

Nucleon TMDs: Clover and DWF results at $m_\pi \approx 300$ MeV

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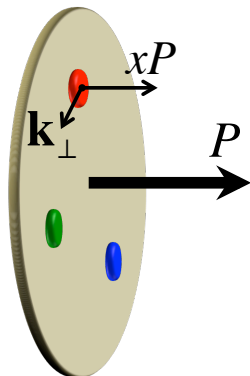
Los Alamos National Laboratory

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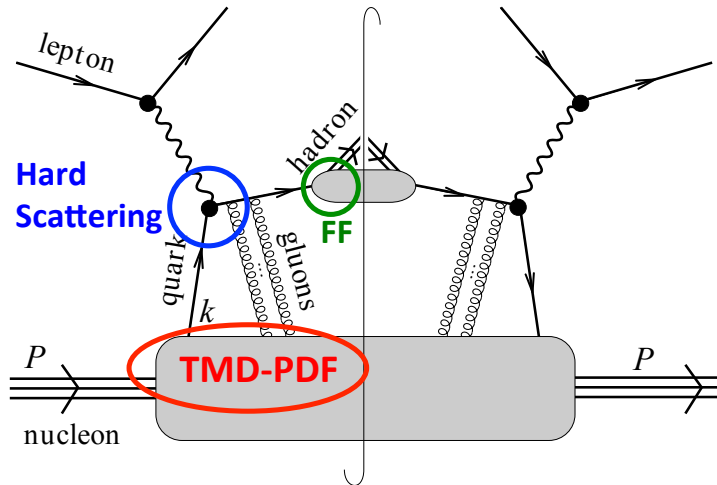
What are TMDs?

- Transverse Momentum Dependent Parton Distribution Functions
- Eg., Unpolarized function $f_1^q(x, k_\perp)$; $\int d^2\mathbf{k}_\perp f_1^q(x, k_\perp) = f_1^q(x)$
- Probability to find a quark carrying the longitudinal momentum fraction x and transverse momentum \mathbf{k}_\perp , in a fast moving nucleon

		Nucleon		
		Unpolarized	L-pol	T-pol
Quark	Unpolarized	f_1		f_{1T}^\perp Sivers
	L-pol		g_{1L} Helicity	g_{1T} Worm-gear
	T-pol	h_1^\perp Boer-Mulders	h_{1L}^\perp Worm-gear	h_1 Transversity h_{1T}^\perp Pretzelosity



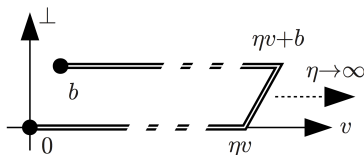
Semi-inclusive Deep Inelastic Scattering (SIDIS)



$$\text{Cross Section} = [\text{TMD-PDF}] \otimes [\text{Hard Scattering}] \otimes [\text{FF}]$$

Lattice Calculation of TMDs

$$\tilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots) = \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \eta v, \eta v + b, b] q(b) | P, S \rangle$$



- Staple link geometry for **SIDIS / DY processes**
- First take v slightly off the light-cone into **space-like** region
- Then approach the **light-cone** as $\hat{\zeta} \equiv \frac{v \cdot P}{|v||P|} \rightarrow \infty$
- **Perturbative evolution** governs the approach the light-cone

[Aybat, Collins, Qiu and Rogers, 2012]

TMDs from Matrix Elements

$$\tilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, P, S, \dots) = \frac{1}{2} \langle P, S | \bar{q}(0) \Gamma \mathcal{U}[0, \eta v, \eta v + b, b] q(b) | P, S \rangle$$

$$\Phi^{[\Gamma]}(x, k_T, \dots) = \int \frac{d^2 b_T}{(2\pi)^2} \int \frac{d(b \cdot P)}{(2\pi) P^+} e^{ix(b \cdot P) - ib_T \cdot k_T} \frac{\tilde{\Phi}_{\text{unsubtr.}}^{[\Gamma]}(b, \dots)}{\tilde{\mathcal{S}}(b^2; \dots)}$$

- $\tilde{\mathcal{S}}$: “Soft factor” needed to subtract divergence from \mathcal{U}
- $\Phi^{[\Gamma]}(x, k_T, \dots)$ can be decomposed by the TMDs as

$$\Phi^{[\gamma^+]}(x, k_T, S, \dots) = f_1(x, k_T) - \frac{\epsilon_{ij} k_i S_j}{m_N} f_{1T}^\perp(x, k_T)$$

- Parameterize in terms of Lorentz-invariant amplitudes:
Transform Euclidean results \implies TMDs in light-cone coordinate

Generalized Shifts

- **Generalized Shifts**

$$\text{Sivers : } \langle \mathbf{k}_y \rangle_{TU}(\mathbf{b}_T^2, \dots) = m_N \frac{\tilde{f}_{1T}^{\perp1}(\mathbf{b}_T^2, \dots)}{\tilde{f}_1^{[1](0)}(\mathbf{b}_T^2, \dots)}$$

$$\text{Boer-Mulders : } \langle \mathbf{k}_y \rangle_{UT}(\mathbf{b}_T^2; \dots) = m_N \frac{\tilde{h}_1^{\perp1}(\mathbf{b}_T^2; \dots)}{\tilde{f}_1^{[1](0)}(\mathbf{b}_T^2; \dots)}$$

$$- \tilde{f}^{[1]} = \int_{-1}^1 dx \tilde{f}(x); \quad \tilde{f}^{(1)}(x, b_T^2) = n! \left(-\frac{2}{m_N^2} \partial_{b_T^2} \right)^n \tilde{f}(x, b_T^2)$$

– **Soft factors and multiplicative renormalization factors cancel**

Lattice Simulation Setup

Fermion	DWF on AsqTad ¹	DWF ²	Clover
Lattices from	MILC	RBC/UKQCD	JLab/W&M
a (fm)	0.12	0.084	0.114
m_π (MeV)	518, 369	297	317
Statistics	3888, 5264	4264	23208

- Understanding systematics regarding
 - Finite lattice spacing
 - Pion mass
 - Chiral/Non-chiral fermion discretization
 - Non-local operator renormalization

¹Musch, *et al.*, PRD 2012

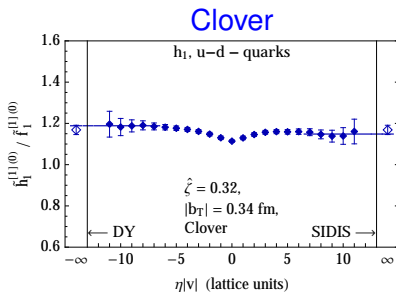
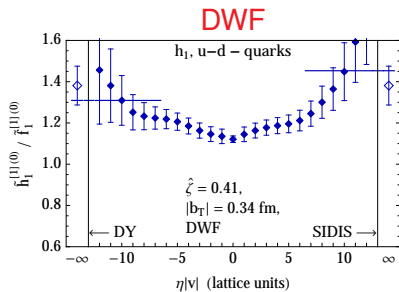
²Engelhardt, Lattice 2014

Results

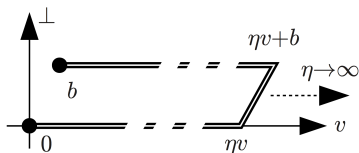
- **T-Even TMDs**
 - Transversity
 - Worm-gear shift g_{1T}
- **T-Odd TMDs**
 - Sivers shift
 - Boer-Mulders shift

		Nucleon		
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Quark	Unpolarized	f_1		f_{1T}^\perp Sivers
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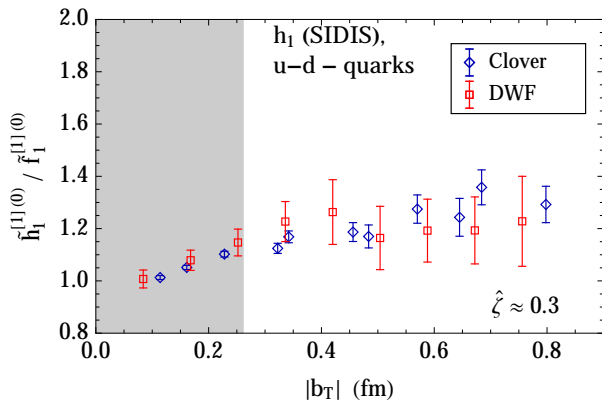
Transversity - Dependence on Staple Extent



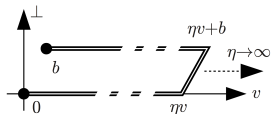
- Infinite staple length $\eta \rightarrow \pm\infty$
- $f^{\text{T-even, SIDIS}} = f^{\text{T-even, DY}}$



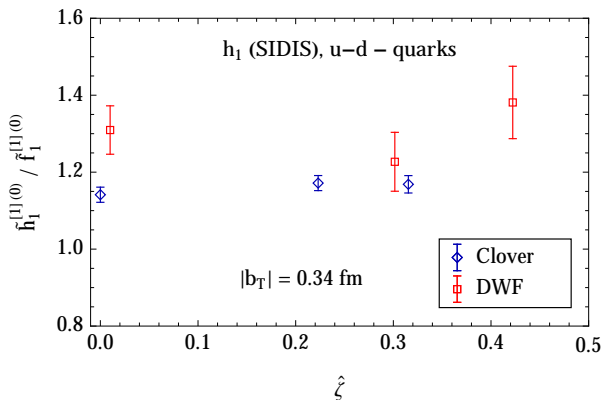
Transversity - Dependence on $|b_T|$



- b_T : Fourier conjugate of k_T

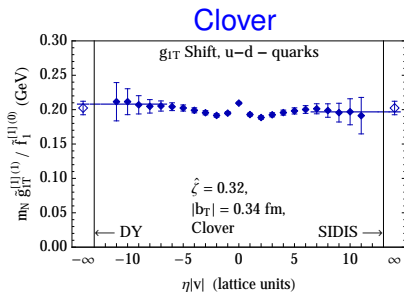
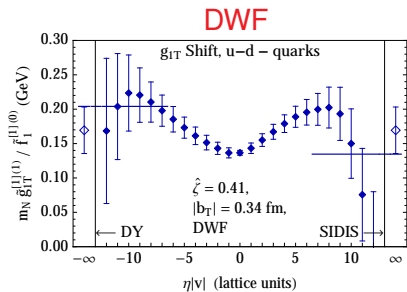


Transversity - Dependence on $\hat{\zeta}$

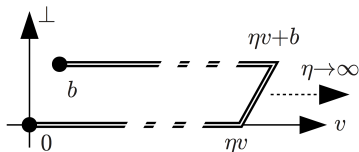


- Collins-Soper parameter $\hat{\zeta} \equiv \frac{v \cdot P}{|v||P|}$
- Light-cone limit : $\hat{\zeta} \rightarrow \infty$

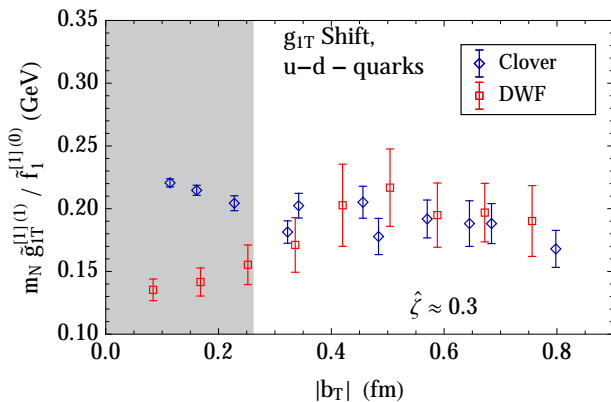
Worm-gear Shift - Dependence on Staple Extent



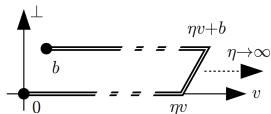
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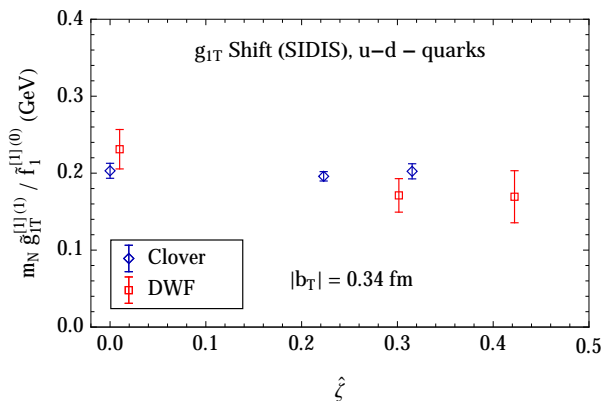
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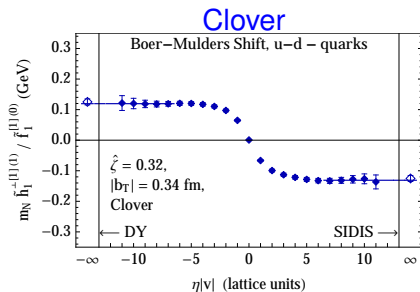
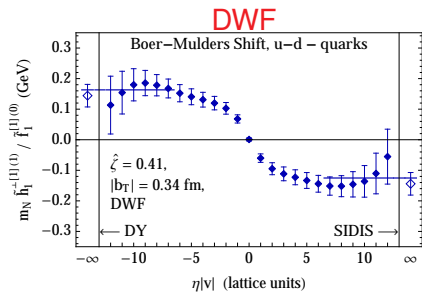


Worm-gear Shift - Dependence on $\hat{\zeta}$

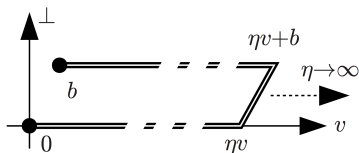


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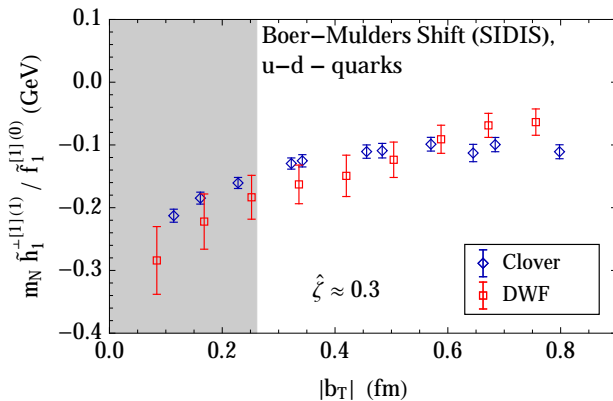
Boer-Mulders Shift - Dependence on Staple Extent



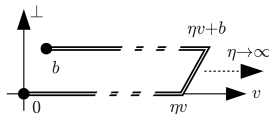
- Infinite staple length $\eta \rightarrow \pm\infty$
- $f^{\text{T-odd, SIDIS}} = -f^{\text{T-odd, DY}}$



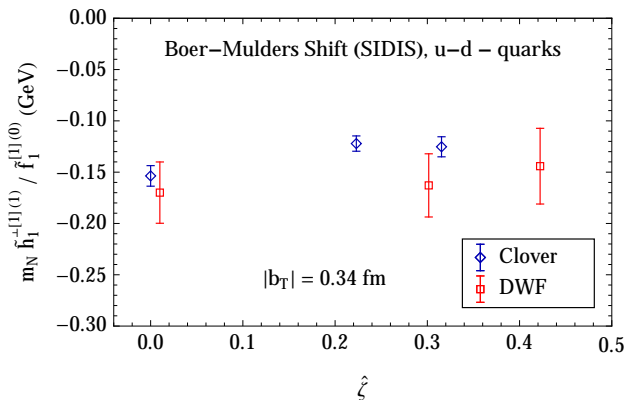
Boer-Mulders Shift - Dependence on $|b_T|$



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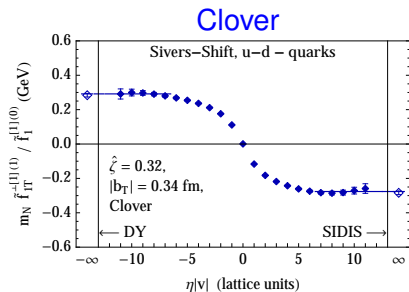
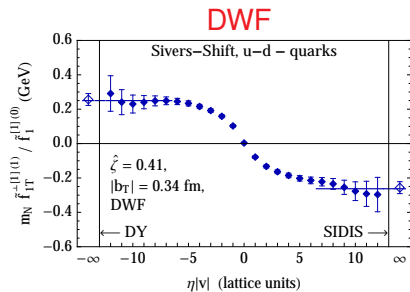


Boer-Mulders Shift - Dependence on $\hat{\zeta}$

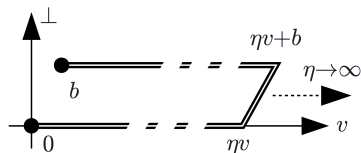


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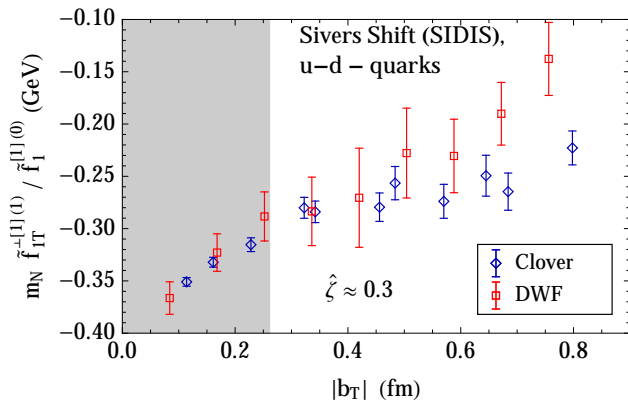
Sivers Shift - Dependence on Staple Extent



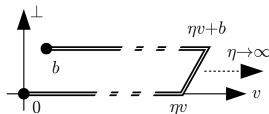
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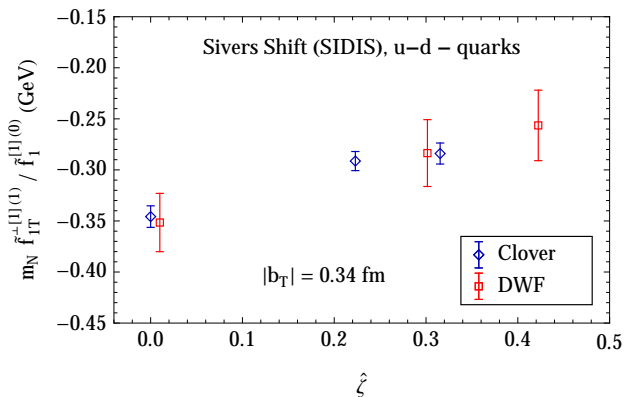
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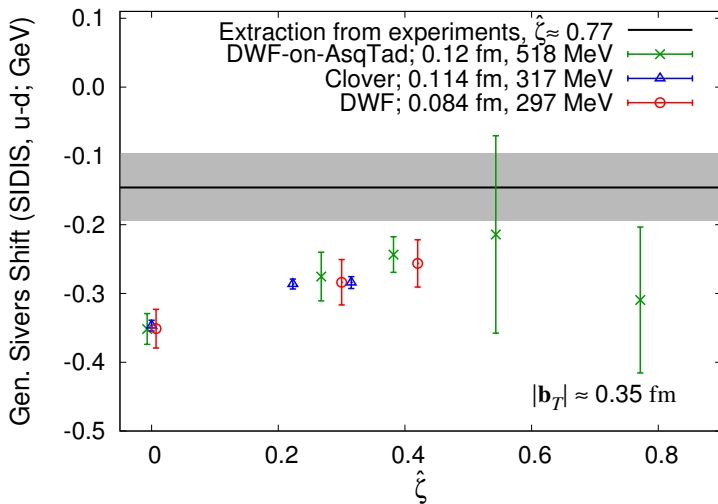


Sivers Shift - Dependence on $\hat{\zeta}$



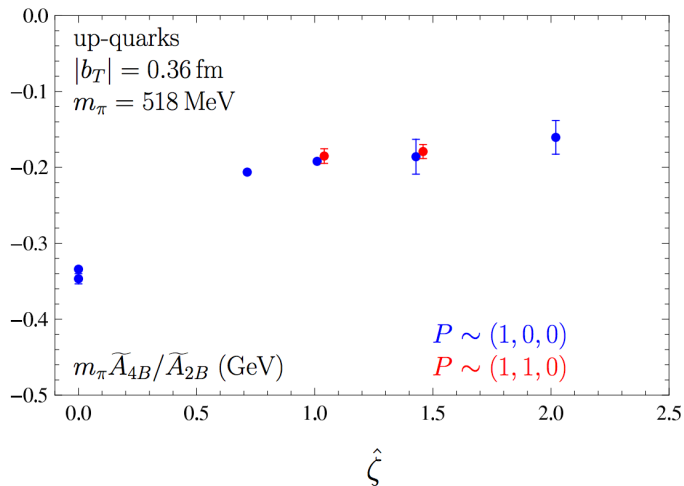
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Comparison with Experiments



- Extraction from exp., based on [Echevarria, *et al.*, 2014] (HERMES, COMPASS, and J-Lab experiments)

Pion Boer-Mulders Study for Large- $\hat{\zeta}$



[Engelhardt, *et al.*, 2013]

Summary

- **Exploration of TMDs** using lattice QCD:
focused on lattice discretization effects at $m_\pi \approx 300 \text{ MeV}$
- **No significant variation detected** between
DWF $a = 0.084 \text{ fm}$ and Clover $a = 0.114 \text{ fm}$ lattices
- Comparison with the extraction from experiments:
Lattice data points tending towards the experimental results
Need more study at higher $\hat{\zeta}$
- See parallel talk on Friday (Room 406, 15:40),
M. Engelhardt, TMDs at $m_\pi \approx 170 \text{ MeV}$