} + c_3(x) m^3$$

Ensemble details

- nHYP N_f=2 dynamical configurations with a=0.121 fm
- nHYP valence $24^3 \times 48$ (m_π=317MeV) $24^3 \times 64$(m_π=227 MeV)
- overlap valence $16^3 \times 32$ for both m_π=317MeV and m_π=227MeV
- For all ensembles we use 100 configurations
- Dirichlet boundary conditions are used in the longest (time) direction and periodic boundary conditions in the other directions
- For each ensemble we run also perform a calculation using periodic boundary conditions in time to serve as reference.

Chiral condensate (nHYP)



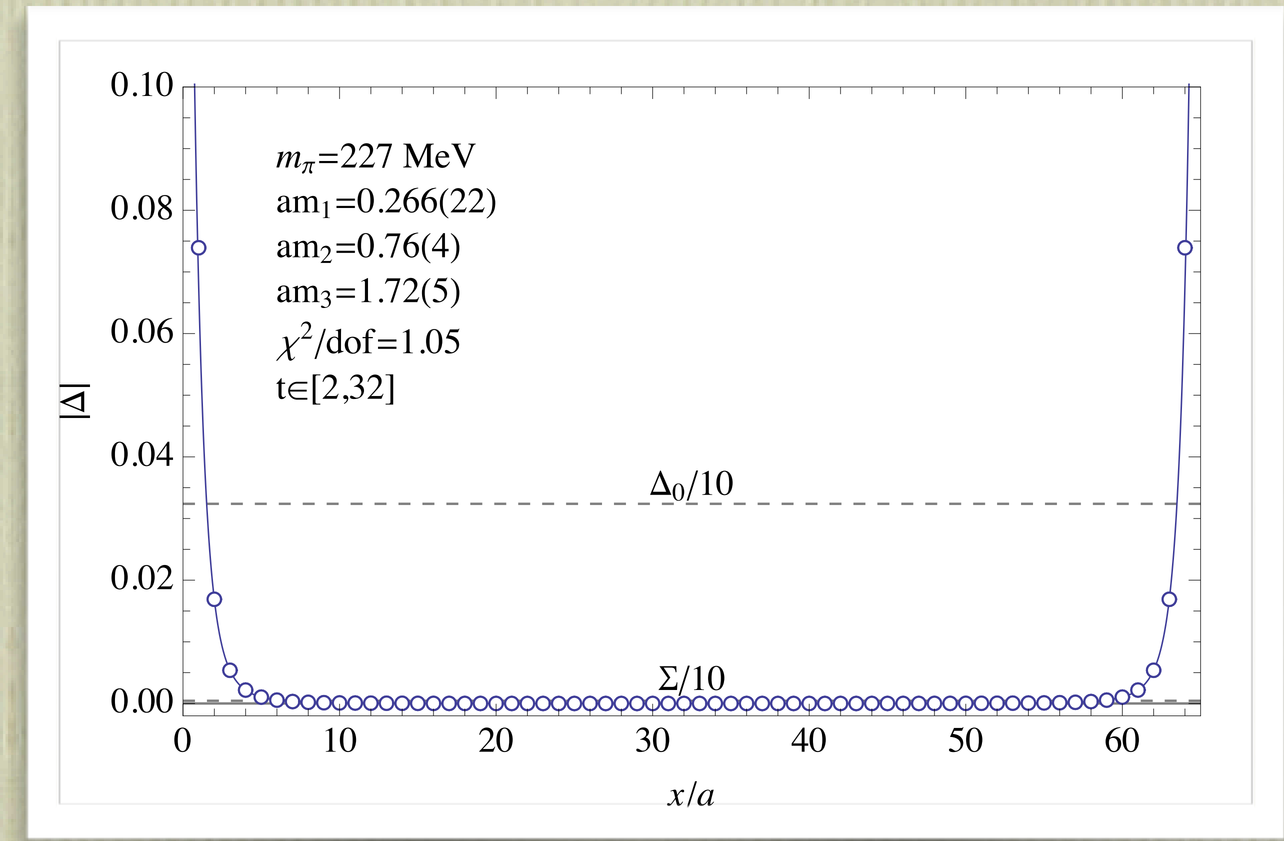


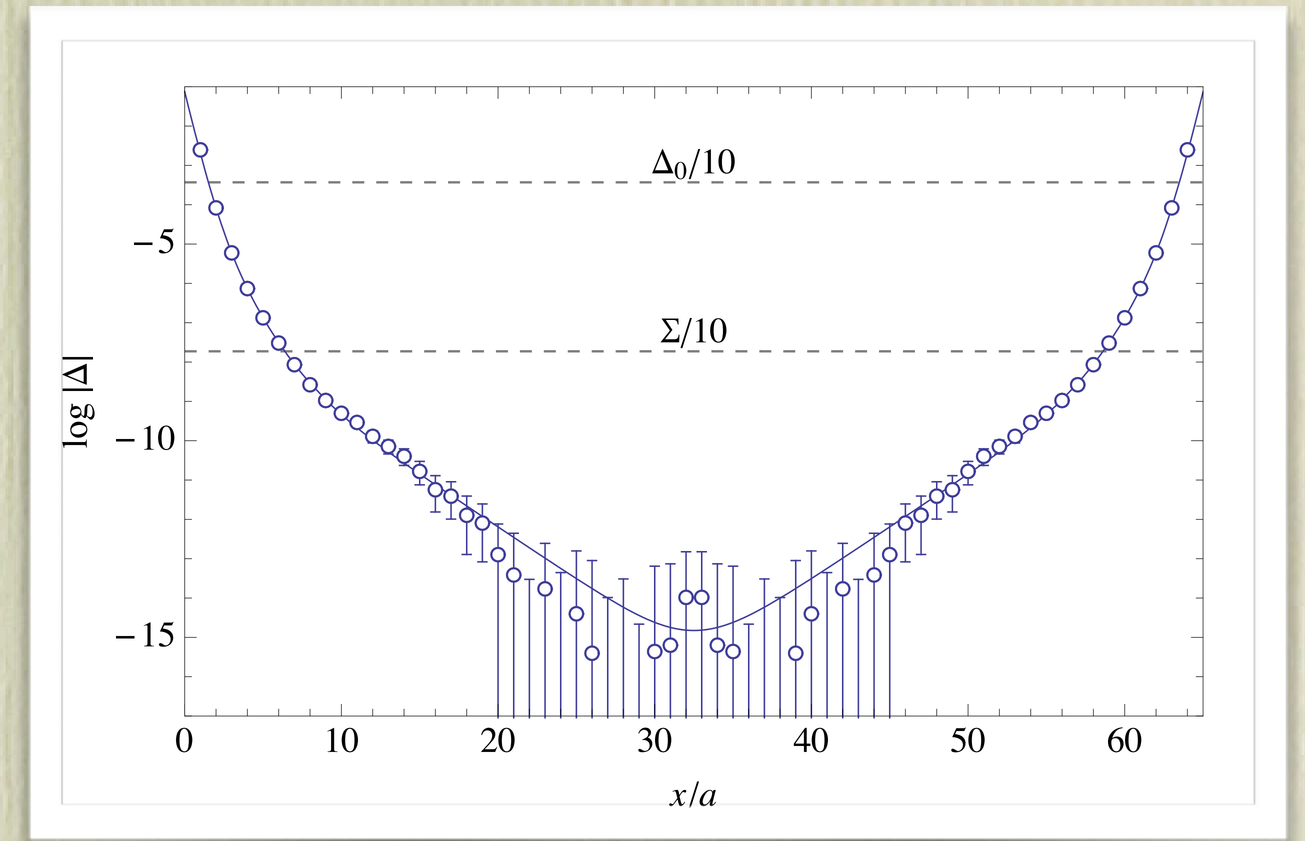
$$\Delta(x) = a^3 \langle \bar{q}q(x) \rangle_{\text{dirichlet}} - a^3 \langle \bar{q}q(x) \rangle_{\text{periodic}} = \left\langle \text{Tr}\, M^{-1}(x) \right\rangle_{\text{periodic}} - \left\langle \text{Tr}\, M^{-1}(x) \right\rangle_{\text{dirichlet}}$$

Engel *et al*, PRL 114 (2015), no. 11 112001 $\quad a^3\Sigma = -a^3 \langle \bar{q}q \rangle = 4.22 \times 10^{-3}$

GMOR (unrenormalized) $\quad a^3\Sigma = -a^3 \langle \bar{q}q \rangle = 4.41 \times 10^{-3}$

Chiral condensate (nHYP)





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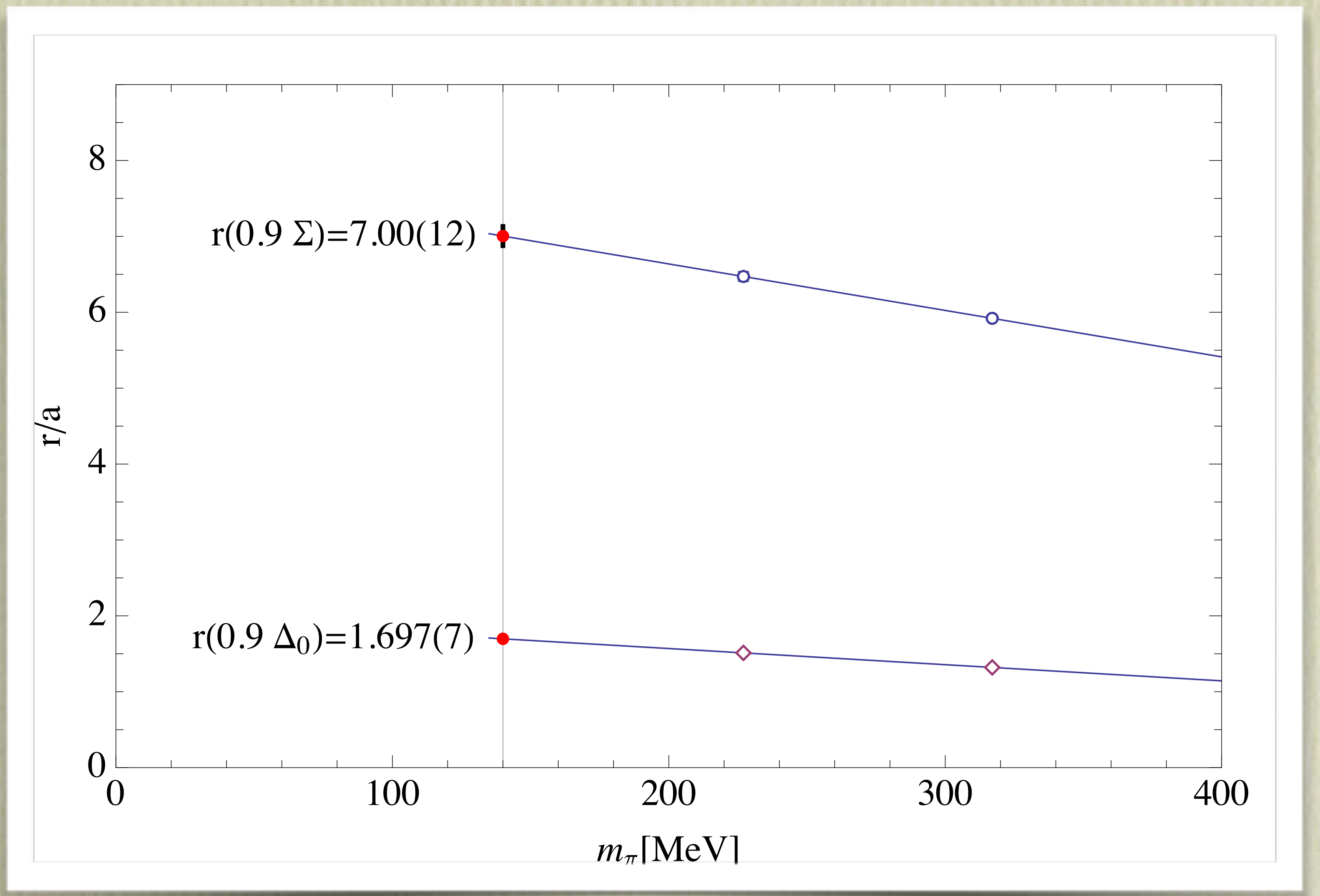
Border thickness



Overlap fermions

- This discretization satisfies a lattice version of the chiral symmetry
- The massive operator has the same eigenvectors as the massless operators

$$D(m) = \rho + \frac{m}{2} + \left(\rho - \frac{m}{2}\right)\gamma_5\epsilon(H) = m + \left(1 - \frac{m}{2\rho}\right)D(0)$$

- The chiral condensate is defined in terms of the "continuum" propagator

$$\langle\bar{q}q\rangle = -\frac{1}{V}\frac{1}{2\rho}\mathrm{Tr}\,\frac{1-\gamma_5\epsilon(H)}{1+\gamma_5\epsilon(H)} = -\frac{1}{V}\mathrm{Tr}\,D_c(m)^{-1} \qquad D_c(m)^{-1} \equiv \left(1 - \frac{1}{2\rho}\right)D(m)^{-1}$$

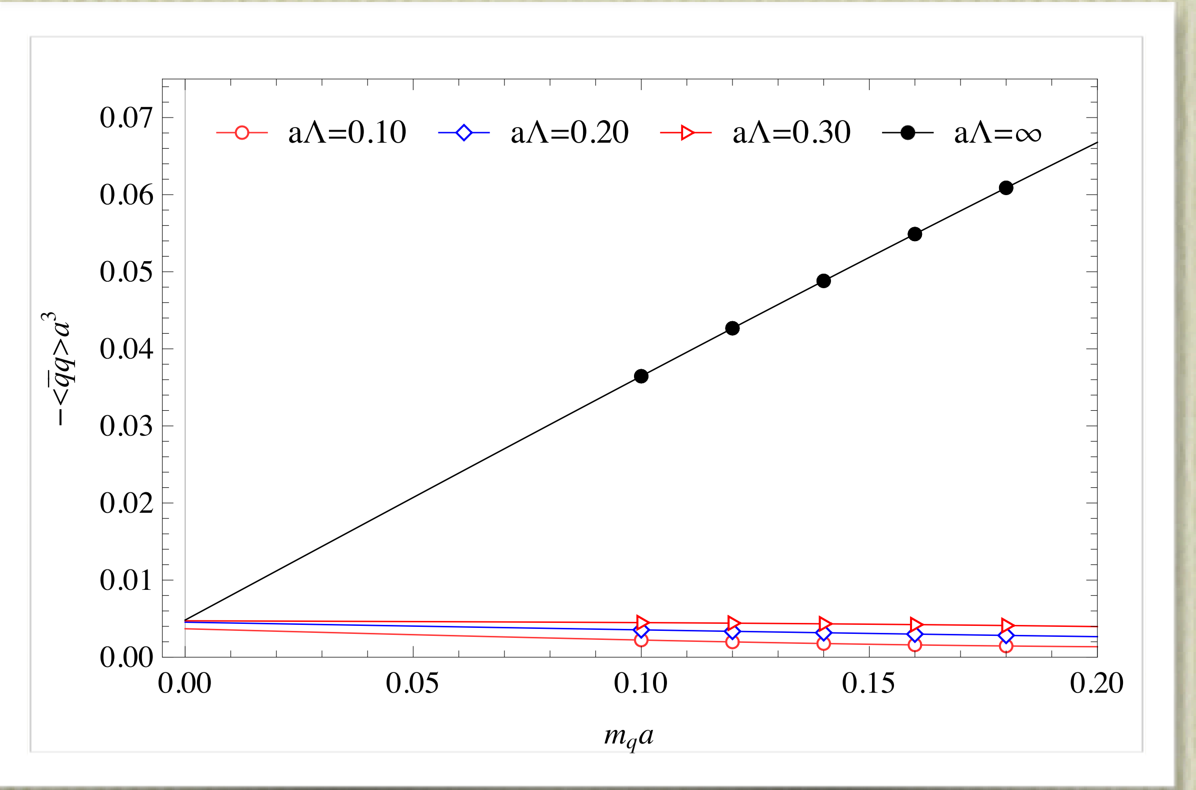
- This operator doesn't have the cubic divergence and the condensate can be extracted from a mass extrapolation

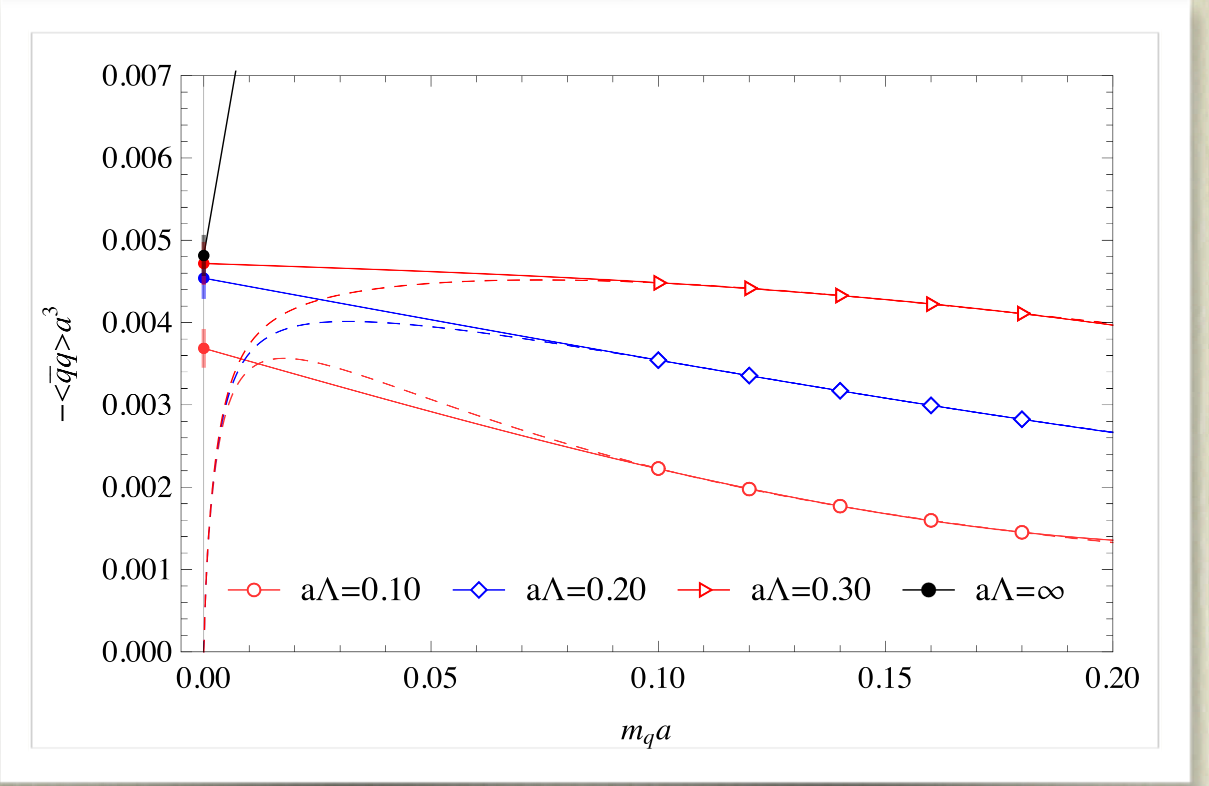
$$\langle\bar{q}q\rangle(m) = \langle\bar{q}q\rangle + c_1 m a^{-2} + c_2 m^2 a^{-1} + c_3 m^3$$

- The quadratic term in mass is usually small and can be neglected in extrapolation

H. Neuberger, Phys.Rev. D57 (1998) 5417

Overlap fermions
eigenmode expansion



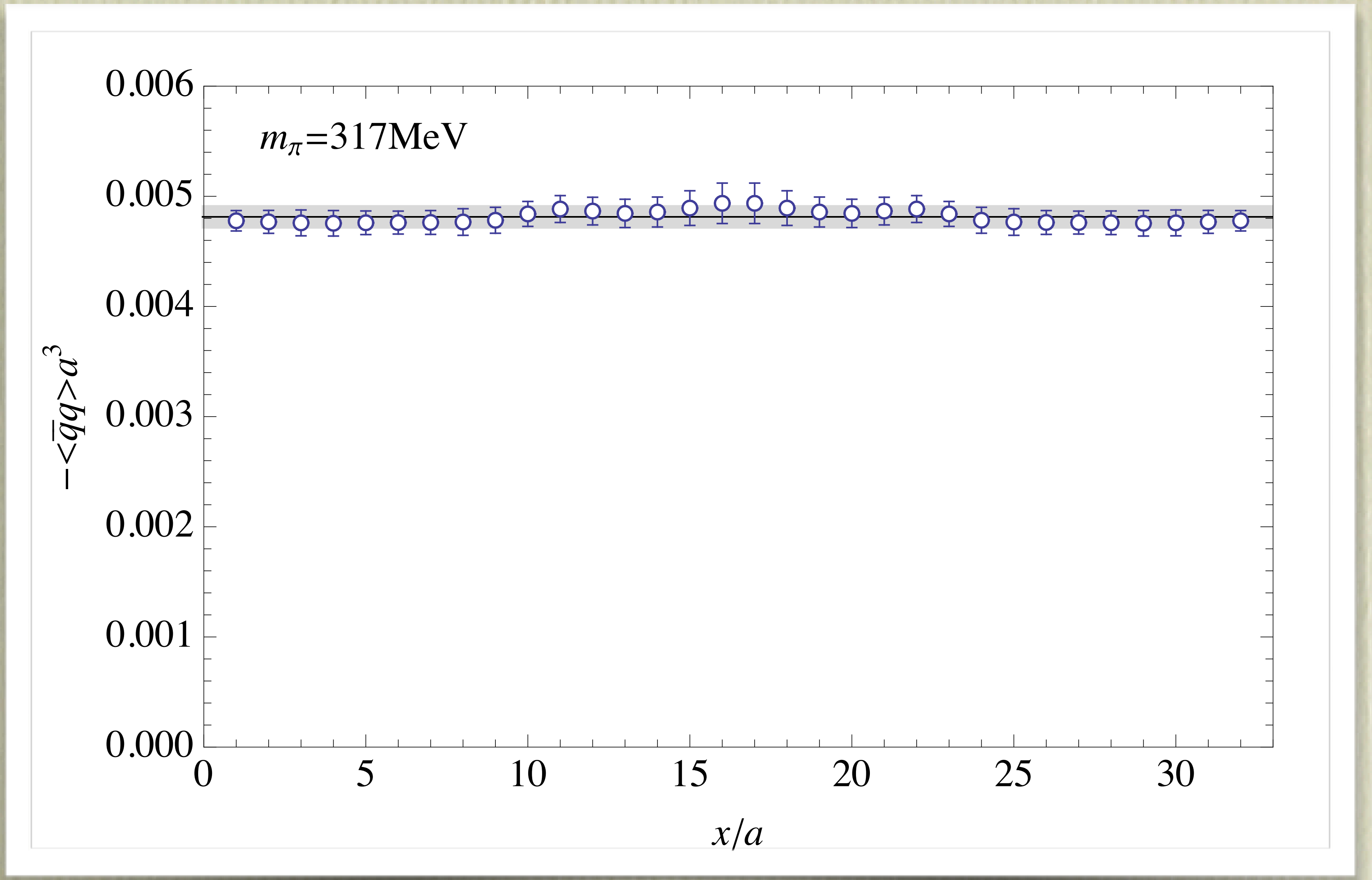


$$D_c(m)^{-1} = \left(1 - \frac{1}{2\rho}D(0)\right) D(m)^{-1} \qquad \langle\bar{q}q(x)\rangle = -\overline{\mathrm{Tr}}_{s,c}\,[D_c^{-1}]_{x,x} \qquad \langle\bar{q}q(x)\rangle^{(\Lambda)} = -\sum_{|\lambda|<\Lambda,\lambda\neq 0} \frac{1}{\lambda}\phi_\lambda(x)^\dagger\phi_\lambda(x)$$

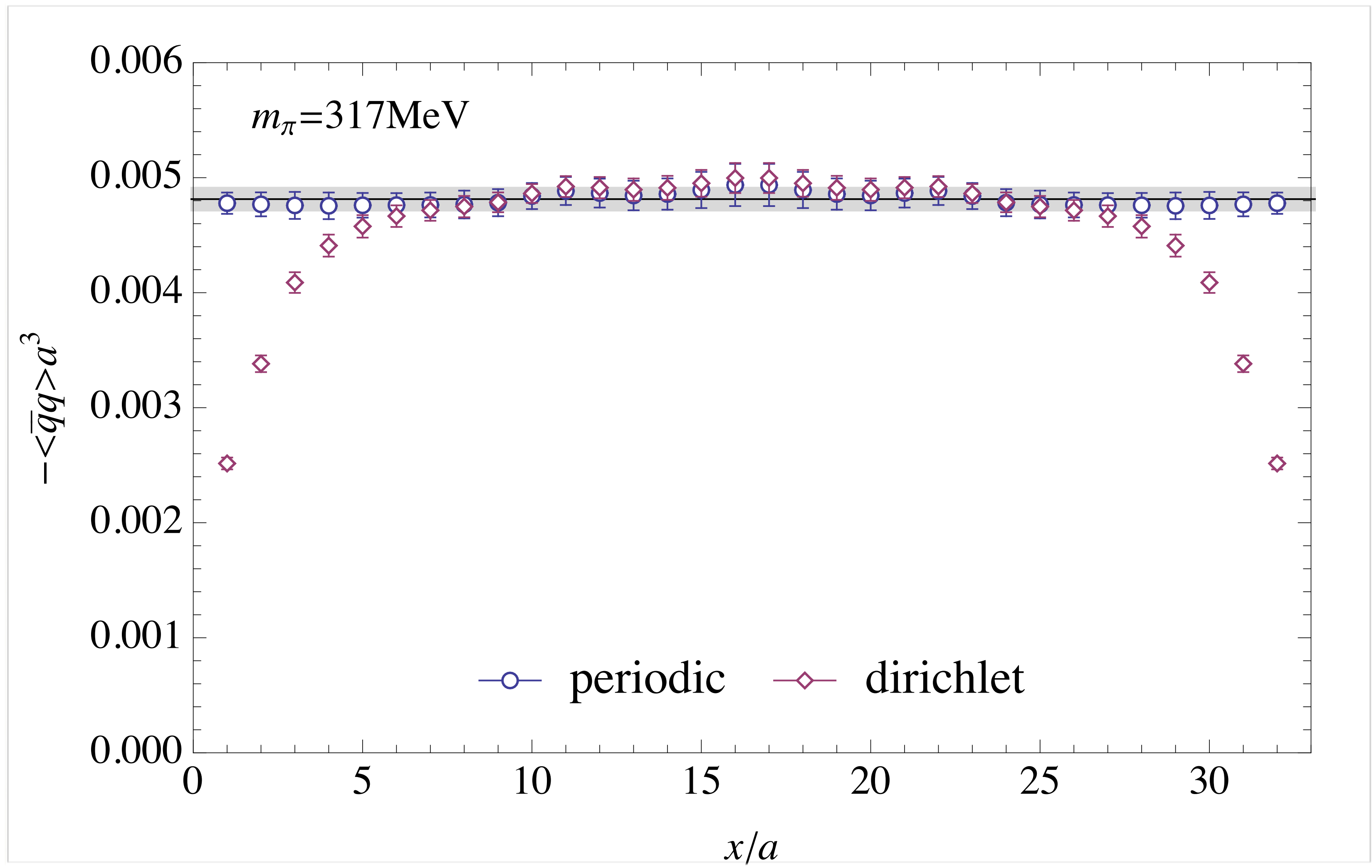
J. Noaki et al., Phys.Rev. D81 (2010) 034502

T. Iritani et al, Phys.Rev. D91 (2015), no. 9 094501

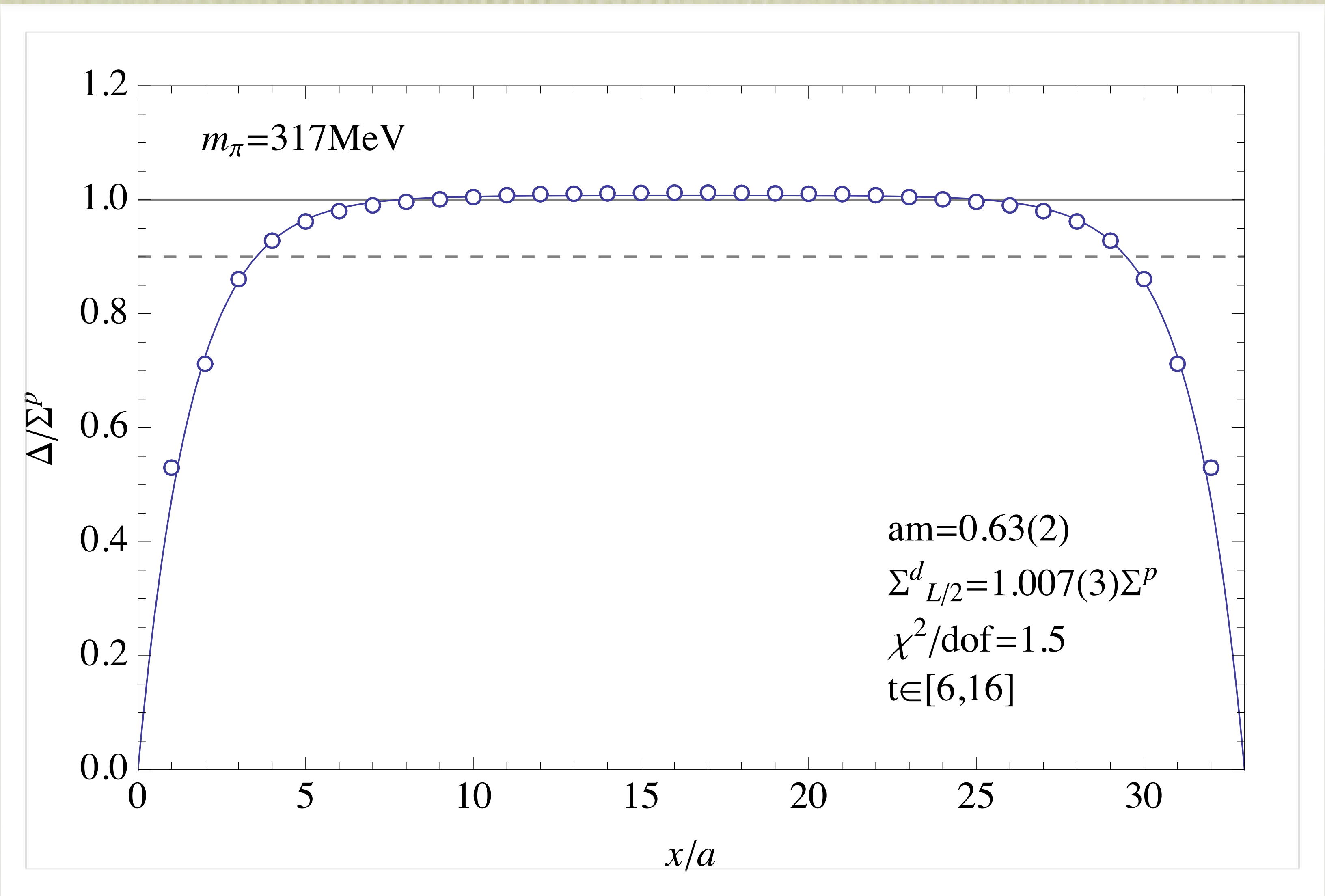
Chiral condensate
periodic boundary conditions



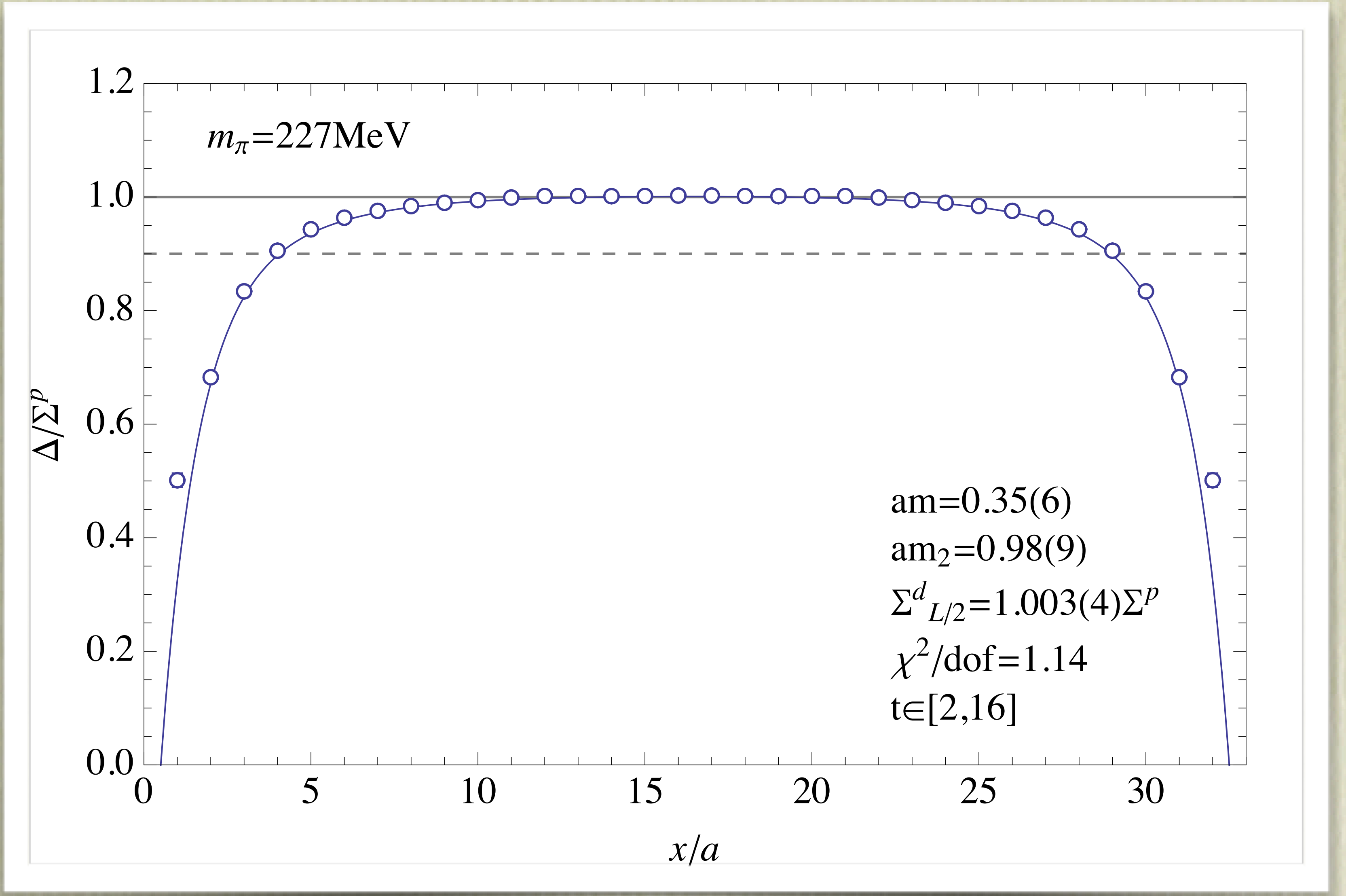
Chiral condensate
Dirichlet boundary conditions



Chiral condensate
Dirichlet boundary conditions



Chiral condensate
Dirichlet boundary conditions



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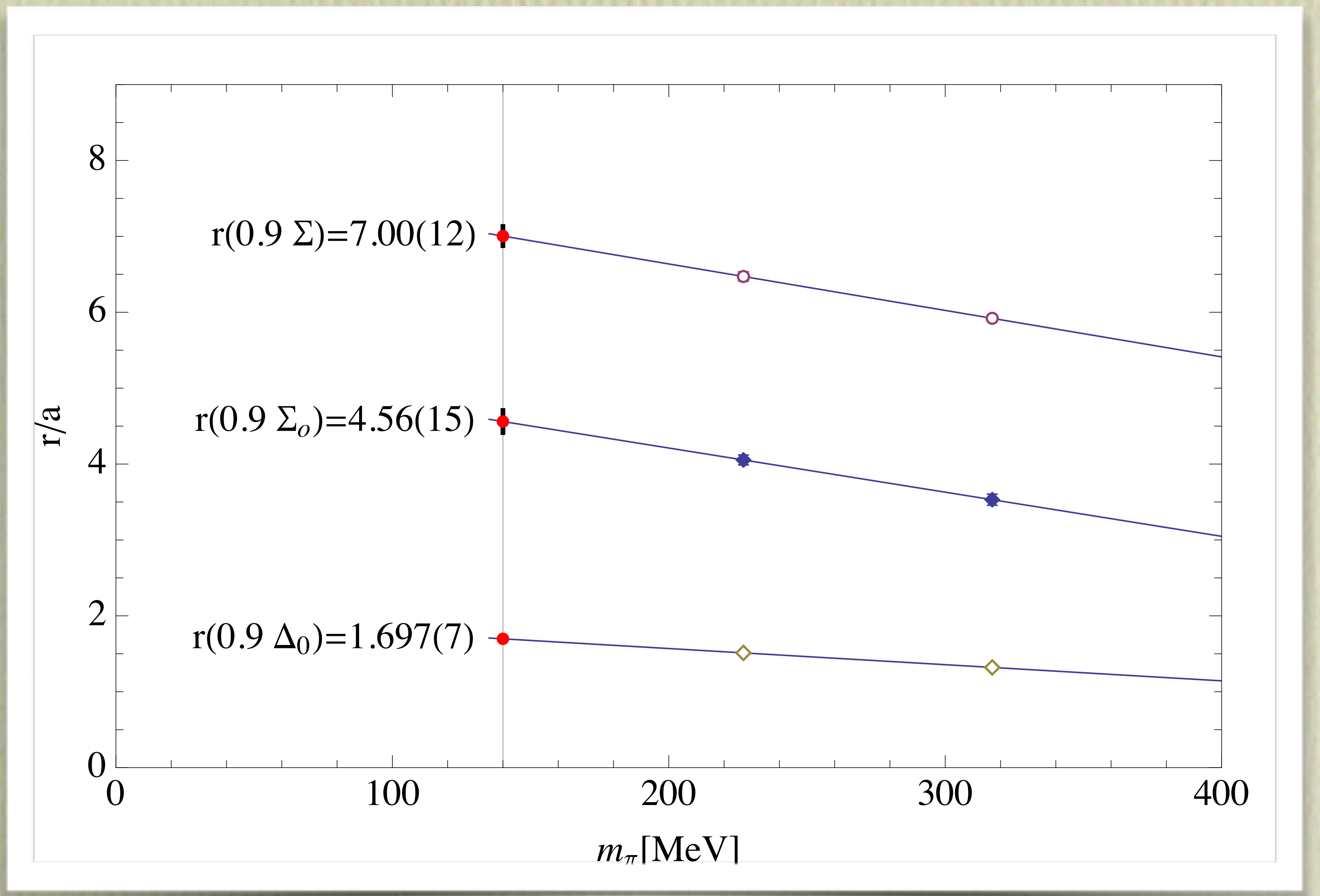

Conclusions

- We calculated the effect of the Dirichlet boundary conditions on the chiral condensate on ensembles directly relevant for our polarizability studies.
- For m_π=227 MeV and m_π=317 MeV the effect of the boundary on Tr M^{-1} is negligible a couple of lattice spacings away from the boundary (approx 1/4 fm).
- To compute the effect on the chiral condensate we used valence overlap fermions.
- The condensate is restored to 90% of its bulk value at 0.5 fm away from the boundary.
- The disagreement with the sigma-model prediction comes from the fact that the correlation in the scalar channel is not saturated by the sigma excitation, as assumed in the model.
- Other scalar excitations are dynamically relevant and their relative coupling are also important. The behavior of the condensate close to the wall is most likely not universal.