# Nucleon Spin Decomposition, GPDs, TMDs

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Spinfest 2015, JPark, Tokai, Japan

## Plan of talk

1. Multidimensional generalization of ordinary Parton Distribution Functions

   - GPDs, TMDs, and Generalized TMDs -

2. The nucleon spin decomposition problem of QCD

   2.1. Introduction to the problem

   2.2. Uniqueness problem of the nucleon spin decomposition

   - the role of Lorentz symmetry –

   2.3. What is "potential angular momentum" ?

   - a lesson from classical and quantum electrodynamics -

   2.4. "Canonical" or "Mechanical" decomposition ?

3. Phenomenology of nucleon spin contents

# 1. Multidimensional generalization of ordinary Parton Distribution Functions

- GPDs, TMDs, and GTMDs -

- GPD : Generalized Parton Distribution
- TMD : Transverse-Momentum-Dependent Distribution
- GTMD : generalized TMD = *F.T.* of Wigner distribution

typical probes of hadron internal structure

↓ elastic scatterings of leptons

↓ deep-inelastic-scatterings

**elastic scatterings of leptons**

cleanest !

Only restricted information on the nucleon structure like charge densities or spin magnetization densities

**deep-inelastic-scatterings**

somewhat sophisticated !

More detailed information on the quark-gluon structure of the nucleon through the measurement of PDFs

Ordinary parton distribution function $q(x)$

one-variable function of $x$ (Bjorken variable), which has the meaning of longitudinal momentum fraction of quarks along the nucleon momentum

By restricting parton's motion in one dimension, a firm theoretical framework of perturbative QCD (or the scheme of collinear factorization) was established. However, this restriction inevitably loses a lot of interesting information on the nucleon internal structures.

Intuitively, it is more natural to consider that the parton's motion in the nucleon is three dimensional. In fact, the existence of such quantities, which contain information of the parton's three dimensional motion, has been long known.

**TMDs**

transverse-momentum-dependent parton distribution functions

- Soper (79)
- Collins, Soper (82)
- Collins, Soper, Sterman (85)

**GPDs**

generalized parton distribution functions

- Mueller et al. (91/94)
- Radyushkin (96)
- Ji (96)

**PDFs :**

gauge link

$$q^{(\Gamma)}(x) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{i\,k^+ z^-} \langle p^+, \mathbf{0}_\perp, \Lambda' \,|\, \bar{\psi}\left(-\frac{z^-}{2}\right) \Gamma\, \mathcal{L}\, \psi\left(\frac{z^-}{2}\right) |\, p^+, \mathbf{0}_\perp, \Lambda\rangle$$

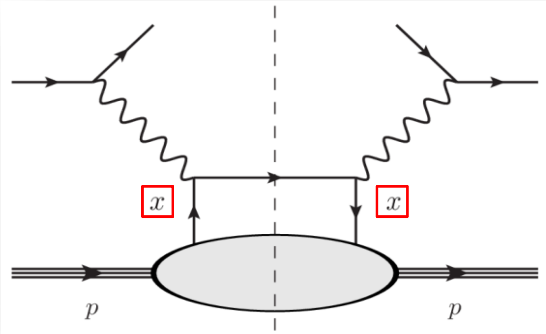
$\left[ z^\pm = \frac{1}{\sqrt{2}}(z^0 \pm z^3) \right]$

$\Gamma$ : Dirac matrix

$\Lambda, \Lambda'$ ; nucleon polarization

$x = \frac{k^+}{p^+}$ ; longitudinal momentum fraction

Bjorken variable

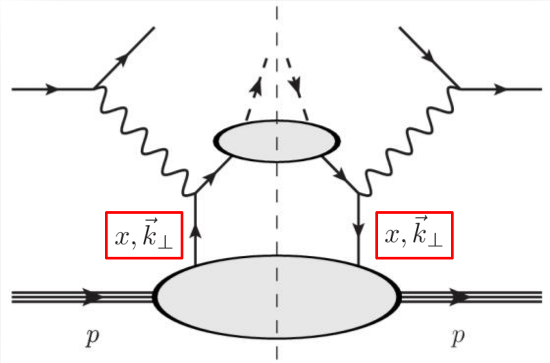



**TMDs :**

natural 3-dimensional generalization of PDFs

$$k = (k^+, k^-, \boldsymbol{k}_\perp), \;\; z = (z^+, z^-, \boldsymbol{z}_\perp)$$

$\boldsymbol{k}_\perp$ : transverse momentum




$$q^{(\Gamma)}(x, \boldsymbol{k}_\perp) \;=\; \frac{1}{2}\int \frac{dz^- \, d^2 z_\perp}{(2\pi)^3} e^{i\,k^+ z^- - i\,\boldsymbol{k}_\perp \cdot \boldsymbol{z}_\perp}$$

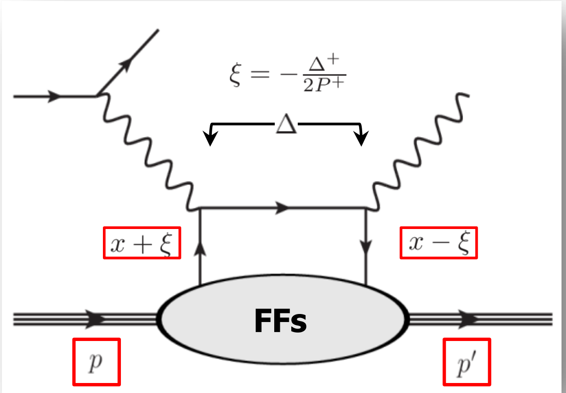
$$\times \langle p^+, \mathbf{0}_\perp, \Lambda' \,|\, \bar{\psi}\left(-\frac{z}{2}\right) \Gamma\, \mathcal{L}\, \psi\left(\frac{z}{2}\right) |\, p^+, \mathbf{0}_\perp, \Lambda\rangle_{z^+=0}$$

**GPDs :**

another 3-dimensional generalization of PDFs

final nucleon mom. $\neq$ initial nucleon mom.

(**off-forward nucleon matrix element**)




$$q^{(\Gamma)}(x,\xi,\Delta_\perp^2) = \frac{1}{2}\int \frac{dz^-}{2\pi}\, e^{i\,k^+ z^-}$$
$$\times \ \langle p'^+, \frac{\Delta_\perp}{2}, \lambda' \,|\, \bar{\psi}\left(-\frac{z^-}{2}\right) \Gamma \mathcal{W} \psi\left(\frac{z^-}{2}\right) |\, p^+, -\frac{\Delta_\perp}{2}, \lambda\rangle_{z^+=0}$$

$$P^+ = \frac{p'^+ + p^+}{2}$$

$x = \frac{k^+}{P^+}$ : average longitudinal momentum fraction

$\Delta_\perp$ : transverse momentum transfer

$\xi$ : skewdness parameter (longitudinal momentum transfer)

TMDs and GPDs are different 3-dimensional generalization of ordinary PDFs and there is no direct relation between them. However, they are said to have brotherly relationship in the sense that they are obtained from common mother distribution called GTMDs (*F.T.* of Wigner distributions)

- Ji (2003), Belitsky, Ji, Yuan (2004)

$$q^{(\Gamma)}(x, \boldsymbol{k}_\perp; \xi, \boldsymbol{\Delta}_\perp^2) = \frac{1}{2} \int \frac{dz^- \, d^2 z_\perp}{(2\pi)^3} \, e^{i x P^+ z^- - i \boldsymbol{k}_\perp \cdot \boldsymbol{z}_\perp} \times \langle p'^+, \frac{\boldsymbol{\Delta}_\perp}{2} | \bar{\psi}\left(-\frac{z}{2}\right) \Gamma \mathcal{W} \psi\left(\frac{z}{2}\right) | p^+, -\frac{\boldsymbol{\Delta}_\perp}{2} \rangle_{z^+=0}$$

⑤

where

$$x = k^+ / P^+, \qquad \xi = \Delta^+ / P^+$$

**GPD**

$$q^{(\Gamma)}(x, \xi, \boldsymbol{\Delta}_\perp^2) = \int d^2 \boldsymbol{k}_\perp \, q^{(\Gamma)}(x, \boldsymbol{k}_\perp; \xi, \boldsymbol{\Delta}_\perp^2)$$

③

**TMD**

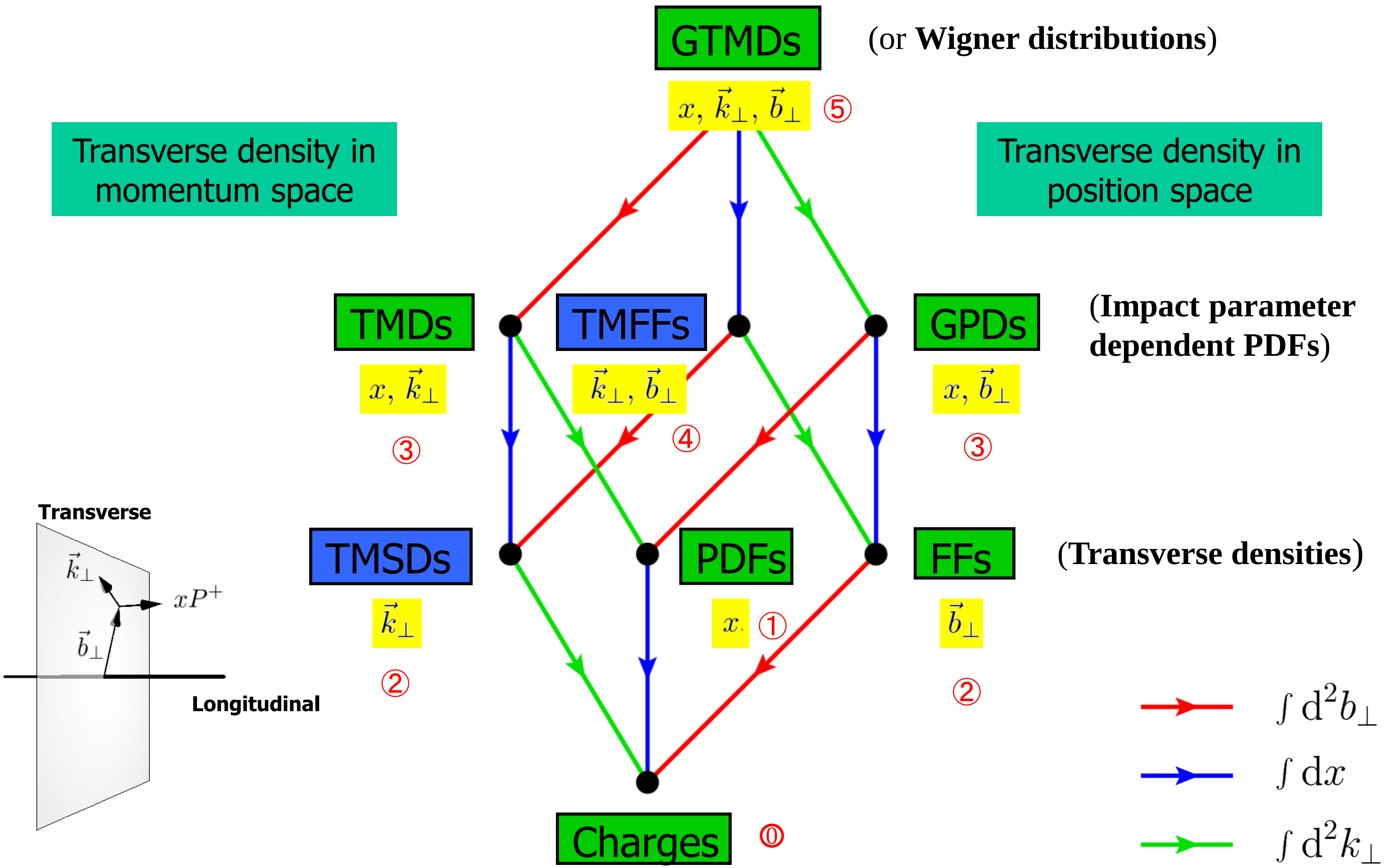
$$q^{(\Gamma)}(x, \boldsymbol{k}_\perp) = q^{(\Gamma)}(x, \boldsymbol{k}_\perp; \xi = \boldsymbol{\Delta}_\perp^2 = 0)$$

③

# Complete picture at $\xi = 0$

| Momentum space | $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ <br> $\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$ | Position space |
|---|---|---|

from Lorce @ Como2013



## (I) Physics of GPDs

Basic theoretical framework of GPDs is already well-established.

- GPDs can be unambiguously defined as off-forward nucleon matrix elements of **bi-local** and **gauge-invariant** **quark-gluon operators on the light-cone.**

- The analysis of them can be done within the framework of **collinear factorization** (ex. through DVCS, Deeply Virtual Meson Production etc.)

- At the leading-twist, **factorization proof** exists. (Collins)

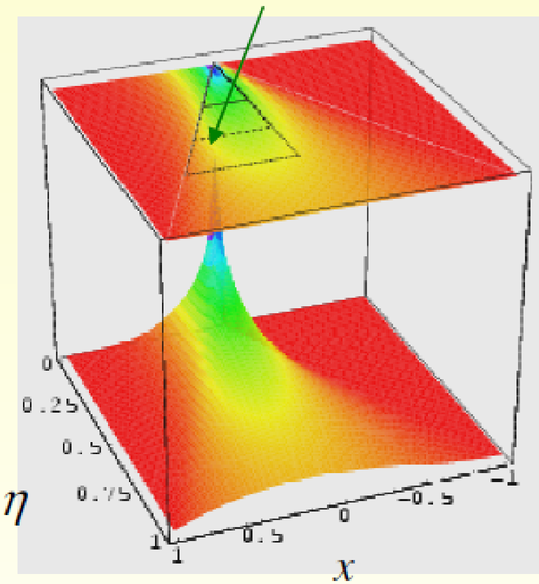
- Fourier transform of GPDs, called **impact parameter space PDFs**, can be interpretated as **probability distributions in the position space** and they are **natural 3-dimensional generalization** **of ordinary PDFs.** (Burkart,2000,2003)

- GPDs offer valuable information to probe **nucleon spin contents** (Ji sum rule)

# *GPD related hard exclusive processes*

from Mueller@Como2013

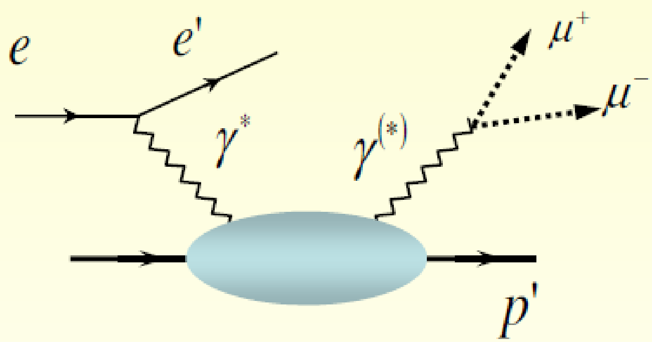
scanned area of the surface as a functions of lepton energy




$ep \to e'p'\mu^+\mu^-$

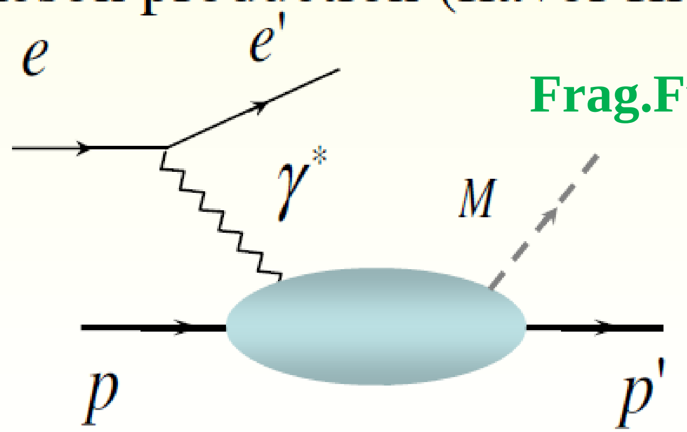
- Deeply virtual Compton scattering (clean probe)

$$ep \to e'p'\gamma$$
$$ep \to e'p'\mu^+\mu^-$$
$$\gamma p \to p'e^-e^+$$




- Deeply virtual meson production (flavor filter)

$$ep \to e'p'\pi$$
$$ep \to e'p'\rho$$
$$ep \to e'n\pi^+$$
$$ep \to e'n\rho^+$$




**Frag.Func**

twist-two observables:

longitudinal cross sections

transverse target spin asymmetries

- etc.

factorization proof for longitudinal cross sections

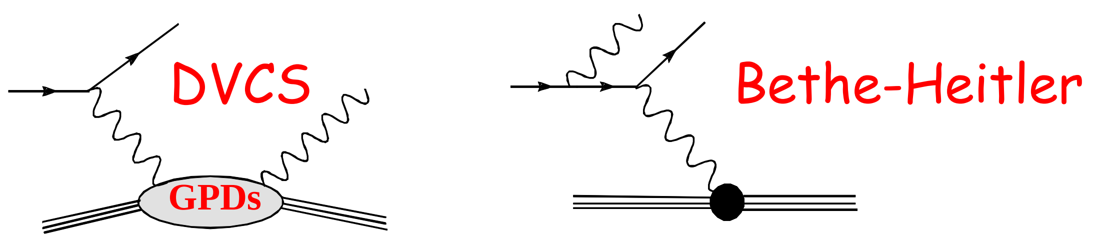
**[Collins, Frankfurt, Strikman (96)]**

- There already exist large compilations of experimental data by HERMES, COMPASS, JLab groups.

- Since GPDs are functions of 3 variable $(x, \xi, t)$ , analyses of measured data necessarily requires appropriate modeling (parametrization) of GPDs.

  ⋆ especially difficult is the modeling of $\xi$-dependence

**LO DVCS + Bethe-Heitler amplitude**




**In general, depend on 8 GPD quantities.**

Real and Imaginary part of $H(x,\xi,t), \ E(x,\xi,t), \ \tilde{H}(x,\xi,t), \ \tilde{E}(x,\xi,t)$

GPDs and the celebrated Ji sum rule

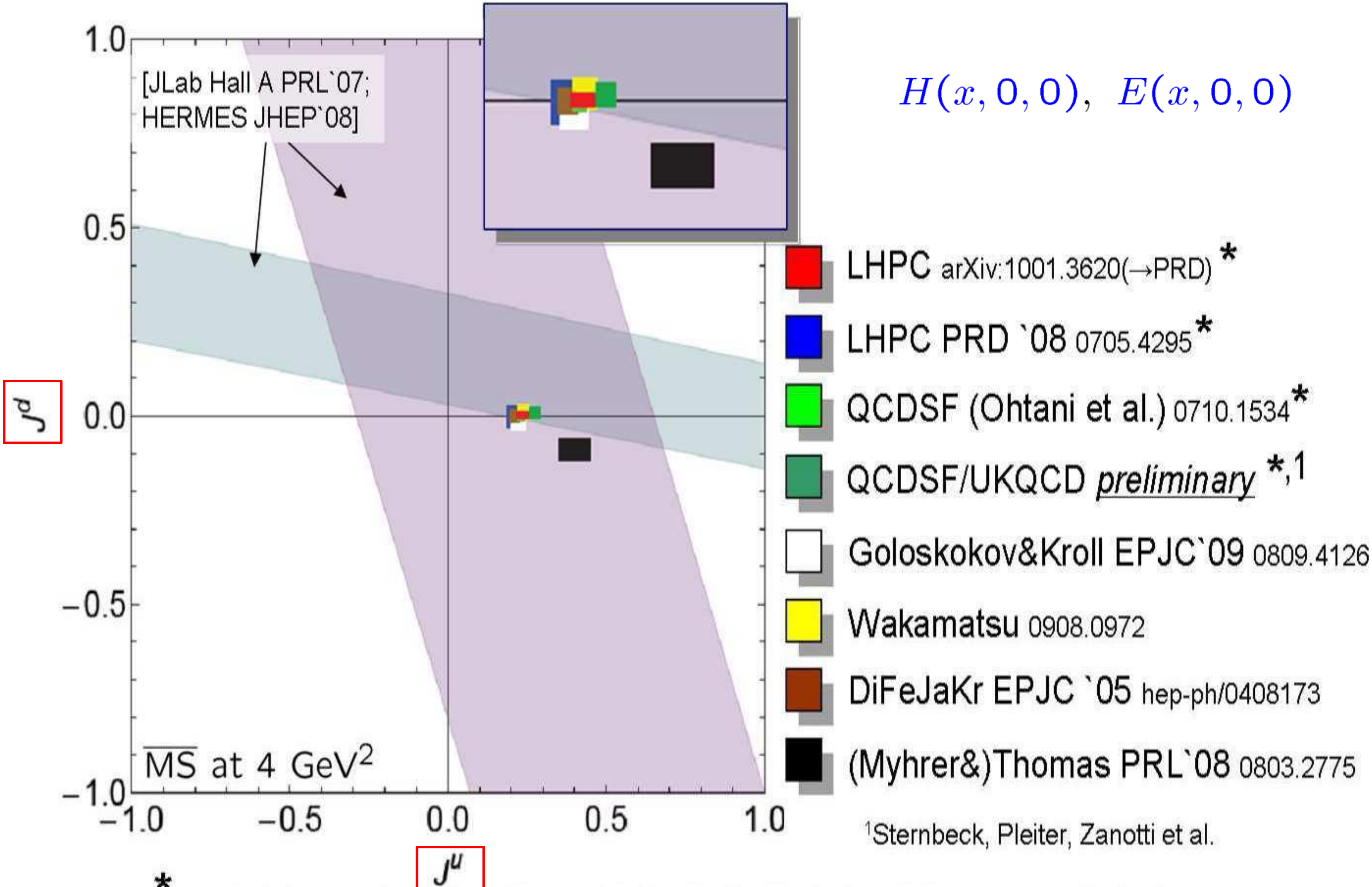
$$J^q = \frac{1}{2}\int x\,[\,H^q(x,0,0) + E^q(x,0,0)\,]\,dx$$
$$J^G = \frac{1}{2}\int x\left[H^G(x,0,0) + E^G(x,0,0)\right]dx$$

For the GPDs of quarks, there are preliminary analyses of $J^u$ and $J^d$ by HERMES Collaboration and also by JLab Hall A Collaboration.

- HERMES Collaboration (F. Ellinghause et al.), Eur. Phys. J. C 46, 729 (2006).
- HERMES Collaboration (Z. Ye), arXiv:hep-ex/0606061.

- JLab Hall A Collaboration (M. Mazouz et al.), PRL 99, 242501 (2007).

GPD extraction of $J^u$ & $J^d$ compared with Lattice and phenomenological data




* [non-singlet, connected only; add. uncertainties due to chiral extrapolations, renormalization]

## impact-parameter dependent PDF

GPDs

$$\rho(x, \boldsymbol{b}_\perp) = \int \frac{d^2 \boldsymbol{\Delta}_\perp}{(2\pi)^2} e^{-i \boldsymbol{\Delta}_\perp \cdot \boldsymbol{b}_\perp} F^q(x, \xi = 0, \boldsymbol{\Delta}_\perp^2)$$

Position space $\quad \boxed{\boldsymbol{b}_\perp \leftrightarrow \boldsymbol{\Delta}_\perp} \quad$ Momentum space

Parton distribution in 3-dimensional phase space $(x, \boldsymbol{b}_\perp)$

Parton distribution in the transverse impact-parameter space as a function of longitudinal momentum fraction $x$.

richer information than ordinary PDFs


- M. Burkardt, Phys. Rev. D62 (2000) 071503
- M. Burkardt, Int. J. Mod. Phys. A18 (2003) 173
- J. P. Ralston and B. Pire, Phys. Rev. D66 (2002) 111501

typical prediction of a simple model

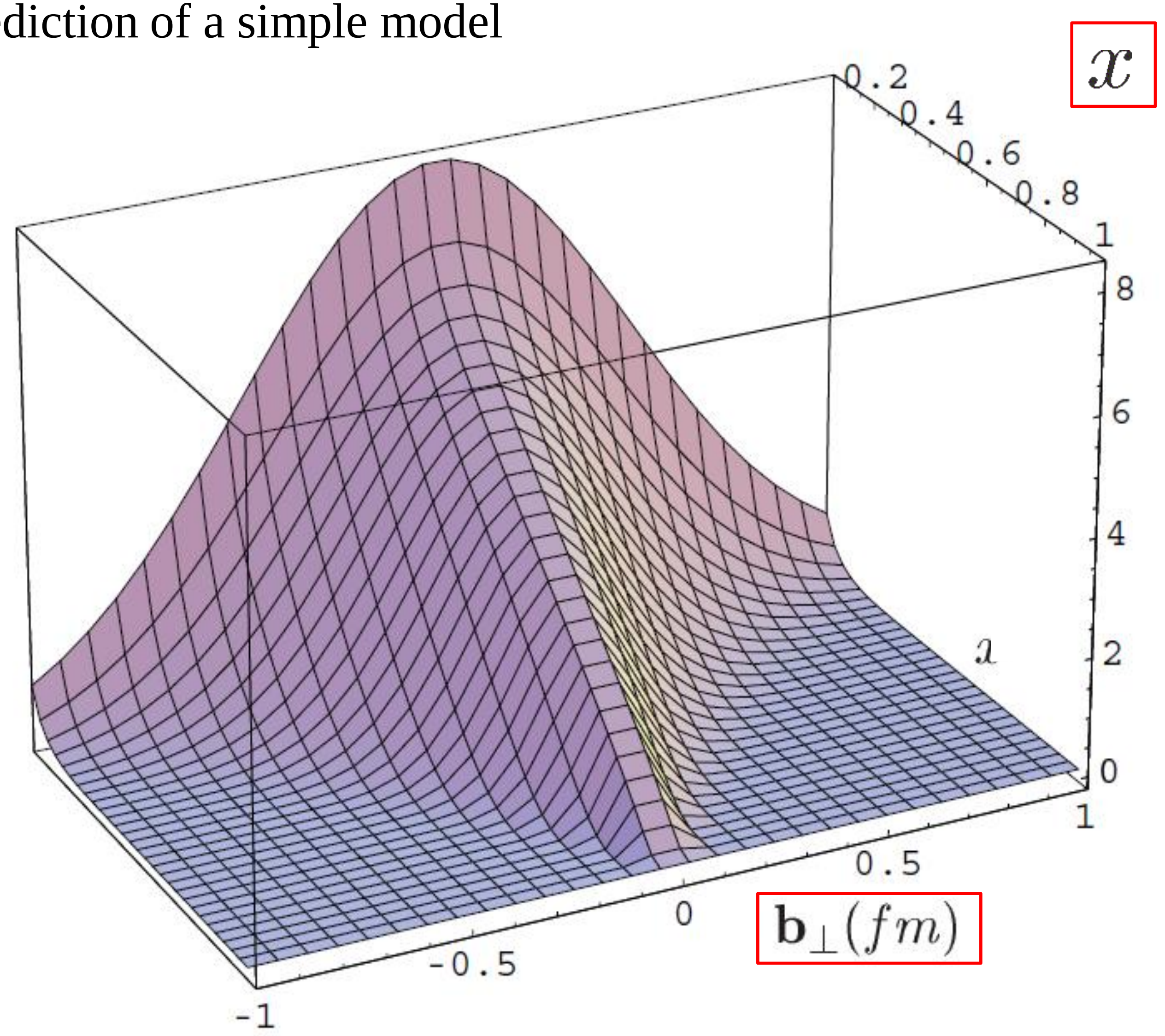



Fig. 1. Impact parameter dependent parton distribution $u(x, \mathbf{b}_\perp)$ for the simple model (31).

## transverse density

$$\int \rho^{(\Gamma)}(x, \boldsymbol{b}_\perp)\, dx \quad \Rightarrow \quad \rho^{(\Gamma)}(\boldsymbol{b}_\perp)$$

For $\Gamma = \hat{Q}\,\gamma^+$, this reduces to the transverse charge density.

$$\begin{aligned}\rho(\boldsymbol{b}_\perp) &= \int \frac{d^2 q_\perp}{(2\,\pi)^2} \frac{e^{-i\,\boldsymbol{q}_\perp \cdot \boldsymbol{b}_\perp}}{2\,p^+} \langle p^+, \frac{\boldsymbol{q}_\perp}{2} \,|\, J^+_{e.m} \,|\, p^+, -\frac{\boldsymbol{q}_\perp}{2} \rangle \\ &= \int_0^\infty \frac{dQ}{2\,\pi}\, Q\, J_0(b_\perp Q)\, F_1(Q^2)\end{aligned}$$

This quantity, given as a 2-dimensional Fourier transform of Dirac F.F. $F_1(Q^2)$, has a meaning of charge density in the infinite momentum frame of the nucleon.

In the IMF, due to the Lorentz contraction along the 3-direction, the nucleon looks like a 2-dimensional object.

Transverse charge density represents charge distribution in this 2-dimensional plane perpendicular to the nucleon momentum.

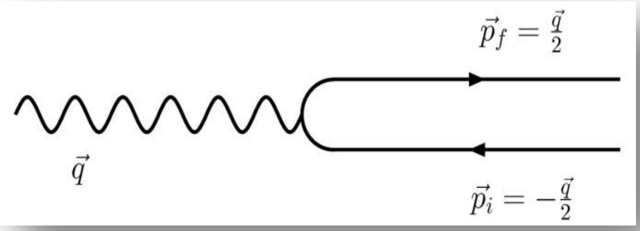
[Cf.] familiar charge density as a 3-dimensional Fourier transform of Sachs F.F.

$$\rho_{ch}(\boldsymbol{r}) = \int \frac{d^3 q}{(2\pi)^2} e^{-i\,\boldsymbol{q}\cdot\boldsymbol{r}} G_E(\boldsymbol{q})$$

Theoretical ground of charge density interpretation :

In the Breit frame : $q^0 = 0 \;\Rightarrow\; Q^2 = \boldsymbol{q}^2$

$$\langle N_{s'}(\boldsymbol{q}/2) \,|\, J^0_{e.m.}(0) \,|\, N_s(-\boldsymbol{q}/2)\rangle = 2\,M\, G_E(\boldsymbol{q}^2)\,\delta_{s',s}$$




$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2)$$

Sachs Dirac Pauli

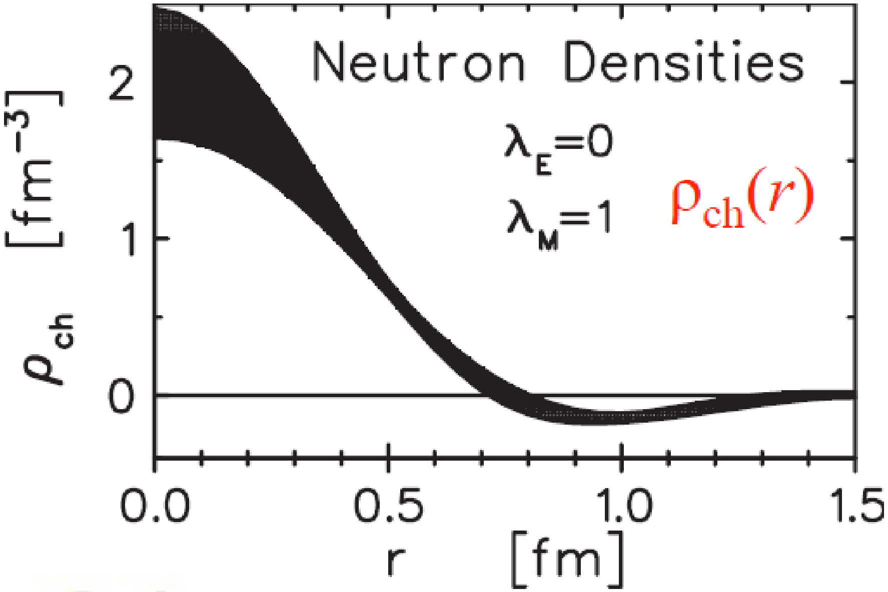
However, the choice of Breit frame is clearly $Q^2$-dependent, and in each Lorentz frame the nucleon receives different Lorentz contraction in relativistic theory.

Charge density interpretation of the F.T. of Sachs F.F. can therefore be justified only in non-relativistic framework, in which the effects of Lorentz contraction are not important !

# Qualitative change in central neutron charge density



from Hoyer@Mainz11

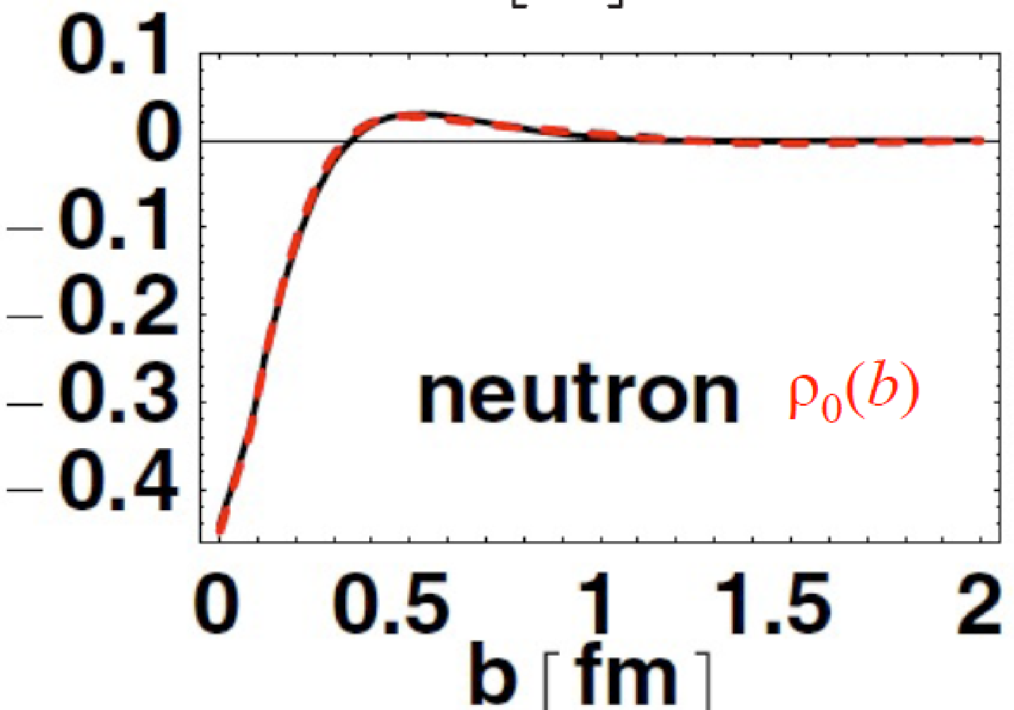



3-dimensional Fourier transform with phenomenological factors (2001)

J. J. Kelly, hep-ph/0111251

$\rho_{ch}(r=0) > 0$

$n \rightarrow p + \pi^-$ : pion cloud




$\rho_0(b=0) < 0$

Transverse Fourier transform (2007)

G. Miller, PRL 99 (2007) 112001

## (II) Physics of TMDs

Until several years ago, theoretical foundation of TMDs was not very solid.

- Intuitively, TMDs as functions of longitudinal momentum fraction $x$ and transverse momentum $\boldsymbol{k}_\perp$ should be a natural extension of ordinary PDFs.

However, various difficulties turned out to appear in the formulation of TMDs, which requires a framework beyond the standard collinear factorization.

Once quantum-loop effects are taken into account, the existence of TMDs satisfying **gauge-invariance** as well as **process independence (factorization)** does not seem to be a trivial matter.

There also existed several other stuffs to be clarified.

- **role of final (or initial) state interaction** ?
- **quantitative relation** between **transverse motion** and longitudinal **OAM** ?

## Basic problem of TMD formulation

a brief reviewal of **standard PDF**

$$q(x) = \frac{1}{2} \int \frac{d\xi^-}{2\pi} e^{-i x P^+ \xi^-} \langle P \,|\, \bar{\psi}(\xi^-, \mathbf{0}_\perp) \gamma^+ \mathcal{L}[\xi^-, \mathbf{0}_\perp; 0, \mathbf{0}_\perp] \psi(0, \mathbf{0}_\perp) \,|\, P \rangle$$

with

$$\mathcal{L}[\xi^-, \boldsymbol{\xi}_\perp; 0^-, \boldsymbol{\xi}_\perp] \equiv P \exp\left( -i g \int_0^{\xi^-} d\xi^- A^+(\xi^-, \boldsymbol{\xi}_\perp) \right) \quad : \quad \texttt{gauge link}$$

The gauge-link is interpreted as taking account of the final-state interaction between an ejected quark and residual spectators, and it works to assure the gauge-invariance of PDFs.

**In the light-cone (LC) gauge** $A^+ = 0$

$$\mathcal{L}[\xi^-, \boldsymbol{\xi}_\perp; 0^-, \boldsymbol{\xi}_\perp] \longrightarrow 1$$

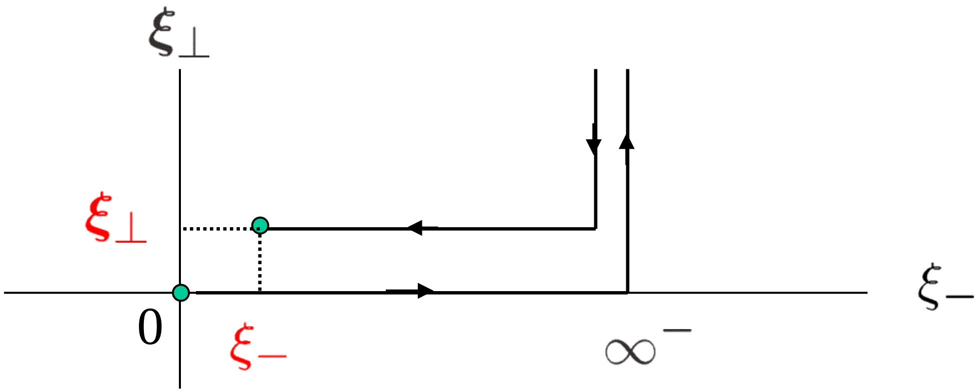
so that **FSI vanishes** and PDF gets naïve probability density interpretation.

Natural extension to TMD ?

$$q(x, \boldsymbol{k}_\perp) = \frac{1}{2}\int \frac{d\xi^-}{2\pi}\int \frac{d^2\xi_\perp}{(2\pi)^2}\, e^{-ixP^+\xi^- + i\boldsymbol{k}_\perp\cdot\boldsymbol{\xi}_\perp}$$
$$\times \langle P\,|\,\bar{\psi}(\xi^-, \boldsymbol{\xi}_\perp)\,\mathcal{L}[\infty^-, \boldsymbol{\infty}_\perp; \xi^-, \boldsymbol{\xi}_\perp]_C^\dagger\,\gamma_+\,\mathcal{L}[\infty^-, \boldsymbol{\infty}_\perp; 0^-, \boldsymbol{0}_\perp]_C\,\psi(0^-, \boldsymbol{0}_\perp)\,|\,P\rangle$$

with

$$\mathcal{L}[\infty^-, \boldsymbol{\infty}_\perp; \xi^-, \boldsymbol{\xi}_\perp]_C \equiv \mathcal{L}[\infty^-, \boldsymbol{\infty}_\perp; \infty^-, \boldsymbol{\xi}_\perp]\,\mathcal{L}[\infty^-, \boldsymbol{\xi}_\perp; \xi^-, \boldsymbol{\xi}_\perp]$$




Gluon propagator in the LC gauge has an ambiguity in the way of avoiding the $1/k_+$ singularity. Customarily, this ambiguity is fixed by imposing the boundary condition at the LC infinity $(\xi^- = \pm\infty)$ for the gluon field :

$$\tilde{A}(+\infty, \tilde{\xi}) = 0 \quad : \quad \text{Advanced boundary cond.}$$
$$\tilde{A}(-\infty, \tilde{\xi}) = 0 \quad : \quad \text{Retarded boundary cond.}$$
$$\tilde{A}(-\infty, \tilde{\xi}) + \tilde{A}(+\infty, \tilde{\xi}) = 0 \quad : \quad \text{Antisymmetric boundary cond.}$$

An important fact is that there is no gauge choice, which simultaneously satisfies

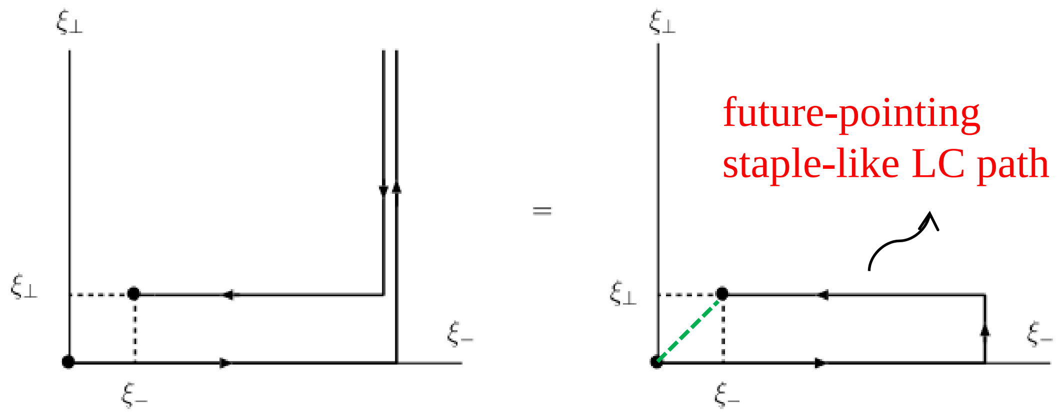
$$\tilde{A}(+\infty, \tilde{\xi}) = 0 \quad \text{and} \quad \tilde{A}(-\infty, \tilde{\xi}) = 0$$

Then, although gauge link along the light-like direction vanishes in the LC gauge, the **gauge link along the transverse direction** survives even at the LC infinity.

Because of this reason, the effect of FSI (or ISI) is manifest even in the LC gauge.

The gauge-link path corresponding to SIDIS process is usually represented as

because of unitarity




This path is different from **straight-line path** connecting $(0, \mathbf{0}_\perp)$ and $(\xi^-, \boldsymbol{\xi}_\perp)$ !

definition of TMD is generally gauge-link-path dependent !

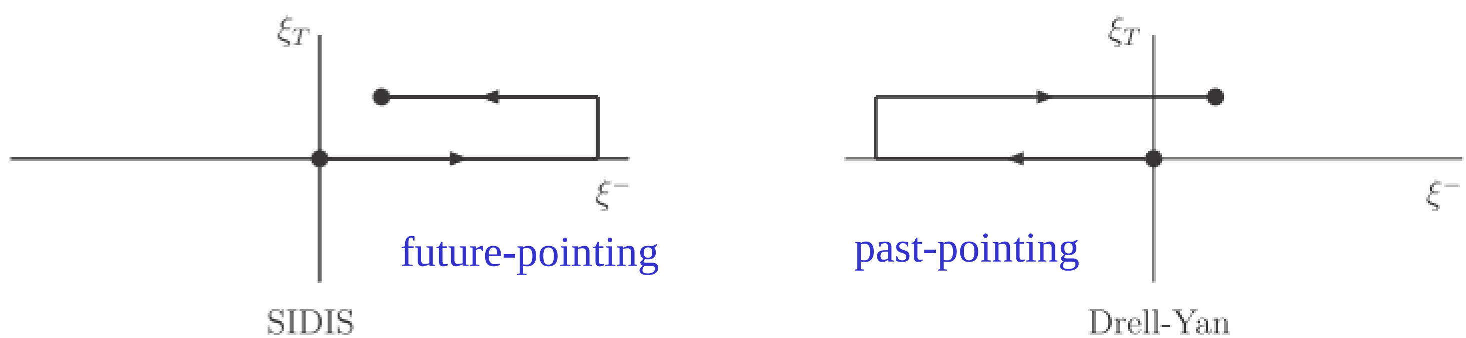
Known important facts

Sivers fnc. Boer-Mulders fnc.

- Two (naïve) **T-odd TMDs** $(f_{1T}^{\perp}, h_{T}^{\perp})$ vanishes if one neglects FSI or ISI. (Brodsky, Hwang, Schmidt, 2002)

- In the LC gauge, gauge link at the infinity is crucial. (Belitsky, Ji, Yuan, 2002)

- Gauge-link corresponding to Drell-Yan process is given by the so-called past-pointing staple-like LC path, corresponding to the fact that the trajectory of the annihilation partner begins from $\xi^{-} = -\infty$ . (Collins, 2002)




Most important pQCD prediction for T-odd TMD (Collins,2002)

$$f(x, \boldsymbol{k}_\perp, \boldsymbol{s}_\perp)|_{\text{future-pointing W}} = f(x, \boldsymbol{k}_\perp, -\boldsymbol{s}_\perp)|_{\text{past-pointing W}}$$

$$f_{1T}^{\perp}(x, k_\perp)|_{\text{SISIS}} = -f_{1T}^{\perp}(x, k_\perp)|_{\text{DY}}$$ sign change !

Trial proof of 1-loop TMD factorization for SIDIS (Ji, Ma, Yuan, 2005)

$$f_1^q(x, \boldsymbol{k}_\perp; \mu_F, \zeta) \overset{\text{def}}{=} \frac{1}{2} \int \frac{d\xi^-}{2\pi} e^{-i x \xi^- P^+} \int \frac{e^{i \boldsymbol{\xi}_\perp \cdot \boldsymbol{k}_\perp}}{(2\pi)^2} \times \langle P \,|\, \bar{\psi}_q(\xi^-, 0, \boldsymbol{\xi}_\perp)\, \mathcal{L}[\infty, \boldsymbol{\infty}_\perp; \xi^-, \boldsymbol{\xi}_\perp]^\dagger \gamma^+ \mathcal{L}[\infty, \boldsymbol{\infty}; 0, \boldsymbol{0}_\perp]\, \psi_q(0)\, P \rangle|_{\xi^+=0}$$

They took off-light-like gauge-link path specified by a 4-vector $v^\mu = (v^-, v^+, \boldsymbol{0}_\perp)$

$$\mathcal{L}[\infty, \boldsymbol{\infty}_\perp; \xi^-, \boldsymbol{\xi}_\perp] = P\, e^{-i g \int_0^\infty d\lambda\, v \cdot A(\lambda v + \xi)}$$

to avoid log. divergence originating from virtual gluon with zero plus-momentum.

This enables them to prove TMD factorization.

Troublingly, however, it turned out that the TMD so defined does not reduce to an ordinary PDF after $\boldsymbol{k}_\perp$ integration !

$$f_1^q(x) \neq \int d^2 \boldsymbol{k}_\perp \, f_1^q(x, \boldsymbol{k}_\perp)$$

Theoretical researches of TMDs in these few years are focused on this fundamental problem.

Owing to efforts by many researchers, the problem has been positively resolved !

- J. C. Collins, *Foundations of Perturbative QCD* (Cambridge University Press, Cam-bridge, 2011).
- J. C. Collins, Acta Phys. Polon., B34:3103–3120, 2003.
- S. M. Aybat and T. C. Rogers, Phys.Rev., D83:114042, 2011.
- I. Cherednikov and N. Stefanis, Phys.Rev., D77:094001, 2008.
- T. Becher and M. Neubert, Eur.Phys.J., C71:1665, 2011.
- M.G. Echevarría, Ahmad Idilbi, Ignazio Scimemi. [arXiv: 1211.1947]

For semi-inclusive-DIS (SIDIS) and Drell-Yan processes, satisfactory definition of TMDs were given, and their evolution equations are also derived.

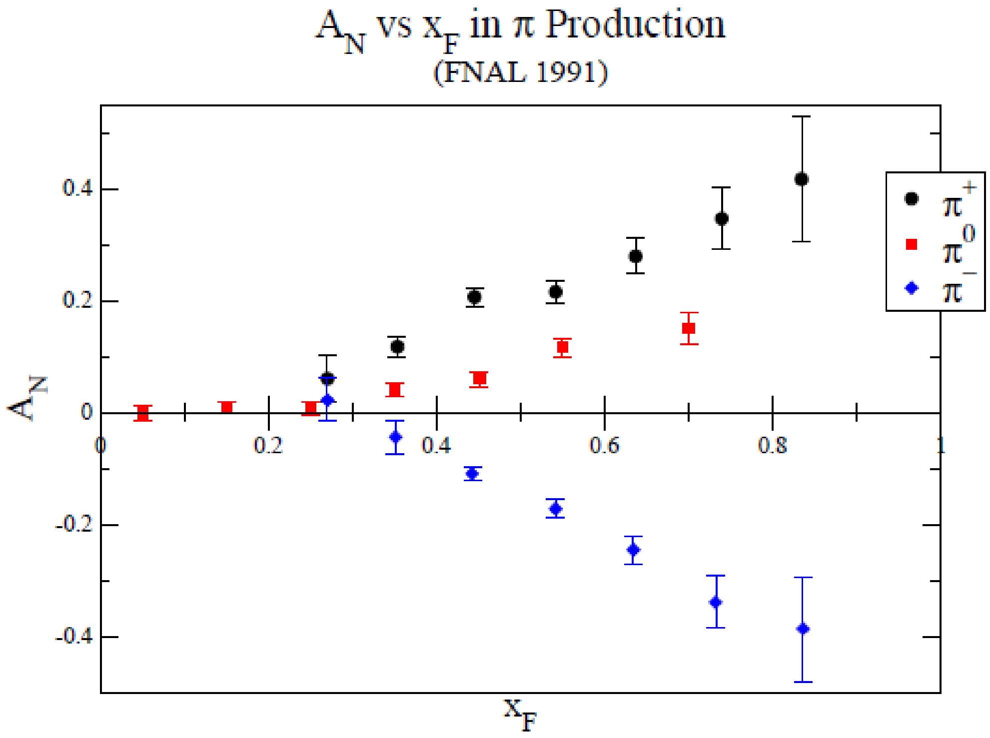
establishment of TMD factorization scheme

However, the universality is likely to be broken, for more complicated processes ?

$$h_1 + h_2 \rightarrow h_3 + h_4 + X$$

Strong interest in TMDs was arose by the discovery of the phenomena called the **single spin asymmetry** (**SSA**) at FNAL.

$$p^{\uparrow} + p \to \pi + X \ : \ \texttt{large}\ \boldsymbol{p}_{\perp}$$




**SSA**

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

SSA represents the asymmetry of the produced hadrons with respect to the direction of nucleon spin.

SSA vanishes in naïve parton model, so that its explanation requires some sort of extension of naïve pQCD framework. ⇨

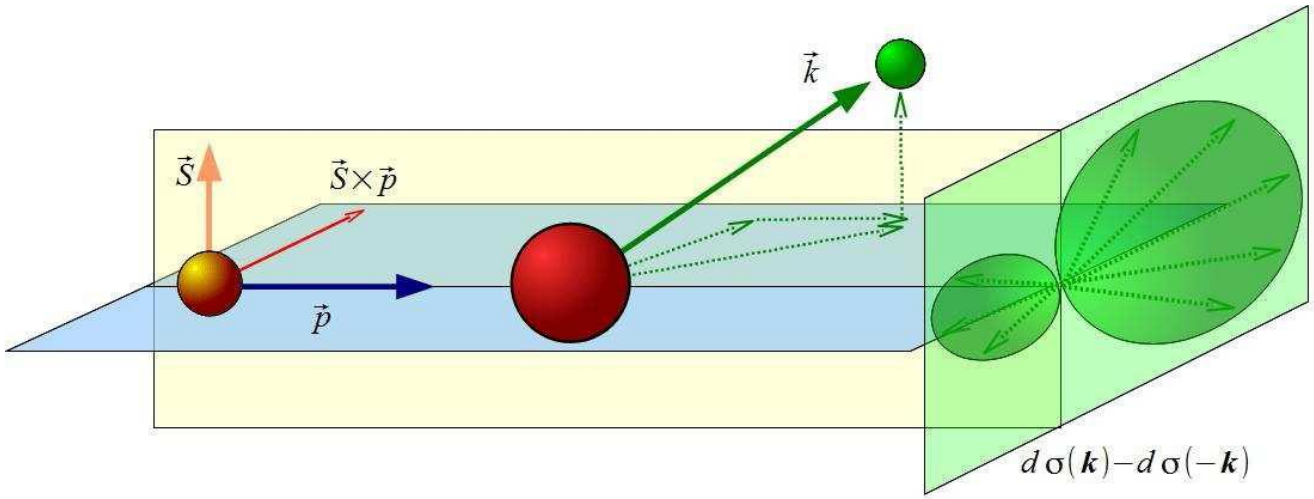
- **TMD factorization approach**

  Sivers, Collins, ・・・

- **Collinear factorization (twist-3) approach**

  Qiu, Sterman, Koike, Tanaka, ・・・

SSA in TMD factorization approach

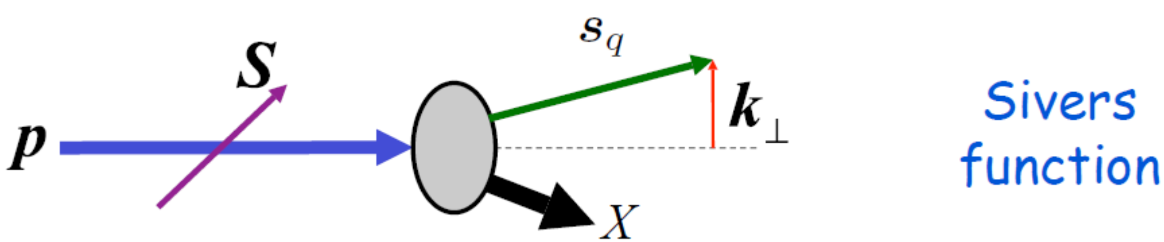



SSA is a reflection of the spin-orbit correlation of the form $(\vec{S}\times\vec{p})\cdot\vec{k}$

SSA is a "naively time-reversal-odd (T-odd) observable", so that it requires the dependence on the spin direction as well as on T-odd imaginary phase.

- T-odd mechanism in the parton distribution (TMD) : **Sivers function**
- T-odd mechanism in the fragmentation function : **Collins function**

## Sivers mechanism in TMD formalism




$$f_{q/p,\boldsymbol{S}}(x,\boldsymbol{k}_\perp) = f_{q/p}(x,k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

$$= f_{q/p}(x,k_\perp) - \frac{k_\perp}{M} f_{1T}^{\perp q}(x,k_\perp)\,\boldsymbol{S}\cdot(\hat{\boldsymbol{p}}\times\hat{\boldsymbol{k}}_\perp)$$

Because the **Sivers function** represents the spin-orbit correlation (correlation between the transverse momentum of quarks and the nucleon spin), it is often claimed that it must be sensitive to the OAM of quarks in the nucleon.

We shall see later that this naïve expectation would be qualitatively OK, but it is not supported in a quantitative sense !

No exact relation exists between Sivers function and quark OAM ?