Gauge fixing and the gluon propagator in renormalizable ξ gauges

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   - R_ξ gauge fixing algorithm
3. Results and comparison with continuum studies
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   - Continuum studies, gluon mass
   - Conclusions and outlook

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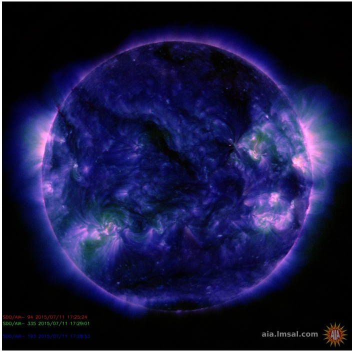
Introduction

Solar Dynamics Observatory



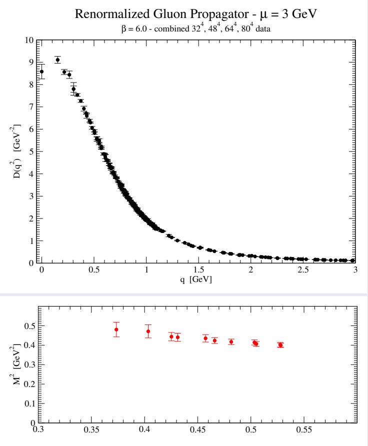
This sun corona image combines 3 different temperatures - NASA.

- Dynamical mass generation in QCD and hadrons accounts for 98% of the visible mass in the universe.
- The quark mass is due to chiral symmetry breaking;
- an effective gluon mass (as the photon mass in a superconductor) is a possible signal of confinement.
- If it exists, the gluon mass should appear in the gluon propagator.
- However Green's functions (propagators and vertices) depend on the gauge fixing.

Cornwall:1981zr, Aguilar:2004sw.

Introduction

Gluon propagator and mass



computed in the Landau gauge

The case of the Landau gauge:

- the transverse part of the gluon propagator saturates in the IR,
- this can be interpreted as evidence for a dynamically generated mass,
- the gluon mass function $m^2(p^2)$ is a monotonically decreasing function, power-law suppressed in the UV.

The question is, what happens in other gauges?
So far , we don't really know...

Oliveira:2010xc.

Introduction

Here we study in Lattice QCD the well known renormalizable-ξ covariant gauges (say, Landau gauge $\xi = 0$, Feynman gauge $\xi = 1$).

- Reliable lattice calculations in renormalizable-ξ (R_ξ) covariant gauges have not been systematically pursued previoulsy.
- R_ξ gauge fixing (GF) lattice implementation has in fact proven to be quite complicated due to a no go result, only recently bypassed.
- However, one still encounters significant convergence problems in realistic lattices, i e for larger GF parameter ξ and lattice size, and smaller number of colors N_c and lattice coupling β.
- We report on our success to GF in a realistic lattice and present the SU(3) gluon propagator in R_ξ gauges for a relatively large lattice volume $(3.25\ \text{fm})^4$ and a GF parameter up to $\xi = 0.5$.

Giusti:1996kf, Cucchieri:2010ku, Bicudo:2015rma.

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R_ξ gauges

In the continuum, R_ξ gauge fixing is achieved by adding to the SU(N_c) action,

$$S_{\rm GF} = \int \mathrm{d}^4 x \left[b^m \Lambda^m - \frac{\xi}{2} (b^m)^2 \right] ,$$

ξ is the GF parameter, $b^m$are Nakanishi-Lautrup multipliers and $\Lambda^m = \Lambda^m[A]$ is the GF condition. Going on-shell, $\xi b^m = \Lambda^m$, and the GF action is Gaussian,

$$S_{\rm GF} = \frac{1}{2\xi} \int \mathrm{d}^4 x \, (\Lambda^m)^2 .$$

R_ξ gauges are obtained when the linear condition is chosen

$$\Lambda^m = \partial^\mu A^m_\mu \, .$$

The gluon propagator is decomposed in transverse / longitudinal components

$$\Delta_{\mu\nu}(q) = (g_{\mu\nu} - q_\mu q_\nu / q^2) \Delta_{\rm T}(q^2) + (q_\mu q_\nu / q^2) \Delta_{\rm L}(q^2) \, .$$

Slavnov-Taylor identities ensure that $q^2 \Delta_{\rm L} = \xi$ to all orders.

R_ξ gauges

In the lattice,

- we use the Wilson action, the gauge links U_μ, are related to the gauge fields,

$$A_\mu(x+\hat{e}_\mu/2) = \left. \frac{U_\mu(x) - U^\dagger_\mu(x)}{2ig_0} \right|_{\text{traceless}} .$$

- In R_ξ gauges, besides the usual integration over the link variables $U_\mu(x)$, one has to integrate over the fields $\Lambda = \sum_m \Lambda^m t^m$
- where each Λ^m is a Gaussian distribution with variance $\frac{2N_c}{\beta}\,\xi$,
- The procedure for GF requires to gauge rotate all link variables,

$$U_\mu(x) \to g(x) U_\mu(x) g^\dagger(x+\hat{e}_\mu)$$

where g are elements of the SU(N_c) gauge group.

R_ξ gauges

- In the Landau gauge case, which is the $\xi \to 0$ limit of the R_ξ gauges studied here, the GF is implemented minimizing the functional,

$$\mathcal{E}_{\mathrm{LG}}[U,g] = -\mathrm{Re}\,\mathrm{Tr}\sum_{x,\mu} g(x)U_\mu(x)g^\dagger(x+\widehat{e}_\mu)\,,$$

  which directly leads to the condition $\nabla\cdot A^m = 0$.

- Contrary to this simple limit, the general case of a non-vanishing ξ was proven to have no suitable GF functional to minimize. The functional,

$$\mathcal{E}_{R_\xi}[U,g] = \mathcal{E}_{\mathrm{LG}}[U,g] + \mathrm{Re}\,\mathrm{Tr}\sum_{x} ig(x)\Lambda(x)\,,$$

  yields a solution close to the correct gauge condition $\nabla\cdot A^m = \Lambda^m$,

- but a no-go theorem prevents it from being exact.

Giusti:1996kf, Cucchieri:2010ku.

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gauge fixing algorithm

A way out from the no-go theorem was recently proposed.

- In practice, the gauge transformation, $g = \prod_j \delta g_j$, is built as a product of a sequence of infinitesimal gauge transformations.
- For each infinitesimal transformation, $\delta g_j = 1 + i \sum_m w^m \, t^m$, we minimize the R_ξ functional with respect to w^m;
- however when moving on to the next infinitesimal transformation δg_{j+1}, the Gaussian distribution Λ^m is maintained unchanged and only the link U_μ is updated through a gauge rotation.
- The variation of the R_ξ functional, a function of the coefficients w^m, is,

$$\mathcal{E}_{R_\xi}[U, \delta g_j] - \mathcal{E}_{R_\xi}[U, 1] = \mathrm{Tr} \sum_{x,\, m} w^m(x) \, t^m \, \Delta(x) \, ,$$

$$\Delta(x) = \sum_\mu g_0 \left[A_\mu(x + \hat{e}_\mu/2) - A_\mu(x - \hat{e}_\mu/2) \right] - \Lambda(x) \, .$$

when $\Delta(x) \to 0$, we reach the desired gauge condition $\nabla \cdot A^m = \Lambda^m$.

gauge fixing algorithm

- Thus, choosing $w^m = \alpha\Delta^m$, where α is a relaxation parameter to be optimized, should reduce Δ with a steepest descent method.
- Our goal is to converge to a vanishing Δ in all lattice points x, impliying,

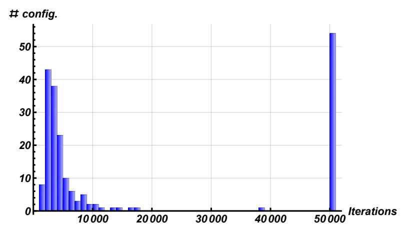
$$\theta = \frac{1}{N_c L^4} \sum_x \mathrm{Tr}\left[\Delta(x)\Delta^\dagger(x)\right] \to 0 \, . \tag{1}$$

- Based on the experience of Landau GF , we need to have $\theta < 10^{-15}$,
- but this turns out, in practice, to be very difficult in a realistic lattice !!!
- We opt to extend our fully parallel and very fast GPU codes for Landau GF, for a thorough optimization of all possible techniques to minimize θ.

Cardoso:2012pv.

gauge fixing algorithm

Minimization convergence for FFT steepest descent, $\xi = 0.3$, $L = 32$



Histogram, with 200 uncorrelated configurations of the number of iterations (stop at 5000) necessary to minimize / converge the R_ξ GF .

We combine three different GF techniques optimized in the Landau case,

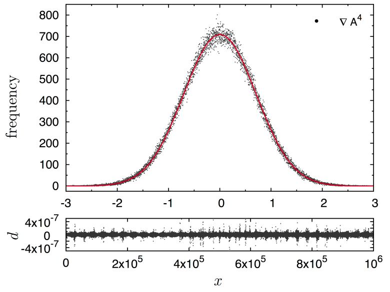
- the Fast Fourier Transform - accelerated steepest descent (FFT),
- Over Relaxation (OVR),
- Stochastic Relaxation (STR).
- Their GF success rate is $\sim$ 75% for $\xi = 0.3$ and $\sim$ 40% for $\xi = 0.5$.

We finally find a solution,

- cycling through FFT, OVR and STR, for our hardest case of $\xi = 0.5$, we increase the convergence success rate up to $\sim$90% ;

gauge fixing algorithm

Check of the GF convergence



- finally, for the remaining 10% cases, we perform a random gauge transformation, and restart the combined algorithm, till convergence is total.
- (*top*) The 32^4 values of $\nabla\cdot A^m$ evaluated for a configuration gauge fixed at $\xi = 0.5$, grouped in 5000 bins, compared with the Gaussian Λ^m with standard deviation $\sqrt{\xi} \simeq 0.316$.
- (*Bottom*) Plot of $d = \nabla\cdot A^4 - \Lambda^4$; the two distributions coincide within $\sqrt{\theta}$ precision.

- Clearly, the GF is achieved is within the desired precision.

Cardoso:2012pv, Bicudo:2015rma, Bicudo et al in preparation,

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Results for the gluon propagator

We now compute the gluon propagator for $\beta = 6.0$ in a 32^4 lattice.

- The definition of linear covariant gauges on the lattice requires an additional integration over the $N_c^2 - 1$ Gaussian distributed Λ^m fields. For the Λ integration, we consider 50 different Λ's for each configuration U_μ.
- Then, for each configuration of $U_\mu(x)$ and $\Lambda(x)$ field, GF is applied.
- The lattice gluon propagator, a two-point correlation function, reads

$$\langle A^m_\mu(\widehat{q}\,)A^n_\nu(\widehat{q}\,')\rangle = \delta^{mn}\Delta_{\mu\nu}(q)L^4\delta(\widehat{q}+\widehat{q}\,')\ , \qquad (2)$$

where we Fourier transform from the positions x to the momenta q.

- The transverse and longitudinal SU(3) propagator form factors are,

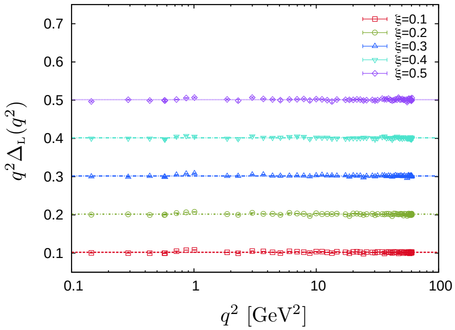
$$\Delta_{\rm T}(q^2) = \frac{1}{24L^4}\sum_{\mu,\nu,m}(\delta_{\mu\nu} - q_\mu q_\nu/q^2)\langle A^m_\mu(\widehat{q}\,)A^m_\nu(-\widehat{q}\,)\rangle,$$

$$\Delta_{\rm L}(q^2) = \frac{1}{8L^4}\sum_{\mu,\nu,m} q_\mu q_\nu/q^2\langle A^m_\mu(\widehat{q}\,)A^m_\nu(-\widehat{q}\,)\rangle. \qquad (3)$$

Leinweber:1998im, Oliveira:2012eh.

Results for the gluon propagator

Gluon longitudinal R_ξ dressing function $q^2\Delta_L$

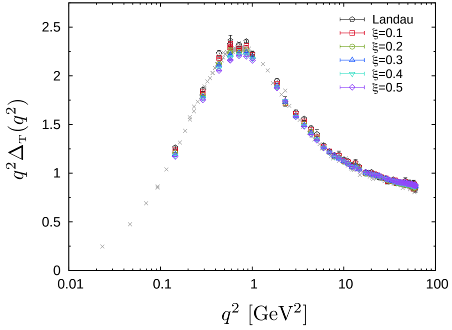


- Analytically, since the R_ξ longitudinal propagator Δ_L remains equal to the tree level one. we should have $q^2\Delta_L \equiv \xi$
- The values of ξ chosen are ξ = 0.1, 0.2, 0.3, 0.4, and 0.5.
- Indeed a fit of the data to a constant yield $\xi =$ 0.103(2), 0.203(2), 0.302(3), 0.402(3) and 0.502(3) respectively.

Oliveira:2012eh.

Results for the gluon propagator

Gluon transverse R_ξ dressing function $q^2\Delta_T$

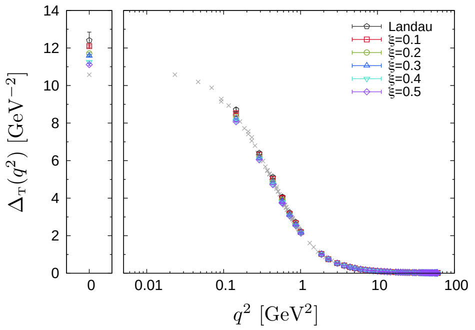


- The R_ξ gluon transverse dressing function $q^2\Delta_T$ is fully dynamical and non-perturbative.
- The simulated volume is $(3.25\text{ fm})^4$, large enough to resolve the onset of the nonperturbative effects.
- for comparison, the Landau gauge results obtained for a symmetric lattice of $L = 80$ and $\beta = 6.0$ (gray crosses) are also plotted.

Cardoso:2011xu, Bali:1992ru.

Results for the gluon propagator

Gluon transverse propagator

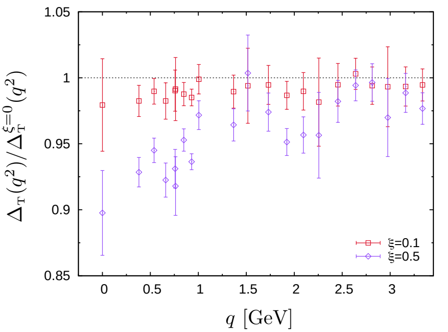


- We show he R_ξ transverse propagator Δ_T (renormalized at $\mu = 4.317$ [GeV]).
- The gray crosses are computed with the Landau gauge for a volume of 80^4 and provide an estimate for the volume effects expected at $q^2 = 0$.

Oliveira:2012eh.

Results for the gluon propagator

Ratio to Landau gauge propagator



- We plot the ratio of the transverse propagator to the Landau gauge propagator $\Delta_T^{\xi=0}$ as a function of the momentum for the two values $\xi = 0.1$ and $\xi = 0.5$.
- The data confirms an IR hierarchy such that Δ_T (slightly) decreases for increasing values of the gauge fixing parameter.
- The maximum difference is of $\sim 10\%$ for $\xi = 0.5$.

Results for the gluon propagator

- As in the Landau gauge, we find the R_ξ transverse propagators show an inflection point, implying that the associated spectral density is not positive definite; it is interpreted as a manifestation of confinement.
- They have a marked tendency to flatten towards the small momentum region, thus providing strong evidence that also in the $\xi \neq 0$ case the behavior of the lower modes of the lattice gluon field are tamed by the dynamical generation of a (momentum-dependent) gluon mass.
- The Landau gauge data on very large volumes suggest that simulations on larger physical volumes suppress the gluon propagator in the infrared region. This would only lead to at most to a decrease of about $\sim 10\%$ of Δ_T at small momenta for the R_ξ transverse propagator.

Oliveira:2012eh.

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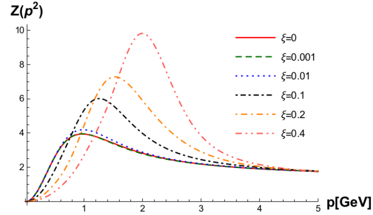
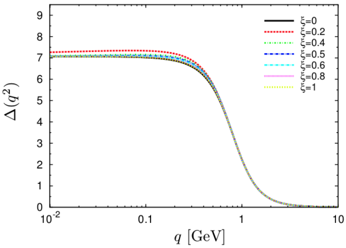
Continuum studies, gluon mass





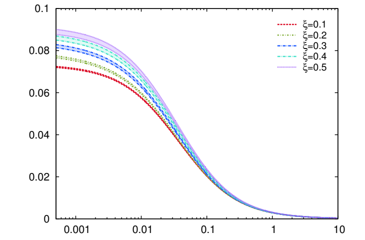
- Our results for the dressing function turn out to differ from the ones of Huber:2015ria.
- On the other hand, our data for the gluon propagator is similar to the one of Aguilar:2015nqa who include Nielsen identities and a gluon mass .
- Applying the same analysis of to our R_ξ lattice propagators, we find that for small ξ, the R_ξ dynamical mass behaves like

$$m^2(q^2) = \left[a(\xi) + c(\xi) \left(\frac{q^2}{\mu^2} \right)^\xi \log \frac{q^2}{\mu^2} \right] m^2_{\xi=0}(q^2),$$

where $m^2_{\xi=0}$ is the Landau gauge gluon mass and to lowest order in the gauge fixing parameter, $a(\xi) = 1 + a_1 \xi$, $c(\xi) = c_{\rm NI} \xi$.

Continuum studies, gluon mass

Reconstructed gluon mass in the R_ξ gauges



- Using as input our $R_\xi = 0$ Landau gauge $L = 32$, $\beta = 6.0$ data for the gluon and ghost propagators we obtain the Landau gauge dynamical mass $m^2_{\xi=0}(q^2)$,
- We solve the renormalization group improved equation as well as the numerical value $c_{\mathrm{NI}} \approx 0.32$.
- We also determine the numerical value of a_1 by requiring that the dynamical mass equals the value of $\Delta_{\mathrm{T}}^{-1}(0)$ for the corresponding value of ξ. We obtain $a_1 \approx 0.26$.
- Then including the statistical errors on the lattice propagators, we plot the gluon running mass for the different R_ξ.

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Conclusions and outlook

- From the numerical point of view, our most intensive task is the gauge fixing due to the large number of GF's required and algorithmic issues.
- Up to large ξ's and volumes, with a proper combination of various steepest descent methods, we solve the GF in R_ξ gauges .
- We also compute the lattice SU(3) gluon propagators in R_ξ gauges, for a lattice volume large enough to access the IR dynamics.
- Our $\Delta_{\mathrm{T}}(q^2)$ propagators are $\sim$ similar for $\xi = 0.0 - 0.5$: an inflection point in the few hundreds MeV region and a saturation in the IR.
- Our propagators are in agreement with very recent continuum analytic studies, and we estimate the dynamically generated R_ξ mass.
- This analysis suggests that dynamical gluon mass generation is a common feature of all R_ξ gauges in SU(3) Yang-Mills theories.

Bicudo:2015rma, Bicudo et al in preparation,

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Edwards:2004sx, Pippig:2011.