12th RIBF Discussion@Univ. of Hong Kong
2015/7/8

proton-neutron pairing vibrations

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Ref: PRC90(2014)031303R

Outline of this lecture:

- ✓ Basics of the vibrational modes of excitation in nuclei

  surface vibration, like-particle pairing vibration, and then···

- ✓ Microscopic framework to describe the vibrations in spin-isospin space

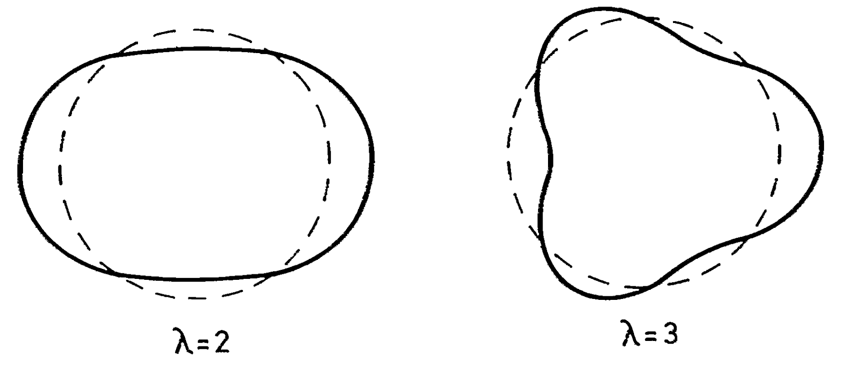
- ✓ Some results in N=Z nuclei: ^{40}Ca - ^{56}Ni

- ✓ Summary and outlook

Vibrational excitations in nuclei

- ✓ Giant resonance: high-frequency vibration of "surface"

  intuitive and classical picture of the collective modes



GQR and HEOR

- ✓ Soft mode: low-frequency vibration associated with "phase transition"

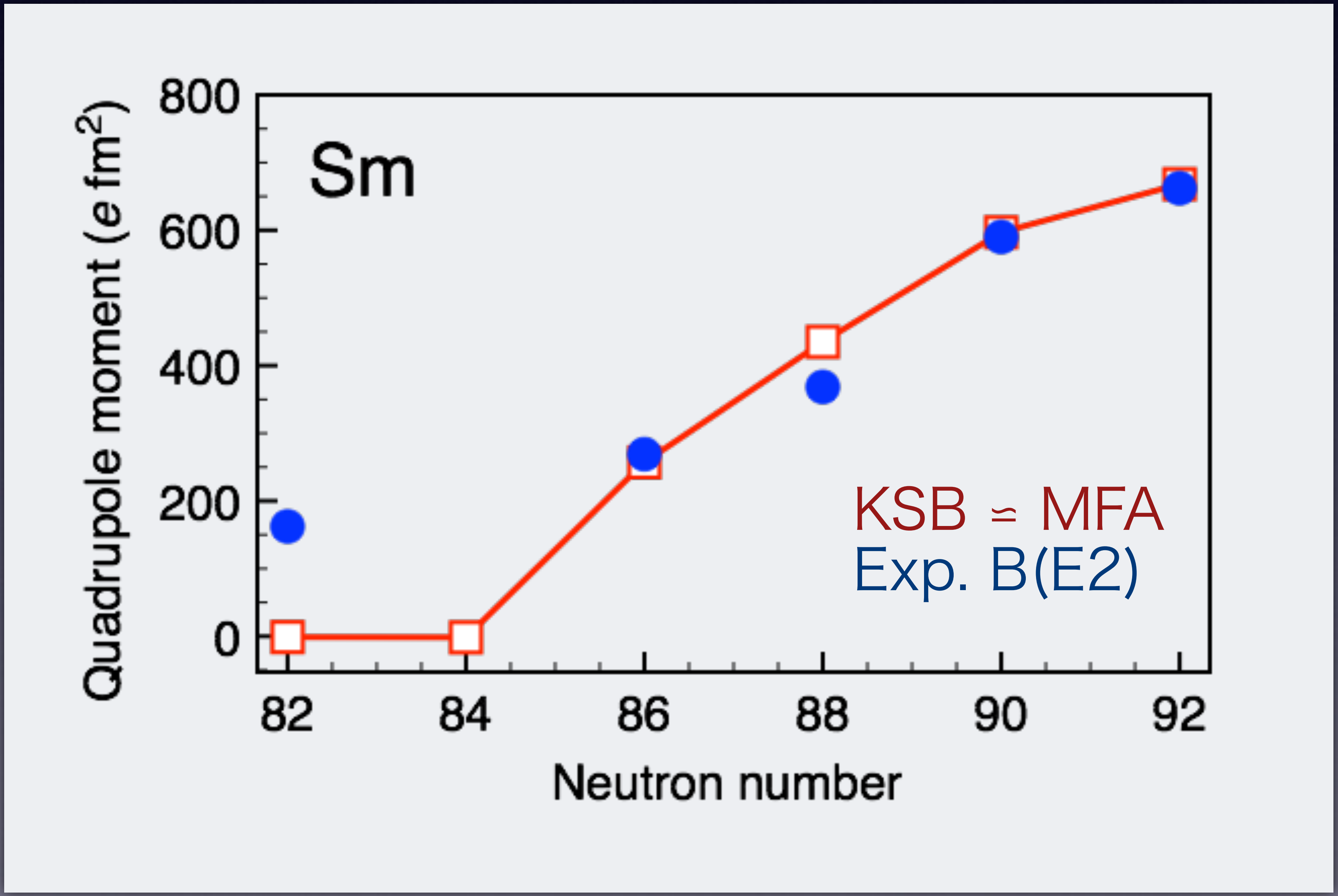
  How do we define "phases" and its transition in finite nuclei?

Quadrupole correlation and associated collective excitation

order parameter characterizing the quadrupole dynamics

$$\hat{Q}_{20} \equiv \int dx r^2 Y_{20}(r) \hat{\psi}^\dagger(x) \hat{\psi}(x) \qquad q \equiv \langle \hat{Q}_{20} \rangle \propto \beta_2$$

$E(2^+), \quad |\langle 2^+ | \hat{Q}_{20} | 0^+ \rangle|^2$:signatures of the collectivity



Quadrupole correlation and associated collective excitation

order parameter characterizing the quadrupole dynamics

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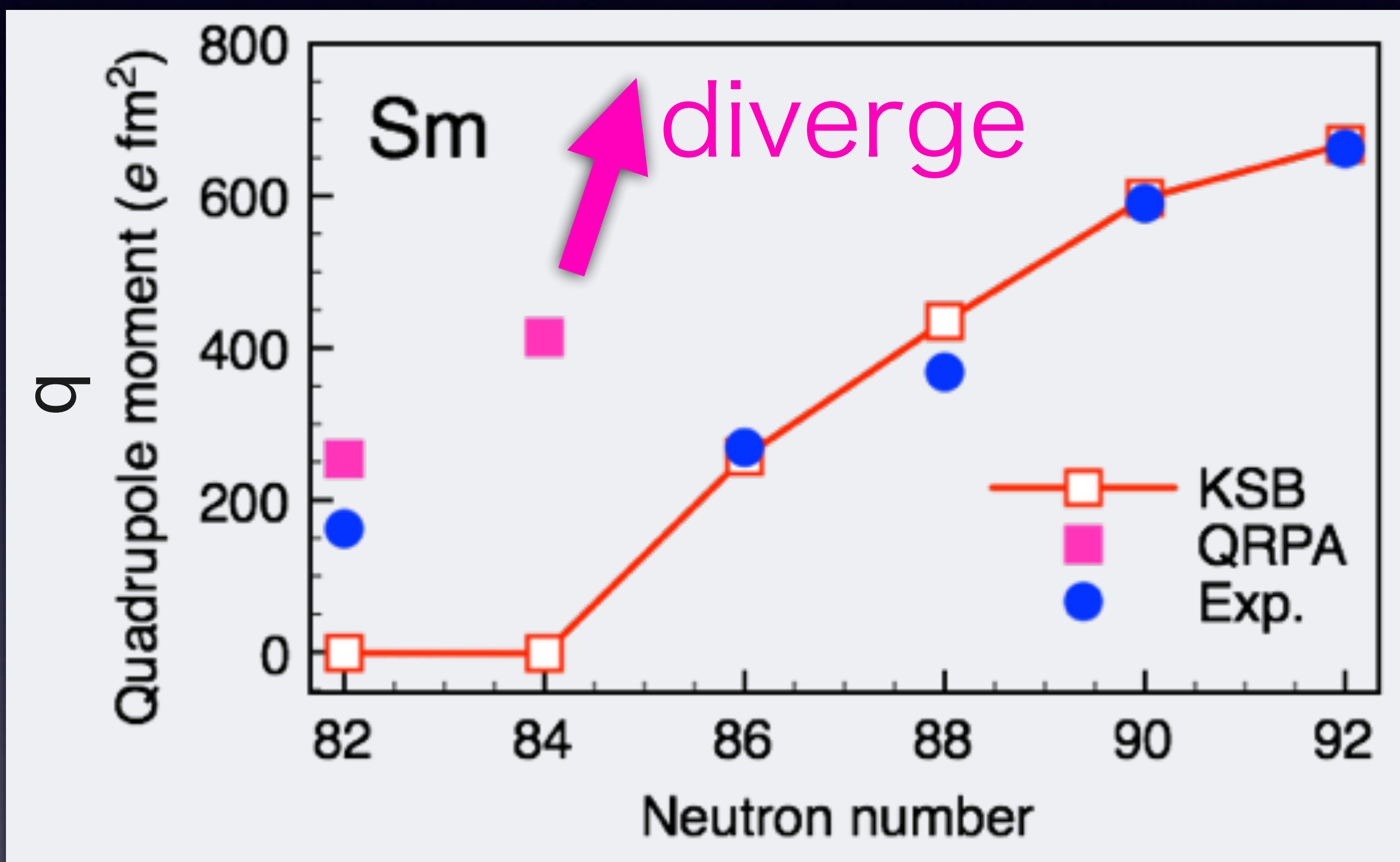
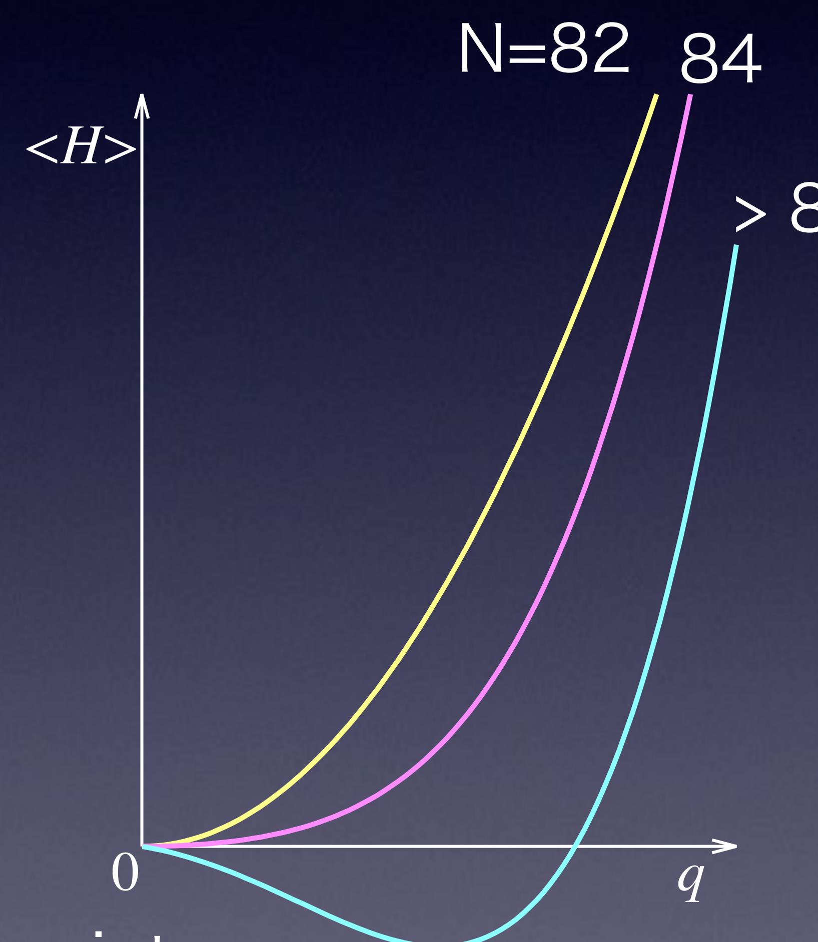
→ low-frequency quadrupole mode: precursory soft mode of the quadrupole deformation



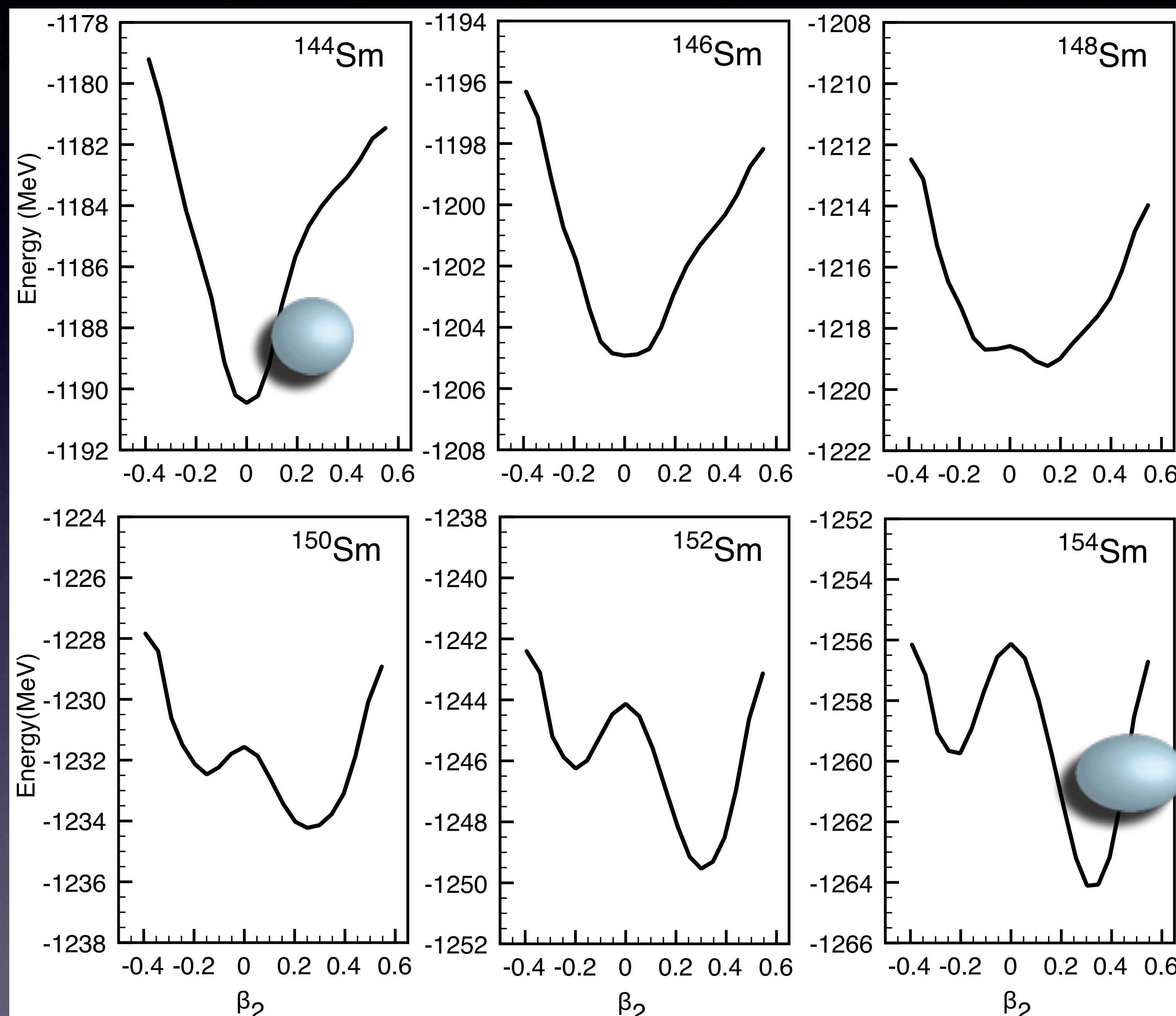


collapse of the RPA solution: location of the critical point
divergence of the transition strength: nature of the “super” phase

anharmonicity and fluctuation
explicitly taken into account around the critical point for a quantitative description

Quadrupole correlation in rare-earth nuclei

shape ‘transition’ as an increase in the neutron number

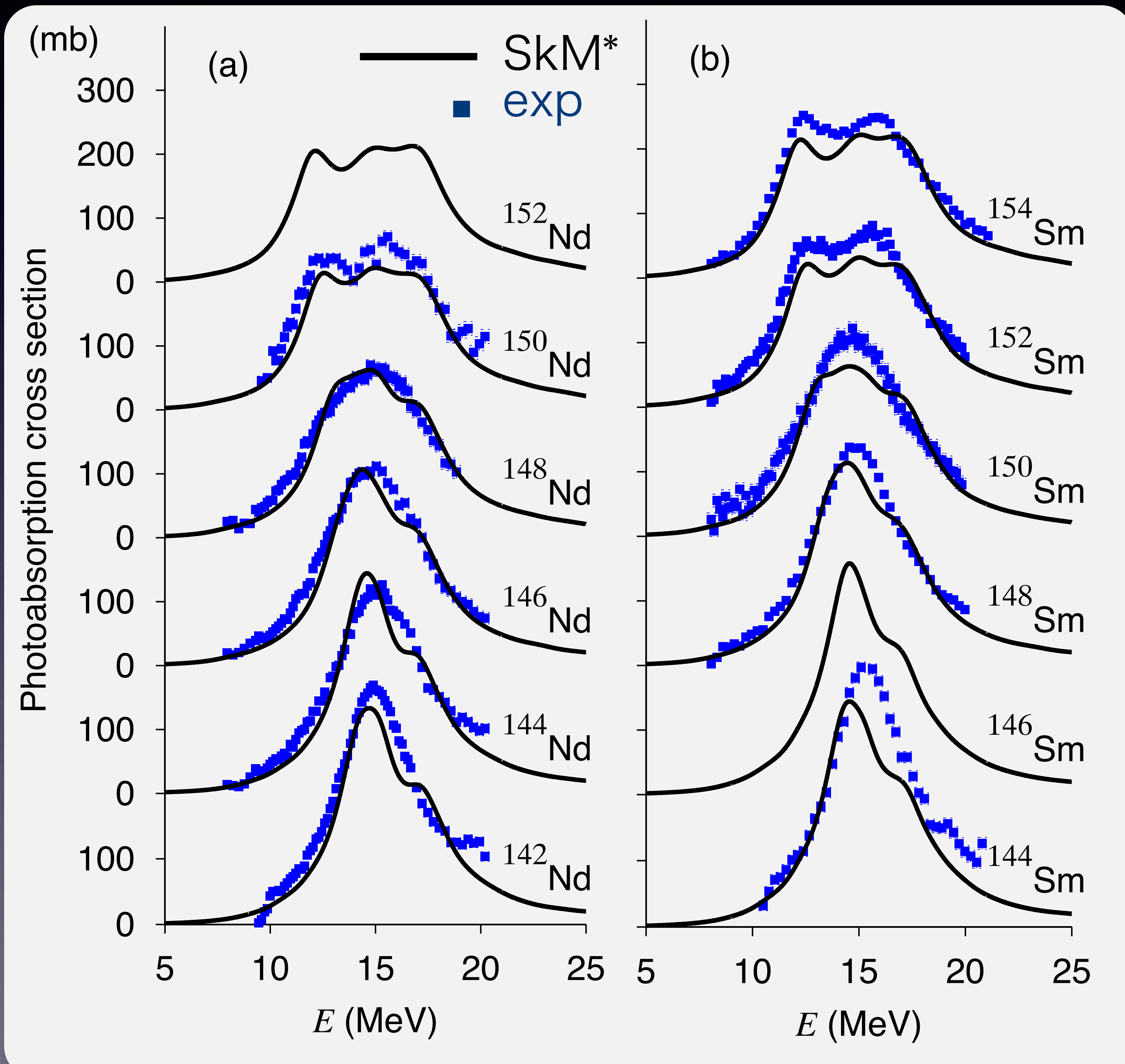


selfconsistent mean-field model taking the breaking of symmetries defines the “phases” of finite nuclear system

Shape evolution seen in photo absorption cross sections

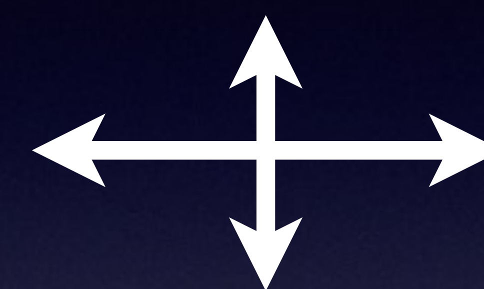
SkM*-KSB-QRPA

KY and T. Nakatsukasa, PRC83(2011)021304R



two eigen frequencies

$$\omega \sim \frac{1}{R}$$



cf. Harakeh & van der Woude, "Giant Resonances"

Pairing vibration and condensation (of neutrons)

cf. Bès and Broglia

neutron-pair operator; a probe to see the collectivity

$$\hat{P}_{T=1,T_z=1,S=0} \equiv \frac{1}{2}\sum_{\sigma,\sigma'}\sum_{\tau,\tau'}\int d\boldsymbol{r}\hat{\psi}(\boldsymbol{r}\sigma\tau)\delta_{\sigma,\sigma'}\langle\tau|\tau_+|\tau'\rangle\hat{\psi}(\boldsymbol{r}\bar{\sigma}'\bar{\tau}') = \sqrt{2}\int d\boldsymbol{r}\hat{\psi}_\nu(\boldsymbol{r}\downarrow)\hat{\psi}_\nu(\boldsymbol{r}\uparrow)$$

$$\hat{\psi}(\boldsymbol{r}\bar{\sigma}\bar{\tau}) = (-2\sigma)(-2\tau)\hat{\psi}(\boldsymbol{r}-\sigma-\tau)$$

pairing condensation: order parameter

$$q \equiv \langle\hat{P}_{T=1,T_z=1,S=0}\rangle = \sqrt{2}\int d\boldsymbol{r}\tilde{\rho}_\nu(\boldsymbol{r})$$

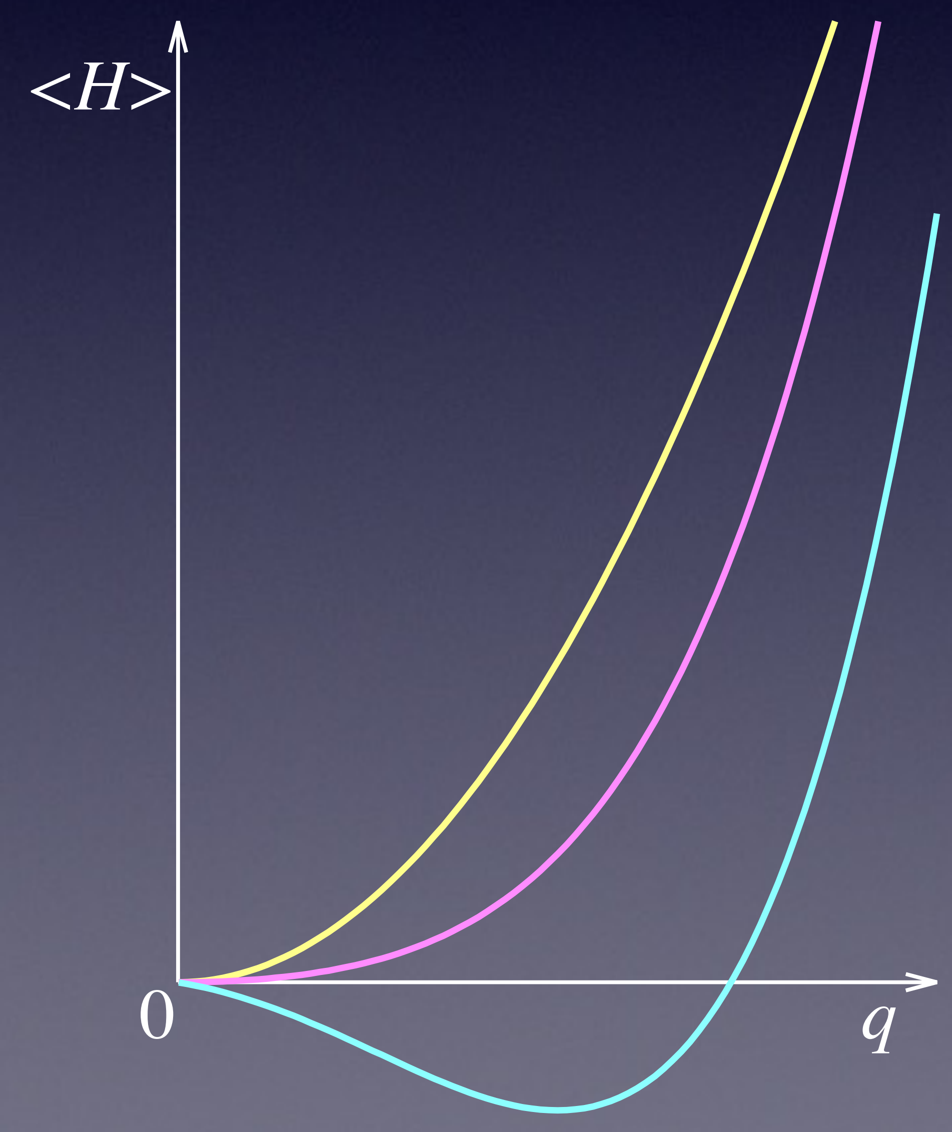
pairing gap: $\Delta \sim \int d\boldsymbol{r}\tilde{h}(\boldsymbol{r})\tilde{\rho}(\boldsymbol{r})$

pairing vibration;
precursory soft mode: $|\lambda\rangle$

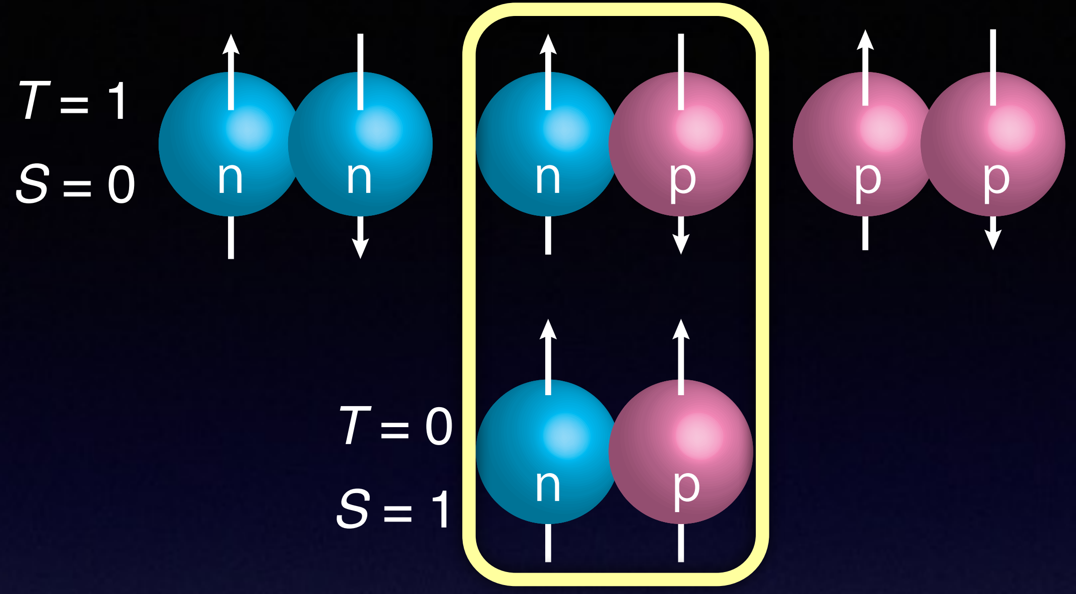
w/ an enhanced transition strength

$$|\langle\lambda|\hat{P}_{T=1,T_z=1,S=0}|\rangle|^2$$

is seen in normal nuclei (q=0)



Proton-neutron pairing collectivity



T=1 (T_z=0), S=0 pair

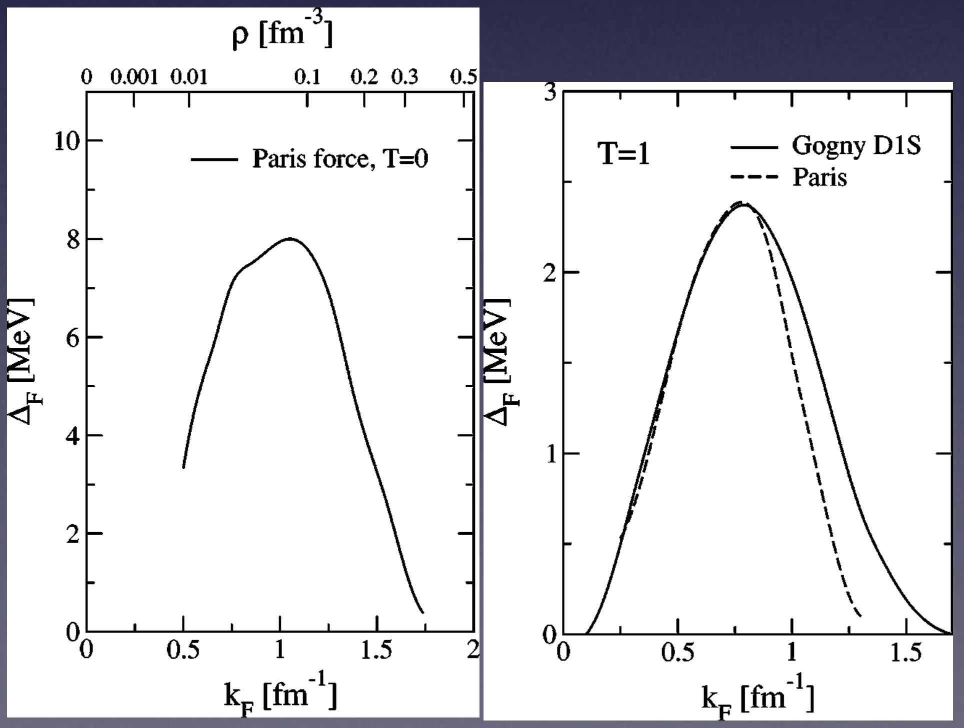
$$\hat{P}_{T=1,T_z=0,S=0} \equiv \frac{1}{2}\sum_{\sigma,\sigma'}\sum_{\tau,\tau'}\int d\boldsymbol{r}\hat{\psi}(\boldsymbol{r}\sigma\tau)\delta_{\sigma,\sigma'}\langle\tau|\tau_0|\tau'\rangle\hat{\psi}(\boldsymbol{r}\bar{\sigma}'\bar{\tau}')$$

strong collectivity is expected as in nn and pp pairings

T=0, S=1 (S_z=0,±1) pair

$$\hat{P}_{T=0,S=1} \equiv \frac{1}{2}\sum_{\sigma,\sigma'}\sum_{\tau,\tau'}\int d\boldsymbol{r}\hat{\psi}(\boldsymbol{r}\sigma\tau)\delta_{\tau,\tau'}\langle\sigma|\boldsymbol{\sigma}|\sigma'\rangle\hat{\psi}(\boldsymbol{r}\bar{\sigma}'\bar{\tau}')$$

$$\Delta_{01} > \Delta_{10}$$



E. Garrido et al., PRC63(2001)037304

many works on the possible occurrence of the condensation, but largely unknown

"no evidence so far"

S. Frauendorf and A. O. Macchiavelli, Prog. Part. Nucl. Phys. 78 (2014) 24

Pairing phase diagram: Pairing vibration and rotation

G.G.Dussel et al., NPA450(1986)164

two-level solvable model: $H = 2N_2 - X_{10} \sum_{\substack{ll'=l_1 l_2 \\ \mu}} D^+_{\mu l} D_{\mu l'} - X_{01} \sum_{\substack{l,l'=l_1 l_2 \\ \mu}} P^+_{\mu l} P_{\mu l'}$

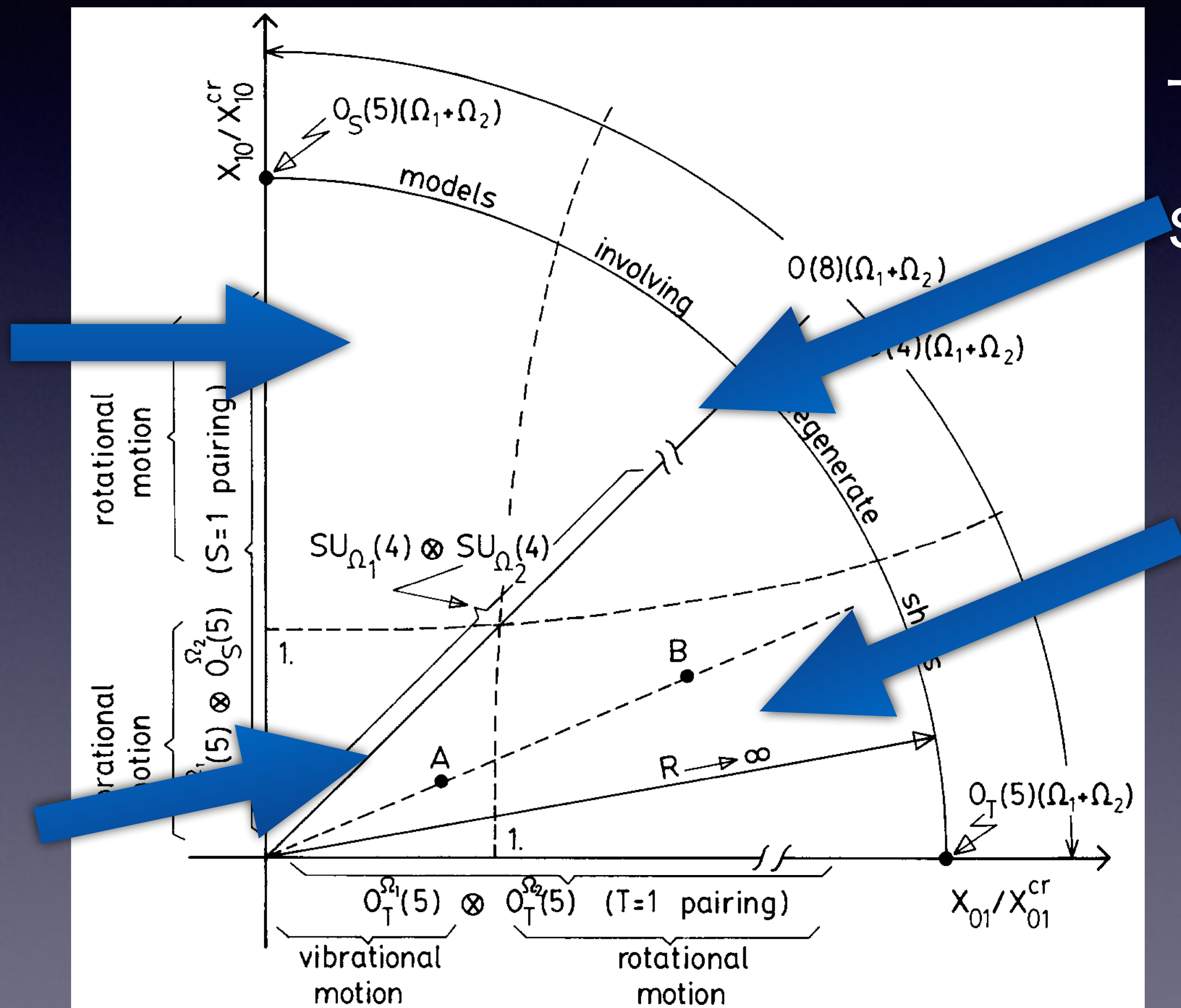


Fig. 1. The two-dimensional space of phases of the model. Various limiting schemes are indicated.

Density functional theory


1998

Kohn

Hohenberg-Kohn theorem (1964)

Existence of the energy density giving the exact g.s. energy of many-body int. system

$$E = \min_{\Psi} \langle \Psi | \hat{H} | \Psi \rangle = \min_{\rho(\boldsymbol{r})} \left[\underline{\min_{\Psi \to \rho(\boldsymbol{r})} \langle \Psi | \hat{H} | \Psi \rangle} \right]$$

$\mathcal{E}[\rho(\boldsymbol{r})]$:EDF

Kohn-Sham theorem (1965)

The exact g.s. of many-body int. system is given as a Slater determinant of the Kohn-Sham orbitals

Kohn-Sham (KS) eq.

$$-\frac{\hbar^2}{2m}\nabla^2\phi_i + v[\rho(\boldsymbol{r})]\phi_i = \epsilon_i\phi_i$$

$$v[\rho(\boldsymbol{r})] = \frac{\delta}{\delta\rho}\{\mathcal{E}[\rho(\boldsymbol{r})] - T_s[\rho(\boldsymbol{r})]\}$$

particle density

$$\rho(\boldsymbol{r}) = \sum_i |\phi_i(\boldsymbol{r})|^2$$

kinetic energy density

$$T_s[\rho(\boldsymbol{r})] = \sum_i |\nabla\phi_i(\boldsymbol{r})|^2$$

cf. HF mean field

$$\Gamma[\rho] \neq v[\rho]$$

gs of many-body system ⟷ single-particle motions in a one-body potential

Skyrme energy-density functional (EDF)

Energy functional: $E = \int d\boldsymbol{r} \mathcal{E}[\rho(\boldsymbol{r})]$ $\qquad \rho(\boldsymbol{r}) \equiv \sum_\sigma \langle \hat{\psi}(\boldsymbol{r}\sigma)\hat{\psi}^\dagger(\boldsymbol{r}\sigma)\rangle$

Energy density: $\mathcal{E} = \mathcal{T} + \mathcal{H}_{\text{Skyrme}} + \mathcal{H}_{\text{em}}$

Skyrme energy density: $\mathcal{H}_{\text{Skyrme}} = \sum_{t=0,1} \sum_{t_3=-t}^{t} \left(\mathcal{H}_{tt_3}^{\text{even}} + \mathcal{H}_{tt_3}^{\text{odd}} \right)$

$$\mathcal{H}_{tt_3}^{\text{even}} = C_t^\rho \, \rho_{tt_3}^2 + C_t^{\Delta\rho} \, \rho_{tt_3} \Delta\rho_{tt_3} + C_t^\tau \, \rho_{tt_3}\tau_{tt_3} + C_t^{\nabla J} \rho_{tt_3} \, \nabla \cdot \mathbf{J}_{tt_3} + C_t^J \overleftrightarrow{J}_{tt_3}^2$$

$$\mathcal{H}_{tt_3}^{\text{odd}} = C_t^s \, \mathbf{s}_{tt_3}^2 + C_t^{\Delta s} \, \mathbf{s}_{tt_3} \cdot \Delta \mathbf{s}_{tt_3} + C_t^T \, \mathbf{s}_{tt_3} \cdot \mathbf{T}_{tt_3} + C_t^{\nabla s} \, (\nabla \cdot \mathbf{s}_{tt_3})^2 + C_t^j \, \mathbf{j}_{tt_3}^2 + C_t^{\nabla j} \, \mathbf{s}_{tt_3} \cdot \nabla \times \mathbf{j}_{tt_3}$$

T-odd densities vanish in g.s of e-e nuclei

T-odd Skyrme energy density is not well constrained,
but plays a role in dynamics

$\mathcal{E}[\rho(\boldsymbol{r}), \tilde{\rho}(\boldsymbol{r})]$ pair correlation is also important

$\tilde{\rho}(\boldsymbol{r}) \equiv \langle \hat{\psi}(\boldsymbol{r}\downarrow)\hat{\psi}(\boldsymbol{r}\uparrow)\rangle$: spin-singlet pair of like-particles

KY, PTEP2013,113D02

Self-consistent pn-QRPA for exploring vibrational modes in spin-isospin space

starting point: Skyrme + pairing EDF $\mathcal{E}[\rho(\boldsymbol{r}), \tilde{\rho}(\boldsymbol{r})]$

T=1 (nn and pp) pairing condensates

variation w.r.t densities

The coordinate-space Kohn-Sham-Bogoliubov-de Gennes eq. for ground states

J. Dobaczewski et al., NPA422(1984)103

$$\begin{pmatrix} h^q(\boldsymbol{r},\sigma) - \lambda^q & \tilde{h}^q(\boldsymbol{r},\sigma) \\ \tilde{h}^q(\boldsymbol{r},\sigma) & -(h^q(\boldsymbol{r},\sigma) - \lambda^q) \end{pmatrix} \begin{pmatrix} \varphi^q_{1,\alpha}(\boldsymbol{r},\sigma) \\ \varphi^q_{2,\alpha}(\boldsymbol{r},\sigma) \end{pmatrix} = E_\alpha \begin{pmatrix} \varphi^q_{1,\alpha}(\boldsymbol{r},\sigma) \\ \varphi^q_{2,\alpha}(\boldsymbol{r},\sigma) \end{pmatrix}$$

$q = \nu, \ \pi$

"s.p." hamiltonian and pair potential: $h^q = \dfrac{\delta \mathcal{E}}{\delta \rho^q}, \qquad \tilde{h}^q = \dfrac{\delta \mathcal{E}}{\delta \tilde{\rho}^q}$

quasiparticle basis $\alpha, \beta \cdots$

The proton-neutron quasiparticle RPA eq. for excited states $[\hat{H}, \hat{O}^\dagger_\lambda]|\Psi_\lambda\rangle = \omega_\lambda \hat{O}^\dagger_\lambda |\Psi_\lambda\rangle$

Collective excitation = coherent superposition of 2qp excitations:

$$\hat{O}^\dagger_\lambda = \sum_{\alpha\beta} X^\lambda_{\alpha\beta} \hat{a}^\dagger_{\alpha,\nu} \hat{a}^\dagger_{\beta,\pi} - Y^\lambda_{\alpha\beta} \hat{a}_{\bar{\beta},\pi} \hat{a}_{\bar{\alpha},\nu}$$

residual interactions derived self-consistently :

$$v_{\rm res}(\boldsymbol{r}_1, \boldsymbol{r}_2) = \frac{\delta^2 \mathcal{E}}{\delta \rho_{1t_3}(\boldsymbol{r}_1) \delta \rho_{1t_3}(\boldsymbol{r}_2)} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 + \frac{\delta^2 \mathcal{E}}{\delta \boldsymbol{s}_{1t_3}(\boldsymbol{r}_1) \delta \boldsymbol{s}_{1t_3}(\boldsymbol{r}_2)} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2$$

Recent progress

EDF-based **self-consistent** pnQRPA for **axially-deformed** nuclei

w/o any free parameters (almost all the) arbitrary nuclei

Skyrme

coordinate-space

↓

suitable for weakly-bound nuclei

PTEP
Prog. Theor. Exp. Phys. **2013**, 113D02 (17 pages)
DOI: 10.1093/ptep/ptt091

Spin–isospin response of deformed neutron-rich nuclei in a self-consistent Skyrme energy-density-functional approach

Kenichi Yoshida*

PHYSICAL REVIEW C **87**, 064302 (2013)

Large-scale calculations of the double-β decay of ^{76}Ge, ^{130}Te, ^{136}Xe, and ^{150}Nd in the deformed self-consistent Skyrme quasiparticle random-phase approximation

M. T. Mustonen[1,2,*] and J. Engel[1,†]

PHYSICAL REVIEW C **90**, 024308 (2014)

Finite-amplitude method for charge-changing transitions in axially deformed nuclei

M. T. Mustonen,[1,*] T. Shafer,[1,†] Z. Zenginerler,[2,‡] and J. Engel[1,§]

PHYSICAL REVIEW C **89**, 044306 (2014)

Gamow-Teller strength in deformed nuclei within the self-consistent charge-exchange quasiparticle random-phase approximation with the Gogny force

M. Martini,[1,2,3] S. Péru,[3] and S. Goriely[1]

Gogny

Interactions employed for pn-pairing vibrations in fp-shell nuclei

KSB(HFB) eq:

SGII + surface pairing

V_0 = -520 MeV fm^3

^{44}Ti

Δn = 1.82 MeV

Δp = 1.87 MeV

pnQRPA eq:

p-h channel: SGII

p-p channel:

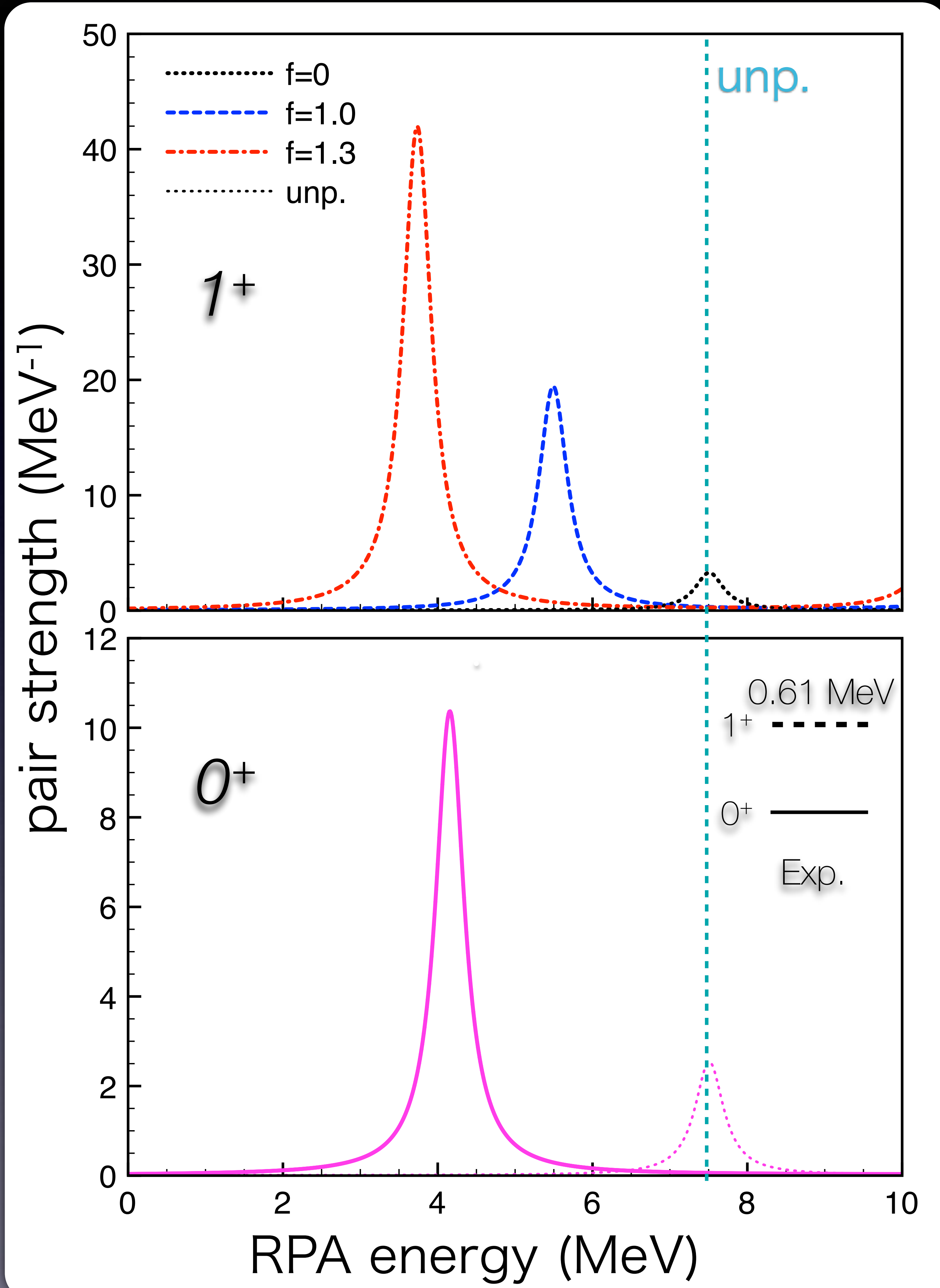
$$v_{\rm pp}^{T=0}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau') = f \times V_0 \frac{1+P_\sigma}{2}\frac{1-P_\tau}{2}\left[1-\frac{\rho(\boldsymbol{r})}{\rho_0}\right]\delta(\boldsymbol{r}-\boldsymbol{r}')$$

$$v_{\rm pp}^{T=1}(\boldsymbol{r}\sigma\tau,\boldsymbol{r}'\sigma'\tau') = V_0 \frac{1-P_\sigma}{2}\frac{1+P_\tau}{2}\left[1-\frac{\rho(\boldsymbol{r})}{\rho_0}\right]\delta(\boldsymbol{r}-\boldsymbol{r}')$$

changing “f” to see an effect of the residual interaction

cf. C. Bai et al., PLB719(2013)116

^{40}Ca ⇩ ^{42}Sc



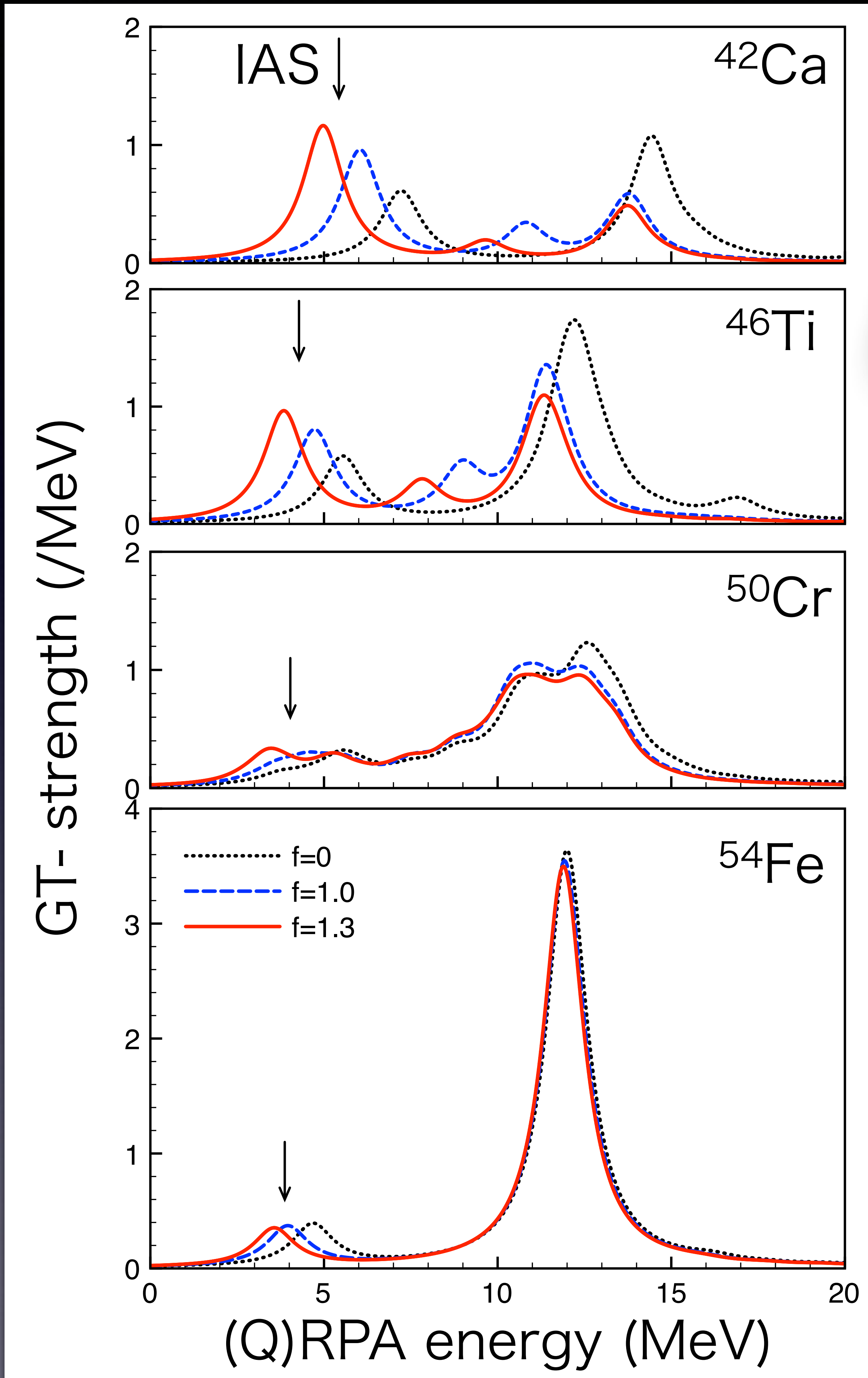
f=1.3

^{42}Sc		$J^\pi = 1^+$	$J^\pi = 0^+$
configuration	$E_\alpha + E_\beta$	$M_{\alpha\beta}^{S=1,S_z=0}$	$M_{\alpha\beta}^{S=0}$
$\pi 1f_{7/2} \otimes \nu 1f_{7/2}$	7.5	1.70	2.85
$\pi 1f_{7/2} \otimes \nu 1f_{5/2}$	15.2	0.62	
$\pi 1f_{5/2} \otimes \nu 1f_{7/2}$	14.7	0.51	
$\pi 2p_{3/2} \otimes \nu 2p_{3/2}$	16.1	0.17	0.22
$\pi 1d_{3/2} \otimes \nu 1d_{3/2}$	4.2	0.25	0.48
$\pi 2s_{1/2} \otimes \nu 2s_{1/2}$	6.6	0.25	
$\pi 1d_{3/2} \otimes \nu 1d_{5/2}$	10.1	0.32	
$\pi 1d_{5/2} \otimes \nu 1d_{3/2}$	10.2	0.32	
$\pi 1d_{5/2} \otimes \nu 1d_{5/2}$	16.1	0.16	0.31

Transition matrix element

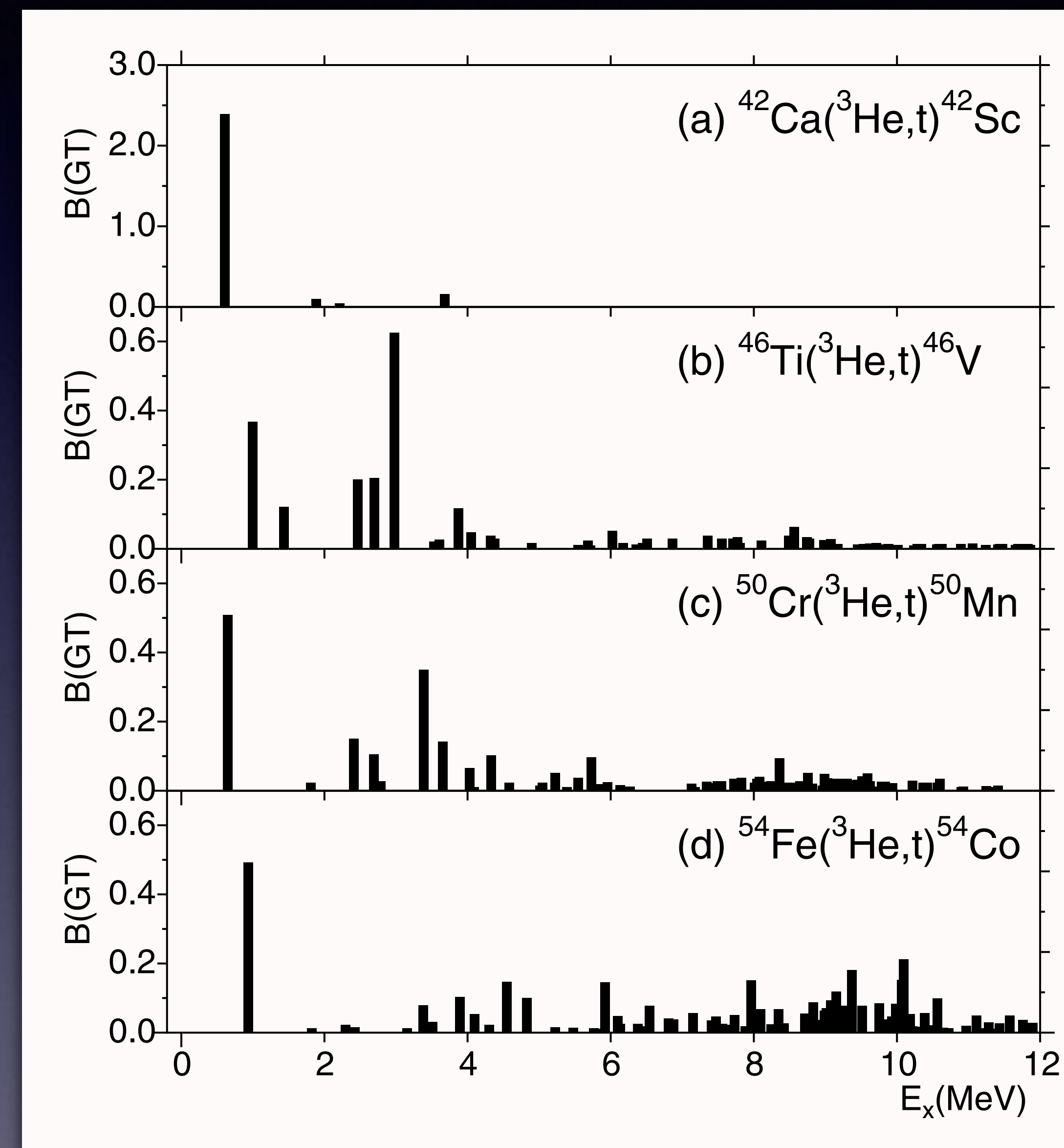
$$\langle \lambda | \hat{P}^\dagger_{T,S} | 0 \rangle = \sum_{\alpha\beta} M^{T,S}_{\alpha\beta}$$

- ✓ coherent superposition of $(f)^2$ excitation
- ✓ sizable hole-hole excitations



pn-pairing more effective

Talk by Fujita this morning:
"Low-energy super GT state" in ^{42}Sc

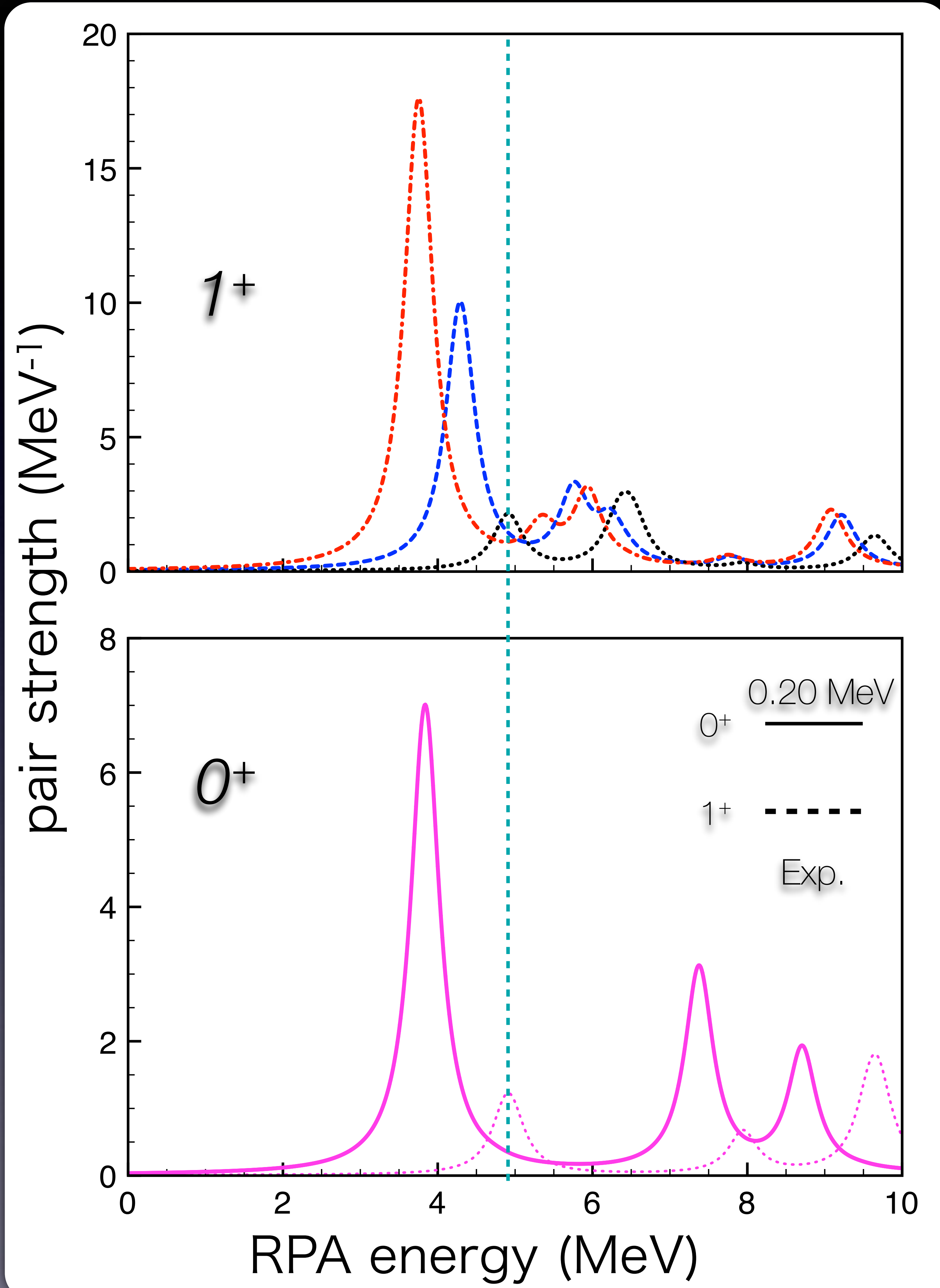


T. Adachi, Y. Fujita et al., NPA788 (2007) 70c

^{56}Ni

⇩

^{58}Cu



f=1.3

^{58}Cu		$J^\pi = 1^+$	$J^\pi = 0^+$
configuration	$E_\alpha + E_\beta$	$M^{S=1,S_z=0}_{\alpha\beta}$	$M^{S=0}_{\alpha\beta}$
$\pi 2p_{3/2} \otimes \nu 2p_{3/2}$	4.5	1.28	1.90
$\pi 2p_{1/2} \otimes \nu 2p_{3/2}$	6.4	0.39	
$\pi 2p_{3/2} \otimes \nu 2p_{1/2}$	6.5	0.37	
$\pi 2p_{1/2} \otimes \nu 2p_{1/2}$	7.9		0.26
$\pi 1f_{5/2} \otimes \nu 1f_{5/2}$	9.7	0.15	0.55
$\pi 1f_{7/2} \otimes \nu 1f_{7/2}$	5.1	0.17	0.50

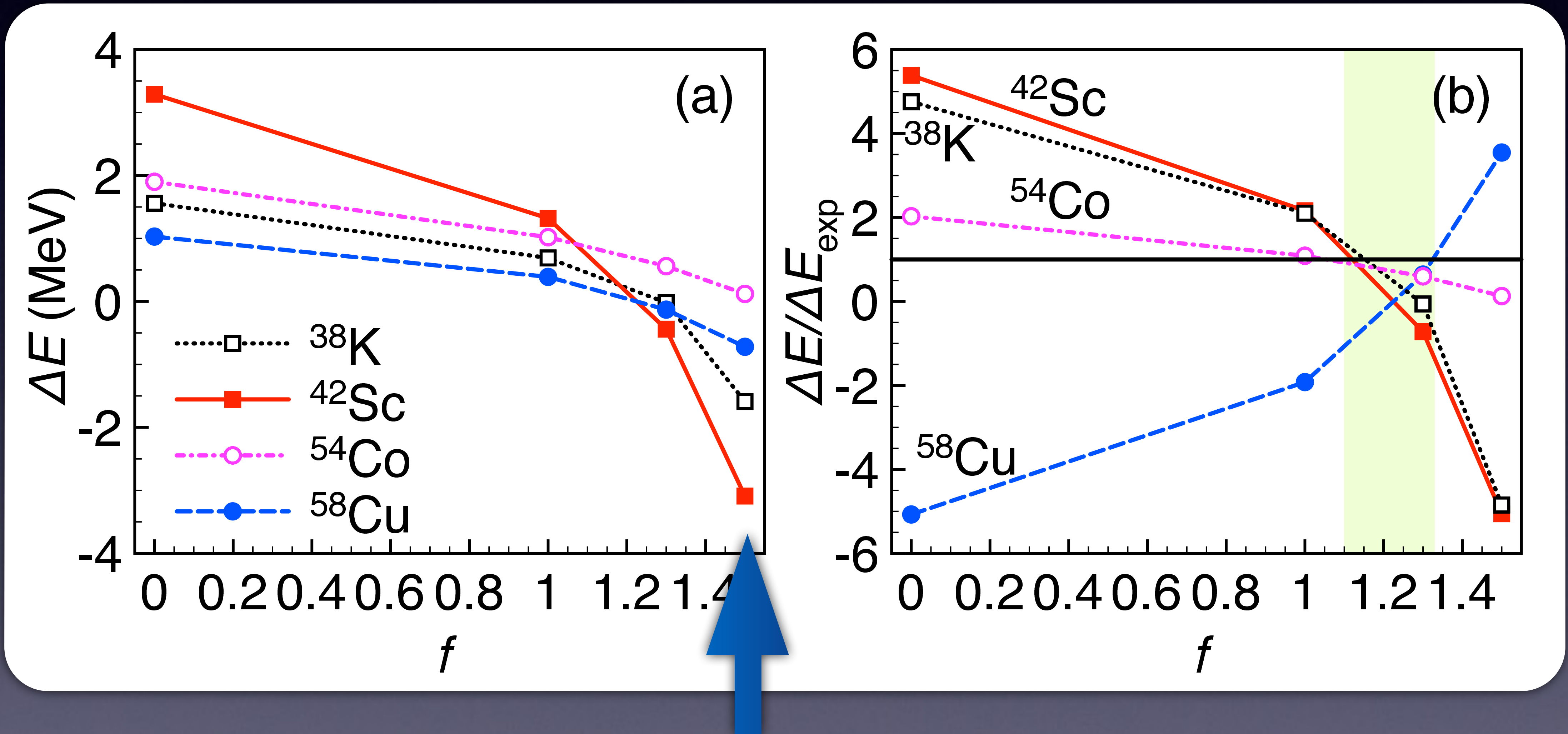
- ✓ coherent superposition of (p)^2 and $(f_{5/2})^2$ excitations
- ✓ $(f_{7/2})^2$ excitation as a ground-state correlation

⬇

weaker collectivity than in ^{40}Ca

Collective pn-pairing vibration mode precursory to the T=0 pairing condensation

$\Delta E = \omega_{1+} - \omega_{0+}$



approaching the critical point to the T=0 pairing condensation

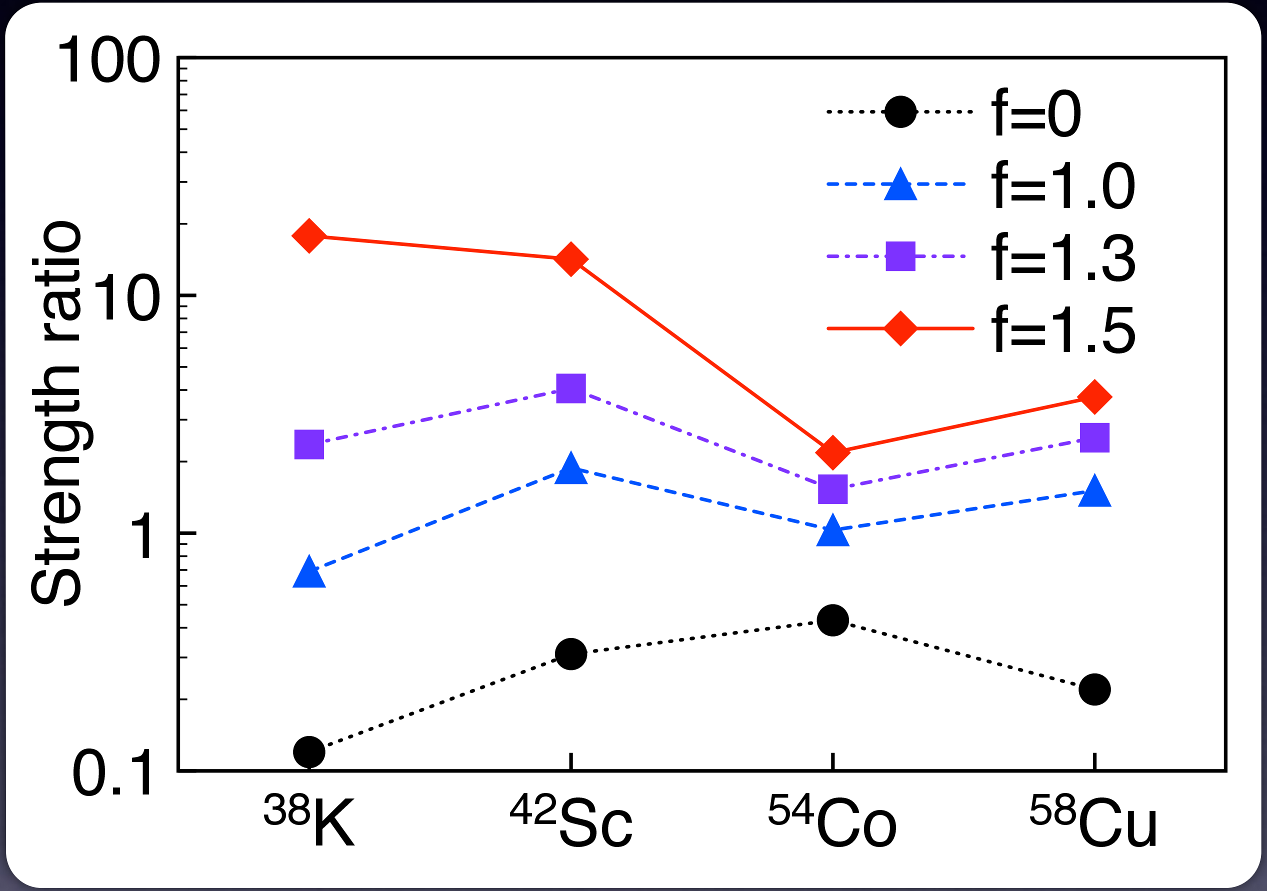
$f_c = 1.53$ (^{40}Ca)

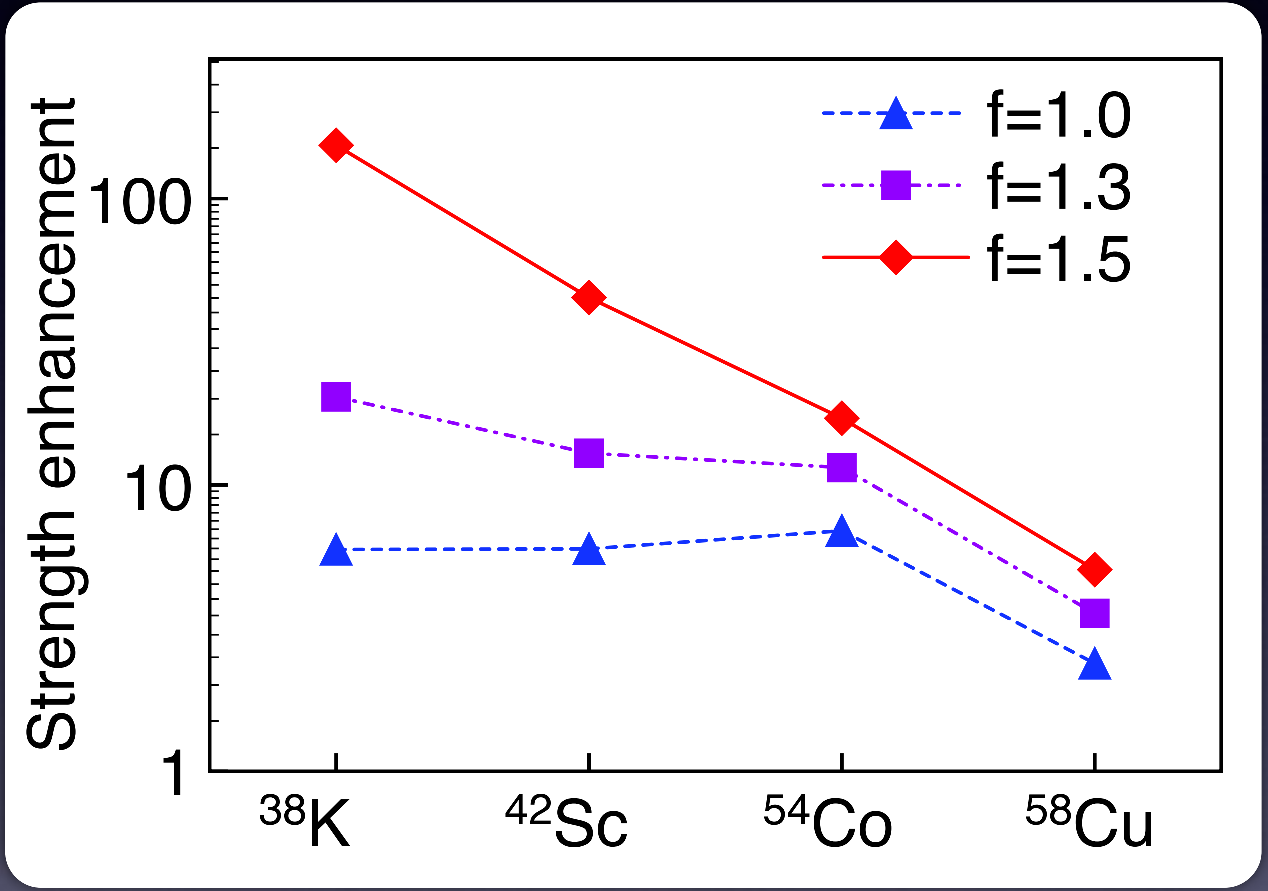
Enhancement of the pair transfer strengths

pair addition and removal

$$\frac{|\langle\lambda|\hat{P}^{\dagger}_{T=0}|0\rangle|^2}{|\langle\lambda|\hat{P}^{\dagger}_{T=1}|0\rangle|^2}$$

$$\frac{|\langle\lambda|\hat{P}^{\dagger}_{T=0}|0\rangle|^2}{|\langle \text{unp.}|\hat{P}^{\dagger}_{T=0}|0\rangle|^2}$$





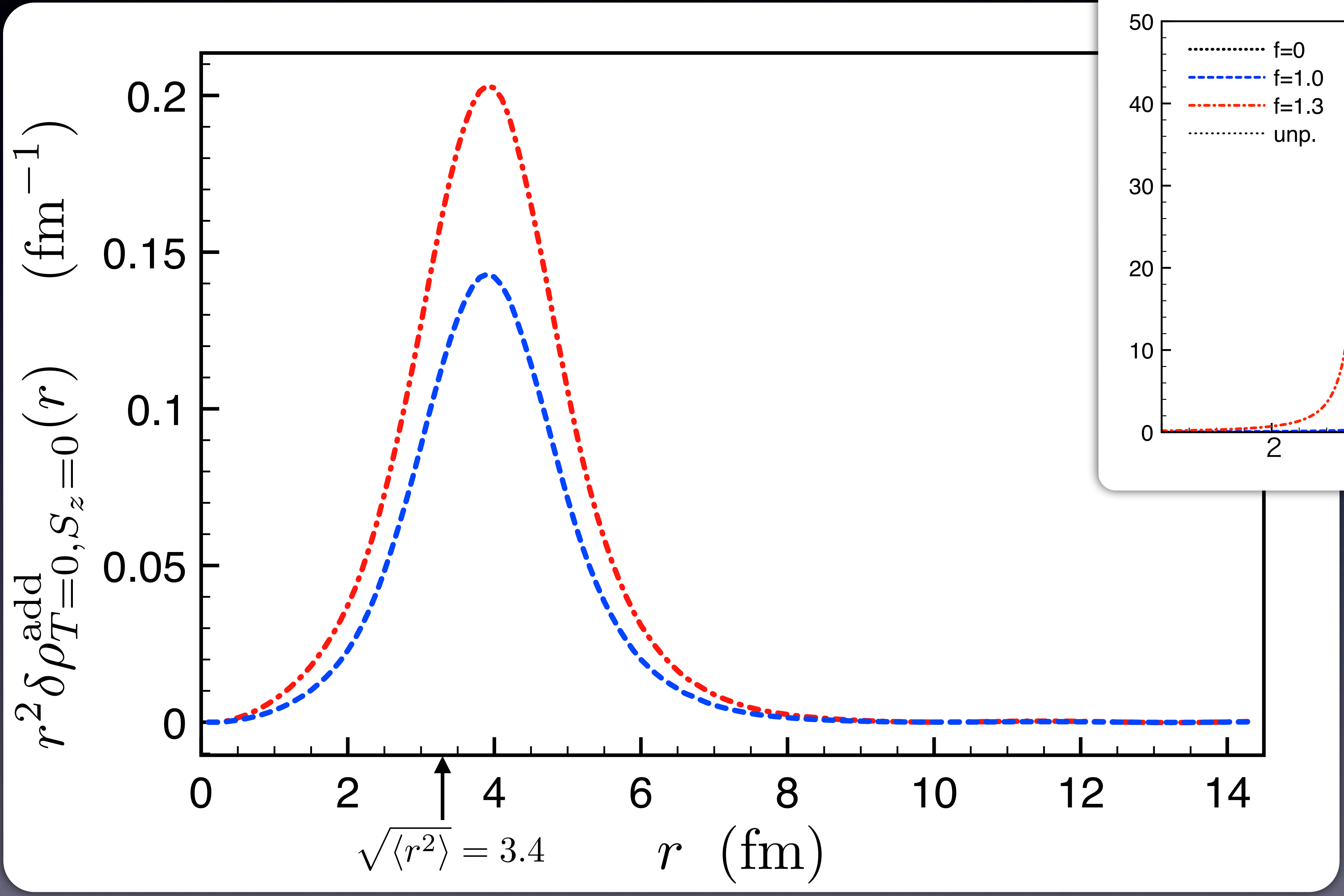
enhancement of the cross section in ^{40}Ca(^{3}He,p)^{42}Sc

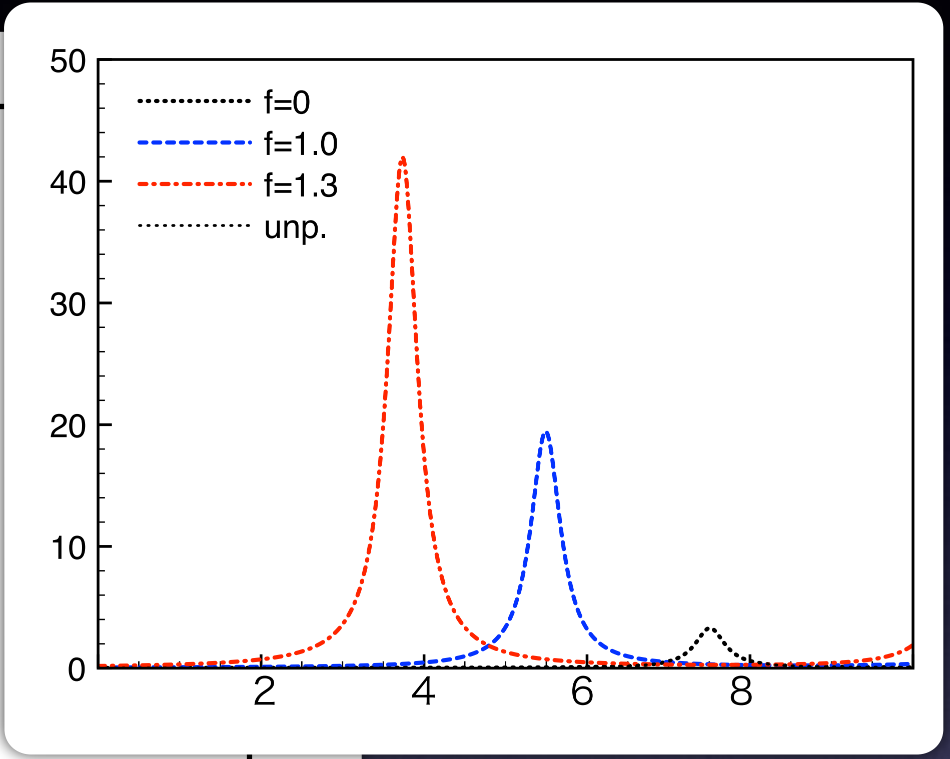
$$\frac{\sigma(1^+)_{\text{exp}}}{\sigma(1^+)_{\text{unp}}} = 23.9$$

F. Pühlhofer, NPA116 (1968) 516

Microscopic transition density of the T=0 pair transfer

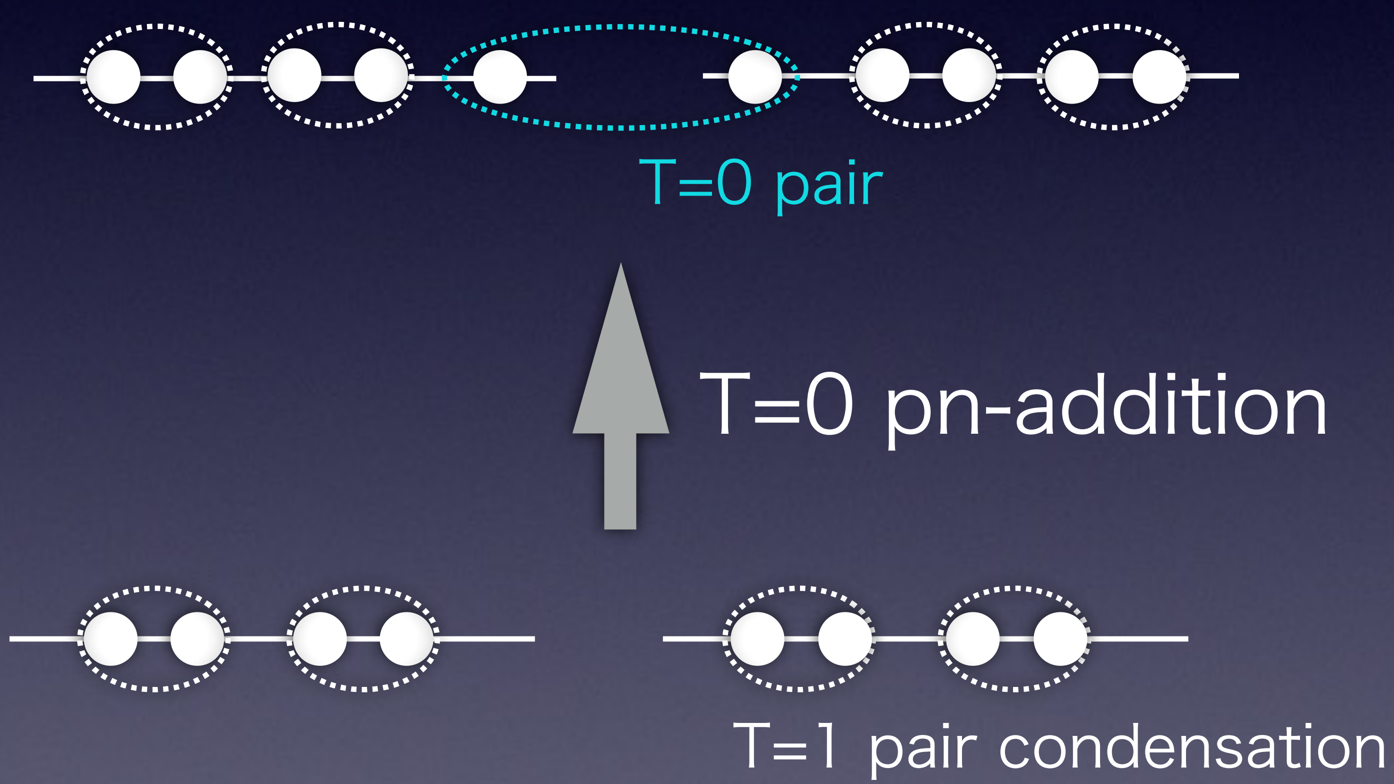
$^{40}Ca\ (0^+) \Rightarrow {}^{42}Sc\ (1^+)$





pn-pairing vibrations in the open-shell nuclei

w/ T=1 pairing condensation



Adding a T=0 pair
needs to destroy (a part of)
the condensed T=1 pairs

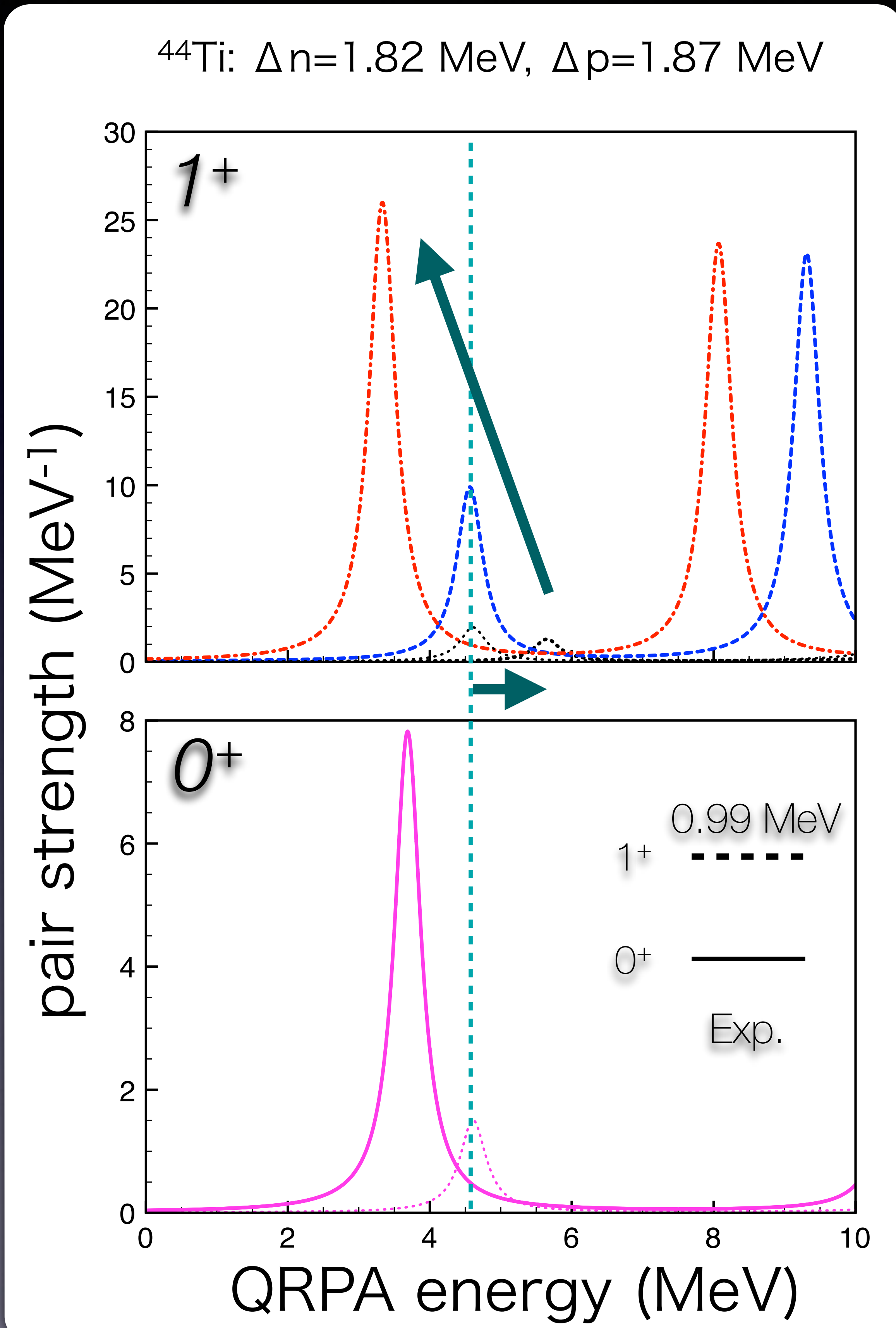
pn-pairing vibrations in the open-shell nuclei

w/ T=1 pairing condensation

^{44}Ti

⇩

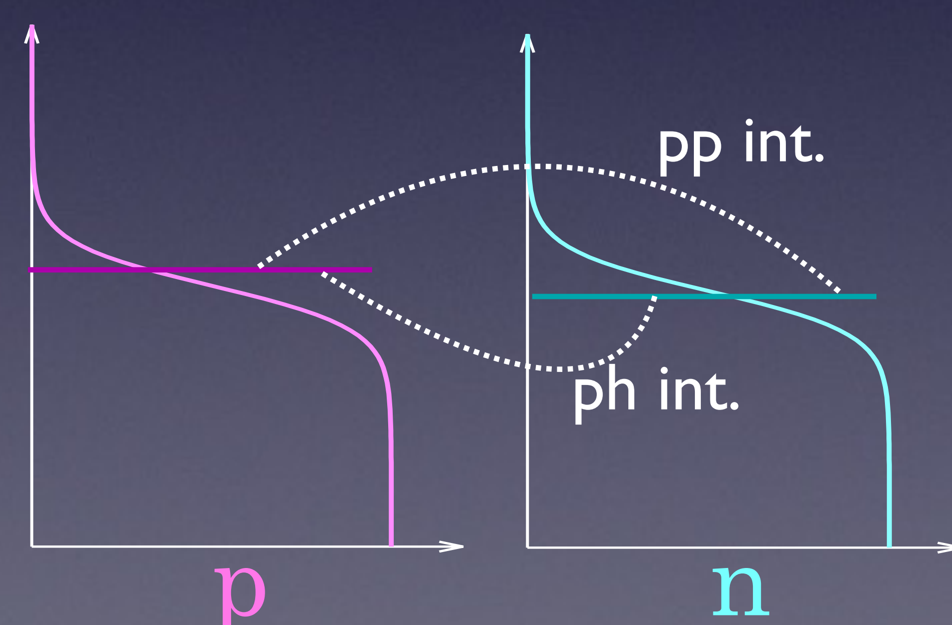
^{46}V



repulsive ph interaction

(GT-type)

attractive pp interaction



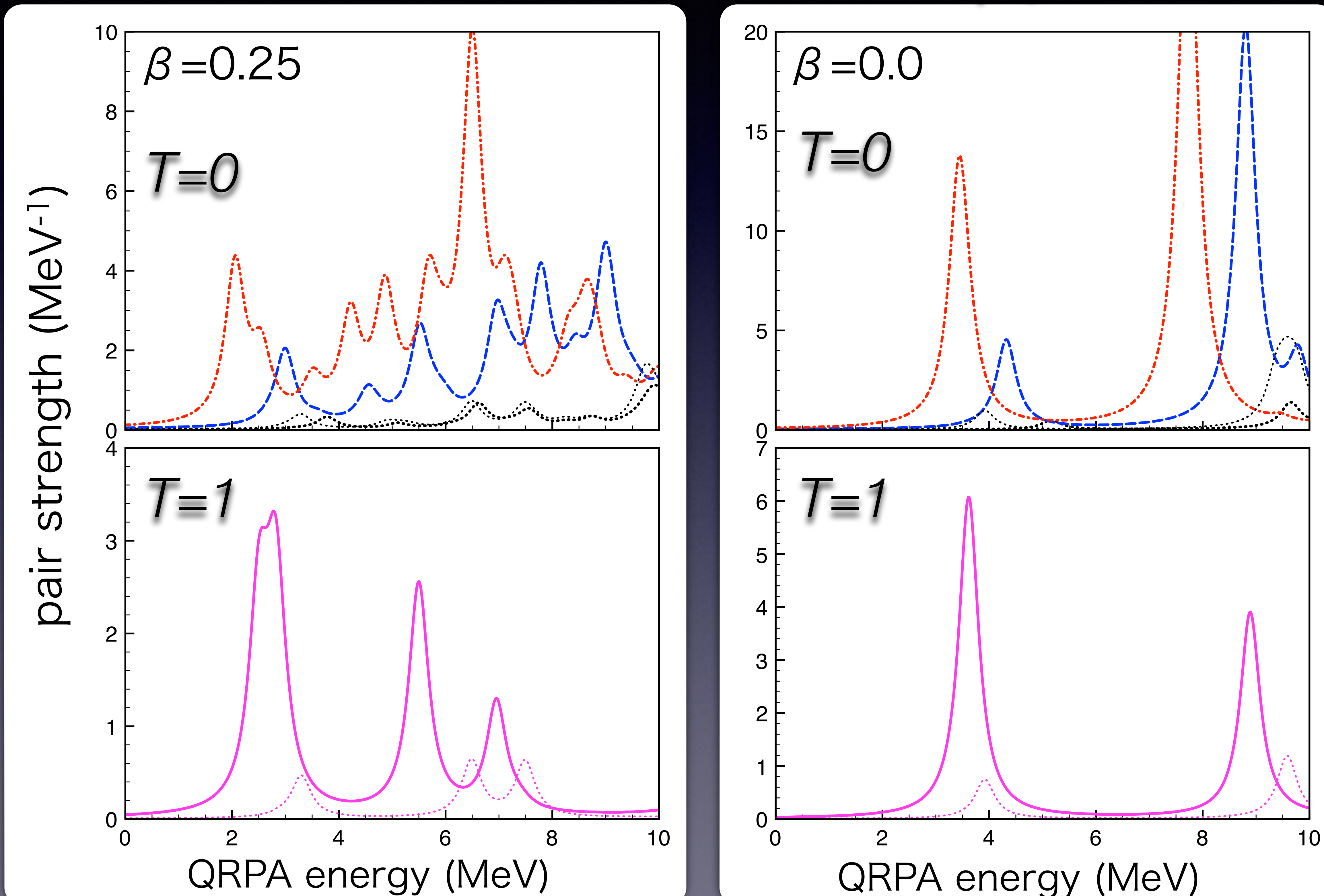
“^{44}Ti + 2qp excitation”

pn-pairing vibrations in the mid-shell nuclei

w/ T=1 pairing condensation and quadrupole def.

^{48}Cr → ^{50}Mn

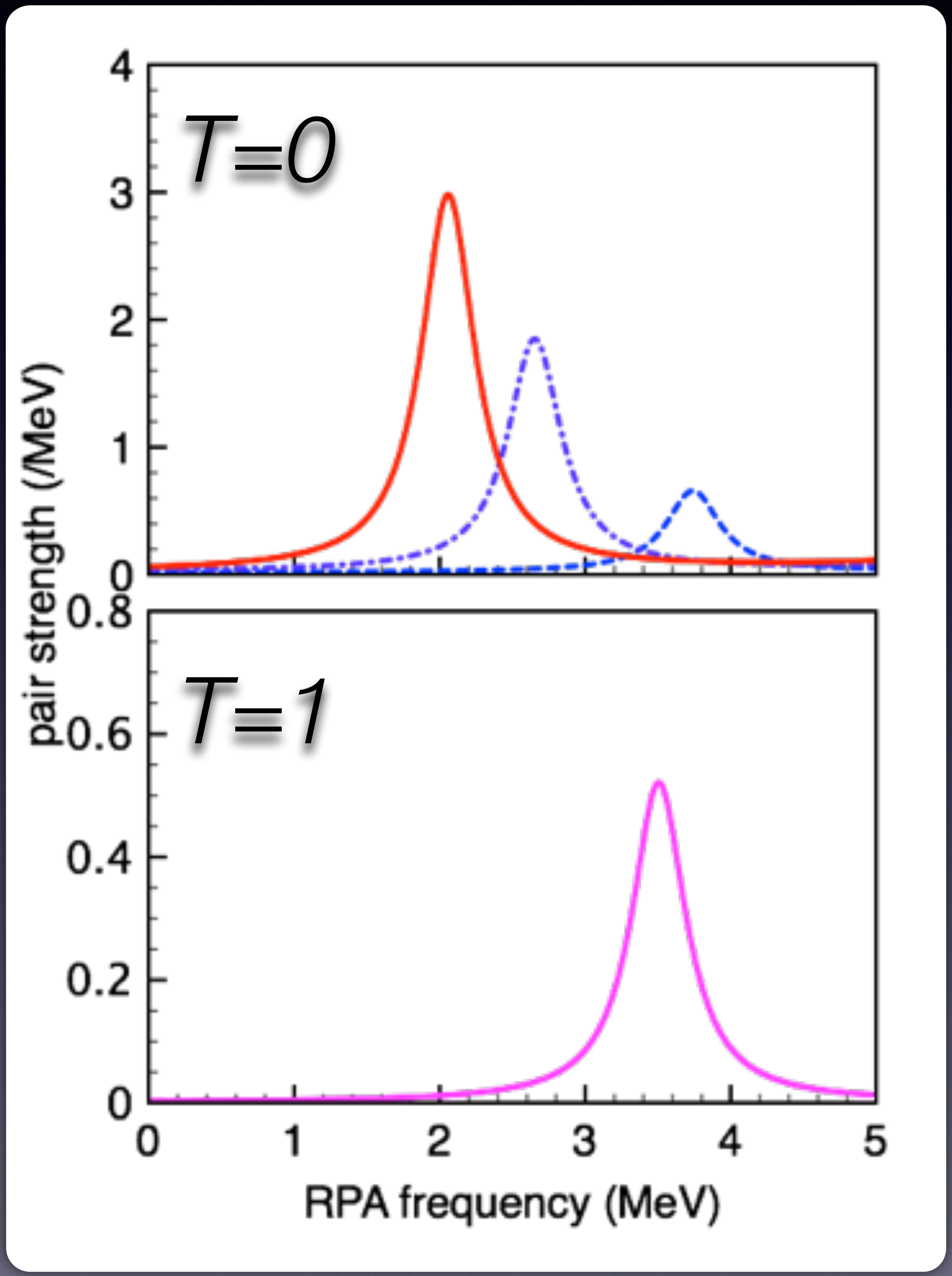
constrained HFB+pnQRPA



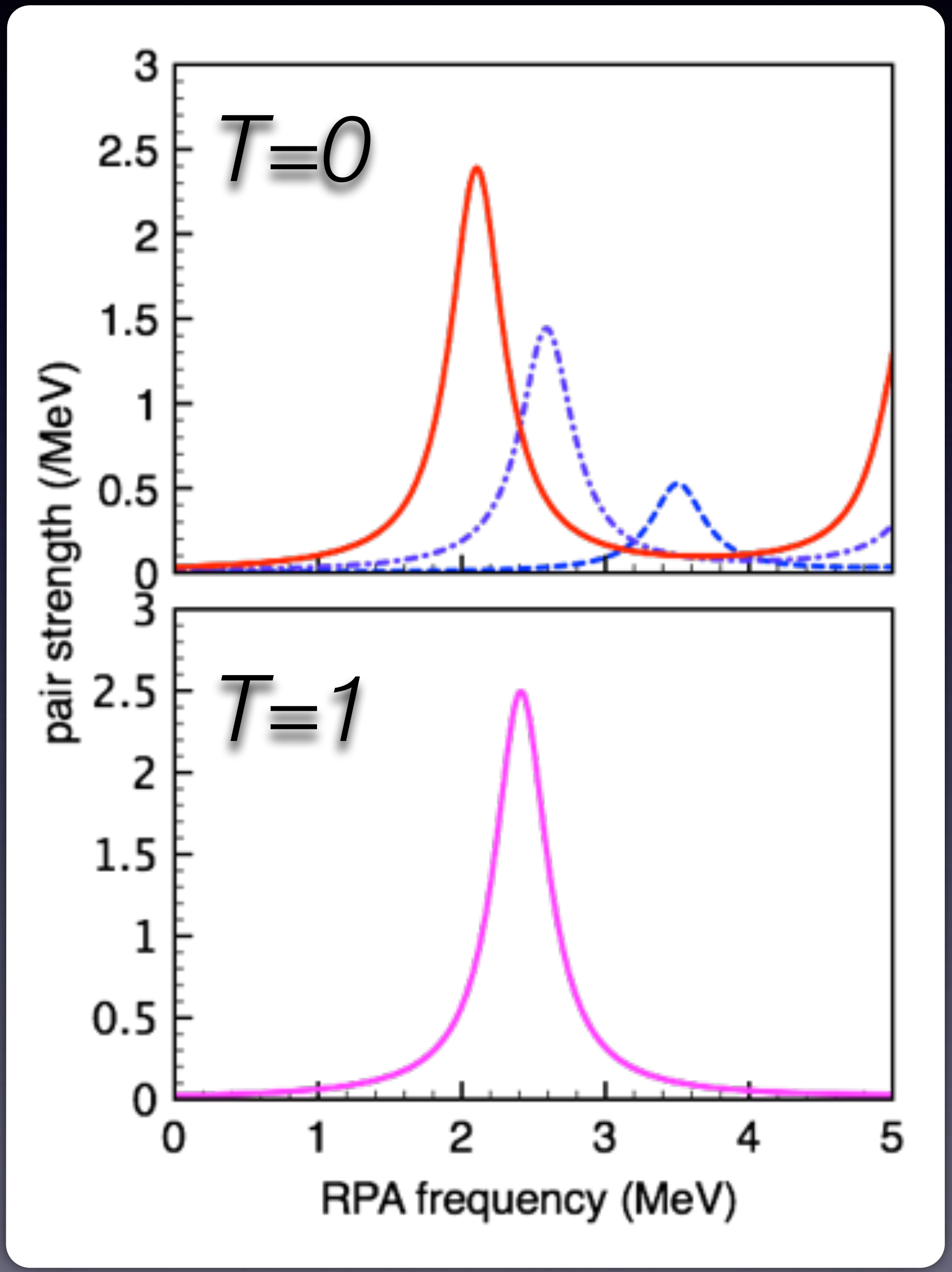
pn-pairing vibrations in sd-shell nuclei

cf. Talks by Lee and Lay;
(^{3}He,p) and (p,^{3}He) exps. in ^{24}Mg, ^{28}Si,···@RCNP

^{24}Mg → ^{26}Al



^{24}Mg → ^{22}Na



Summary and outlook

Nuclear energy-density functional method for spin-isospin response

Microscopic and powerful framework to study a rich variety of nuclear collective dynamics

Possible occurrence of a new kind of collective mode associated with the spin-triplet pairing condensation

In LS-closed nuclei, the spin-orbit partners have a coherent contribution to the collective mode

We can study the T=0 pairing in nuclei even if they are in the “normal” phase.

Comparison with the experiments

- ✓ strength of the T=0 pairing interaction
- ✓ cross sections of deuteron transfer/knock-out reactions