$B - \overline{B}$ mixing with domain-wall light quarks and relativistic b-quarks

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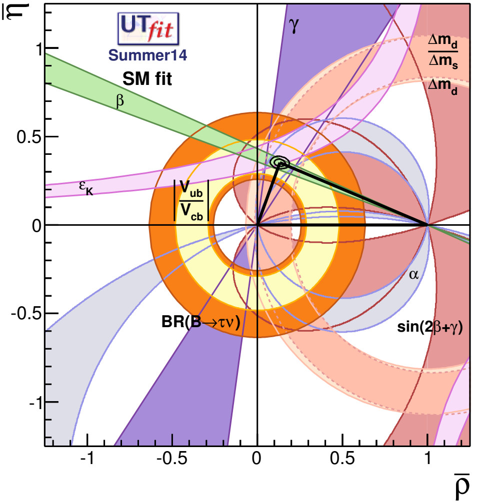
Motivation: CKM unitarity triangle fit

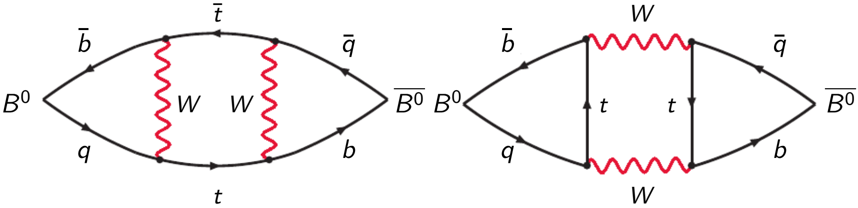


constraint from $B - \overline{B}$ mixing

dominant uncertainty from lattice QCD

http://utfit.roma1.infn.it, http://ckmfitter.in2p3.fr, http://www.latticeaverages.org

Motivation: B^0–$\overline{B^0}$ mixing



- Dominant contribution in SM: box diagram with top quarks
- Allows us to determine the CKM matrix elements

Motivation: B^0–$\overline{B^0}$ mixing

- Conventionally parametrized by

$$\left.\begin{array}{r}|V_{td}^* V_{tb}|\,\mathrm{for}\,B_d-\mathrm{mixing}\\ |V_{ts}^* V_{tb}|\,\mathrm{for}\,B_s-\mathrm{mixing}\end{array}\right\}\Delta M_q = \frac{G_F^2 m_W^2}{6\pi^2}\eta_B S_0 M_{B_q} f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2$$

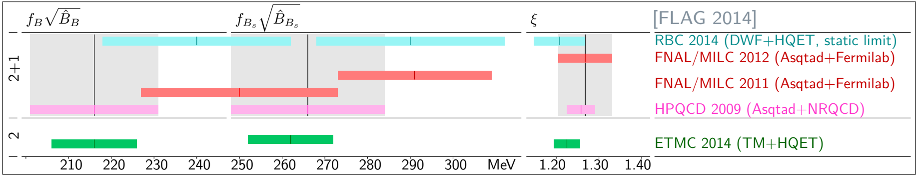
- Nonperturbative contribution: $f_q^2 B_{Bq}$
- Define the $SU(3)$ breaking ratio $\xi^2 = f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$

- CKM matrix elements are extracted by

$$\frac{\Delta M_s}{\Delta M_d} = \frac{M_{B_s}}{M_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$

- Experimental error of ΔM_q is better than a percent
lattice uncertainty for ξ is about 3%

Published results



- Fermilab/MILC (2+1 flavor), HPQCD (2+1+1 flavor), and RBC (2+1 flavor, static) are working on updates

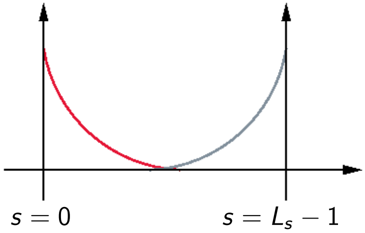
Our Project

- Use domain-wall light quarks and nonperturbatively tuned relativistic b-quarks to compute at few-percent precision
  - B^0–$\overline{B^0}$ mixing (T. Kawanai, O.W.)
  - Decay constants f_B and f_{B_s} (R. Van de Water, O.W.) [PRD 91 (2015) 054502]
  - $B \to \pi \ell \nu$ and $B_s \to K \ell \nu$ form factors (T. Kawanai) [PRD 91 (2015) 074510]
  - $g_{B^* B \pi}$ coupling constant (B. Samways) [arXiv:1506.06413]
  - Rare B decays (E. Lizarazo)

- f_B, f_{B_s}, and semi-leptonic form factors
  - $O(a)$ improvement at 1-loop and mostly nonperturbative renormalization
  - Correction factors and coefficients computed at 1-loop (C. Lehner)
- B mixing
  - Tree-level $O(a)$ improvement
  - Perturbative or mostly nonperturbative renormalization

2+1 Flavor Domain-Wall Gauge Field Configurations

- Domain-wall fermions for the light quarks (u, d, s) with $M_\pi > M_\pi^{\text{phys}}$
  [Kaplan PLB 288 (1992) 342], [Shamir NPB 406 (1993) 90]
- Möbius DWF for new ensembles with $M_\pi \approx M_\pi^{\text{phys}}$
  [Brower et al. 2004],[Brower et al. 2012]
- Iwasaki gauge action [Iwasaki UTHEP (1983) 118]
- Configurations generated by RBC and UKQCD collaborations



2+1 Flavor Domain-Wall Iwasaki ensembles

L	a^{-1}(GeV)	am_l	am_s	M_π(MeV)	# configs.	#sources	
24	1.785	0.005	0.040	338	1636	1	[PRD 78 (2008) 114509]
24	1.785	0.010	0.040	434	1419	1	[PRD 78 (2008) 114509]
32	2.383	0.004	0.030	301	628	2	[PRD 83 (2011) 074508]
32	2.383	0.006	0.030	362	889	2	[PRD 83 (2011) 074508]
32	2.383	0.008	0.030	411	544	2	[PRD 83 (2011) 074508]
48	1.730	0.00078	0.0362	139	40	81/1*	[arXiv:1411.7017]
64	2.359	0.000678	0.02661	139	—	—	[arXiv:1411.7017]
48	∼2.8			∼230	??		[in production]

* All mode averaging: 81 "sloppy" and 1 "exact" solve [Blum et al. 2012]

- Lattice spacing determined from combined analysis [arXiv:1411.7017]
- a: ~ 0.11 fm, ~ 0.08 fm, ~ 0.07 fm

Relativistic Heavy Quark Action for the *b*-Quarks

- Relativistic Heavy Quark action developed by Christ, Li, and Lin
  [Christ e t al. PRD 76 (2007) 074505], [Lin and Christ PRD 76 (2007) 074506]

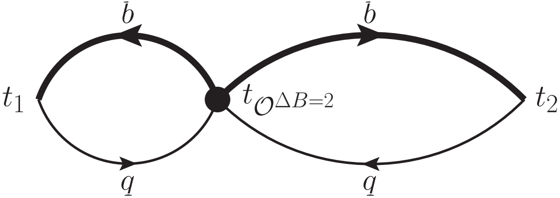
- Allows to tune the three parameters ($m_0 a$, c_P, ζ) nonperturbatively
  [PRD 86 (2012) 116003]

- Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933]
  by tuning all parameters of the clover action non-perturbatively;
  close relation to the Tsukuba formulation [S. Aoki et al. PTP 109 (2003) 383]

- Heavy quark mass is treated to all orders in $(m_b a)^n$
- Expand in powers of the spatial momentum through $O(\vec{p}a)$
  - Resulting errors will be of $O(\vec{p}^2 a^2)$
  - Allows computation of heavy-light quantities with discretization errors of the same size as in light-light quantities

- Applies for all values of the quark mass
- Has a smooth continuum limit

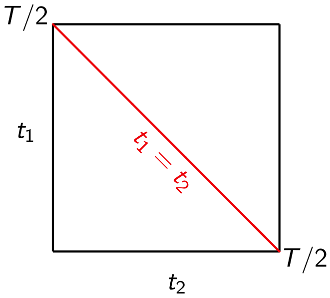
$B^0 - \overline{B^0}$ mixing matrix element calculation



- Fix location of four-quark operator $t_{\mathcal{O}}$
- Vary location of B-mesons over time slices t_1, $t_2 \leq T/2$
- Need: one point-source light quark and one point-source heavy quark originating from operator location
- Project out zero-momentum component using a Gaussian sink
- Tree-level $O(a)$-improvement of operators via HQ field rotation
- We measured all five operators, but focus on $\mathcal{O}_1$ (Standard Model)

Data structure of 3-point functions

- Varying $1 \leq t_1, t_2 \leq T/2$ results in a $T/2 \times T/2$ matrix of measurements
  - $t_1 = t_2$ sits on the diagonal
  - $t_1 - t_2 = k$ defines the k^{th} super-/sub-diagonal

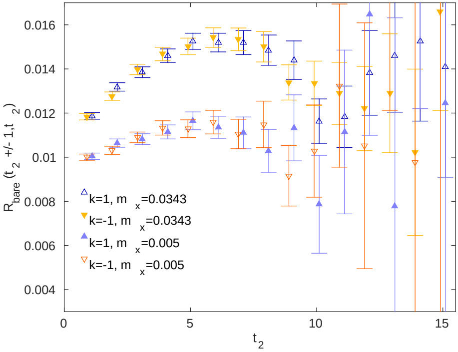


- For symmetry reasons super- and diagonals should be equal allowing us to symmetrize the matrix

Checking symmetry of our 3-point data

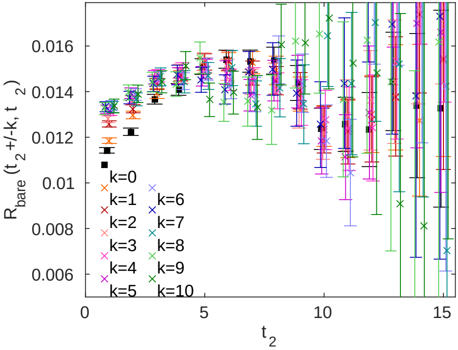
- Compute ratios related to the bag parameter $\hat{B}_{B_q}$

$$R_{\text{bare}}(t_1, t_2) = \frac{C_3^{\text{sm-pt-sm}}(t_{\mathcal{O}}, t_1, t_2)}{\sqrt{C_2^{\text{sm-sm}}(-t_1)C_2^{\text{sm-sm}}(t_2)}} \cdot \frac{2M_{B_q}}{\sqrt{\exp(-M_{B_q}(-t_1+t_2))}}$$



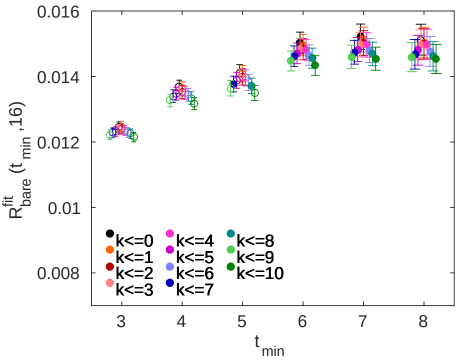
- Data shown on $24^3 \times 64$, $m_l = 0.005$
- $m_x = m_l$ and close to the phys. strange quark

Overlaying ratios for different (off-)diagonals k



- Data points shown with small offsets, $m_x = 0.0343$
- Noise increases for large k
- Combining different k may help to smoothen the plateau / improve our signal

Comparing combined fits with different k_{max}



- Data points shown with small offsets, $m_x = 0.0343$
- Simultaneous, correlated fits varying t_{min} for fixed $t_{\text{max}} = 16$
- Open symbols indicate a p-value $< 5\%$
- Warning: adding more noisier off-diagonals fools the χ^2/dof (p-value)

Extracting R_{bare} and ratios for different ensembles

Sorry, last night a poor in internet connection
did not allow me to download the needed data.

Conclusion

- Finally, $B - \overline{B}$ mixing code is verified and ready for production
- Basic data analysis is sorted out
- Extracted mixing matrix elements look sensible
- Combined fits to different separations improve our estimates

Outlook

- Obtain renormalized bag parameters and corresponding ratios
- Compute mixing matrix elements for other ensembles
- Perform combined chiral-continuum extrapolation with 2(3) lattice spacings and in total 6(7) different light sea quark masses including physical pions
- *After* properly re-tuning the RHQ parameters

Need for re-tune the RHQ parameters

- Originally we tuned the RHQ parameters using [Y. Aoki et al. (2008)]

$$a_{24}^{-1} = 1.729(25)\text{ GeV and } a_{24} m_s^{24} = 0.0348(11)$$

- The tuning is performed in the B_s system with close-to-physical strange quark propagators $am_{s'} = 0.0343$ [PRD 86 (2012) 116003]
- To include the new 48^3 ensemble (physical pions) we need up-to-date values:

$$a_{24}^{-1} = 1.7848(50)\text{ GeV} \neq a_{48}^{-1} = 1.7295(38)\text{ GeV (both at } \beta = 2.13)$$

  - New analysis includes DSDR and MDWF ensembles as well as refined analysis strategy [arXiv:1411.7017]

- This requires to account for the change in the bare strange quark mass

$$a_{24} m_s = 0.03224(18)$$