SCALE SETTING ON THE CLS 2+1 LATTICES

Stefan Schaefer

Mattia Bruno, Tomasz Korzec, Rainer Sommer

Lattice 2015, Kobe, Japan

CLS large volume simulations

A major goal is Λ parameter in physical units → TALK BY SINT

Scale determination from large volume simulations

CLS effort

$N_\mathrm{f} = 2 + 1$ flavors of NP improved Wilson fermions
Lüscher–Weisz gauge action

c_sw determined two years ago BULAVA, S.S.'13

open boundary conditions → no topological freezing as $a \to 0$

gauge field generation with openQCD code LÜSCHER, S.S.'12

account of simulations published BRUNO ET AL'15

three lattice spacings $a = 0.05\,\mathrm{fm}, \ldots, 0.085\,\mathrm{fm}$

Next step: scale setting via PS decay constants

Ensembles

Chiral trajectory

Generated along lines with QCDSF'10

$$\mathrm{tr}(M_\mathrm{q}) = \sum_f m_{\mathrm{q,f}} = \mathrm{const}$$

Guarantees constant improved coupling

$$\tilde{g}_0^2 = g_0(1 + \frac{1}{3} b_\mathrm{g} a \mathrm{tr} M_\mathrm{q}) = \mathrm{const}$$

Note that BHATTACHARYA ET AL'06

$$\mathrm{tr}(M^R) = Z_\mathrm{m} r_\mathrm{m} \left[(1 + a\bar{d}_\mathrm{m} \mathrm{tr} M_\mathrm{q}) \mathrm{tr} M_\mathrm{q} + a d_\mathrm{m} \mathrm{tr}(M_\mathrm{q}^2) \right]$$

Tuning variables

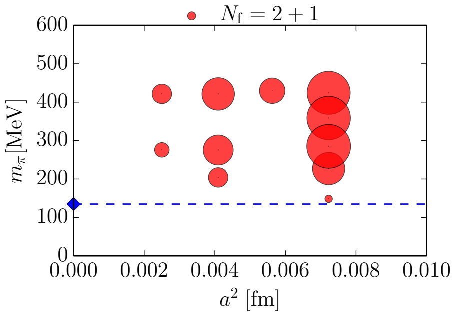
$$\phi_2 = 8 t_0 m_\pi^2 \qquad \phi_4 = 8 t_0 (m_\mathrm{K}^2 + \frac{1}{2} m_\pi^2)$$

Match at symmetric point $m_\mathrm{u} = m_\mathrm{d} = m_\mathrm{s}$ and $m_\pi \approx 420$ MeV

$$\phi_4 = 1.15$$

Chiral trajectory fixed from there on by $\mathrm{tr}(M_\mathrm{q})$ =const.

The ensembles

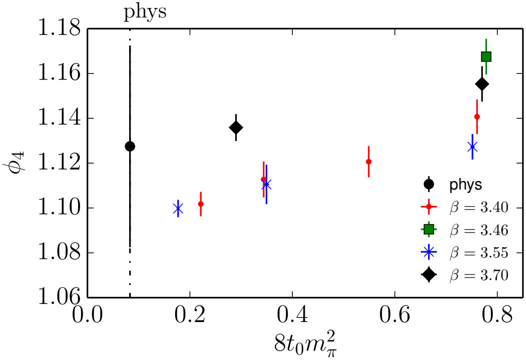


Target at $\tau_\mathrm{md} > 50\,\tau_\mathrm{exp}$

Area of circle $\propto \tau_\mathrm{md}/\tau_\mathrm{exp}$, for largest circles ≈ 120

Main chiral trajectory, some more ensembles available

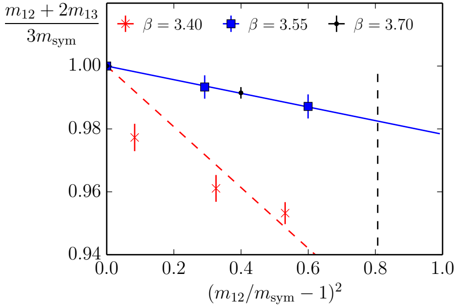
Chiral trajectory



$$\phi_2 = 8t_0 m_\pi^2 \qquad \phi_4 = 8t_0(m_K^2 + \frac{1}{2}m_\pi^2)$$

Discretization effects

How constant is the quark mass's sum?



$$m_{ij} = (\partial_0 f_{\rm A}^{ij}(x_0) + a c_{\rm A} \partial_0 \tilde{\partial}_0 f_{\rm P}^{ij}(x_0))/2 f_{\rm P}(x_0)$$

$m_{\rm sym} = m_{12} + 2m_{13}$ at symmetric point.

Non-perturbative $c_{\rm A}$ BULAVA ET AL'15

Expect linear $\mathrm{O}(a)$ effects linear in $(1 - \frac{m_{12}}{m_{\rm sym}})^2$.

Significant violations at coarsest lattice spacing.

Mass corrections

Tuning is never perfect
$\rightarrow$ want to shift expectation values to different $(m_{\rm u}, m_{\rm s})$

If shifts are not large, compute derivative

$$\frac{\partial}{\partial m_{\rm f}}\langle A\rangle = \Big\langle \frac{\partial A}{\partial m_{\rm f}}\Big\rangle - \Big\langle (A - \langle A\rangle)\,(\frac{\partial S}{\partial m_{\rm f}} - \langle\frac{\partial S}{\partial m_{\rm f}}\rangle)\Big\rangle\,.$$

1 – Stochastic estimate of the derivative of

$$\partial_{m_{\rm f}} S_{\rm f}(m_{\rm f}) = -\partial_{m_{\rm f}} {\rm tr}\log(D + m_{\rm f}) = -{\rm tr}(D + m_{\rm f})^{-1}$$

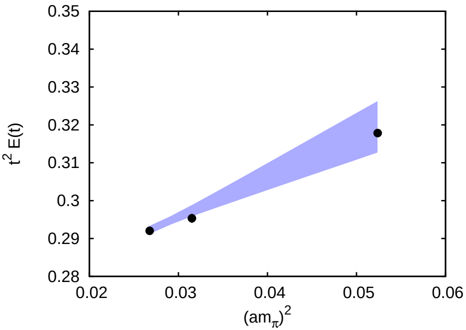
2 – Analytic derivatives of 2 pt functions via

$$\partial_{m_{\rm f}}\frac{1}{D + m_{\rm f}} = -\frac{1}{(D + m_{\rm f})^2}$$

Use first term in expansion to correct measured quantities

$$f(\langle\vec{A}(m')\rangle) \rightarrow f(\langle\vec{A}(m)\rangle) + (m' - m)\,\partial_m f(\langle\vec{A}(m)\rangle)$$

Mass derivative: examples



Energy density at fixed Wilson flow time t ($\approx t_0$).

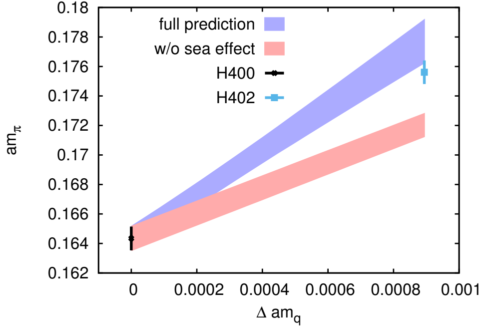
96×32^3 lattice, prediction from $m_\pi \approx 420\,$MeV

Moving along $m_\mathrm{u} = m_\mathrm{d} = m_\mathrm{s}$ line.

Significant step in quark mass possible.
4 MeV shift leads to 2$\times$ stat. error, 30 MeV shift to 10$\times$.

Mass derivative: examples

Pion mass



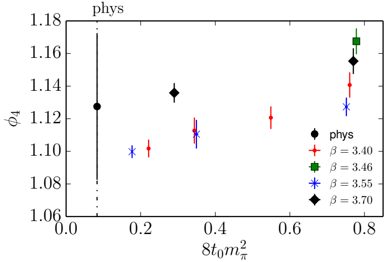
Moving along $m_\mathrm{u} = m_\mathrm{d} = m_\mathrm{s}$ line.

Roughly $\mathrm{O}(4\,\mathrm{MeV})$ change in quark mass
Error doubled.

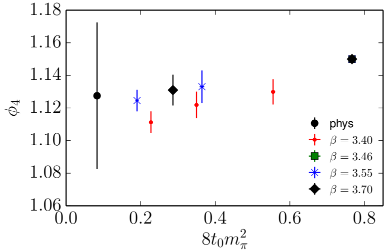
Effect from the sea quarks signficant

Curvature negligible

Chiral trajectory



Raw data



After correction

At the moment, most mass corrections extrapolated taken from fit.

Scale setting

Use light pseudoscalar decay constants

$$f_{\pi\mathrm{K}} = \frac{2}{3}\left[f_{\mathrm{K}} + \frac{1}{2} f_{\pi}\right]$$
$$= f\left[1 + \frac{16\, B\, \mathrm{tr}(M)}{3f^2}(L_5 + 3L_4) + \mathrm{logs}\right]$$

In NLO ChPT combination const up to known log corrections.

Two strategies

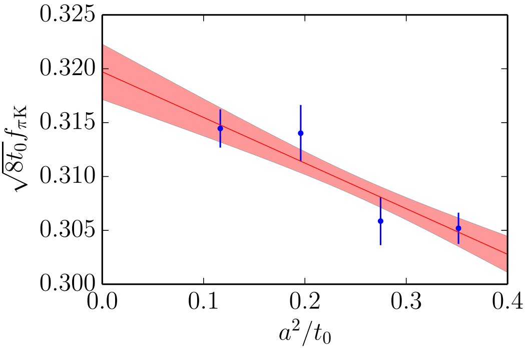
1 – Set the scale via t_0

adapted to tuning strategy
ambiguity due to different flow definitions
get physical value of t_0 from $\sqrt{8t_0}\, f_{\pi\mathrm{K}}$ in continuum

2 – Set the scale via $f_{\pi\mathrm{K}}$

good experience in $N_{\mathrm{f}} = 2$
need to deal with corrections in mass
chiral trajectories at different β no longer match

Discretization effects



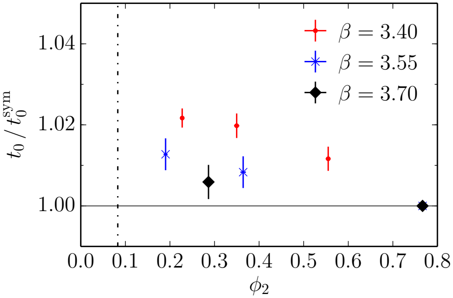
Relative discretization effects between the two scales $\sqrt{8t_0}\, f_{\pi\mathrm{K}}$.

At $m_\mathrm{u} = m_\mathrm{d} = m_\mathrm{s}$ with $\phi_4 = 1.15$

Scales by t_0 and $f_{\pi\mathrm{K}}$ differ at $a \approx 0.085$ fm by $\approx 5\%$

High precision Z_A from chirally rotated SF DALLA BRIDA, KORZEC

Chiral corrections



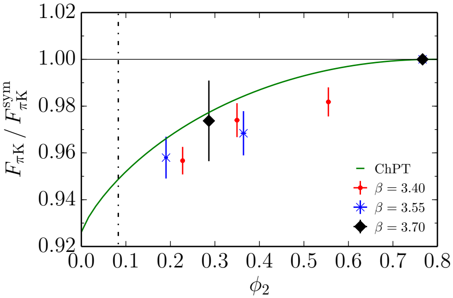
Chiral corrections are $\mathrm{O}(2\%)$ Bär, Golterman'14

$$t_0(\phi_2) = t_0(0)(1 + k_1 \frac{2m_K^2 + m_\pi^2}{4\pi f} + \dots)$$

On our trajectory $2m_K^2 + m_\pi^2 \approx$const
Spoiled by $O(am)$ effects + higher order corrections.

Chiral corrections

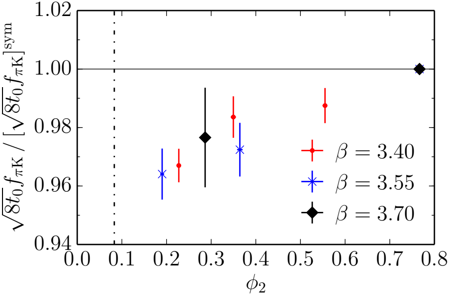
Decay constants



Chiral corrections to physical quark masses of $O(5\%)$

NLO SU(3) ChPT prediction: no free parameters
works within 20% of the chiral effect
... at $m_\pi \approx 420$ MeV stretching range of validity

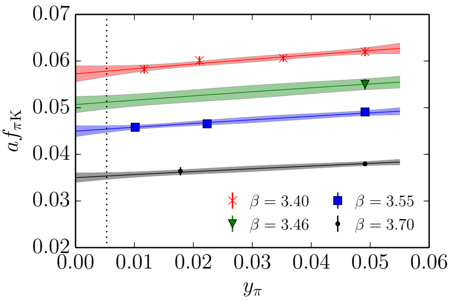
Chiral corrections



Chiral corrections are $\mathrm{O}(5\%)$

Depending on model assumptions they are under control on the 1%–2% level.

Lattice spacing



Measurements shifted to chiral trajectories which go through

$$y_\pi = \frac{m_\pi^2}{(4\pi f_{\pi K})^2} = y_\pi^{\text{phys}} \qquad \text{and} \qquad y_K = \frac{m_K^2}{(4\pi f_{\pi K})^2} = y_K^{\text{phys}}$$

Increased uncertainties with current data sets
→ lattice spacings at 2% level

Conclusions

CLS effort is paying off:
scale setting for running coupling possible

Mass corrections to get on defined line of constant physics are important

Possible without new simulations by measuring the derivatives

4 MeV shift in all three quark masses leads to doubling of the stat error in m_π.

$$\sqrt{8t_0} = 0.4122(78)\,\mathrm{fm}$$

Goal of 1% accuracy not yet reached

Still room for improvement