PDF from Hadronic Tensor on the Lattice

- Lattice Formulation of Hadronic Tensor in DIS
- Parton Degrees of Freedom and OPE
- Connected Sea Partons and Gottfried Sum Rule Violation
- Numerical Challenges and source method

Lattice 2015, Kobe

July 15, 2015

Hadronic Tensor in Euclidean Path-Integral Formalism

- Deep inelastic scattering

  In Minkowski space

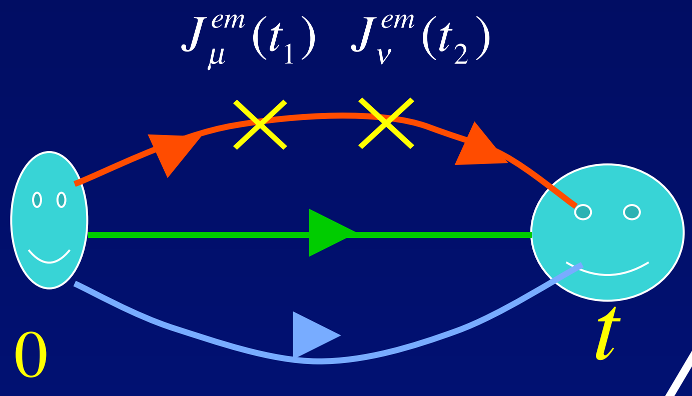
$$\frac{d^2\sigma}{dE' d\Omega} = \frac{\alpha^2}{q^4}\left(\frac{E'}{E}\right) l^{\mu\nu} W_{\mu\nu}$$

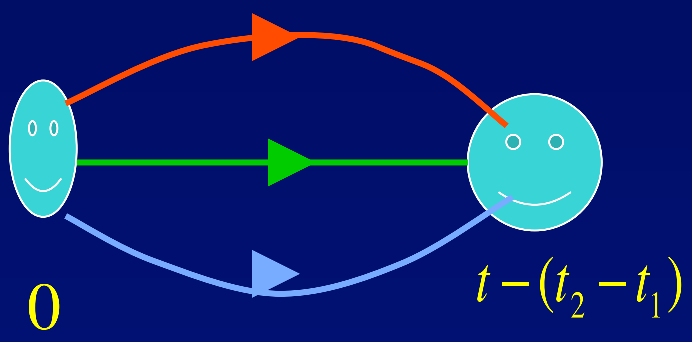
$$T_{\mu\nu}(q^2,\nu) = \frac{1}{2M_N}\int \frac{d^4x}{(2\pi)^4}\, e^{iq\cdot x} < P\,|\,T[J_\mu^{em}(x) J_\nu^{em}(0)\,|\,P>,$$

$$W_{\mu\nu}(q^2,\nu) = \frac{1}{\pi}\mathrm{Im}\,T_{\mu\nu} = \frac{(2\pi)^3}{2M_N}\sum_n \delta^4(p_n - p + q) < P\,|\,J_\mu^{em}\,|\,n >< n\,|\,J_\nu^{em}\,|\,P>$$

- Euclidean path-integral

K.F. Liu, PRD 62, 074501 (2000)





- Euclidean $W_{\mu\nu}$

$$\frac{\frac{2E_P V}{2M_N} < O_N(\vec{p},t) \int \frac{d^3x}{2\pi} \; e^{-i\vec{q}\cdot\vec{x}} J_\mu^{em}(\vec{x},t_2) J_\nu^{em}(\vec{0},t_1) O_N^+(\vec{p},0) >}{< O_N(\vec{p},t-(t_1-t_2)) O_N(\vec{p},0) >}$$

$$\xrightarrow[t-t_2 >> 1/\Delta E_P,\; t_1 >> 1/\Delta E_P]{}$$

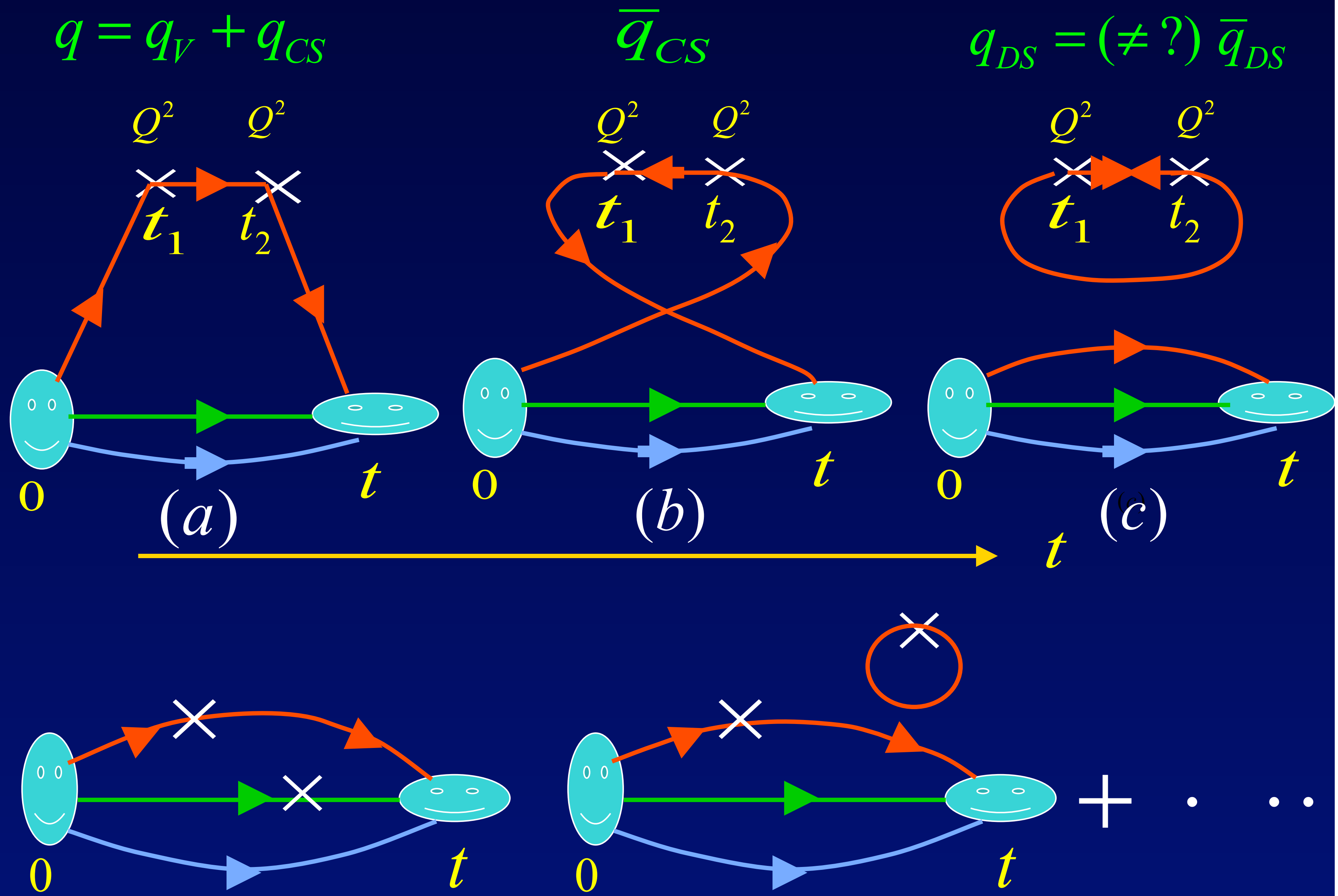
$$\tilde{W}_{\mu\nu}(q^2, \tau = t_2 - t_1) = \frac{1}{2M_N} \sum_n (2\pi)^2 \delta^3(\vec{p}_n - \vec{p} + \vec{q}) < P | J_\mu^{em} | n >$$

$$\times < n | J_\nu^{em} | N > \; e^{-(E_n - E_P)\tau};$$

- Minkowski $W_{\mu\nu}$ from Laplace transform

$$W_{\mu\nu}(q^2,\nu) = \frac{1}{i} \int_{c-i\infty}^{c+i\infty} d\tau \; e^{\nu\tau} \; \tilde{W}_{\mu\nu}(q^2,\tau)$$

$$= \lim_{\varepsilon\to 0} \frac{1}{\pi} \mathrm{Im} \int_{-\infty}^{\infty} d\tau \; \mathrm{e}^{\nu\tau + i\varepsilon} \; \tilde{W}_{\mu\nu}(q^2,\tau)$$



Cat's ears diagrams are suppressed by O(1/Q²).

- $$W_{\mu\nu}(p,q) = -W_1(q^2,\nu)(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}) + W_2(q^2,\nu)(p_\mu - \frac{p\cdot q}{q^2}q_\mu)(p_\nu - \frac{p\cdot q}{q^2}q_\nu)$$

- Large momentum frame

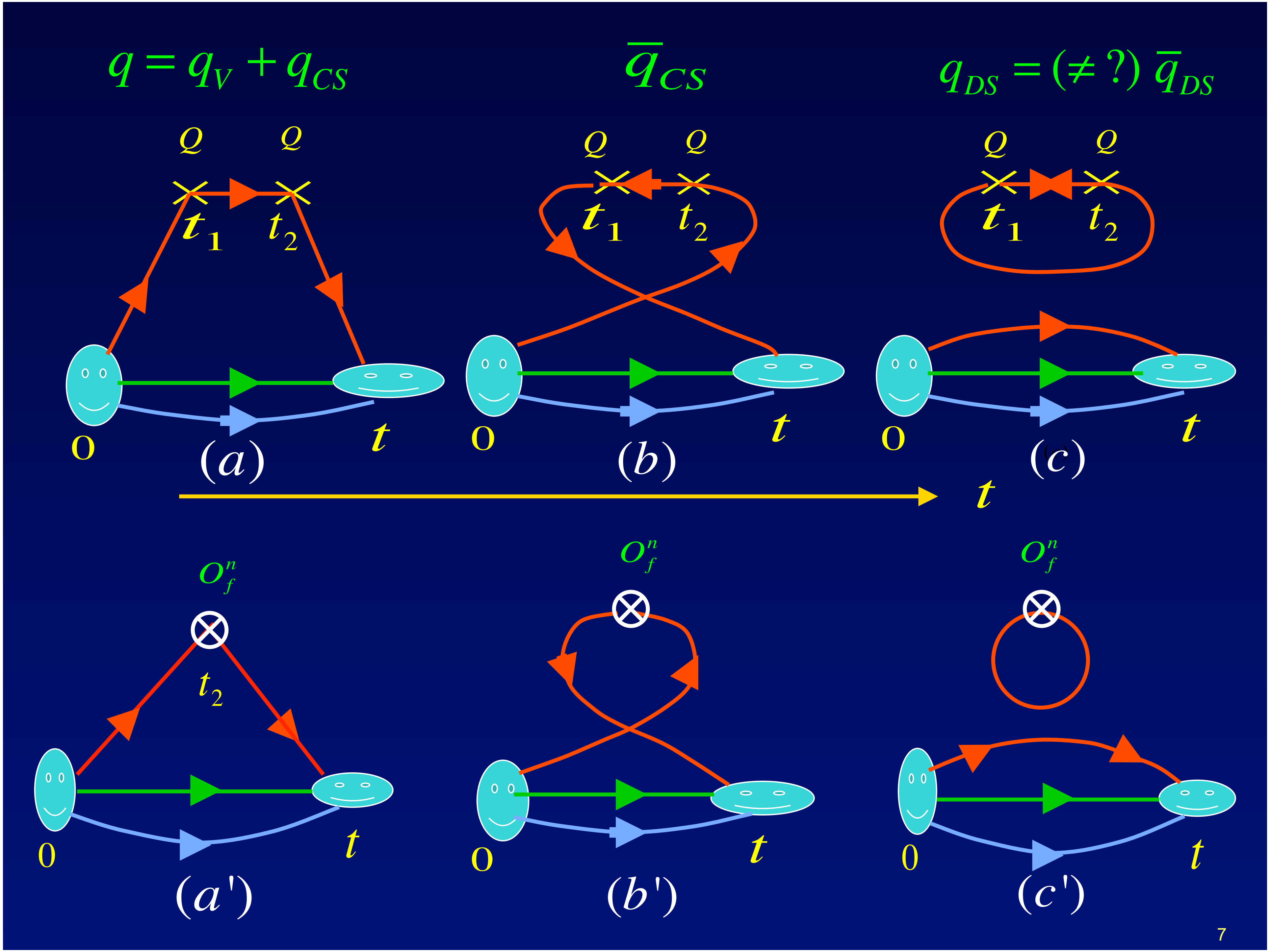
$$\nu W_2(q^2,\nu) \xrightarrow[|\vec{p}|>>|\vec{q}|]{} F_2(x,Q^2) = x\sum_i e_i^2 (q_i(x,Q^2) + \bar{q}_i\left(x,Q^2\right)); \quad x = \frac{Q^2}{2p\cdot q}$$

- Parton degrees of freedom: valence, connected sea and disconnected sea

u	d	s
$u_V(x) + u_{CS}(x)$	$d_V(x) + d_{CS}(x)$	
$\bar{u}_{CS}(x)$	$\bar{d}_{CS}(x)$	
$u_{DS}(x) + \bar{u}_{DS}(x)$	$d_{DS}(x) + \bar{d}_{DS}(x)$	$s_{DS}(x) + \bar{s}_{DS}(x)$

Properties of this separation

- Gauge invariant
- Topologically distinct as far as the quark lines are concerned
- Frame dependent
- $|\vec{p}| \gg M_N$ to have parton interpretation
- Parton model has a natural interpretation in the large momentum frame where the intermediate $q\bar{q}$ pair states induced by the currents are suppressed.

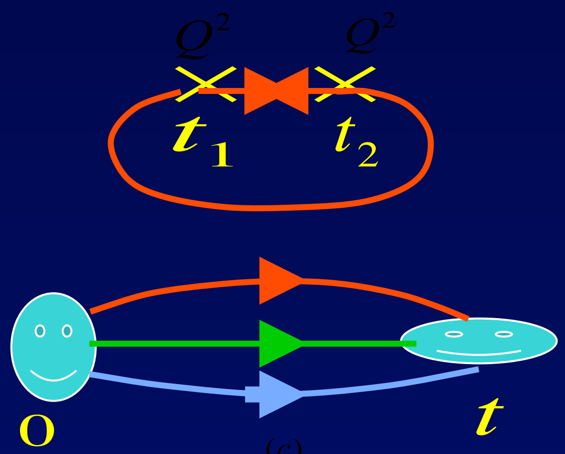


$q = q_V + q_{CS}$
$\bar{q}_{CS}$
$q_{DS} = (\neq ?)\ \bar{q}_{DS}$
Q Q
t_1 t_2
0
t
(a) (b) (c)
O_f^n
(a') (b') (c')

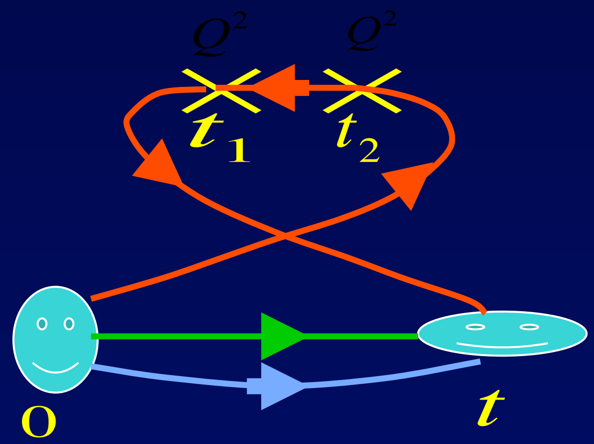
2) Gottfried Sum Rule Violation

$$S_G(0,1;Q^2) = \frac{1}{3} + \frac{2}{3}\int_0^1 dx\ (\bar{u}_P(x) - \bar{d}_P(x));\quad S_G(0,1;Q^2) = \frac{1}{3} \text{(Gottfried Sum Rule)}$$

NMC: $S_G(0,1;4\ \text{GeV}^2) = 0.240 \pm 0.016$ (5σ from GSR)



two flavor traces ($\bar{u}_{DS} = \bar{d}_{DS}$)



one flavor trace ($\bar{u}_{CS} \neq \bar{d}_{CS}$)

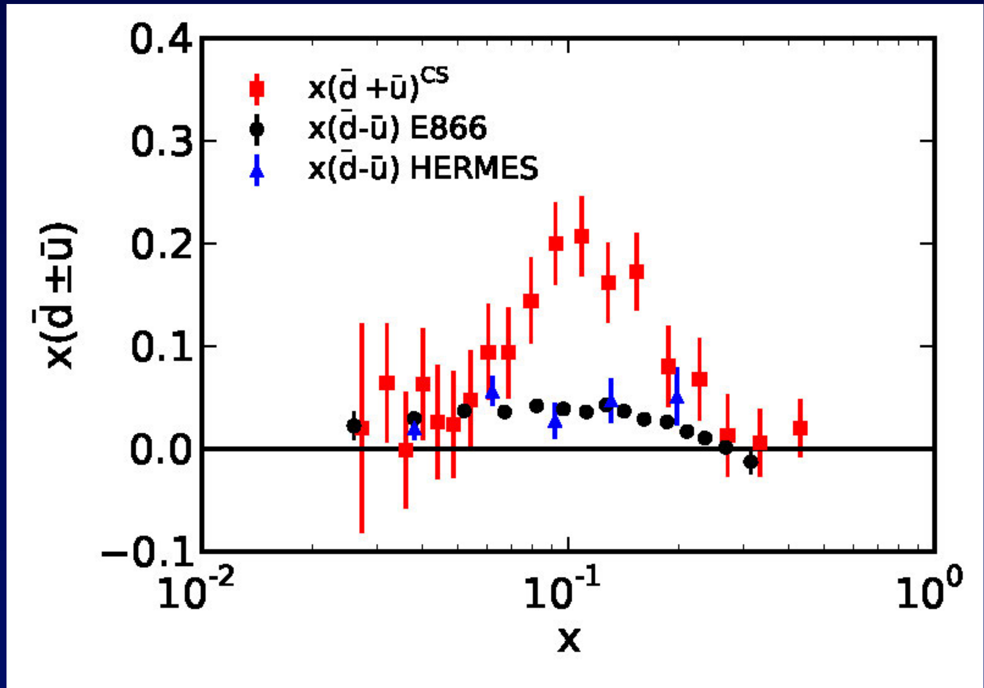
K.F. Liu and S.J. Dong, PRL 72, 1790 (1994)

$$Sum = \frac{1}{3} + \frac{2}{3}\int_0^1 dx\ (\bar{u}_{CS}(x) - \bar{d}_{CS}(x)),$$

$$= \frac{1}{3} + \frac{2}{3}\left[n_{\bar{u}_{CS}} - n_{\bar{d}_{CS}}\right]\ (1 + O(\alpha_s^2))$$

Connected Sea Partons

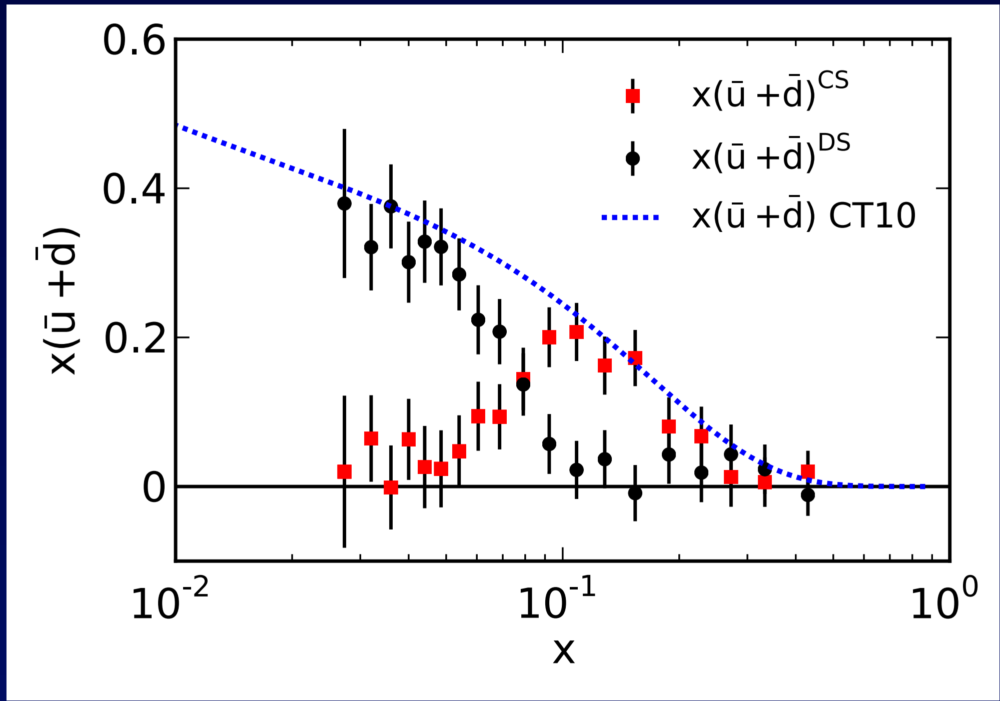
K.F. Liu, W.C. Chang, H.Y. Cheng, J.C. Peng, PRL 109, 252002 (2012)



$$x(\bar{d}+\bar{u})_{CS}(x) = x(\bar{d}+\bar{u})(x) - \frac{1}{R}x(s+\bar{s})(x);$$

CT10 lattice expt

$$R = \frac{\langle x \rangle_s}{\langle x \rangle_u (DI)} \text{(lattice)} \sim 0.857 \text{ (T. Doi, M. Sun)}$$



$$q_V\,,\ q_{CS}\ ,\ \bar{q}_{CS} \sim_{x\to 0}\ x^{-\alpha_R}\,(x^{-1/2})$$

$$q_{DS}\ ,\ \bar{q}_{DS} \sim_{x\to 0}\ x^{-1}$$

Numerical Challenges

- Lattice calculation of the hadronic tensor – no renormalization, continuum and chiral limits, direct comparison with expts ⟹ PDF from continuum factorization theorem
- Fourier transform $W_{\mu\nu}(q^2,\nu) = \lim_{\varepsilon \to 0} \frac{1}{\pi} \mathrm{Im} \int_{-\infty}^{\infty} d\tau \; e^{\nu\tau + i\varepsilon} \; \tilde{W}_{\mu\nu}(\vec{q},\tau)$
- Improved maximum entropy method

  (A. Rothkopf – 1110.6285)
- Fitting with multiple states: $\tilde{W}_{\mu\nu}(\vec{q},\tau) = \sum_n W_n e^{-(E_n - E_p)\tau}$
- Fictitious heavy quark to obtain moments from OPE

  (W. Detmold and D. Lin, 0507007)

Kinematics

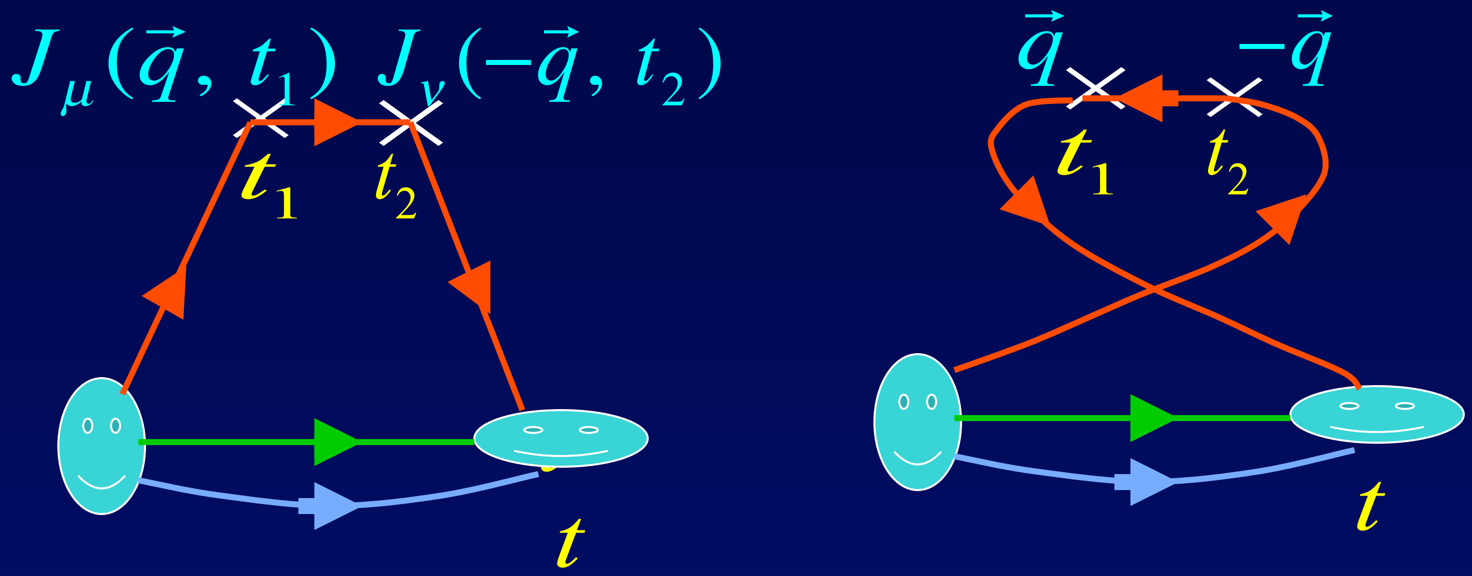
- Bjorken x $\quad x = \dfrac{Q^2}{2p\cdot q} = \dfrac{\vec{q}^{\,2} - \nu^2}{2(\nu E_p - \vec{p}\cdot\vec{q})}$
- Decay at large τ → ν - $(E_n - E_p) < 0$
- Range of x $\quad -\vec{q} \parallel \vec{p} \qquad Q^2 = 2\ \text{GeV}^2$

$$|\vec{p}| = 3\ \text{GeV}, |\vec{q}| = 2\ \text{GeV}, \nu = -1.4\ \text{GeV} \Rightarrow x = 0.64$$

$$|\vec{p}| = 3\ \text{GeV}, |\vec{q}| = 2\ \text{GeV}, \nu = 1.4\ \text{GeV} \Rightarrow x = 0.096$$

Source method

- Reduce the 4-pt fn to 3-pt fn



- Sequential source for $J_\mu(\vec{q},\, t_1)$
- Quark propagator from t_1 to t

$$\lim_{\theta\to 0}\frac{d}{d\theta}\left(\frac{1}{D+\theta x}\right)=\frac{1}{D}\,x\,\frac{1}{D} \qquad x=\sum_{t_2=t_1+1}^{t} J_\nu(-\vec{q},\, t_2)\, e^{\nu(t_2-t_1)}$$

- Exchange insertion → use source with definite momentum

Summary

- Hadronic tensor with the source method is numerically equivalent to 3-point function calculation.
- Improved maximum entropy to go to the Minkowski space.
- Large momentum frame, but NO renormalization!
- Other applications: NEDM with CP violating fermion bilinear terms, radiative correction of parity violating e p scattering.

Comments

- *The results are the same as derived from the conventional operator product expansion.*
- *The OPE turns out to be Taylor expansion of functions in the path-integral formalism.*
- *Contrary to conventional OPE, the path-integral formalism admits separation of CI and DI.*
- *For* O_f^n *with definite n, there is only one CI and one DI in the three-point function, i.e. (a') is the same as (b'). Thus, one cannot separate quark contribution from that of antiquark in matrix elements.*

X. Ji, PRL, 110, 262002 (2013)

§ Take the large-P_z limit:

$$q(x,\mu^2,P^z) \ = \int \frac{dz}{4\pi} e^{izk^z} \langle P|\overline{\psi}(z)\gamma^z \exp\left(-ig\int_0^z dz' A^z(z')\right)\psi(0)|P\rangle$$
$$+\mathcal{O}\left(\Lambda^2/(P^z)^2, M^2/(P^z)^2\right)$$

$x = k^z/P^z$

Lattice *z* coordinate

Nucleon momentum $P^\mu = \{P^0, 0, 0, P^z\}$

Product of lattice gauge links

- At $P^z \longrightarrow \infty$ limit, twist-2 parton distribution is recovered
- For finite P^z, corrections are needed

Xiangdong Ji, this Thursday; HWL et al in progress

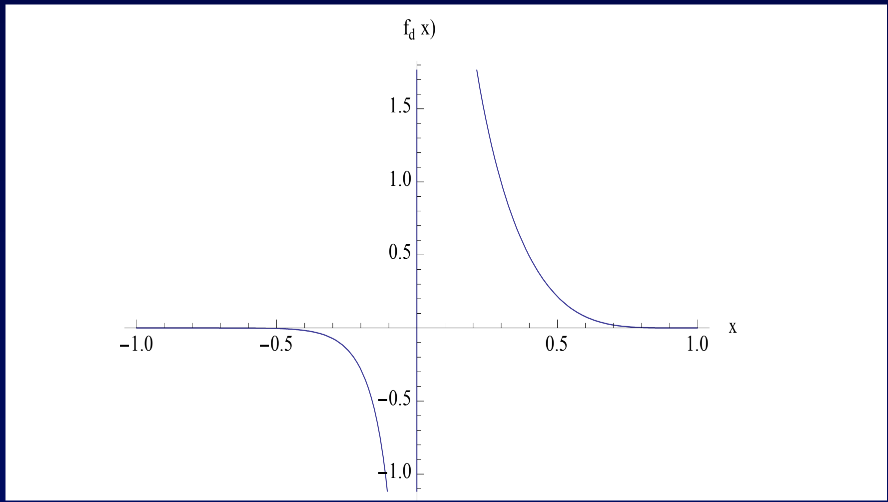
Theoretical Issues

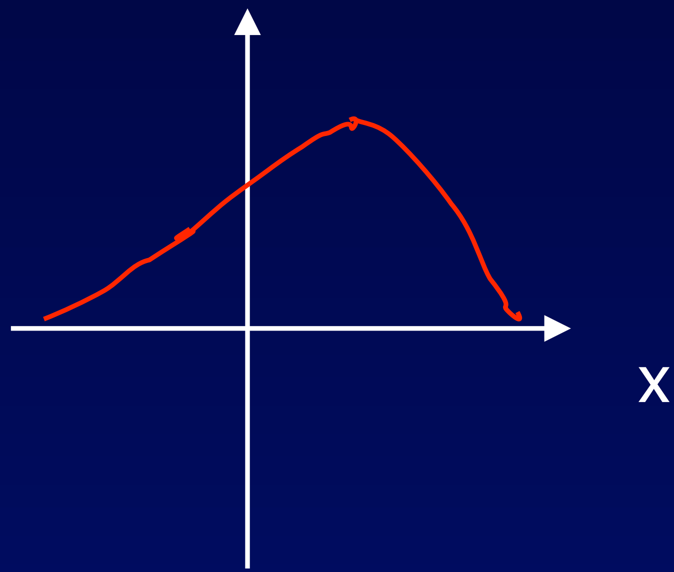
- Relatively simple numerically
- Renormalization of quasi-distribution and factorization

$$\tilde{q}(x,\mu^2,P_z) = \int_0^1 \frac{dy}{y} Z\left(\frac{x}{y},\frac{\mu}{P_z}\right) q(y,\mu^2) + O\left(\frac{\Lambda^2}{P_z^2},\frac{M^2}{P_z^2}\right)$$

  - Perturbative lattice renormalization ?
  - RI/MOM renormalization scheme for the CI ?

- When will $x_B = x_F$?





d and $\bar{d}$ from CTEQ6 (JW Chen)

$\bar{d}(|x|) = -d(-|x|)$

present P_z

- Renormalization for the disconnected insertion

Summary

- Hadronic tensor calculation is numerically tough, but theoretically interpretation is easy.
- Parton distribution function calculation at IMF is numerically easy, but theoretical interpretation is not straightforward.